Appendix

Appendix A: Equilibrium Solutions for Different Cases.

In this proof, we derive the equilibrium profits resulting from each model. We set the inverse demand function as:

$$p_i = m - (q_i + \lambda q_j), (i, j \in \{o, t, d\}, i \neq j \text{ and } i = t \vee j = t) \tag{1}$$

Because the profit functions of the supplier and the e-platform are always expressed as concave quadratic equations with respect to the decision variable (i.e., quantity), all the unreleased second-order conditions are satisfied in the following profit-maximization problem. Therefore, we omit the description of some of the second-order conditions for profit maximization.

Model N:

The e-platform only resells the supplier's products in this scenario, bringing $\lambda = 0$ and $i = t$ to Equation (1) to obtain p_t, we can derive the supplier's and the e-platform's profit function as follows:

$$\pi_S^N = wq_t \tag{2}$$

$$\pi_P^N = (p_t - w)q_t \tag{3}$$

Solving $\frac{\partial \pi_P^N}{\partial q_t} = m - 2q - w = 0$, from which we can derive the optimal reselling quantity as a function of the wholesale price $q_t(w) = \frac{m-w}{2}$. The supplier's optimization problem in Stage 1 is characterized by the first-order condition of Equation $\frac{\partial \pi_S^N}{\partial w} = \frac{1}{2}(m - 2w) = 0$, from which we can derive the optimal wholesale price of product $w^N = \frac{m}{2}$. Substituting the above optimal wholesale price into the above optimal reselling quantity function, we can derive the optimal reselling quantity: $q_t^N = \frac{m}{4}$.

With the above equilibrium wholesale price and reselling quantity, inverse demand function in Equation (1), the supplier's profit function in Equation (2) and the e-platform's profit function in Equation (3), we have all equilibrium solutions in Table 1.

Model D:

The supplier not only wholesales his products to the e-platform for reselling, but also sells products through a direct channel in this scenario, we bring $i = t$, $j = d$ and $i = d$, $j = t$ to Equation (1) to obtain p_t and p_d, respectively. We can derive the supplier's and the e-platform's profit function as follows:

$$\pi_S^D = wq_t + p_d q_d - F \tag{4}$$

$$\pi_P^D = (p_t - w)q_t \tag{5}$$

Solving $\frac{\partial \pi_P^D}{\partial q_t} = m - 2q_t - w - \lambda q_d = 0$, from which we can derive the optimal reselling quantity as a function of the wholesale price and the direct selling quantity $q_t(w, q_d) = \frac{(m-w-\lambda q_d)}{2}$. The supplier's optimization problem in Stage 3 is characterized by the first-order condition of Equation (4):

$$\frac{\partial \pi_S^D}{\partial w} = \frac{1}{2}(m - 2w) = 0 \tag{6}$$

$$\frac{\partial \pi_S^D}{\partial q_d} = m - \frac{m\lambda}{2} + (-2 + \lambda^2)q_d = 0 \tag{7}$$

The Hessian matrix for the profit function of the supplier is:

$$\begin{bmatrix} -1 & 0 \\ 0 & -2 + \lambda^2 \end{bmatrix}$$

The Hessian matrix is positive. From Equation (6) and (7), we can derive the optimal wholesale price of product $w^D = \frac{m}{2}$ and the optimal direct selling quantity $q_d^D = \frac{m(-2+\lambda)}{2(-2+\lambda^2)}$. Substituting the above optimal wholesale price and optimal direct selling quantity into the above optimal reselling quantity function, we can derive the optimal reselling quantity: $q_t^D = \frac{m(-1+\lambda)}{2(-2+\lambda^2)}$. With the above equilibrium wholesale price, direct selling quantity and reselling quantity, inverse demand function in Equation (1), the supplier's profit function in Equation (6) and e-platform's profit function in Equation (7), we have all equilibrium solutions in Table 1.

Model A:

The supplier not only wholesales his products to the e-platform for reselling, but also sells products through an agency channel on the e-platform by paying commissions in this scenario, we bring $i = t$, $j = o$ and $i = o$, $j = t$ to Equation (1) to obtain p_t and p_d, respectively. We can derive the supplier's and the e-platform's profit function as follows:

$$\pi_S^A = wq_t + (1-\alpha)p_o q_o \tag{8}$$

$$\pi_P^A = (p_t - w)q_t + \alpha p_o q_o \tag{9}$$

Solving $\frac{\partial \pi_P^A}{\partial q_t} = m - 2q - w - (1+\alpha)\lambda q_o = 0$, from which we can derive the optimal reselling quantity as a function of the wholesale price and the agency selling quantity $q_t(w, q_o) = \frac{(m-w-(1+\alpha)\lambda q_o)}{2}$. The supplier's optimization problem in Stage 3 is characterized by the first-order condition of Equation (8):

$$\frac{\partial \pi_S^A}{\partial w} = \frac{1}{2}(m - 2(w + \alpha\lambda q_o)) = 0 \tag{10}$$

$$\frac{\partial \pi_S^A}{\partial q_o} = \frac{1}{2}m(-1+\alpha)(-2+\lambda) - w\alpha\lambda + (-2 + 2\alpha + \lambda^2 - \alpha^2\lambda^2)q_o = 0 \tag{11}$$

The Hessian matrix for the profit function of the supplier is:

$$\begin{bmatrix} -1 & 0 \\ 0 & (-1+\alpha)(2-(1+\alpha)\lambda^2) \end{bmatrix}$$

The Hessian matrix is positive. From Equation (10) and (11), we can derive the optimal wholesale price of product $w^A = \frac{m(-2-2\alpha^2\lambda+\lambda^2+\alpha(2+2\lambda-\lambda^2))}{2(-2+2\alpha+\lambda^2)}$ and the optimal agency selling quantity $q_o^A = \frac{m(-2+2\alpha+\lambda)}{2(-2+2\alpha+\lambda^2)}$. Substituting the above optimal wholesale price and optimal agency selling quantity into the above optimal reselling quantity function, we can derive the optimal reselling quantity: $q_t^A = \frac{m(1-\alpha)(-1+\lambda)}{2(-2+2\alpha+\lambda^2)}$.

With the above equilibrium wholesale price, agency selling quantity and reselling quantity, from inverse demand function in Equation (1), the supplier's profit function in Equation (8) and the e-platform's profit function in Equation (9), we have all equilibrium solutions in Table 1.

Model $\bar{N}$:

The supplier wholesales his products to the e-platform for reselling, and the e-platform provides EW service as a countermeasure in this scenario, we bring $\lambda = 0$ and $i = t$ to Equation (1) obtain p_t. We can derive the supplier's and the e-platform's profit function as follows:

$$D_e = q_t - p_e + vT_e \tag{12}$$

$$\pi_S^{\bar{N}} = wq_t \tag{13}$$

$$\pi_P^{\bar{N}} = (m - q_t - w)q_t + (p_e - \rho f)D_e - \varphi T_e^2 \tag{14}$$

In Stage 4, the e-platform needs to decide on the EW price, the EW period and the reselling price. Hence the e-platform's optimization problem in Stage 4 is characterized by the first-order condition of Equation (14):

$$\frac{\partial \pi_P^{\bar{N}}}{\partial q_t} = m + p_e - 2q - w - f\rho = 0 \tag{15}$$

$$\frac{\partial \pi_P^{\bar{N}}}{\partial p_e} = -2p_e + q + T_e v + f\rho = 0 \tag{16}$$

$$\frac{\partial \pi_P^{\bar{N}}}{\partial T_e} = vp_e - fv\rho - 2\varphi T_e = 0 \tag{17}$$

The Hessian matrix for the profit function of the supplier is:

$$\begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & v \\ 0 & v & -2\varphi \end{bmatrix}$$

The Hessian matrix is non-positive when $v^2 < 3\varphi$ is met. From Equation (15), (16) and (17) which we can derive the optimal reselling quantity, EW price, EW period as functions of the wholesale price and the direct selling quantity $q(w) = \frac{mv^2 - v^2 w - 4m\varphi + 4w\varphi + 2f\rho\varphi}{2v^2 - 6\varphi}$, $p_e(w) = \frac{f\rho(v^2-\varphi)+(-m+w)\varphi}{v^2-3\varphi}$, $T_e(w) = \frac{v(-m+w+2f\rho)}{2(v^2-3\varphi)}$. The supplier's optimization problem in Stage 3 is characterized by the first-order condition of Equation (13):

$$\frac{\partial \pi_S^{\bar{N}}}{\partial w} = \frac{mv^2 - 2v^2 w - 4m\varphi + 8w\varphi + 2f\rho\varphi}{2v^2 - 6\varphi} = 0 \tag{18}$$

From Equation (18), we can derive the optimal wholesale price of product $w^{\bar{N}} = \frac{mv^2 - 4m\varphi + 2f\rho\varphi}{2v^2 - 8\varphi}$. Substituting the above optimal wholesale price into the above optimal reselling quantity function, EW price function and EW period function, we can derive the optimal reselling quantity, EW price and EW period:

$$q_t^{\bar{N}} = \frac{mv^2 - 4m\varphi + 2f\rho\varphi}{4v^2 - 12\varphi} \tag{19}$$

$$p_e^{\bar{N}} = \frac{m\varphi(-v^2 + 4\varphi) + 2f\rho(v^4 - 5v^2\varphi + 5\varphi^2)}{2(v^2 - 4\varphi)(v^2 - 3\varphi)} \tag{20}$$

$$T_e^{\bar{N}} = \frac{v(-mv^2 + 4fv^2\rho + 4m\varphi - 14f\rho\varphi)}{4(v^2 - 4\varphi)(v^2 - 3\varphi)} \tag{21}$$

With the above equilibrium wholesale price, reselling quantity, EW price and EW period, thus, inverse demand function in Equation (1), EW demand function in (12), the supplier's profit function in Equation (13) and the e-platform's profit function in Equation (14), we have all equilibrium solutions in Table 2.

Model $\bar{D}$:

The supplier not only wholesales his products to the e-platform for reselling, but also sells products , and the e-platform provides EW service as a countermeasure in this scenario, we bring $i = t$, $j = d$ and $i = d$, $j = t$ to Equation (1) obtain p_t and p_d, respectively. We can derive the supplier's and the e-platform's profit function as follows:

$$\pi_S^{\bar{D}} = wq_t + (m - q_d - \lambda q_t)q_d - F \tag{22}$$

$$\pi_P^{\bar{D}} = (m - q_t - \lambda q_d - w)q_t + (p_e - \rho f)D_e - \varphi T_e^2 \tag{23}$$

In Stage 4, the e-platform needs to decide on the EW price, the EW period and the reselling price. Hence the e-platform's optimization problem in Stage 4 is characterized by the first-order condition of Equation (23):

$$\frac{\partial \pi_P^{\bar{D}}}{\partial q_t} = m + p_e - 2q - w - \lambda q_d - f\rho = 0 \tag{24}$$

$$\frac{\partial \pi_P^{\bar{D}}}{\partial p_e} = -2p_e + q + vT_e + f\rho = 0 \tag{25}$$

$$\frac{\partial \pi_P^{\bar{D}}}{\partial T_e} = vp_e - fv\rho - 2\varphi T_e = 0 \tag{26}$$

The Hessian matrix for the profit function of the supplier is:

$$\begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & v \\ 0 & v & -2\varphi \end{bmatrix}$$

The Hessian matrix is non-positive when $v^2 < 3\varphi$ is met. From Equation (24), (25) and (26) which we can derive the optimal reselling quantity, EW price, EW period as functions of the wholesale price and the direct selling quantity $q_t(w,q_d) = \frac{-v^2(w+\lambda q_d)+m(v^2-4\varphi)+2(2w+2\lambda q_d+f\rho)\varphi}{2(v^2-3\varphi)}$, $p_e(w,q_d) = \frac{f\rho(v^2-\varphi)+(-m+w+\lambda q_d)\varphi}{v^2-3\varphi}$, $T_e(w,q_d) = \frac{v(-m+w+\lambda q_d+2f\rho)}{2(v^2-3\varphi)}$. The supplier's optimization problem in Stage 3 is characterized by the first-order condition of Equation (22):

$$\frac{\partial \pi_S^{\bar{D}}}{\partial w} = \frac{mv^2 - 2v^2w - 4m\varphi + 8w\varphi + 2f\rho\varphi}{2v^2 - 6\varphi} = 0 \tag{27}$$

$$\frac{\partial \pi_S^{\bar{D}}}{\partial q_d} = m - \frac{m\lambda}{2} + (-2+\lambda^2)q_d = 0 \tag{28}$$

The Hessian matrix for the profit function of the supplier is:

$$\begin{bmatrix} -\frac{v^2-4\varphi}{v^2-3\varphi} & 0 \\ 0 & \frac{v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi}{v^2-3\varphi} \end{bmatrix}$$

The Hessian matrix is positive when $v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi > 0$ is met. From Equation (27) and (28), we can derive the optimal wholesale price of product $w^{\bar{D}} = \frac{mv^2-4m\varphi+2f\rho\varphi}{2v^2-8\varphi}$ and the optimal direct selling quantity $q_d^{\bar{D}} = \frac{2f\lambda\rho\varphi+m(v^2(-2+\lambda)+6\varphi-4\lambda\varphi)}{2(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)}$. Substituting the above optimal wholesale price and optimal direct selling quantity into the above optimal reselling quantity function, EW price function and EW period function, we can derive the optimal reselling quantity, EW price and EW period:

$$q_t^{\bar{D}} = \frac{m(-1+\lambda)(v^2-4\varphi) - 2f\rho\varphi}{2(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)} \tag{29}$$

$$p_e^{\bar{D}} = \frac{-m(-1+\lambda)(v^2-4\varphi)\varphi + f\rho(v^4(-2+\lambda^2)+2v^2(5-3\lambda^2)\varphi+2(-5+4\lambda^2)\varphi^2)}{(v^2-4\varphi)(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)} \tag{30}$$

$$T_e^{\bar{D}} = \frac{v(-m(-1+\lambda)(v^2-4\varphi)+2f\rho(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi))}{2(v^2-4\varphi)(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)} \tag{31}$$

With the above equilibrium wholesale price, direct selling quantity, reselling quantity, EW price and EW period, thus, inverse demand function in Equation (1), EW demand function in (12), the supplier's profit function in Equation (22) and the e-platform's profit function in Equation (23), we have all equilibrium solutions in Table 3.

Model $\bar{A}$:

The supplier not only wholesales his products to the e-platform for reselling, but also sell their products through an agency channel on the e-platform by paying commissions, and the e-platform provides EW service as a countermeasure in this scenario, we bring $i=t$, $j=o$ and $i=o$, $j=t$ to Equation (1) obtain p_t and p_o, respectively. We can derive the supplier's and the e-platform's profit function as follows:

$$\pi_S^{\bar{A}} = wq_t + (1-\alpha)p_oq_o \tag{32}$$

$$\pi_P^{\bar{A}} = q(p_t - w) + \alpha p_oq_o + (p_e - \rho f)D_e - \varphi T_e^2 \tag{33}$$

In Stage 4, the e-platform needs to decide on the EW price, the EW period and the reselling price. Hence the e-platform's optimization problem in stage 4 is characterized by the first-order condition of Equation (33):

$$\frac{\partial \pi_P^{\bar{A}}}{\partial q_t} = m + p_e - 2q - w - (1+\alpha)\lambda q_o - f\rho = 0 \tag{34}$$

$$\frac{\partial \pi_P^{\bar{A}}}{\partial p_e} = -2p_e + q + vT_e + f\rho = 0 \tag{35}$$

$$\frac{\partial \pi_P^{\bar{A}}}{\partial T_e} = p_ev - fv\rho - 2\varphi T_e = 0 \tag{36}$$

The Hessian matrix for the profit function of the supplier is:

$$\begin{bmatrix} -2 & 1 & 0 \\ 1 & -2 & v \\ 0 & v & -2\varphi \end{bmatrix}$$

The Hessian matrix is non-positive when $v^2 < 3\varphi$ is met. From Equation (34), (35) and (36) which we can derive the optimal reselling quantity, EW price, EW period as functions of the wholesale price and the direct selling quantity $q_t(w, q_o) = \frac{-v^2(w+(1+\alpha)\lambda q_o)+m(v^2-4\varphi)+2(2w+2(1+\alpha)\lambda q_o+f\rho)\varphi}{2(v^2-3\varphi)}$, $p_e(w, q_o) = \frac{f\rho(v^2-\varphi)+(-m+w+(1+\alpha)\lambda q_o)\varphi}{v^2-3\varphi}$, $T_e(w, q_o) = \frac{v(-m+w+(1+\alpha)\lambda q_o+2f\rho)}{2(v^2-3\varphi)}$. The supplier's optimization problem in stage 3 is characterized by the first-order condition of Equation (32):

$$\frac{\partial \pi_S^{\bar{A}}}{\partial w} = \frac{-2v^2(w+\alpha\lambda q_o)+m(v^2-4\varphi)+2(4w+4\alpha\lambda q_o+f\rho)\varphi}{2(v^2-3\varphi)} = 0 \tag{37}$$

$$\frac{\partial \pi_S^{\bar{A}}}{\partial q_o} = \frac{\begin{array}{c} m(-1+\alpha)(v^2(-2+\lambda)+(6-4\lambda)\varphi)-2q_o(-1+\alpha)(v^2(-2+(1+\alpha)\lambda^2)+ \\ 6\varphi-4(1+\alpha)\lambda^2\varphi)+2\lambda(-v^2w\alpha+4w\alpha\varphi+f(-1+\alpha)\rho\varphi) \end{array}}{2(v^2-3\varphi)} = 0 \tag{38}$$

The Hessian matrix for the profit function of the supplier is:

$$\begin{bmatrix} -\frac{v^2-4\varphi}{v^2-3\varphi} & 0 \\ 0 & -\frac{(-1+\alpha)(v^2(-2+(1+\alpha)\lambda^2)+6\varphi-4(1+\alpha)\lambda^2\varphi)}{v^2-3\varphi} \end{bmatrix}$$

The Hessian matrix is positive when $v^2(-2+2\alpha+\lambda^2)-2(-3+3\alpha+2\lambda^2)\varphi > 0$ is met. From Equation (37) and (38), we can derive the optimal wholesale price of product $w^{\bar{A}} = \frac{(1-\alpha)(2f\rho\varphi(v^2(-2+\lambda^2)+6\varphi-4\lambda^2\varphi)}{(v^2-4\varphi)z_1} + \frac{m(v^2-4\varphi)(v^2(-2+2\alpha\lambda+\lambda^2)-2(-3+3\alpha\lambda+2\lambda^2)\varphi))}{(v^2-4\varphi)(z_1}$ and the optimal direct selling quantity $q_o^{\bar{A}} = \frac{mv^2(-2+2\alpha+\lambda)}{z_1} - \frac{2m(-3+3\alpha+2\lambda)\varphi+2f\lambda\rho\varphi}{z_1}$. Where $z1 = 2v^2(-2+2\alpha+\lambda^2)-4(-3+3\alpha+2\lambda^2)\varphi$. Substituting the above optimal wholesale price and optimal direct selling quantity into the above optimal reselling quantity function, EW price function and EW period function, we can derive the optimal reselling quantity, EW price and EW period:

$$q_t^{\bar{A}} = \frac{(1-\alpha)(m(-1+\lambda)(v^2-4\varphi)-2f\rho\varphi)}{2v^2(-2+2\alpha+\lambda^2)-4(-3+3\alpha+2\lambda^2)\varphi} \tag{39}$$

$$p_e^{\bar{A}} = \frac{m(-1+\alpha)(-1+\lambda)(v^2-4\varphi)\varphi+f\rho(v^4(-2+2\alpha+\lambda^2)-2v^2(-5+5\alpha+3\lambda^2)\varphi+2(-5+5\alpha+4\lambda^2)\varphi^2)}{(v^2-4\varphi)(v^2(-2+2\alpha+\lambda^2)-2(-3+3\alpha+2\lambda^2)\varphi)} \tag{40}$$

$$T_e^{\bar{A}} = \frac{v(m(-1+\alpha)(-1+\lambda)(v^2-4\varphi)+2f\rho(v^2(-2+2\alpha+\lambda^2)+(7-7\alpha-4\lambda^2)\varphi))}{2(v^2-4\varphi)(v^2(-2+2\alpha+\lambda^2)-2(-3+3\alpha+2\lambda^2)\varphi)} \tag{41}$$

With the above equilibrium wholesale price, direct selling quantity, reselling quantity, EW price and EW period, thus, inverse demand function in Equation (1), EW demand function in (12), the supplier's profit function in Equation (32) and the e-platform's profit function in Equation (33), we have all equilibrium solutions in Table 4.

Table 1, Table 2, Table 3, Table 4 summarizes the equilibrium solutions for five different models in Section 4: no encroachment scenario (Model N), direct encroachment without EW service (Model D), agency encroachment without EW service (Model A), no encroachment with EW service (Model $\bar{N}$), direct encroachment with EW service (Model $\bar{D}$), agency encroachment with EW service (Model $\bar{A}$).

Table 1: Equilibrium solutions of Model N, D, A.

Notations	Model N	Model D	Model A
w^s	$\frac{m}{2}$	$\frac{m}{2}$	$\frac{m(-2-2\alpha^2\lambda+\lambda^2+\alpha(2+2\lambda-\lambda^2))}{2(-2+2\alpha+\lambda^2)}$
q^s	$\frac{m}{4}$	$\frac{m(-1+\lambda)}{2(-2+\lambda^2)}$	$-\frac{m(-1+\alpha)(-1+\lambda)}{2(-2+2\alpha+\lambda^2)}$
q_d^s/q_o^s	–	$\frac{m(-2+\lambda)}{2(-2+\lambda^2)}$	$\frac{m(-2+2\alpha+\lambda)}{2(-2+2\alpha+\lambda^2)}$
p_t^s	$\frac{3m}{4}$	$\frac{m(-3+\lambda+\lambda^2)}{2(-2+\lambda^2)}$	$\frac{m(-3-\alpha(-3+\lambda)+\lambda+\lambda^2)}{2(-2+2\alpha+\lambda^2)}$
p_d^s/p_o^s	–	$\frac{m}{2}$	$\frac{m(-2+\lambda^2+\alpha(2-\lambda+\lambda^2))}{2(-2+2\alpha+\lambda^2)}$
π_S^s	$\frac{m^2}{8}$	$-F+\frac{m^2(-3+2\lambda)}{4(-2+\lambda^2)}$	$-\frac{m^2(-1+\alpha)(-3+2\alpha+2\lambda)}{4(-2+2\alpha+\lambda^2)}$
π_P^s	$\frac{m^2}{16}$	$\frac{m^2(-1+\lambda)^2}{4(-2+\lambda^2)^2}$	$\frac{m^2(4\alpha^3+(-1+\lambda)^2+\alpha^2(-7-2\lambda+5\lambda^2)+\alpha(2+4\lambda-7\lambda^2+2\lambda^3))}{4(-2+2\alpha+\lambda^2)^2}$

Note: "s" means superscript, representing model as no/direct/agency encroachment, respectively.

Table 2: Equilibrium solutions of Model $\bar{N}$.

Notations	Model $\bar{N}$
$w^{\bar{N}}$	$\frac{mv^2-4m\varphi+2f\rho\varphi}{2v^2-8\varphi}$
$q^{\bar{N}}$	$\frac{mv^2-4m\varphi+2f\rho\varphi}{4v^2-12\varphi}$
$p_e^{\bar{N}}$	$\frac{m\varphi(-v^2+4\varphi)+2f\rho(v^4-5v^2\varphi+5\varphi^2)}{2(v^2-4\varphi)(v^2-3\varphi)}$
$T_e^{\bar{N}}$	$\frac{v(-mv^2+4fv^2p+4m\varphi-14f\rho\varphi)}{4(v^2-4\varphi)(v^2-3\varphi)}$
$p_t^{\bar{N}}$	$\frac{3mv^2-8m\varphi-2f\rho\varphi}{4v^2-12\varphi}$
$D_e^{\bar{N}}$	$-\frac{\varphi(m(v^2-4\varphi)+2f\rho(-2v^2+7\varphi))}{2(v^2-4\varphi)(v^2-3\varphi)}$
$\pi_S^{\bar{N}}$	$\frac{(m(v^2-4\varphi)+2f\rho\varphi)^2}{8(v^2-4\varphi)(v^2-3\varphi)}$
$\pi_P^{\bar{N}}$	$\frac{m^2(v^2-4\varphi)^2+4fm\rho(v^2-4\varphi)\varphi+4f^2\rho^2\varphi(-4v^2+13\varphi)}{16(v^2-4\varphi)(v^2-3\varphi)}$

Table 3: Equilibrium solutions of Model $\bar{D}$.

Notations	Model $\bar{D}$
$w^{\bar{D}}$	$\frac{mv^2-4m\varphi+2f\rho\varphi}{2v^2-8\varphi}$
$q^{\bar{D}}$	$\frac{2f\lambda\rho\varphi+m(v^2(-2+\lambda)+6\varphi-4\lambda\varphi)}{2(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)}$
$q_d^{\bar{D}}$	$\frac{m(-1+\lambda)(v^2-4\varphi)-2f\rho\varphi)}{2(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)}$
$p_e^{\bar{D}}$	$\frac{-m(-1+\lambda)(v^2-4\varphi)\varphi+f\rho(v^4(-2+\lambda^2)+2v^2(5-3\lambda^2)\varphi+2(-5+4\lambda^2)\varphi^2)}{(v^2-4\varphi)(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)}$
$T_e^{\bar{D}}$	$\frac{v(-m(-1+\lambda)(v^2-4\varphi)+2f\rho(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi))}{2(v^2-4\varphi)(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)}$
$p_t^{\bar{D}}$	$\frac{mv^2(-3+\lambda+\lambda^2)-2m(-4+\lambda+2\lambda^2)\varphi-2f(-1+\lambda^2)\rho\varphi}{2(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)}$
$p_t^{\bar{D}}$	$\frac{m}{2}$
$D_e^{\bar{D}}$	$\frac{\varphi(-m(-1+\lambda)(v^2-4\varphi)+2f\rho(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi))}{(v^2-4\varphi)(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)}$
$\pi_S^{\bar{D}}$	$\frac{4fm(-1+\lambda)\rho(v^2-4\varphi)\varphi-4f^2\rho^2\varphi^2+m^2(v^2-4\varphi)(v^2(-3+2\lambda)+2(5-4\lambda)\varphi)-4F(v^2-4\varphi)(v^2(-2+\lambda^2)+6\varphi-4\lambda^2\varphi)}{4(v^2-4\varphi)(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)}$
$\pi_P^{\bar{D}}$	$q^{\bar{D}}(p_t^{\bar{D}}-w^{\bar{D}})+D_e^{\bar{D}}(p_e^{\bar{D}}-f\rho)-\varphi T_e^{\bar{D}}$

Table 4: Equilibrium solutions of Model $\bar{A}$.

Notations	Model $\bar{A}$
$w^{\bar{A}}$	$\frac{(1-\alpha)(2f\rho\varphi(v^2(-2+\lambda^2)+6\varphi-4\lambda^2\varphi)+m(v^2-4\varphi)(v^2(-2+2\alpha\lambda+\lambda^2)-2(-3+3\alpha\lambda+2\lambda^2)\varphi))}{2(v^2-4\varphi)(v^2(-2+2\alpha+\lambda^2)-2(-3+3\alpha+2\lambda^2)\varphi)}$
$q^{\bar{A}}$	$\frac{(1-\alpha)(m(-1+\lambda)(v^2-4\varphi)-2f\rho\varphi)}{2v^2(-2+2\alpha+\lambda^2)-4(-3+3\alpha+2\lambda^2)\varphi}$
$q_o^{\bar{A}}$	$\frac{mv^2(-2+2\alpha+\lambda)-2m(-3+3\alpha+2\lambda)\varphi+2f\lambda\rho\varphi}{2v^2(-2+2\alpha+\lambda^2)-4(-3+3\alpha+2\lambda^2)\varphi}$
$p_e^{\bar{A}}$	$\frac{m(-1+\alpha)(-1+\lambda)(v^2-4\varphi)\varphi+f\rho(v^4(-2+2\alpha+\lambda^2)-2v^2(-5+5\alpha+3\lambda^2)\varphi+2(-5+5\alpha+4\lambda^2)\varphi^2)}{(v^2-4\varphi)(v^2(-2+2\alpha+\lambda^2)-2(-3+3\alpha+2\lambda^2)\varphi)}$
$T_e^{\bar{A}}$	$\frac{v(m(-1+\alpha)(-1+\lambda)(v^2-4\varphi)+2f\rho(v^2(-2+2\alpha+\lambda^2)+(7-7\alpha-4\lambda^2)\varphi))}{2(v^2-4\varphi)(v^2(-2+2\alpha+\lambda^2)-2(-3+3\alpha+2\lambda^2)\varphi)}$
$p_o^{\bar{A}}$	$\frac{mv^2(-2+\lambda^2+\alpha(2-\lambda+\lambda^2))-2m(-3+2\lambda^2+\alpha(3-2\lambda+2\lambda^2))\varphi-2f\alpha\lambda\rho\varphi}{2v^2(-2+2\alpha+\lambda^2)-4(-3+3\alpha+2\lambda^2)\varphi}$
$p_t^{\bar{A}}$	$\frac{mv^2(-3-\alpha(-3+\lambda)+\lambda+\lambda^2)+2m(4+\alpha(-4+\lambda)-\lambda-2\lambda^2)\varphi-2f(-1+\alpha+\lambda^2)\rho\varphi}{2v^2(-2+2\alpha+\lambda^2)-4(-3+3\alpha+2\lambda^2)\varphi}$
$D_e^{\bar{A}}$	$\frac{\varphi(m(-1+\alpha)(-1+\lambda)(v^2-4\varphi)+2f\rho(v^2(-2+2\alpha+\lambda^2)+(7-7\alpha-4\lambda^2)\varphi))}{(v^2-4\varphi)(v^2(-2+2\alpha+\lambda^2)-2(-3+3\alpha+2\lambda^2)\varphi)}$
$\pi_S^{\bar{A}}$	$\frac{(1-\alpha)(4fm(-1+\lambda)\rho(v^2-4\varphi)\varphi-4f^2\rho^2\varphi^2+m^2(v^2-4\varphi)(v^2(-3+2\alpha+2\lambda)-2(-5+3\alpha+4\lambda)\varphi))}{4(v^2-4\varphi)(v^2(-2+2\alpha+\lambda^2)-2(-3+3\alpha+2\lambda^2)\varphi)}$
$\pi_P^{\bar{A}}$	$q^{\bar{A}}(p_t^{\bar{A}}-w^{\bar{A}})+\alpha p_o^{\bar{A}}q_o^{\bar{A}}+D_e^{\bar{A}}(p_e^{\bar{A}}-f\rho)-\varphi {T_e^{\bar{A}}}^2$

Appendix B: Proof of Lemma and Proposition.

Proof of Lemma 1 The equilibrium solutions of model N, model D and model A are known from Appendix A. Therefore, it is easy to conclude the following.

(1) $w^N = \frac{m}{2}$, $w^D = \frac{m}{2}$, $w^A = \frac{m(-2-2\alpha^2\lambda+\lambda^2+\alpha(2+2\lambda-\lambda^2))}{2(-2+2\alpha+\lambda^2)}$, $w^N - w^A = \frac{m\alpha\lambda(-2+2\alpha+\lambda)}{2(-2+2\alpha+\lambda^2)} > 0$, so it is easy to see that $w^N = w^D > w^A$.

(2) $q_t^N = \frac{m}{4}$, $q_t^D = \frac{m(-1+\lambda)}{2(-2+\lambda^2)}$, $q_t^A = \frac{m(1-\alpha)(-1+\lambda)}{2(-2+2\alpha+\lambda^2)}$, $q_t^N - q_t^A = \frac{m\lambda(-2+2\alpha+\lambda)}{4(-2+2\alpha+\lambda^2)} > 0$, $q_t^A - q_t^D = \frac{m\alpha(1-\lambda)\lambda^2}{2(-2+\lambda^2)(-2+2\alpha+\lambda^2)} < 0$, so it is easy to see that $q_t^N > q_t^A > q_t^D$.

(3) $q_o^D + q_t^D = \frac{m(-3+2\lambda)}{2(-2+\lambda^2)}$, $q_o^A + q_t^A = -\frac{m(3+\alpha(-3+\lambda)-2\lambda)}{2(-2+2\alpha+\lambda^2)}$, $q_o^D + q_t^D - (q_o^A + q_t^A) = \frac{m\alpha(-2+\lambda)(-1+\lambda)\lambda}{2(-2+\lambda^2)(-2+2\alpha+\lambda^2)} > 0$. $q_o^A + q_t^A - q_t^N = -\frac{m(-2+\lambda)(-2+2\alpha+\lambda)}{4(-2+2\alpha+\lambda^2)} > 0$, so it is easy to see that $q_d^D + q_t^D > q_o^A + q_t^A > q_t^N$. $\frac{\partial(q_d^D+q_t^D-(q_o^A+q_t^A))}{\partial\alpha} = \frac{m(-2+\lambda)(-1+\lambda)\lambda}{2(-2+2\alpha+\lambda^2)^2} > 0$, thus $(q_d^D + q_t^D - (q_o^A + q_t^A))$ increases with α.

Derivation of Critical Thresholds We follow the upper and lower bound rules defined in the main paper to solve the upper and lower bounds of f for each model when the EW service is provided.

(1) We first prove whether the e-platform provides EW service under no encroachment scenario, the e-platform only provides EW service when it is beneficial to do so. We can derive upper and lower bounds on f for the provision of EW service by the e-platform by calculating them separately for two cases. When f is too low so that the reselling price is lower than the wholesale price, e.g., $p_t^{\bar{N}} - w^{\bar{N}} = \frac{mv^4-6v^2(m+f\rho)\varphi+4(2m+5f\rho)\varphi^2}{4(v^2-4\varphi)(v^2-3\varphi)} > 0$, we can get

$$\hat{f}_1^N = \frac{mv^4 - 6mv^2\varphi + 8m\varphi^2}{6v^2\rho\varphi - 20\rho\varphi^2} \tag{42}$$

Here we find that the denominator of $\hat{f}_1^N$ is negative, however, it is uncertain whether $mv^4 - 6mv^2\varphi + 8m\varphi^2$ is positive or negative, so $\hat{f}_1^N$ is not necessarily positive. When $\varphi > \frac{v^2}{2}$, $\hat{f}_1^N < 0$, thus when $\frac{v^2}{3} < \varphi < \frac{v^2}{2}$, $\hat{f}_1^N > 0$. When f is too high so that the EW period falls to zero with the rising cost of EW, e.g., $D_e^{\bar{N}} = -\frac{\varphi(m(v^2-4\varphi)+2f\rho(-2v^2+7\varphi))}{2(v^2-4\varphi)(v^2-3\varphi)} > 0$, we can get

$$\hat{f}_2^N = \frac{mv^2 - 4m\varphi}{4v^2\rho - 14\rho\varphi} \tag{43}$$

Also, we prove that while $v^2 < 3\varphi$, $\hat{f}_2^N - \hat{f}_1^N = -\frac{m(v^2-4\varphi)^2(v^2-3\varphi)}{\rho\varphi(6v^4-41v^2\varphi+70\varphi^2)} > 0$. By solving that $\pi_P^{\bar{N}} - \pi_P^N = 0$, we can obtain two imaginary roots $f = \frac{(v^2-4\varphi)(m\rho\varphi+(v^2-3\varphi)\sqrt{-\frac{3m^2\rho^2\varphi^2}{v^4-7v^2\varphi+12\varphi^2}})}{2\rho^2(4v^2-13\varphi)\varphi}$ and $f =$

$\frac{(v^2-4\varphi)(m\rho\varphi+(-v^2+3\varphi)\sqrt{-\frac{3m^2\rho^2\varphi^2}{v^4-7v^2\varphi+12\varphi^2}})}{2\rho^2(4v^2-13\varphi)\varphi}$. When f between the two roots, $\pi_P^{\bar{N}} > \pi_P^N$. Thus when $\frac{v^2}{3} < \varphi < \frac{v^2}{2}$, $f < \hat{f}_1^N$ or $f > \hat{f}_2^N$ and $\varphi > \frac{v^2}{2}$, $f > \hat{f}_2^N$, the e-platform will not provide EW service.

(2) Under model $\bar{D}$, similar to no encroachment scenario, we discuss the upper and lower bounds of EW service provision by the e-platform under agency encroachment. When f is too low so that the reselling price is lower than the wholesale price, e.g., $p_t^{\bar{D}} - w^{\bar{D}} = \frac{2f\rho\varphi(v^2(3-2\lambda^2)+2(-5+4\lambda^2)\varphi)}{2(v^2-4\varphi)(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)} + \frac{m(-1+\lambda)(v^4-6v^2\varphi+8\varphi^2)}{2(v^2-4\varphi)(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)} > 0$, we can get

$$\hat{f}_1^D = \frac{m(1-\lambda)(v^2-4\varphi)(v^2-2\varphi)}{2\rho\varphi(v^2(3-2\lambda^2)+2(-5+4\lambda^2)\varphi)} \tag{44}$$

Here we find that the denominator of $\hat{f}_1^D$ is negative, and $(v^2(3-2\lambda^2)+2(-5+4\lambda^2)\varphi)+(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi) = (1-\lambda^2)(v^2-4\varphi) < 0$, however, it is uncertain whether $v^2-2\varphi$ is positive or negative, so $\hat{f}_1^D$ is not necessarily positive. Thus when $\frac{v^2}{3} < \varphi < \frac{v^2}{2}$, $\hat{f}_1^D > 0$. when f is too high so that the EW period falls to zero with the rising cost of EW, e.g., $D_e^{\bar{D}} = \frac{\varphi(-m(-1+\lambda)(v^2-4\varphi)+2f\rho(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi))}{(v^2-4\varphi)(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)} > 0$, we can get

$$\hat{f}_2^D = \frac{m(-1+\lambda)(v^2-4\varphi)}{2\rho(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi)} \tag{45}$$

Also we prove that $\hat{f}_2^D - \hat{f}_1^D = \frac{m(1-\lambda)(v^2-4\varphi)^2(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)}{2\rho\varphi(-v^2(-2+\lambda^2)+(-7+4\lambda^2)\varphi)(v^2(3-2\lambda^2)+2(-5+4\lambda^2)\varphi)} > 0$. By solving that $\pi_P^{\bar{D}} - \pi_P^D = 0$, we can obtain two imaginary roots

$$f = \frac{(v^2-4\varphi)(-m(-1+\lambda)\rho(v^2-3\varphi)\varphi+(v^2(-2+\lambda^2)+6\varphi-4\lambda^2\varphi)^2 z_5}{2\rho^2\varphi(v^4(-2+\lambda^2)^2+v^2(-25+28\lambda^2-8\lambda^4)\varphi+(39+16\lambda^2(-3+\lambda^2))\varphi^2)} \tag{46}$$

$$f = -\frac{(v^2-4\varphi)(m(-1+\lambda)\rho(v^2-3\varphi)\varphi+(v^2(-2+\lambda^2)+6\varphi-4\lambda^2\varphi)^2 z_5}{2\rho^2\varphi(v^4(-2+\lambda^2)^2+v^2(-25+28\lambda^2-8\lambda^4)\varphi+(39+16\lambda^2(-3+\lambda^2))\varphi^2)} \tag{47}$$

Where $z_5 = \sqrt{\frac{m^2(-1+\lambda)^2\rho^2\varphi^2(v^2(-3+\lambda^4)+(9-4\lambda^4)\varphi)}{(v^2-4\varphi)(v^2(-2+\lambda^2)^2-2(6-7\lambda^2+2\lambda^4)\varphi)^2}}$. When f between the two roots, $\pi_P^{\bar{D}} > \pi_P^D$. Thus when $\frac{v^2(-2+\lambda^2)}{-(6-4\lambda^2)} < \varphi < \frac{v^2}{2}$, $f < \hat{f}_1^D$, $f > \hat{f}_2^D$, the e-platform will not provide EW service.

(3) Under model $\bar{A}$, similar to direct encroachment scenario, we discuss the upper and lower bounds of EW service provision by the e-platform under agency encroachment. When f is too low so that the reselling price is lower than the wholesale price, e.g., $p_t^{\bar{A}} - w^{\bar{A}} = 0$, we can get

$$\hat{f}_1^A = \frac{m(v^2-4\varphi)(v^2(-1+\alpha+\lambda+\alpha\lambda(-3+2\alpha+\lambda))-2(-1+\alpha+\lambda+\alpha\lambda(-4+3\alpha+2\lambda))\varphi)}{2\rho\varphi(v^2(3(-1+\alpha)-(-2+\alpha)\lambda^2)+2(5-5\alpha+2(-2+\alpha)\lambda^2)\varphi)} \tag{48}$$

Let $A = (v^2(3(-1+\alpha)-(-2+\alpha)\lambda^2)+2(5-5\alpha+2(-2+\alpha)\lambda^2)\varphi)$, $A - (v^2(-2+2\alpha+\lambda^2) - 2(-3+3\alpha+2\lambda^2)\varphi) = (1-\alpha)(-1+\lambda^2)(v^2-4\varphi) > 0$, thus $A > 0$. However, it is uncertain whether the numerator is positive or negative, so $\hat{f}_1^A$ is not necessarily positive. When $\varphi > \frac{v^2(-1+\alpha+\lambda+\alpha\lambda(-3+2\alpha+\lambda))}{2(-1+\alpha+\lambda+\alpha\lambda(-4+3\alpha+2\lambda))}$, $\hat{f}_1^A < 0$. Thus when $\frac{v^2}{3} < \varphi < \frac{v^2(-1+\alpha+\lambda+\alpha\lambda(-3+2\alpha+\lambda))}{2(-1+\alpha+\lambda+\alpha\lambda(-4+3\alpha+2\lambda))}$, $\hat{f}_1^A > 0$. When f is too high so that the EW period falls to zero with the rising cost of EW, e.g., $D_e^{\bar{A}} = \frac{\varphi(m(-1+\alpha)(-1+\lambda)(v^2-4\varphi)+2f\rho(v^2(-2+2\alpha+\lambda^2)+(7-7\alpha-4\lambda^2)\varphi))}{(v^2-4\varphi)(v^2(-2+2\alpha+\lambda^2)-2(-3+3\alpha+2\lambda^2)\varphi)} > 0$, we can get

$$\hat{f}_2^A = \frac{m(1-\alpha)(-1+\lambda)(v^2-4\varphi)}{2v^2(-2+2\alpha+\lambda^2)\rho-2(-7+7\alpha+4\lambda^2)\rho\varphi} \tag{49}$$

We can easily find that when $\alpha < 1-\lambda+\frac{\lambda^2}{2}$, $\frac{v^2(-1+\alpha+\lambda+\alpha\lambda(-3+2\alpha+\lambda))}{4(-1+\lambda)+\alpha(4+\lambda(-11+7\alpha+4\lambda))} < \frac{v^2(-2+2\alpha+\lambda^2)}{-6+6\alpha+4\lambda^2}$, it means that, when $\varphi > \frac{v^2(-2+2\alpha+\lambda^2)}{-6+6\alpha+4\lambda^2}$, $\hat{f}_1^A < \hat{f}_2^A$. By solving that $\pi_P^{\bar{A}} - \pi_P^A = 0$, we can also obtain two imaginary roots, when f between the two roots, $\pi_P^{\bar{A}} > \pi_P^A$. When f exceeds the upper and lower bounds $\hat{f}_1^A$ and $\hat{f}_2^A$, the e-platform will not provide EW service, where $\frac{v^2}{3} < \varphi < \frac{v^2(-1+\alpha+\lambda+\alpha\lambda(-3+2\alpha+\lambda))}{2(-1+\alpha+\lambda+\alpha\lambda(-4+3\alpha+2\lambda))}$ is the condition that $\hat{f}_1^A$ greater than 0.

Then we compare the thresholds $\hat{f}_1^D$, $\hat{f}_2^D$, $\hat{f}_1^N$, $\hat{f}_1^N$ for whether the e-platform adopts EW service under direct encroachment: First of all, it's easy to prove that $\hat{f}_2^D < \hat{f}_2^N$. If $v^2 > 2\varphi$, then $\hat{f}_1^D < \hat{f}_1^N$, and if $v^2 < 2\varphi$, $\hat{f}_1^D$, $\hat{f}_1^N$ are both less than zero, and $v^2 > 2\varphi$ is the condition that makes $\hat{f}_1^D$ and $\hat{f}_1^N$ greater than 0. Let $\hat{f}_2^D = \hat{f}_1^N$, we can obtain

$$\hat{\varphi}_{d1} = -\frac{v^2(14 - 3\lambda(1 + 2\lambda) + \sqrt{4 + \lambda(-4 + (1 - 2\lambda)^2\lambda)})}{-48 + 4\lambda(5 + 4\lambda)} \tag{50}$$

When $\frac{v^2(-2+\lambda^2)}{-(6-4\lambda^2)} < \varphi < \hat{\varphi}_{d1}$, $\hat{f}_2^D < \hat{f}_1^N$. Then we compare the thresholds $\hat{f}_1^A$, $\hat{f}_2^A$, $\hat{f}_1^N$, $\hat{f}_2^N$ for whether the e-platform adopts EW service under agency encroachment: It is easy to prove that $\hat{f}_2^A < \hat{f}_2^N$ when $\alpha < 1 - \lambda + \frac{\lambda^2}{2}$. Let $\hat{f}_1^A = \hat{f}_1^N$, we can get $\varphi = \frac{v^2(-1+2\alpha)}{-2+6\alpha}$ or $\varphi = \frac{v^2(-3+3\alpha+2\lambda)}{2(-5+5\alpha+4\lambda)}$, when $\alpha < \frac{1}{3}$, $\frac{v^2(-1+2\alpha)}{-2+6\alpha} > \frac{v^2}{2}$, when $\frac{1}{3} < \alpha < \frac{1}{2}$, $\frac{v^2(-1+2\alpha)}{-2+6\alpha} < 0$, when $\frac{1}{2} < \alpha < 1 - \lambda + \frac{\lambda^2}{2}$, $\frac{v^2(-1+2\alpha)}{-2+6\alpha} < \frac{v^2}{4}$. We let

$$\hat{\varphi}_{\alpha 1} = \frac{v^2(-3 + 3\alpha + 2\lambda)}{2(-5 + 5\alpha + 4\lambda)} \tag{51}$$

When $\varphi < \hat{\varphi}_{\alpha 1}$, $\hat{f}_1^A > \hat{f}_1^N$, when $\varphi > \hat{\varphi}_{\alpha 1}$, $\hat{f}_1^A < \hat{f}_1^N$. If $\alpha < (-1 + \lambda)^2$, $\hat{\varphi}_{\alpha 1} < \frac{v^2(-2+2\alpha+\lambda^2)}{2(-3+3\alpha+2\lambda^2)}$, thus $\hat{f}_1^A < \hat{f}_1^N$. If $\alpha > 1 - \lambda$, thus $\hat{f}_1^A > \hat{f}_1^N$. Let $\hat{f}_2^A = \hat{f}_1^N$, we can get

$$\hat{\varphi}_{\alpha 2} = \frac{z_6 + v^2(14 - 3\lambda(1 + 2\lambda) + \alpha(-14 + 3\lambda)}{48 - 4\lambda(5 + 4\lambda) + 4\alpha(-12 + 5\lambda)} \tag{52}$$

Where $z_6 = \sqrt{4(-1 + \alpha)^2 - 4(-1 + \alpha)^2\lambda + (-1 + \alpha)(-1 + 9\alpha)\lambda^2 + 4(-1 + \alpha)\lambda^3 + 4\lambda^4}$. When $0 < \alpha < (-1 + \lambda)^2$, $\hat{\varphi}_{\alpha 2} > \frac{v^2(-2+2\alpha+\lambda^2)}{2(-3+3\alpha+2\lambda^2)}$, thus when $\varphi < \hat{\varphi}_{\alpha 2}$, $\hat{f}_1^N > \hat{f}_2^A$. When $\alpha < (-1 + \lambda)^2 < \alpha < 1 - \lambda + \frac{\lambda^2}{2}$, $\hat{\varphi}_{\alpha 2} < \frac{v^2(-2+2\alpha+\lambda^2)}{2(-3+3\alpha+2\lambda^2)}$, thus $\hat{f}_2^A > \hat{f}_1^N$.

Proof of Lemma 2 Under no encroachment, $\frac{\partial w^{\bar{N}}}{\partial f} = \frac{\rho\varphi}{v^2-4\varphi} < 0$, $\frac{\partial q_t^{\bar{N}}}{\partial f} = \frac{\rho\varphi}{2v^2-6\varphi} < 0$, $\frac{\partial p_t^{\bar{N}}}{\partial f} = -\frac{\rho\varphi}{2v^2-6\varphi} > 0$, $\frac{\partial p_e^{\bar{N}}}{\partial f} = \frac{\rho(v^4-5v^2\varphi+5\varphi^2)}{(v^2-4\varphi)(v^2-3\varphi)}$, $\frac{\partial T_e^{\bar{N}}}{\partial f} = \frac{v\rho(2v^2-7\varphi)}{2(v^4-7v^2\varphi+12\varphi^2)} < 0$, $\frac{\partial D_e^{\bar{N}}}{\partial f} = \frac{\rho(2v^2-7\varphi)\varphi}{(v^2-4\varphi)(v^2-3\varphi)} < 0$. $\frac{\partial w^{\bar{N}}}{\partial \varphi} = \frac{fv^2\rho}{(v^2-4\varphi)^2} > 0$, $\frac{\partial q_t^{\bar{N}}}{\partial \varphi} = -\frac{v^2(m-2f\rho)}{4(v^2-3\varphi)^2}$, $\frac{\partial p_t^{\bar{N}}}{\partial \varphi} = \frac{v^2(m-2f\rho)}{4(v^2-3\varphi)^2}$, $\frac{\partial p_e^{\bar{N}}}{\partial \varphi} = \frac{fv^2\rho}{(v^2-4\varphi)^2} - \frac{v^2(m-2f\rho)}{2(v^2-3\varphi)^2}$, $\frac{\partial T_e^{\bar{N}}}{\partial \varphi} = \frac{2fv\rho}{(v^2-4\varphi)^2} - \frac{3v(m-2f\rho)}{4(v^2-3\varphi)^2}$, $\frac{\partial D_e^{\bar{N}}}{\partial \varphi} = \frac{fv^2\rho}{(v^2-4\varphi)^2} - \frac{v^2(m-2f\rho)}{2(v^2-3\varphi)^2}$. If m tends to infinity, $\frac{\partial q_t^{\bar{N}}}{\partial \varphi} < 0$, $\frac{\partial p_t^{\bar{N}}}{\partial \varphi} > 0$, $\frac{\partial p_e^{\bar{N}}}{\partial \varphi} < 0$, $\frac{\partial T_e^{N}}{\partial \varphi} < 0$, $\frac{\partial D_e^{\bar{N}}}{\partial \varphi} < 0$. When $\varphi < \frac{(5+\sqrt{5})v^2}{10}$, $\frac{\partial p_e^{\bar{N}}}{\partial f} < 0$, otherwise, $\frac{\partial p_e^{\bar{N}}}{\partial f} > 0$.

Under direct encroachment, $\frac{\partial w^{\bar{D}}}{\partial f} = \frac{\rho\varphi}{v^2-4\varphi} < 0$, $\frac{\partial q_d^{\bar{D}}}{\partial f} = \frac{\lambda\rho\varphi}{v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi} > 0$, $\frac{\partial p_t^{\bar{D}}}{\partial f} = \frac{(1-\lambda^2)\rho\varphi}{v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi} > 0$, $\frac{\partial q_t^{\bar{D}}}{\partial f} = \frac{\rho\varphi}{2v^2-6\varphi} < 0$, $\frac{\partial D_e^{\bar{D}}}{\partial f} = \frac{2\rho\varphi(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi)}{(v^2-4\varphi)(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)} < 0$, $\frac{\partial T_e^{\bar{D}}}{\partial f} = \frac{v\rho(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi)}{(v^2-4\varphi)(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)} < 0$, $\frac{\partial p_e^{\bar{D}}}{\partial f} = \frac{\rho(v^4(-2+\lambda^2)+2v^2(5-3\lambda^2)\varphi+2(-5+4\lambda^2)\varphi^2)}{(v^2-4\varphi)(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)}$. $\frac{\partial w^{\bar{D}}}{\partial \varphi} = \frac{fv^2\rho}{(v^2-4\varphi)^2} > 0$, $\frac{\partial q_d^{\bar{D}}}{\partial \varphi} = \frac{v^2\lambda(m-m\lambda+f(-2+\lambda^2)\rho)}{(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)^2}$, $\frac{\partial q_t^{\bar{D}}}{\partial \varphi} = \frac{v^2(m(-1+\lambda)-f(-2+\lambda^2)\rho)}{(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)^2}$, $\frac{\partial p_t^{\bar{D}}}{\partial \varphi} = \frac{v^2(1-\lambda^2)(m-m\lambda+f(-2+\lambda^2)\rho)}{(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)^2}$, $\frac{\partial p_e^{\bar{D}}}{\partial \varphi} = v^2(\frac{f\rho}{(v^2-4\varphi)^2}+\frac{(-2+\lambda^2)(m-m\lambda+f(-2+\lambda^2)\rho)}{(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)^2})$, $\frac{\partial T_e^{\bar{D}}}{\partial \varphi} = \frac{2fv\rho}{(v^2-4\varphi)^2} + \frac{v(-3+2\lambda^2)(m-m\lambda+f(-2+\lambda^2)\rho)}{(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)^2}$, $\frac{\partial D_e^{\bar{D}}}{\partial \varphi} = v^2(\frac{f\rho}{(v^2-4\varphi)^2} + \frac{(-2+\lambda^2)(m-m\lambda+f(-2+\lambda^2)\rho)}{(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)^2})$. If m tends to infinity, $\frac{\partial q_t^{\bar{D}}}{\partial \varphi} < 0$, $\frac{\partial p_d^{\bar{D}}}{\partial \varphi} > 0$, $\frac{\partial p_e^{\bar{D}}}{\partial \varphi} < 0$, $\frac{\partial T_e^{\bar{D}}}{\partial \varphi} < 0$, $\frac{\partial D_e^{\bar{D}}}{\partial \varphi} < 0$. When $\varphi < \frac{v^2(5-3\lambda^2)+\sqrt{v^4(5-4\lambda^2+\lambda^4)}}{10-8\lambda^2}$, $\frac{\partial p_e^{\bar{D}}}{\partial f} < 0$, otherwise, $\frac{\partial p_e^{\bar{D}}}{\partial f} > 0$.

Under agency encroachment, $\frac{\partial w^{\bar{A}}}{\partial f} = \frac{(1-\alpha)\rho\varphi(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)}{(v^2-4\varphi)z_3} < 0$, $\frac{\partial q_o^{\bar{A}}}{\partial f} = \frac{2\lambda\rho\varphi}{2z_3} > 0$, $\frac{\partial q_t^{\bar{A}}}{\partial f} = \frac{2(-1+\alpha)\rho\varphi}{2z_3} < 0$, $\frac{\partial p_t^{\bar{A}}}{\partial f} = \frac{(1-\alpha-\lambda^2)\rho\varphi}{z_3} > 0$, $\frac{\partial p_e^{\bar{A}}}{\partial f} = \frac{\rho(v^4(-2+2\alpha+\lambda^2)-2v^2(-5+5\alpha+3\lambda^2)\varphi+2(-5+5\alpha+4\lambda^2)\varphi^2)}{(v^2-4\varphi)z_3}$, $\frac{\partial p_o^{\bar{A}}}{\partial f} = \frac{\alpha\lambda\rho\varphi}{-z_3} < 0$, $\frac{\partial T_e^{\bar{A}}}{\partial f} = \frac{v\rho(v^2(-2+2\alpha+\lambda^2)+(7-7\alpha-4\lambda^2)\varphi)}{(v^2-4\varphi)z_3} < 0$, $\frac{\partial D_e^{\bar{A}}}{\partial f} = \frac{2\rho\varphi(v^2(-2+2\alpha+\lambda^2)+(7-7\alpha-4\lambda^2)\varphi)}{(v^2-4\varphi)z_3} < 0$. $\frac{\partial w^{\bar{A}}}{\partial \varphi} = v^2(\frac{f\rho}{(v^2-4\varphi)^2} + \frac{\alpha\lambda^2(m(-1+\alpha+\lambda-\alpha\lambda)-f(-2+2\alpha+\lambda^2)\rho)}{z_3^2})$, $\frac{\partial q_o^{\bar{A}}}{\partial \varphi} = \frac{v^2\lambda(m(-1+\alpha)(-1+\lambda)+f(-2+2\alpha+\lambda^2)\rho)}{z_3^2}$, $\frac{\partial q_t^{\bar{A}}}{\partial \varphi} = \frac{2(-1+\alpha)\rho\varphi}{2z_3} < 0$, $\frac{\partial p_t^{\bar{A}}}{\partial \varphi} = \frac{v^2(1-\alpha-\lambda^2)(m(-1+\alpha)(-1+\lambda)+f(-2+2\alpha+\lambda^2)\rho)}{z_3^2}$, $\frac{\partial p_e^{\bar{A}}}{\partial \varphi} = \frac{z_4}{(v^2-4\varphi)^2z_3^2}$, $\frac{\partial D_e^{\bar{A}}}{\partial \varphi} = \frac{z_4}{(v^2-4\varphi)^2z_3^2}$, $\frac{\partial p_o^{\bar{A}}}{\partial \varphi} = -\frac{v^2\alpha\lambda(m(-1+\alpha)(-1+\lambda)+f(-2+2\alpha+\lambda^2)\rho)}{z_3^2}$, $\frac{\partial T_e^{\bar{A}}}{\partial \varphi} = \frac{2fv\rho}{(v^2-4\varphi)^2}+\frac{v(-3+3\alpha+2\lambda^2)(m(-1+\alpha)(-1+\lambda)+f(-2+2\alpha+\lambda^2)\rho)}{z_3^2}$, where $z_3 = (v^2(-2 + 2\alpha + \lambda^2) - 2(-3 + 3\alpha + 2\lambda^2)\varphi) > 0$ $z_4 = v^2(m(-1 + \alpha)(-1 + \lambda)(-2 + 2\alpha + \lambda^2)(v^2 - 4\varphi)^2 + 2f\rho(v^4(-2+2\alpha+\lambda^2)^2 - 2v^2(-2+2\alpha+\lambda^2)(-7+7\alpha+4\lambda^2)\varphi + 2(25(-1+\alpha)^2 + 28(-1+\alpha)\lambda^2 + 8\lambda^4)\varphi^2))$.

If m tends to infinity, $\frac{\partial w^{\bar{A}}}{\partial \varphi} < 0$, $\frac{\partial q_o^{\bar{A}}}{\partial \varphi} > 0$, $\frac{\partial q_t^{\bar{A}}}{\partial \varphi} < 0$, $\frac{\partial p_t^{\bar{A}}}{\partial \varphi} > 0$, $\frac{\partial p_o^{\bar{A}}}{\partial \varphi} < 0$, $\frac{\partial p_e^{\bar{A}}}{\partial \varphi} < 0$, $\frac{\partial T_e^{\bar{A}}}{\partial \varphi} < 0$, $\frac{\partial D_e^{\bar{A}}}{\partial \varphi} < 0$. When $\varphi < \frac{v^2(-5+5\alpha+3\lambda^2)-\sqrt{v^4(5(-1+\alpha)^2+4(-1+\alpha)\lambda^2+\lambda^4)}}{2(-5+5\alpha+4\lambda^2)}$, $\frac{\partial p_e^{\bar{A}}}{\partial f} < 0$, otherwise, $\frac{\partial p_e^{\bar{A}}}{\partial f} > 0$.

Proof of Proposition 1 In this proposition, we compare the EW price, EW period, and EW demand of all scenarios. $p_e^{\bar{N}} - p_e^{\bar{A}} = \frac{\lambda\varphi(-mv^2(-2+2\alpha+\lambda)+2m(-3+3\alpha+2\lambda)\varphi-2f\lambda\rho\varphi)}{2(v^2-3\varphi)(v^2(-2+2\alpha+\lambda^2)-2(-3+3\alpha+2\lambda^2)\varphi)}$,
$T_e^{\bar{N}} - T_e^{\bar{A}} = -\frac{v\lambda(mv^2(-2+2\alpha+\lambda)-2m(-3+3\alpha+2\lambda)\varphi+2f\lambda\rho\varphi)}{4(v^2-3\varphi)(v^2(-2+2\alpha+\lambda^2)-2(-3+3\alpha+2\lambda^2)\varphi)}$,
$D_e^{\bar{N}} - D_e^{\bar{A}} = -\frac{\lambda\varphi(mv^2(-2+2\alpha+\lambda)-2m(-3+3\alpha+2\lambda)\varphi+2f\lambda\rho\varphi)}{2(v^2-3\varphi)(v^2(-2+2\alpha+\lambda^2)-2(-3+3\alpha+2\lambda^2)\varphi)}$.

Let $p_e^{\bar{N}} - p_e^{\bar{A}} = T_e^{\bar{N}} - T_e^{\bar{A}} = D_e^{\bar{N}} - D_e^{\bar{A}} = 0$, we can get a threshold $f = \frac{m(-v^2(-2+2\alpha+\lambda)+2(-3+3\alpha+2\lambda)\varphi)}{2\lambda\rho\varphi}$, we always have $\hat{f}_1^N - \frac{m(-v^2(-2+2\alpha+\lambda)+2(-3+3\alpha+2\lambda)\varphi)}{2\lambda\rho\varphi} = \frac{m(v^2-3\varphi)(v^2(-3+3\alpha+2\lambda)-2(-5+5\alpha+4\lambda)\varphi)}{\lambda\rho\varphi(-3v^2+10\varphi)} > 0$, here let $(v^2(-3+3\alpha+2\lambda)-2(-5+5\alpha+4\lambda)\varphi) = 0$, we can get $\varphi = \frac{v^2(-3+2\lambda)}{-10+8\lambda} < \frac{v^2(-2+\lambda^2)}{-(6-4\lambda^2)}$. Thus $(v^2(-3+3\alpha+2\lambda)-2(-5+5\alpha+4\lambda)\varphi) > 0$. $\hat{f}_1^A - \frac{m(-1+\lambda)(v^2-4\varphi)}{2\rho\varphi} = \frac{m(v^2(-3+3\alpha+2\lambda)-2(-5+5\alpha+4\lambda)\varphi)(v^2(-2+2\alpha+\lambda^2)-2(-3+3\alpha+2\lambda^2)\varphi)}{2\lambda\rho\varphi(v^2(3(-1+\alpha)-(-2+\alpha)\lambda^2)+2(5-5\alpha+2(-2+\alpha)\lambda^2)\varphi)} > 0$, where the denominator $(v^2(3(-1+\alpha)-(-2+\alpha)\lambda^2)+2(5-5\alpha+2(-2+\alpha)\lambda^2)\varphi) > (v^2(-3+3\alpha+2\lambda)-2(-5+5\alpha+4\lambda)\varphi) > 0$, thus, $\frac{m(-v^2(-2+2\alpha+\lambda)+2(-3+3\alpha+2\lambda)\varphi)}{2\lambda\rho\varphi} < \max\{\hat{f}_1^N, \hat{f}_1^A\}$, we can derive that, $p_e^{\bar{N}} > p_e^{\bar{A}}$, $T_e^{\bar{N}} > T_e^{\bar{A}}$ and $D_e^{\bar{N}} > D_e^{\bar{A}}$.

Similarly, $p_e^{\bar{A}} - p_e^{\bar{D}} = \frac{\alpha\lambda^2\varphi(m(-1+\lambda)(v^2-4\varphi)-2f\rho\varphi)}{z_{16}}$, $T_e^{\bar{A}} - T_e^{\bar{D}} = \frac{v\alpha\lambda^2(m(-1+\lambda)(v^2-4\varphi)-2f\rho\varphi)}{2z_{16}}$, $D_e^{\bar{A}} - D_e^{\bar{D}} = \frac{\alpha\lambda^2\varphi(m(-1+\lambda)(v^2-4\varphi)-2f\rho\varphi)}{z_{16}}$, where $z_{16} = (v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)(v^2(-2+2\alpha+\lambda^2)-2(-3+3\alpha+2\lambda^2)\varphi)$. Let $p_e^{\bar{A}} - p_e^{\bar{D}} = T_e^{\bar{A}} - T_e^{\bar{D}} = D_e^{\bar{A}} - D_e^{\bar{D}} = 0$, we can get a threshold $f = \frac{m(-1+\lambda)(v^2-4\varphi)}{2\rho\varphi}$, it can always obtain that $\frac{m(-1+\lambda)(v^2-4\varphi)}{2\rho\varphi} - \hat{f}_2^D = -\frac{m(-1+\lambda)(v^2-4\varphi)(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)}{2\rho\varphi(-v^2(-2+\lambda^2)+(-7+4\lambda^2)\varphi)} > 0$, $\frac{m(-1+\lambda)(v^2-4\varphi)}{2\rho\varphi} - \hat{f}_2^A = -\frac{m(-1+\lambda)(v^2-4\varphi)(v^2(-2+2\alpha+\lambda^2)-2(-3+3\alpha+2\lambda^2)\varphi)}{2\rho\varphi(-v^2(-2+2\alpha+\lambda^2)+(-7+7\alpha+4\lambda^2)\varphi)} > 0$, thus, $\frac{m(-1+\lambda)(v^2-4\varphi)}{2\rho\varphi} > \max\{\hat{f}_2^D, \hat{f}_2^A\}$, we can derive that, $p_e^{\bar{A}} > p_e^{\bar{D}}$, $T_e^{\bar{A}} > T_e^{\bar{D}}$ and $D_e^{\bar{A}} > D_e^{\bar{D}}$.

Proof of Proposition 2 These results can be obtained from Derivation of Critical Thresholds.

Proof of Proposition 3

(1) By solving that $\pi_S^{\bar{N}} - \pi_S^N = \frac{(m(v^2-4\varphi)+2f\rho\varphi)^2}{8(v^2-4\varphi)(v^2-3\varphi)} - \frac{m^2}{8} = 0$, we can obtain

$$\hat{f}_3^N = \frac{-m\rho(v^2-4\varphi) - \sqrt{m^2\rho^2(v^2-4\varphi)(v^2-3\varphi)}}{2\rho^2\varphi} \tag{53}$$

It is easy to prove that $\hat{f}_1^N < \hat{f}_3^N < \hat{f}_2^N$, thus when $\hat{f}_1^N < f < \hat{f}_3^N$, the supplier can profit from the provision of EW service.

(2) We first demonstrate the impact of EW service on the supplier's direct encroachment, solving $\pi_S^{\bar{D}} - \pi_S^D = \frac{\varphi(-m^2(-1+\lambda)^2(v^2-4\varphi)+2fm(-1+\lambda)(-2+\lambda^2)\rho(v^2-4\varphi)-2f^2(-2+\lambda^2)\rho^2\varphi)}{2(-2+\lambda^2)(v^2-4\varphi)(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)} = 0$, we can get

$$\hat{f}_3^D = \frac{z_7 + m(-1+\lambda)(-2+\lambda^2)\rho(v^2-4\varphi)}{2(-2+\lambda^2)\rho^2\varphi} \tag{54}$$

Where $z_7 = \sqrt{m^2(-1+\lambda)^2(-2+\lambda^2)\rho^2(v^2-4\varphi)(v^2(-2+\lambda^2)+6\varphi-4\lambda^2\varphi)}$. When $f > \hat{f}_3^D$, $\pi_S^{\bar{D}} - \pi_S^D < 0$. So we need to determine the size of $\hat{f}_3^D$ and $\hat{f}_1^D$, $\hat{f}_2^D$. Firstly, it is easily to prove that $\hat{f}_3^D < \hat{f}_2^D$. Then we compare $\hat{f}_1^D$ and $\hat{f}_3^D$, let $\hat{f}_1^D = \hat{f}_3^D$, we can get a threshold

$$\hat{\varphi}_{d2} = \frac{-2\sqrt{v^4(2-\lambda^2)} + v^2(-13+8\lambda^2)}{-46+32\lambda^2} \tag{55}$$

Here, $\hat{\varphi}_{d2} > \frac{v^2(-2+\lambda^2)}{-(6-4\lambda^2)}$, when $\frac{v^2(-2+\lambda^2)}{-(6-4\lambda^2)} < \varphi < \hat{\varphi}_{d2}$, it can be obtained that $\hat{f}_3^D < \hat{f}_1^D$, the supplier cannot profit from EW service at this time. When $\varphi > \hat{\varphi}_{d2}$, it can be obtained that $\hat{f}_3^D > \hat{f}_1^D$, at this time if $\hat{f}_1^D < f < \hat{f}_3^D$, the supplier cannot profit from EW service.

(3) Then we demonstrate the impact of EW service on the supplier's agency encroachment, similar to direct encroachment, solving $\pi_S^{\bar{A}} - \pi_S^{A} = (1-\alpha)\varphi(m^2(-1+\alpha)(-1+\lambda)^2(v^2-4\varphi) + 2fm(-1+\lambda)(-2+2\alpha+\lambda^2)\rho(v^2-4\varphi) - 2f^2(-2+2\alpha+\lambda^2)\rho^2\varphi)2(-2+2\alpha+\lambda^2)(v^2-4\varphi)(v^2(-2+2\alpha+\lambda^2) - 2(-3+3\alpha+2\lambda^2)\varphi)$, we can get

$$\hat{f}_3^A = \frac{z_8 + m(-1+\lambda)(-2+2\alpha+\lambda^2)\rho(v^2-4\varphi)}{2(-2+2\alpha+\lambda^2)\rho^2\varphi} \tag{56}$$

Where $z_8 = \sqrt{m^2(-1+\lambda)^2(-2+2\alpha+\lambda^2)\rho^2(v^2-4\varphi)(v^2(-2+2\alpha+\lambda^2)-2(-3+3\alpha+2\lambda^2)\varphi)}$. When $f > \hat{f}_3^A$, $\pi_S^{\bar{A}} < \pi_S^{A}$. We need to determine the size of $\hat{f}_3^A$ and $\hat{f}_1^A$, $\hat{f}_2^A$. Firstly, it is easy to prove that $\hat{f}_3^A < \hat{f}_2^A$. Then we compare $\hat{f}_1^A$ and $\hat{f}_3^A$, Let $\hat{f}_1^A = \hat{f}_3^A$, we can get a threshold

$$\hat{\varphi}_{\alpha3} = (\sqrt{z_9} + v^2(26(-1+\alpha)^2 + 4(-1+\alpha)^2(-13+14\alpha)\lambda + 2(-1+\alpha)(-5+\alpha(52+7(-5+\alpha)\alpha))\lambda^2 + 16(-2+\alpha)(-1+\alpha)\lambda^3 + (-16+\alpha(70+3\alpha(-19+5\alpha)))\lambda^4 + 8(-2+\alpha)\alpha\lambda^5))/(92(-1+\alpha)^2 + 8(-1+\alpha)^2(-23+24\alpha)\lambda + 4(-1+\alpha)(-7+\alpha(91+12(-5+\alpha)\alpha))\lambda^2 + 64(-2+\alpha)(-1+\alpha)\lambda^3 + 8(-8+\alpha(34+\alpha(-27+7\alpha)))\lambda^4 + 32(-2+\alpha)\alpha\lambda^5) \tag{57}$$

Where $z_9 = v^4(-1+\alpha)^2(2+(-2+\alpha)\lambda)^2(-2+2\alpha+\lambda^2)(4(-1+\alpha)+8(-1+\alpha)^2\lambda + 2(-2+\alpha(8+(-5+\alpha)\alpha))\lambda^2 - 4\alpha\lambda^3 + \alpha^2\lambda^4)$. Here, $\hat{\varphi}_{\alpha3} > \frac{v^2(-2+2\alpha+\lambda^2)}{2(-3+3\alpha+2\lambda^2)}$, when $\frac{v^2(-2+2\alpha+\lambda^2)}{2(-3+3\alpha+2\lambda^2)} < \varphi < \hat{\varphi}_{\alpha3}$, it can be obtained that $\hat{f}_3^A < \hat{f}_1^A$, the supplier cannot profit from EW service at this time. When $\varphi > \hat{\varphi}_{\alpha3}$, it can be obtained that $\hat{f}_3^A > \hat{f}_1^A$, at this time if $\hat{f}_1^A < f < \hat{f}_3^A$, the supplier cannot profit from EW service.

Proof of Proposition 4 Under direct encroachment,

(1) Solving for $\pi_S^D - \pi_S^N = 0$, we can get obtain that

$$\hat{F}_1 = -\frac{m^2(-2+\lambda)^2}{8(-2+\lambda^2)} \tag{58}$$

When $F > \hat{F}_1$, $\pi_S^D < \pi_S^N$, thus the supplier will not choose to encroach on the direct channel. The conditions for the situation which the e-platform will not provide EW service under no encroachment and direct encroachment scenario are $\begin{cases} \varphi < \frac{v^2}{2},\ 0 < f < \hat{f}_1^D; \\ f > \hat{f}_2^N; \\ \varphi < \hat{\varphi}_{d1},\ \hat{f}_2^D < f < \hat{f}_1^N. \end{cases}$.

(2) Solving for $\pi_S^{\bar{D}} - \pi_S^N = 0$, we can get

$$\hat{f}_1 = \frac{2m(-1+\lambda)\rho(v^2-4\varphi)\varphi - \sqrt{2}\sqrt{(-8F+m^2)\rho^2(v^2-4\varphi)\varphi^2(v^2(-2+\lambda^2)+6\varphi-4\lambda^2\varphi)}}{4\rho^2\varphi^2} \tag{59}$$

$\hat{f}_1$ exists only when $F > \frac{m^2}{8}$. When $\hat{f}_1^D < f < \min\{\hat{f}_1, \hat{f}_1^N, \hat{f}_2^D\}$ and $\varphi < \frac{v^2}{2}$, it suggests that direct encroachment makes the supplier better.

(3) Solving for $\pi_S^D - \pi_S^{\bar{N}} = 0$, we can get

$$\hat{f}_2 = -\frac{(v^2-4\varphi)(m\rho\varphi + \sqrt{2}(v^2-3\varphi)\sqrt{-\frac{(m^2(3-2\lambda)+4F(-2+\lambda^2))\rho^2\varphi^2}{(-2+\lambda^2)(v^4-7v^2\varphi+12\varphi^2)}})}{2\rho^2\varphi^2} \tag{60}$$

When $\hat{f}_2 < f < \hat{f}_2^N$, it suggests that direct encroachment makes the supplier worse off. When $\max\left\{\hat{f}_1^N, \hat{f}_2^D\right\} < f < \hat{f}_2$, it suggests that direct encroachment makes the supplier better.

(4) Solving for $\pi_S^{\bar{D}} - \pi_S^{\bar{N}} = 0$, we can get:

$$\hat{f}_3 = \frac{m(-v^2(-2+\lambda) - 6\varphi + 4\lambda\varphi)}{2\lambda\rho\varphi} + \frac{2F(v^2-4\varphi)}{\sqrt{-\frac{2F\lambda^2\rho^2(v^2-4\varphi)^2\varphi^2}{(v^2-3\varphi)(v^2(-2+\lambda^2)+6\varphi-4\lambda^2\varphi)}}} \tag{61}$$

Solving that $\hat{f}_3 = \hat{f}_2^A$, we can obtain

$$\hat{F}_2 = -\frac{m^2(v^4(-2+\lambda)(-2+2\alpha+\lambda^2) - z_{10}}{8(v^2-3\varphi)(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)(v^2(-2+2\alpha+\lambda^2)+(7-7\alpha-4\lambda^2)\varphi)^2} \tag{62}$$

Where $z_{10} = v^2(26+\alpha(-26+\lambda(14+\lambda)) + \lambda(-14+\lambda(-15+8\lambda)))\varphi + 2((-7+4\lambda)(-3+2\lambda^2) + \alpha(-21+2\lambda(6+\lambda)))\varphi^2)^2$. When $\hat{f}_3 < f < \hat{f}_2^D$ and $F < \hat{F}_2$, we can obtain that $\pi_S^{\bar{D}} > \pi_S^{\bar{N}}$.

Proof of Proposition 5 Under agency encroachment,

(1) Solving for $\pi_S^A - \pi_S^N = -\frac{m^2(-2+2\alpha+\lambda)^2}{8(-2+2\alpha+\lambda^2)} > 0$, thus agency encroachment in favor of the supplier.

(2) Solving for $\pi_S^{\bar{A}} - \pi_S^N = 0$, we can get

$$\hat{f}_4 = \frac{z_{11} + 2m(1-\alpha)(1-\lambda)\rho(v^2-4\varphi)\varphi}{4(-1+\alpha)\rho^2\varphi^2} \tag{63}$$

Where $z_{11} = \sqrt{2m^2(1-\alpha)(1-2\alpha)\rho^2(v^2-4\varphi)\varphi^2(v^2(-2+2\alpha+\lambda^2) - 2(-3+3\alpha+2\lambda^2)\varphi)}$ and only exists when $\alpha > \frac{1}{2}$. Let $\hat{f}_4 = \hat{f}_1^N$, we can only get two roots of α, which both less than $\frac{1}{2}$. It suggests that agency encroachment makes the supplier better.

(3) Solving for $\pi_S^A - \pi_S^{\bar{N}} = 0$, we can get

$$\hat{f}_5 = \frac{(v^2-4\varphi)(-m\rho\varphi + \sqrt{-\frac{2m^2(-1+\alpha)(-3+2\alpha+2\lambda)\rho^2(v^2-3\varphi)\varphi^2}{(-2+2\alpha+\lambda^2)(v^2-4\varphi)}}}{2\rho^2\varphi^2} \tag{64}$$

When $0 < \alpha < 1 - \lambda + \frac{\lambda^2}{2}$ and $f > \hat{f}_5$, we can obtain that $\pi_S^A > \pi_S^{\bar{N}}$. From Lemma 4 we can learn that when $\varphi < \hat{\varphi}_{\alpha 1}$, $\hat{f}_1^A > \hat{f}_1^N$, thus $\hat{f}_1^N < f < \hat{f}_1^A$ will only happen if $(-1+\lambda)^2 < \alpha < 1 - \lambda + \frac{\lambda^2}{2}$ and $\varphi < \hat{\varphi}_{\alpha 1}$. When $f > \hat{f}_5$, we can obtain $\pi_S^A > \pi_S^{\bar{N}}$, when $f < \hat{f}_5$, $\pi_S^A < \pi_S^{\bar{N}}$.

(4) Solving for $\pi_S^{\bar{A}} - \pi_S^{\bar{N}} = \frac{(mv^2(-2+2\alpha+\lambda) - 2m(-3+3\alpha+2\lambda)\varphi + 2f\lambda\rho\varphi)^2}{-8(v^2-3\varphi)(v^2(-2+2\alpha+\lambda^2) - 2(-3+3\alpha+2\lambda^2)\varphi)} > 0$, thus agency encroachment in favor of the supplier.

Proof of Proposition 6 and 7 Under direct encroachment:

(1) Solving that $\pi_P^D - \pi_P^N = \frac{m^2\lambda(-8+8\lambda-\lambda^3)}{16(-2+\lambda^2)^2} < 0$, we can prove that direct encroachment harms the e-platform if the e-platform won't provide EW service.

(2) Solving that $\pi_P^{\bar{D}} - \pi_P^N$ we can obtain two roots:

$$\hat{f}_{K1} = \frac{(v^2(-2+\lambda^2) + 6\varphi - 4\lambda^2\varphi)^2 z_{12} + (v^2-4\varphi)(-2m(-1+\lambda)\rho(v^2-3\varphi)\varphi}{4p^2\varphi(v^4(-2+\lambda^2)^2 + v^2(-25+28\lambda^2-8\lambda^4)\varphi + (39+16\lambda^2(-3+\lambda^2))\varphi^2)} \tag{65}$$

$$\hat{f}_{K2} = -\frac{(v^2(-2+\lambda^2) + 6\varphi - 4\lambda^2\varphi)^2 z_{12} + (v^2-4\varphi)(2m(-1+\lambda)\rho(v^2-3\varphi)\varphi}{4\rho^2\varphi(v^4(-2+\lambda^2)^2 + v^2(-25+28\lambda^2-8\lambda^4)\varphi + (39+16\lambda^2(-3+\lambda^2))\varphi^2)} \tag{66}$$

Where $z_{12} = \sqrt{\frac{m^2\rho^2\varphi(-v^4\lambda(8-8\lambda+\lambda^3) + v^2(-3+8\lambda(7-7\lambda+\lambda^2))\varphi + (9-16\lambda(6-6\lambda+\lambda^3))\varphi^2)}{(v^2-4\varphi)(v^2(-2+\lambda^2)+6\varphi-4\lambda^2\varphi)^2}}$. When $\hat{f}_{K1} < f < \hat{f}_{K2}$, the e-platform can benefit from direct encroachment if the e-platform provides EW service after encroachment. It is easily to prove that, when $\varphi = \frac{v^2(-2+\lambda^2)}{-(6-4\lambda^2)}$, $\hat{f}_{K1} = \hat{f}_{K2} = \hat{f}_1^D = \hat{f}_2^D$, $\hat{f}_{K1} - \hat{f}_1^D$ first increases and then decreases with φ. When φ approaches infinity, $\hat{f}_{K1} - \hat{f}_1^D < 0$. Therefore, there exists $\hat{\varphi}_1 > \frac{v^2(-2+\lambda^2)}{-(6-4\lambda^2)}$, when $\varphi < \hat{\varphi}_1$, $\hat{f}_{K1} > \hat{f}_1^D$. Combining the conditions of Proposition 3, we can prove that direct encroachment can benefit the e-platform only when $\hat{f}_1^D < f < \min\left\{\hat{f}_{K1}, \hat{f}_1, \hat{f}_1^N, \hat{f}_2^D\right\}$ and $\frac{v^2(-2+\lambda^2)}{-(6-4\lambda^2)} < \varphi < \hat{\varphi}_1$.

(3) Solving that $\pi_S^{D} - \pi_S^{\bar{N}} = 0$, we can obtain two roots, for the two square roots to be defined, $\varphi < \frac{v^2\lambda(8-8\lambda+\lambda^3)}{3+\lambda(26-29\lambda+4\lambda^3)}$ needs to hold, but $\frac{v^2\lambda(8-8\lambda+\lambda^3)}{3+\lambda(26-29\lambda+4\lambda^3)} < \frac{v^2(-2+\lambda^2)}{-(6-4\lambda^2)}$, thus $\pi_S^{D} < \pi_S^{\bar{N}}$.

(4) Solving that $\pi_S^{\bar{D}} - \pi_S^{\bar{N}} = 0$, we can obtain two roots $f = \frac{m(-v^2(-2+\lambda)+2(-3+2\lambda)\varphi)}{2\lambda\rho\varphi}$ and $f = \frac{m(v^2-4\varphi)(v^2(-4+2\lambda+\lambda^2)-2(-6+3\lambda+2\lambda^2)\varphi)}{2\rho\varphi(-v^2(-4+\lambda^2)+4(-3+\lambda^2)\varphi)}$. We compare $\hat{f}_1^D$ and $\frac{m(-v^2(-2+\lambda)+2(-3+2\lambda)\varphi)}{2\lambda\rho\varphi}$, when $\varphi > \frac{v^2(-2+\lambda^2)}{-(6-4\lambda^2)}$, $\hat{f}_1^D > \frac{m(-v^2(-2+\lambda)+2(-3+2\lambda)\varphi)}{2\lambda\rho\varphi}$. Then we compare $\hat{f}_2^D$ and $\frac{m(v^2-4\varphi)(v^2(-4+2\lambda+\lambda^2)}{2\rho\varphi(-v^2(-4+\lambda^2)+4(-3+\lambda^2)\varphi)} + \frac{m(v^2-4\varphi)(-2(-6+3\lambda+2\lambda^2)\varphi)}{2\rho\varphi(-v^2(-4+\lambda^2)+4(-3+\lambda^2)\varphi)}$. Only when $\varphi < \frac{v^2(-4+\lambda(2+\lambda))}{-12+\lambda(5+4\lambda)}$, $\frac{m(v^2-4\varphi)(v^2(-4+2\lambda+\lambda^2)-2(-6+3\lambda+2\lambda^2)\varphi)}{2\rho\varphi(-v^2(-4+\lambda^2)+4(-3+\lambda^2)\varphi)} < \hat{f}_2^D$, but $\frac{v^2(-4+\lambda(2+\lambda))}{-12+\lambda(5+4\lambda)} < \frac{v^2(-2+\lambda^2)}{-(6-4\lambda^2)}$, thus $\hat{f}_2^D < \frac{m(v^2-4\varphi)(v^2(-4+2\lambda+\lambda^2)-2(-6+3\lambda+2\lambda^2)\varphi)}{2\rho\varphi(-v^2(-4+\lambda^2)+4(-3+\lambda^2)\varphi)}$. Therefore, when f is not between these two roots, $\pi_S^{\bar{D}} > \pi_S^{\bar{N}}$. When f between these two roots, $\pi_S^{\bar{D}} < \pi_S^{\bar{N}}$.

Under agency encroachment:

(1) Solving that $\pi_P^{A} - \pi_P^{N} = \frac{m^2(-2+2\alpha+\lambda)(8\alpha^2-\lambda(-4+\lambda(2+\lambda))+2\alpha(-4+\lambda(-4+5\lambda)))}{16(-2+2\alpha+\lambda^2)^2} = 0$ we can obtain

$$\hat{\alpha}_1 = \frac{1}{8}(4+4\lambda-5\lambda^2) - \frac{1}{8}\sqrt{16-8\lambda^2-32\lambda^3+25\lambda^4} \tag{67}$$

When $\hat{\alpha}_1 < \alpha < 1 - \frac{\lambda}{2}$, we can obtain that $\pi_P^{A} > \pi_P^{N}$, which means that the e-platform can benefit from the supplier's agency encroachment. Solving that $\pi_P^{\bar{A}} - \pi_P^{N} = 0$, we can obtain two roots $\hat{f}_{K3}$ and $\hat{f}_{K4}$. when $\varphi = \frac{v^2(-2+2\alpha+\lambda^2)}{2(-3+3\alpha+2\lambda^2)}$, $\hat{f}_{K3} = \hat{f}_{K4} = \hat{f}_1^A = \hat{f}_2^A$, $\hat{f}_{K3} - \hat{f}_2^A$ first increases and then decreases with φ. When φ approaches infinity, $\hat{f}_{K3} - \hat{f}_2^A < 0$. Therefore, there exists $\hat{\varphi}_2 > \frac{v^2(-2+2\alpha+\lambda^2)}{2(-3+3\alpha+2\lambda^2)}$, when $\varphi < \hat{\varphi}_2$, $\hat{f}_{K3} > \hat{f}_2^A$. Combining the conditions of Proposition 3, we can prove that agency encroachment can benefit the e-platform only when $f > \hat{f}_2^N$ or $f < \min\{\hat{f}_1^N, \hat{f}_1^A\}$ and $\hat{\alpha}_1 < \alpha < 1 - \lambda + \frac{\lambda^2}{2}$.

(2) Solving that $\pi_P^{A} - \pi_P^{\bar{N}} = 0$, we can obtain two roots $\hat{f}_{K5}$ and $\hat{f}_{K6}$. when $\varphi = \frac{v^2(-2+2\alpha+\lambda^2)}{2(-3+3\alpha+2\lambda^2)}$, $\hat{f}_{K3} = \hat{f}_{K4} = \hat{f}_1^A = \hat{f}_2^A$,when $\varphi > \frac{v^2(-2+2\alpha+\lambda^2)}{2(-3+3\alpha+2\lambda^2)}$, $\hat{f}_{K5} < \hat{f}_1^A < \hat{f}_2^A < \hat{f}_{K2}$. Thus, $\pi_P^{A} < \pi_P^{\bar{N}}$. Combining the conditions of Proposition 3, we can prove that agency encroachment can benefit the e-platform only when $\max\{0, \hat{f}_1^A\} < f < \min\{\hat{f}_1^N, \hat{f}_2^A, \hat{f}_{K3}\}$, $\max\{\hat{\varphi}_{\alpha 1}, \frac{v^2(-2+2\alpha+\lambda^2)}{2(-3+3\alpha+2\lambda^2)}\} < \varphi < \frac{v^2}{2}$, $\alpha < 1 - \lambda$.

(3) Solving that $\pi_P^{\bar{A}} - \pi_P^{\bar{N}} = 0$, we can obtain two roots $\hat{f}_{K7}$ and $\hat{f}_{K8}$. when $\varphi = \frac{v^2(-2+2\alpha+\lambda^2)}{2(-3+3\alpha+2\lambda^2)}$, $\hat{f}_{K3} = \hat{f}_{K4} = \hat{f}_1^A = \hat{f}_2^A$,when $\varphi > \frac{v^2(-2+2\alpha+\lambda^2)}{2(-3+3\alpha+2\lambda^2)}$, $\hat{f}_1^A < \hat{f}_{K7} < \hat{f}_2^A < \hat{f}_{K8}$. Thus, when $\max\left\{0, \hat{f}_1^A\right\} < \hat{f}_{K7}$, $\pi_P^{\bar{A}} > \pi_P^{\bar{N}}$. Combining the conditions of Proposition 3, we can prove that agency encroachment can benefit the e-platform only when $\max\left\{0, \hat{f}_1^N, \hat{f}_1^A\right\} < f < \hat{f}_{K7}$.

Proof of Proposition 8 Table B.1 to Table B.5 summarizes the equilibrium solutions for six different scenarios where the initial channel is the agency channel: no encroachment scenario (Model $\dot{N}$), direct encroachment without EW service (Model $\dot{D}$), resell encroachment without EW service (Model $\dot{T}$), no encroachment with EW service (Model $\dot{\bar{N}}$), direct encroachment with EW service (Model $\dot{\bar{D}}$), resell encroachment with EW service (Model $\dot{\bar{T}}$).

Table B.1: Equilibrium solutions of model Model $\dot{N}$ and Model $\dot{N}$.

Notations	Model $\dot{N}$	Model $\dot{D}$
q_o	$\frac{m}{2}$	$\frac{m(-2+2\alpha+2\lambda-\alpha\lambda)}{4(-1+\alpha)+(-2+\alpha)^2\lambda^2}$
q_d/q_t	–	$\frac{m(-1+\alpha)(2+(-2+\alpha)\lambda)}{4(-1+\alpha)+(-2+\alpha)^2\lambda^2}$
p_o	$\frac{m}{2}$	$-\frac{m(-1+\lambda)(-2(1+\lambda)+\alpha(2+\lambda))}{4(-1+\alpha)+(-2+\alpha)^2\lambda^2}$
p_d/p_t	–	$\frac{m(-1+\alpha)(-1+\lambda)(-2+(-2+\alpha)\lambda)}{4(-1+\alpha)+(-2+\alpha)^2\lambda^2}$
π_S	$\frac{m^2(1-\alpha)}{4}$	$-F + \frac{m^2(-2+\alpha)(-1+\alpha)(-1+\lambda)}{4(-1+\alpha)+(-2+\alpha)^2\lambda^2}$
π_P	$\frac{m^2\alpha}{4}$	$\frac{m^2\alpha(2+\alpha(-2+\lambda)-2\lambda)(-1+\lambda)(-2(1+\lambda)+\alpha(2+\lambda))}{(4(-1+\alpha)+(-2+\alpha)^2\lambda^2)^2}$

Table B.2: Equilibrium solutions of model Model $\dot{T}$.

Notations	Model $\dot{T}$
$w^{\dot{T}}$	$-\frac{m(-1+\alpha)(-2+2\alpha\lambda+\lambda^2)}{2(-2+2\alpha+\lambda^2)}$
$q_o^{\dot{T}}$	$\frac{m(-2+2\alpha+\lambda)}{2(-2+2\alpha+\lambda^2)}$
$q_t^{\dot{T}}$	$-\frac{m(-1+\alpha)(-1+\lambda)}{2(-2+2\alpha+\lambda^2)}$
$p_o^{\dot{T}}$	$\frac{m}{2}$
$\pi_S^{\dot{T}}$	$-\frac{m^2(-1+\alpha)(-3+2\alpha+2\lambda)}{4(-2+2\alpha+\lambda^2)}$
$\pi_P^{\dot{T}}$	$\frac{m^2(4\alpha^3+(-1+\lambda)^2+\alpha^2(1+\lambda)(-7+5\lambda)+\alpha(2+\lambda(4+\lambda(-7+2\lambda))))}{4(-2+2\alpha+\lambda^2)^2}$

Table B.3: Equilibrium solutions of model Model $\dot{\dot{N}}$.

Notations	Model $\dot{\dot{N}}$
$q_o^{\dot{\dot{N}}}$	$\frac{m}{2}$
$p_o^{\dot{\dot{N}}}$	$\frac{m}{2}$
$p_e^{\dot{\dot{N}}}$	$\frac{f\rho(v^2-2\varphi)-m\varphi}{v^2-4\varphi}$
$T_e^{\dot{\dot{N}}}$	$\frac{v(2f\rho-m)}{2(v^2-4\varphi)}$
$D_e^{\dot{\dot{N}}}$	$-\frac{(m-2f\rho)\varphi}{v^2-4\varphi}$
$\pi_S^{\dot{\dot{N}}}$	$-\frac{1}{4}m^2(-1+\alpha)$
$\pi_P^{\dot{\dot{N}}}$	$\frac{4fm\rho\varphi-4f^2\rho^2\varphi+m^2(v^2\alpha-(1+4\alpha)\varphi)}{4(v^2-4\varphi)}$

Table B.4: Equilibrium solutions of model Model $\dot{\dot{D}}$.

Notations	Model $\dot{\dot{D}}$
$q_o^{\dot{\dot{D}}}$	$\frac{m(-2+2\alpha+2\lambda-\alpha\lambda)}{4(-1+\alpha)+(-2+\alpha)^2\lambda^2}$
$q_d^{\dot{\dot{D}}}$	$\frac{m(-1+\alpha)(2+(-2+\alpha)\lambda)}{4(-1+\alpha)+(-2+\alpha)^2\lambda^2}$
$p_o^{\dot{\dot{D}}}$	$-\frac{m(-1+\lambda)(-2(1+\lambda)+\alpha(2+\lambda))}{4(-1+\alpha)+(-2+\alpha)^2\lambda^2}$
$p_d^{\dot{\dot{D}}}$	$\frac{m(-1+\alpha)(-1+\lambda)(-2+(-2+\alpha)\lambda)}{4(-1+\alpha)+(-2+\alpha)^2\lambda^2}$
$p_e^{\dot{\dot{D}}}$	$\frac{f\rho(v^2-2\varphi)(4(-1+\alpha)+(-2+\alpha)^2\lambda^2)+2m\varphi(2+\alpha(-2+\lambda)-2\lambda)}{(4(-1+\alpha)+(-2+\alpha)^2\lambda^2)(v^2-4\varphi)}$
$T_e^{\dot{\dot{D}}}$	$\frac{v(m(2-2\alpha+(\alpha-2)\lambda)+f\rho(4\alpha-4+(\alpha-2)^2\lambda^2))}{(4\alpha-4+(\alpha-2)^2\lambda^2)(v^2-4\varphi)}$
$D_e^{\dot{\dot{D}}}$	$\frac{2\varphi(m(2-2\alpha+(\alpha-2)\lambda)+f\rho(4\alpha-4+(\alpha-2)^2\lambda^2))}{(4\alpha-4+(\alpha-2)^2\lambda^2)(v^2-4\varphi)}$
$\pi_S^{\dot{\dot{D}}}$	$-F+\frac{m^2(-2+\alpha)(-1+\alpha)(-1+\lambda)}{4(-1+\alpha)+(-2+\alpha)^2\lambda^2}$
$\pi_P^{\dot{\dot{D}}}$	$\alpha q_o^{\dot{\dot{D}}}p_o^{\dot{\dot{D}}}+D_e^{\dot{\dot{D}}}(p_e^{\dot{\dot{D}}}-f\rho)-\varphi {T_e^{\dot{\dot{D}}}}^2$

Table B.5: Equilibrium solutions of model Model $\dot{\bar{D}}$.

Notations	Model $\dot{\bar{T}}$
$w^{\dot{\bar{T}}}$	$-\frac{m(-1+\alpha)(-2+2\alpha\lambda+\lambda^2)}{2(-2+2\alpha+\lambda^2)}$
$q_o^{\dot{\bar{T}}}$	$\frac{m(-2+2\alpha+\lambda)}{2(-2+2\alpha+\lambda^2)}$
$q_d^{\dot{\bar{T}}}$	$-\frac{m(-1+\alpha)(-1+\lambda)}{2(-2+2\alpha+\lambda^2)}$
$p_o^{\dot{\bar{T}}}$	$\frac{m(-2+\lambda^2+\alpha(2+(-1+\lambda)\lambda))}{2(-2+2\alpha+\lambda^2)}$
$p_o^{\dot{\bar{T}}}$	$\frac{m(-3-\alpha(-3+\lambda)+\lambda+\lambda^2)}{2(-2+2\alpha+\lambda^2)}$
$p_e^{\dot{\bar{T}}}$	$\frac{f\rho(v^2-2\varphi)(-2+2\alpha+\lambda^2)-m(-2+2\alpha+\lambda)\varphi}{(-2+2\alpha+\lambda^2)(v^2-4\varphi)}$
$T_e^{\dot{\bar{T}}}$	$\frac{v(2f\rho(-2+2\alpha+\lambda^2)-m(-2+2\alpha+\lambda))}{2(-2+2\alpha+\lambda^2)(v^2-4\varphi)}$
$D_e^{\dot{\bar{T}}}$	$\frac{(-m(-2+2\alpha+\lambda)+2f(-2+2\alpha+\lambda^2)\rho)\varphi}{(-2+2\alpha+\lambda^2)(v^2-4\varphi)}$
$\pi_S^{\dot{\bar{T}}}$	$\frac{m^2(1-\alpha)(-3+2\alpha+2\lambda)}{4(-2+2\alpha+\lambda^2)}$
$\pi_P^{\dot{\bar{T}}}$	$\alpha q_o^{\dot{\bar{T}}} p_o^{\dot{\bar{T}}} + (p_t^{\dot{\bar{T}}} - w^{\dot{\bar{T}}})q_o^{\dot{\bar{T}}} + D_e^{\dot{\bar{T}}}(p_e^{\dot{\bar{T}}} - f\rho) - \varphi {T_e^{\dot{\bar{T}}}}^2$

From the above equilibrium solutions we can easily find, under no encroachment scenario, $q_o^{\dot{N}} = q_o^{\dot{\bar{N}}}$, $q_t^{\dot{N}} = q_t^{\dot{\bar{N}}}$, $\pi_S^{\dot{N}} = \pi_S^{\dot{\bar{N}}}$, under direct encroachment we have $q_o^{\dot{D}} = q_o^{\dot{\bar{D}}}$, $q_t^{\dot{D}} = q_t^{\dot{\bar{D}}}$, $\pi_S^{\dot{D}} = \pi_S^{\dot{\bar{D}}}$, and under agency encroachment we have $q_o^{A} = q_o^{\dot{\bar{A}}}$, $q_t^{\dot{A}} = q_t^{\dot{\bar{A}}}$, $\pi_S^{\dot{A}} = \pi_S^{\dot{\bar{A}}}$.

Proof of Proposition 9 Table B.7 to Table B.9 summarize the equilibrium solutions for three different scenarios where the supplier enters into a partial cost-sharing contract with the e-platform: no encroachment scenario (Model $\hat{N}$), direct encroachment scenario (Model $\hat{D}$), agency encroachment scenario (Model $\hat{A}$).

Table B.6: Equilibrium solutions of model Model $\hat{N}$.

Notations	Model $\hat{N}$
$w^{\hat{N}}$	$\frac{2f\rho\varphi((1-2e)v^2-3(-1+e)^2\varphi)+m(v^4+(-7+8e)v^2\varphi+12(-1+e)^2\varphi^2)}{2v^4+(-14+15e)v^2\varphi+24(-1+e)^2\varphi^2}$
$q_t^{\hat{N}}$	$\frac{m(v^2+4(-1+e)\varphi)^2+2f\rho\varphi(v^2-4(-1+e)^2\varphi)}{4v^4+2(-14+15e)v^2\varphi+48(-1+e)^2\varphi^2}$
$p_t^{\hat{N}}$	$m-\frac{m(v^2+4(-1+e)\varphi)^2+2f\rho\varphi(v^2-4(-1+e)^2\varphi)}{4v^4+2(-14+15e)v^2\varphi+48(-1+e)^2\varphi^2}$
$p_e^{\hat{N}}$	$\frac{2fv^4\rho+v^2((-1+e)m+(-10+11e)f\rho)\varphi+2(-1+e)^2(2m+5f\rho)\varphi^2}{2v^4+(-14+15e)v^2\varphi+24(-1+e)^2\varphi^2}$
$T_e^{\hat{N}}$	$-\frac{v^3(m-4f\rho)+2(-1+e)v(2m-7f\rho)\varphi}{4v^4+2(-14+15e)v^2\varphi+48(-1+e)^2\varphi^2}$
$D_e^{\hat{N}}$	$\frac{(-1+e)\varphi(v^2(m-4f\rho)+2(-1+e)(2m-7f\rho)\varphi)}{2v^4+(-14+15e)v^2\varphi+24(-1+e)^2\varphi^2}$
$\pi_S^{\hat{N}}$	$\frac{m^2(v^2+4(-1+e)\varphi)^2+4fm\rho\varphi(v^2-4(-1+e)^2\varphi)+4f^2\rho^2\varphi(\varphi+e^2\varphi-2e(v^2+\varphi))}{8v^4+4(-14+15e)v^2\varphi+96(-1+e)^2\varphi^2}$
$\pi_P^{\hat{N}}$	$q_t^{\hat{N}}(p_t^{\hat{N}} - w^{\hat{N}}) + D_e^{\hat{N}}(p_e^{\hat{N}} - f\rho) - \varphi(1-e){T_e^{\hat{N}}}^2$

Table B.7: Equilibrium solutions of model Model $\hat{D}$.

Notations	Model $\hat{D}$
$w^{\hat{D}}$	$\frac{(2f\rho\varphi(-((-1+2e)v^2(-2+\lambda^2))-2(-1+e)^2(-3+2\lambda^2)\varphi)+m(v^4(-2+\lambda^2)+v^2(14-16e+e\lambda+8(-1+e)\lambda^2)\varphi+8(-1+e)^2(-3+2\lambda^2)\varphi^2))}{2(v^4(-2+\lambda^2)+v^2(14-15e+8(-1+e)\lambda^2)\varphi+8(-1+e)^2(-3+2\lambda^2)\varphi^2)}$
$q_t^{\hat{D}}$	$\frac{m(-1+\lambda)(v^2+4(-1+e)\varphi)^2+2f\rho\varphi(-v^2+4(-1+e)^2\varphi)}{2(v^4(-2+\lambda^2)+v^2(14-15e+8(-1+e)\lambda^2)\varphi+8(-1+e)^2(-3+2\lambda^2)\varphi^2)}$
$q_d^{\hat{D}}$	$\frac{(mv^4(-2+\lambda)+v^2(m(14-15e+8(-1+e)\lambda)+2f\lambda\rho)\varphi+8(-1+e)^2(m(-3+2\lambda)-f\lambda\rho)\varphi^2)}{2(v^4(-2+\lambda^2)+v^2(14-15e+8(-1+e)\lambda^2)\varphi+8(-1+e)^2(-3+2\lambda^2)\varphi^2)}$
$p_t^{\hat{D}}$	$\frac{(-2f(-1+\lambda^2)\rho\varphi(v^2-4(-1+e)^2\varphi)+m(v^4(-3+\lambda+\lambda^2)+v^2(20-22e-6\lambda+7e\lambda+8(-1+e)\lambda^2)\varphi+8(-1+e)^2(-4+\lambda+2\lambda^2)\varphi^2))}{2(v^4(-2+\lambda^2)+v^2(14-15e+8(-1+e)\lambda^2)\varphi+8(-1+e)^2(-3+2\lambda^2)\varphi^2)}$
$p_d^{\hat{D}}$	$\frac{m}{2}$
$p_e^{\hat{D}}$	$\frac{((-1+e)m(-1+\lambda)\varphi(v^2+4(-1+e)\varphi)+f\rho(v^4(-2+\lambda^2)+v^2(10-11e+6(-1+e)\lambda^2)\varphi+2(-1+e)^2(-5+4\lambda^2)\varphi^2))}{v^4(-2+\lambda^2)+v^2(14-15e+8(-1+e)\lambda^2)\varphi+8(-1+e)^2(-3+2\lambda^2)\varphi^2}$
$T_e^{\hat{D}}$	$\frac{v^3(m-m\lambda+2f(-2+\lambda^2)\rho)-2(-1+e)v(2m(-1+\lambda)+f(7-4\lambda^2)\rho)\varphi}{2(v^4(-2+\lambda^2)+v^2(14-15e+8(-1+e)\lambda^2)\varphi+8(-1+e)^2(-3+2\lambda^2)\varphi^2)}$
$D_e^{\hat{D}}$	$\frac{(-1+e)\varphi(v^2(m(-1+\lambda)-2f(-2+\lambda^2)\rho)+2(-1+e)(2m(-1+\lambda)+f(7-4\lambda^2)\rho)\varphi)}{v^4(-2+\lambda^2)+v^2(14-15e+8(-1+e)\lambda^2)\varphi+8(-1+e)^2(-3+2\lambda^2)\varphi^2}$
$\pi_S^{\hat{D}}$	$w^{\hat{D}}q_t^{\hat{D}}+p_d^{\hat{D}}q_d^{\hat{D}}-F-\varphi e{T_e^{\hat{D}}}^2$
$\pi_P^{\hat{D}}$	$q_t^{\hat{D}}(p_t^{\hat{D}}-w^{\hat{D}})+D_e^{\hat{D}}(p_e^{\hat{D}}-f\rho)-\varphi(1-e){T_e^{\hat{D}}}^2$

Table B.8: Equilibrium solutions of model Model $\hat{A}$.

Notations	Model $\hat{A}$
$w^{\hat{A}}$	$\frac{2f\varphi(v^2(-2(-1+2e)(-1+\alpha)-(-1+2e+\alpha)\lambda^2)+2(-1+e)^2(-1+\alpha)(-3+2\lambda^2)\varphi)+m(-1+\alpha)(-v^4(-2+2\alpha\lambda+\lambda^2)-v^2(14-16e+(e-14\alpha+15e\alpha)\lambda+8(-1+e)\lambda^2)\varphi-8(-1+e)^2(-3+3\alpha\lambda+2\lambda^2)\varphi^2)}{2(v^4(-2+2\alpha+\lambda^2)+v^2((-14+15e)(-1+\alpha)+8(-1+e)\lambda^2)\varphi+8(-1+e)^2(-3+3\alpha+2\lambda^2)\varphi^2)}$
$p_t^{\hat{A}}$	$\frac{(2f(-1+\alpha+\lambda^2)\rho\varphi(-v^2+4(-1+e)^2\varphi)+m(v^4(-3-\alpha(-3+\lambda)+\lambda+\lambda^2)+v^2(2(-10+11e)(-1+\alpha)-(-6+7e)(-1+\alpha)\lambda+8(-1+e)\lambda^2)\varphi-8(-1+e)^2(4+\alpha(-4+\lambda)-\lambda(1+2\lambda))\varphi^2))}{2(v^4(-2+2\alpha+\lambda^2)+v^2((-14+15e)(-1+\alpha)+8(-1+e)\lambda^2)\varphi+8(-1+e)^2(-3+3\alpha+2\lambda^2)\varphi^2)}$
$p_o^{\hat{A}}$	$\frac{2f\alpha\lambda\rho\varphi(-v^2+4(-1+e)^2\varphi)+m(v^4(-2+\lambda^2+\alpha(2+(-1+\lambda)\lambda))+v^2((-14+15e)(-1+\alpha)-8(-1+e)\alpha\lambda+8(-1+e)(1+\alpha)\lambda^2)\varphi+8(-1+e)^2(-3+2\lambda^2+\alpha(3+2(-1+\lambda)\lambda))\varphi^2))}{(2(v^4(-2+2\alpha+\lambda^2)+v^2((-14+15e)(-1+\alpha)+8(-1+e)\lambda^2)\varphi+8(-1+e)^2(-3+3\alpha+2\lambda^2)\varphi^2)}$
$p_e^{\hat{A}}$	$\frac{(-((-1+e)m(-1+\alpha)(-1+\lambda)\varphi(v^2+4(-1+e)\varphi))+f\rho(v^4(-2+2\alpha+\lambda^2)+v^2((-10+11e)(-1+\alpha)+6(-1+e)\lambda^2)\varphi+2(-1+e)^2(-5+5\alpha+4\lambda^2)\varphi^2))}{(v^4(-2+2\alpha+\lambda^2)+v^2((-14+15e)(-1+\alpha)+8(-1+e)\lambda^2)\varphi+8(-1+e)^2(-3+3\alpha+2\lambda^2)\varphi^2)}$
$T_e^{\hat{A}}$	$\frac{v(m(-1+\alpha)(-1+\lambda)(v^2+4(-1+e)\varphi)+2f\rho(v^2(-2+2\alpha+\lambda^2)+(-1+e)(-7+7\alpha+4\lambda^2)\varphi))}{2(v^4(-2)+2\alpha+\lambda^2)+v^2((-14+15e)(-1+\alpha)+8(-1+e)\lambda^2)\varphi+8(-1+e)^2(-3+3\alpha+2\lambda^2)\varphi^2)}$
$D_e^{\hat{A}}$	$\frac{-(-1+e)\varphi(m(-1+\alpha)(-1+\lambda)(v^2+4(-1+e)\varphi)+2f\rho(v^2(-2+2\alpha+\lambda^2)+(-1+e)(-7+7\alpha+4\lambda^2)\varphi))}{(v^4(-2+2\alpha+\lambda^2)+v^2((-14+15e)(-1+\alpha)+8(-1+e)\lambda^2)\varphi+8(-1+e)^2(-3+3\alpha+2\lambda^2)\varphi^2)}$
$\pi_S^{\hat{A}}$	$\frac{-((4fm(-1+\alpha)(-1+\lambda)p\varphi(v^2-4(-1+e)^2\varphi)+4f^2p^2\varphi(ev^2(-2+2\alpha+\lambda^2)-(-1+e)^2(-1+\alpha)\varphi)+m^2(-1+\alpha)(v^4(-3+2\alpha+2\lambda)+v^2(22-23e-14\alpha+15e\alpha+16(-1+e)\lambda)\varphi+8(-1+e)^2(-5+3\alpha+4\lambda)\varphi^2))}{(4(v^4(-2+2\alpha+\lambda^2)+v^2((-14+15e)(-1+\alpha)+8(-1+e)\lambda^2)\varphi+8(-1+e)^2(-3+3\alpha+2\lambda^2)\varphi^2)))}$
$\pi_P^{\hat{A}}$	$q_t^{\hat{A}}(p_t^{\hat{A}}-w^{\hat{A}})+\alpha p_o^{\hat{A}}q_o^{\hat{A}}+D_e^{\hat{A}}(p_e^{\hat{A}}-f\rho)-\varphi(1-e){T_e^{\hat{A}}}^2$

For Figure 5(A) in the main paper, solving that $\pi_P^{\hat{N}}-\pi_P^{\bar{N}}=0$, we can get two roots $\hat{f}_{PN1}$ and $\hat{f}_{PN2}$, which $\hat{f}_{PN1}<\hat{f}_{PN2}$. Solving that $\pi_S^{\hat{N}}-\pi_S^{\bar{N}}=0$, we can get two roots $\hat{f}_{SN1}$ and $\hat{f}_{SN2}$. Consider the following parameters: $m=8$, $v=3.5$, $\alpha=0.2$, $\varphi=6.5$, $\rho=0.6$, $\lambda=0.5$. When $\hat{f}_{PN1}<\hat{f}_{SN2}$, both parties will sign the partial cost-sharing contract. For Figure 5(B) in the main paper, solving that $\pi_P^{\hat{D}}-\pi_P^{\bar{D}}=0$, we can get two roots $\hat{f}_{PD1}$ and $\hat{f}_{PD2}$, which $\hat{f}_{PD1}<\hat{f}_{PD2}$. Solving that $\pi_S^{\hat{D}}-\pi_S^{\bar{D}}=0$, we can get two roots $\hat{f}_{SD1}$ and $\hat{f}_{SD2}$. Consider the following parameters:$m=8$, $v=3.5$, $\alpha=0.2$, $\varphi=6.5$, $\rho=0.6$, $\lambda=0.5$. When $\hat{f}_{PD1}<\hat{f}_{SD2}$, both parties will sign the partial cost-sharing contract. For Figure 5(C) in the main paper, solving that $\pi_P^{\hat{A}}-\pi_P^{\bar{A}}=0$, we can get two roots $\hat{f}_{PA1}$ and $\hat{f}_{PA2}$, which $\hat{f}_{PA1}<\hat{f}_{PA2}$. Solving that $\pi_S^{\hat{A}}-\pi_S^{\bar{A}}=0$, we can get two roots $\hat{f}_{SA1}$ and $\hat{f}_{SA2}$. Consider the following parameters: $m=8$, $v=3.5$, $\alpha=0.2$, $\varphi=6.5$, $\rho=0.6$, $\lambda=0.5$. When $\hat{f}_{PA1}<\hat{f}_{SA2}$, both parties will sign the partial cost-sharing contract.

Proof of Proposition 10 Table B.10 to Table B.12 summarize the equilibrium solutions for three different scenarios when EW service is provided by the supplier: no encroachment scenario (Model $\widetilde{N}$), direct encroachment scenario (Model $\widetilde{D}$), agency encroachment scenario (Model $\widetilde{A}$).

Table B.9: Equilibrium solutions of model Model $\widetilde{N}$.

Notations	Model $\widetilde{N}$
$w^{\widetilde{N}}$	$\frac{mv^2-3m\varphi-2f\rho\varphi}{2v^2-7\varphi}$
$q_t^{\widetilde{N}}$	$\frac{mv^2-4m\varphi+2f\rho\varphi}{4v^2-14\varphi}$
$p_t^{\widetilde{N}}$	$\frac{3mv^2-10m\varphi-2f\rho\varphi}{4v^2-14\varphi}$
$p_e^{\widetilde{N}}$	$\frac{2fv^2\rho-m\varphi-3f\rho\varphi}{2v^2-7\varphi}$
$T_e^{\widetilde{N}}$	$-\frac{v(m-4f\rho)}{4v^2-14\varphi}$
$D_e^{\widetilde{N}}$	$-\frac{(m-4f\rho)\varphi}{2v^2-7\varphi}$
$\pi_S^{\widetilde{N}}$	$\frac{m^2(v^2-4\varphi)+4fm\rho\varphi-8f^2\rho^2\varphi}{8v^2-28\varphi}$
$\pi_P^{\widetilde{N}}$	$\frac{(m(v^2-4\varphi)+2f\rho\varphi)^2}{4(2v^2-7\varphi)^2}$

Table B.10: Equilibrium solutions of model Model $\widetilde{D}$.

Notations	Model $\widetilde{D}$
$w^{\widetilde{D}}$	$\frac{mv^2(-2+\lambda^2)+m(6+\lambda-4\lambda^2)\varphi-2f(-2+\lambda^2)\rho\varphi}{2(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi)}$
$q_t^{\widetilde{D}}$	$\frac{m(-1+\lambda)(v^2-4\varphi)-2f\rho\varphi)}{2(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi)}$
$q_d^{\widetilde{D}}$	$\frac{mv^2(-2+\lambda)+m(7-4\lambda)\varphi+2f\lambda\rho\varphi}{2(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi)}$
$p_t^{\widetilde{D}}$	$\frac{mv^2(-3+\lambda+\lambda^2)+m(10-3\lambda-4\lambda^2)\varphi-2f(-1+\lambda^2)\rho\varphi}{2(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi)}$
$p_d^{\widetilde{D}}$	$\frac{m}{2}$
$p_e^{\widetilde{D}}$	$\frac{-m(-1+\lambda)\varphi+f\rho(v^2(-2+\lambda^2)+(3-2\lambda^2)\varphi)}{v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi}$
$T_e^{\widetilde{D}}$	$\frac{v(m-m\lambda+2f(-2+\lambda^2)\rho)}{2(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi)}$
$D_e^{\widetilde{D}}$	$\frac{(m-m\lambda+2f(-2+\lambda^2)\rho)\varphi}{v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi}$
$\pi_S^{\widetilde{D}}$	$\frac{4fm(-1+\lambda)\rho\varphi-4f^2(-2+\lambda^2)\rho^2\varphi+m^2(v^2(-3+2\lambda)+(17-8\lambda)\varphi)-4F(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi)}{4(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi)}$
$\pi_P^{\widetilde{D}}$	$\frac{(m(-1+\lambda)(v^2-4\varphi)-2f\rho\varphi)^2}{4(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi)^2}$

Table B.11: Equilibrium solutions of model Model $\widetilde{A}$.

Notations	Model $\widetilde{D}$
$w^{\widetilde{D}}$	$-\frac{2f(-2+\lambda^2+\alpha(2+\lambda^2))\rho\varphi+m(-1+\alpha)(v^2(-2+2\alpha\lambda+\lambda^2)+(6+\lambda-7\alpha\lambda-4\lambda^2)\varphi)}{2(v^2(-2+2\alpha+\lambda^2)+(7-7\alpha-4\lambda^2)\varphi)}$
$q_t^{\widetilde{D}}$	$-\frac{(-1+\alpha)(m(-1+\lambda)(v^2-4\varphi)-2f\rho\varphi)}{2(v^2(-2+2\alpha+\lambda^2)+(7-7\alpha-4\lambda^2)\varphi)}$
$q_o^{\widetilde{D}}$	$\frac{mv^2(-2+2\alpha+\lambda)+m(7-7\alpha-4\lambda)\varphi+2f\lambda\rho\varphi}{2(v^2(-2+2\alpha+\lambda^2)+(7-7\alpha-4\lambda^2)\varphi)}$
$p_t^{\widetilde{D}}$	$\frac{mv^2(-3-\alpha(-3+\lambda)+\lambda+\lambda^2)+m(10-3\lambda-4\lambda^2+\alpha(-10+3\lambda))\varphi-2f(-1+\alpha+\lambda^2)\rho\varphi}{2(v^2(-2+2\alpha+\lambda^2)+(7-7\alpha-4\lambda^2)\varphi)}$
$p_o^{\widetilde{D}}$	$\frac{mv^2(-2+\lambda^2+\alpha(2-\lambda+\lambda^2))+m(7-4\lambda^2+\alpha(-7+4\lambda-4\lambda^2))\varphi-2f\alpha\lambda\rho\varphi}{2(v^2(-2+2\alpha+\lambda^2)+(7-7\alpha-4\lambda^2)\varphi)}$
$p_e^{\widetilde{D}}$	$\frac{m(-1+\alpha)(-1+\lambda)\varphi+f\rho(v^2(-2+2\alpha+\lambda^2)+(3-3\alpha-2\lambda^2)\varphi)}{v^2(-2+2\alpha+\lambda^2)+(7-7\alpha-4\lambda^2)\varphi}$
$T_e^{\widetilde{D}}$	$\frac{v(m(-1+\alpha)(-1+\lambda)+2f(-2+2\alpha+\lambda^2)\rho)}{2(v^2(-2+2\alpha+\lambda^2)+(7-7\alpha-4\lambda^2)\varphi)}$
$D_e^{\widetilde{D}}$	$\frac{(m(-1+\alpha)(-1+\lambda)+2f(-2+2\alpha+\lambda^2)\rho)\varphi}{v^2(-2+2\alpha+\lambda^2)+(7-7\alpha-4\lambda^2)\varphi}$
$\pi_S^{\widetilde{D}}$	$-\frac{4fm(-1+\alpha)(-1+\lambda)\rho\varphi+4f^2(-2+2\alpha+\lambda^2)\rho^2\varphi+m^2(-1+\alpha)(v^2(-3+2\alpha+2\lambda)+(11-7\alpha-8\lambda)\varphi)}{4(v^2(-2+2\alpha+\lambda^2)+(7-7\alpha-4\lambda^2)\varphi)}$
$\pi_P^{\widetilde{D}}$	$q_t^{\widetilde{D}}(p_t^{\widetilde{D}}-w^{\widetilde{D}})+\alpha p_o^{\widetilde{D}}q_o^{\widetilde{D}}$

Comparing the profit margins of whether the supplier offers EW service.

$$\pi_S^{\widetilde{N}} - \pi_S^N = -\frac{(m-4f\rho)^2\varphi}{8(2v^2-7\varphi)} > 0 \tag{68}$$

$$\pi_S^D - \pi_S^D = -\frac{(m-m\lambda+2f(-2+\lambda^2)\rho)^2\varphi}{4(-2+\lambda^2)(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi)} > 0 \tag{69}$$

$$\pi_S^A - \pi_S^A = -\frac{(m(-1+\alpha)(-1+\lambda)+2f(-2+2\alpha+\lambda^2)\rho)^2\varphi}{4(-2+2\alpha+\lambda^2)(v^2(-2+2\alpha+\lambda^2)+(7-7\alpha-4\lambda^2)\varphi)} > 0 \tag{70}$$

Comparison of the profitability of the e-platform when the supplier provides EW service in various scenarios.

$$\pi_P^{\widetilde{N}} - \pi_P^N = -\frac{(m-4f\rho)\varphi(4mv^2-15m\varphi+4f\rho\varphi)}{16(2v^2-7\varphi)^2} \tag{71}$$

When $\frac{m}{4\rho} < f < \frac{m(-4v^2+15\varphi)}{4\rho\varphi}$, $\pi_P^{\widetilde{N}} - \pi_P^N < 0$, thus $\pi_P^{\widetilde{N}} - \pi_P^N > 0$ base on $f < \hat{f}_{t1}$.

$$\pi_P^{\widetilde{D}} - \pi_P^D = \frac{(m(-1+\lambda)-2f(-2+\lambda^2)\rho)\varphi(-2f(-2+\lambda^2)\rho\varphi+m(-1+\lambda)(2v^2(-2+\lambda^2)+(15-8\lambda^2)\varphi))}{4(-2+\lambda^2)^2(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi)^2} \tag{72}$$

When $\frac{m-m\lambda}{4\rho-2\lambda^2\rho} < f < \frac{m(-1+\lambda)(2v^2(-2+\lambda^2)+(15-8\lambda^2)\varphi)}{2(-2+\lambda^2)\rho\varphi}$, $\pi_P^{\widetilde{D}} - \pi_P^D < 0$, thus $\pi_P^{\widetilde{D}} - \pi_P^D > 0$ base on $f < \hat{f}_{t2}$.

$$\pi_P^{\widetilde{A}} - \pi_P^A = \frac{(1+\alpha^2-\alpha(2+\lambda^2))(m(-1+\alpha)(-1+\lambda)+2f(-2+2\alpha+\lambda^2)\rho)\varphi z_{17}}{4(-2+2\alpha+\lambda^2)^2(v^2(-2+2\alpha+\lambda^2)+(7-7\alpha-4\lambda^2)\varphi)^2} \tag{73}$$

Where $z_{17} = (2f(-2+2\alpha+\lambda^2)\rho\varphi + m(-1+\lambda)(-2v^2(-2+2\alpha+\lambda^2)+(-15+15\alpha+8\lambda^2)\varphi))$. When $-\frac{m(-1+\alpha)(-1+\lambda)}{2(-2+2\alpha+\lambda^2)\rho} < f < \frac{m(-1+\lambda)(2v^2(-2+2\alpha+\lambda^2)+(15-15\alpha-8\lambda^2)\varphi)}{2(-2+2\alpha+\lambda^2)\rho\varphi}$, $\pi_P^{\widetilde{A}} - \pi_P^A < 0$, thus $\pi_P^{\widetilde{A}} - \pi_P^A > 0$ base on $f < \hat{f}_{t3}$.

Proof of Proposition 11

$$p_e^{\widetilde{N}} - p_e^{\bar{N}} = -\frac{\varphi^2(m(v^2-4\varphi)+2f\rho\varphi)}{2(2v^2-7\varphi)(v^2-4\varphi)(v^2-3\varphi)} \tag{74}$$

$$T_e^{\widetilde{N}} - T_e^{\bar{N}} = -\frac{v\varphi(m(v^2-4\varphi)+2f\rho\varphi)}{4(2v^2-7\varphi)(v^2-4\varphi)(v^2-3\varphi)} \tag{75}$$

$$D_e^{\widetilde{N}} - D_e^{\bar{N}} = -\frac{\varphi^2(m(v^2-4\varphi)+2f\rho\varphi)}{2(2v^2-7\varphi)(v^2-4\varphi)(v^2-3\varphi)} \tag{76}$$

When $f < -\frac{m(v^2-4\varphi)}{2\rho\varphi}$, $p_e^{\widetilde{N}} - p_e^{\bar{N}} < 0$, $T_e^{\widetilde{N}} - T_e^{\bar{N}} < 0$, $D_e^{\widetilde{N}} - D_e^{\bar{N}} < 0$. We can obtain $\hat{f}_{t1} = \frac{m}{4\rho}$, $\hat{f}_{t1} < -\frac{m(v^2-4\varphi)}{2\rho\varphi}$, thus $p_e^{\widetilde{N}} - p_e^{\bar{N}} < 0$, $T_e^{\widetilde{N}} - T_e^{\bar{N}} < 0$, $D_e^{\widetilde{N}} - D_e^{\bar{N}} < 0$.

$$p_e^D - p_e^{\bar{D}} = \frac{\varphi^2(-m(-1+\lambda)(v^2-4\varphi)+2f\rho\varphi)}{(v^2-4\varphi)(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi)} \tag{77}$$

$$T_e^{\widetilde{D}} - T_e^{\bar{D}} = \frac{v\varphi(-m(-1+\lambda)(v^2-4\varphi)+2f\rho\varphi)}{2(v^2-4\varphi)(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi)} \tag{78}$$

$$D_e^{\widetilde{D}} - D_e^{\bar{D}} = \frac{\varphi^2(-m(-1+\lambda)(v^2-4\varphi)+2f\rho\varphi)}{(v^2-4\varphi)(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi)(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi)} \tag{79}$$

When $f < \frac{m(-1+\lambda)(v^2-4\varphi)}{2\rho\varphi}$, $p_e^{\widetilde{D}} - p_e^{\bar{D}} < 0$, $T_e^{\widetilde{D}} - T_e^{\bar{D}} < 0$, $D_e^{\widetilde{D}} - D_e^{\bar{D}} < 0$. We can obtain $\hat{f}_{t2} = \frac{m-m\lambda}{4\rho-2\lambda^2\rho}$, $\hat{f}_{t2} < \frac{m(-1+\lambda)(v^2-4\varphi)}{2\rho\varphi}$, thus $p_e^{\widetilde{D}} - p_e^{\bar{D}} < 0$, $T_e^{\widetilde{D}} - T_e^{\bar{D}} < 0$, $D_e^{\widetilde{D}} - D_e^{\bar{D}} < 0$.

$$p_e^{\widetilde{A}} - p_e^{\bar{A}} = \frac{(-1+\alpha)^2\varphi^2(-m(-1+\lambda)(v^2-4\varphi)+2f\rho\varphi)}{z_{18}} \tag{80}$$

$$T_e^{\widetilde{A}} - T_e^{\bar{A}} = -\frac{v(-1+\alpha)^2\varphi(m(-1+\lambda)(v^2-4\varphi)-2f\rho\varphi)}{2z_{18}} \tag{81}$$

$$D_e^{\widetilde{A}} - D_e^{\bar{A}} = \frac{(-1+\alpha)^2\varphi^2(-m(-1+\lambda)(v^2-4\varphi)+2f\rho\varphi)}{z_{18}} \tag{82}$$

where $z_{18} = (v^2-4\varphi)(v^2(-2+2\alpha+\lambda^2)+(7-7\alpha-4\lambda^2)\varphi)(v^2(-2+2\alpha+\lambda^2)-2(-3+3\alpha+2\lambda^2)\varphi)$. When $f < \frac{m(-1+\lambda)(v^2-4\varphi)}{2\rho\varphi}$, $p_e^{\widetilde{A}} - p_e^{\bar{A}} < 0$, $T_e^{\widetilde{A}} - T_e^{\bar{A}} < 0$, $D_e^{\widetilde{A}} - D_e^{\bar{A}} < 0$. We can obtain $\hat{f}_{t3} = -\frac{m(-1+\alpha)(-1+\lambda)}{2(-2+2\alpha+\lambda^2)\rho}$, $\hat{f}_{t3} < \frac{m(-1+\lambda)(v^2-4\varphi)}{2\rho\varphi}$, thus $p_e^{\widetilde{A}} - p_e^{\bar{A}} < 0$, $T_e^{\widetilde{A}} - T_e^{\bar{A}} < 0$, $D_e^{\widetilde{A}} - D_e^{\bar{A}} < 0$.

Proof of Proposition 12 We consider the impact of channel encroachment on consumer surplus and social welfare. The consumer surplus and social welfare under no encroachment without EW service (Model N), direct encroachment without EW service (Model D), and agency encroachment without EW service (Model A) are given by

$$CS^N = \frac{m^2}{32} \tag{83}$$

$$SW^N = \frac{7m^2}{32} \tag{84}$$

$$CS^D = \frac{m^2\big(5+2(-3+\lambda)\lambda\big)}{8(-2+\lambda^2)^2} \tag{85}$$

$$SW^D = \frac{m^2\big(19+2\lambda(-9+\lambda(-1+2\lambda))\big)}{8(-2+\lambda^2)^2} - F \tag{86}$$

$$CS^A = \frac{m^2\big(5+2(-3+\lambda)\lambda-2\alpha(5+(-4+\lambda)\lambda)+\alpha^2(5+(-2+\lambda)\lambda)\big)}{8(-2+2\alpha+\lambda^2)^2} \tag{87}$$

$$SW^A = \frac{m^2\big(19(-1+\alpha)^2-2(-1+\alpha)(-9+7\alpha)\lambda+(-2+\alpha(-6+7\alpha))\lambda^2+4\lambda^3\big)}{8(-2+2\alpha+\lambda^2)^2} \tag{88}$$

Comparing consumer surplus under direct encroachment and no encroachment, we obtain $CS^D - CS^N = \frac{m^2\left(-16+24\lambda-12\lambda^2+\lambda^4\right)}{-32(-2+\lambda^2)^2} > 0$. Therefore, direct encroachment improves consumer surplus. Further, comparing social welfare yields $SW^D - SW^N = \frac{m^2(2-\lambda)\left(24+(-2+\lambda)\lambda(12+7\lambda)\right)}{32(-2+\lambda^2)^2} - F$. Hence, when $F < F_{S1}$, we have $SW^D - SW^N > 0$, where $F_{S1} = \frac{m^2(2-\lambda)\left(24+(-2+\lambda)\lambda(12+7\lambda)\right)}{32(-2+\lambda^2)^2}$.

Comparing consumer surplus under agency encroachment and no encroachment, we obtain $CS^A - CS^N = \frac{m^2(-2+2\alpha+\lambda)\left(-8-(-2+\lambda)\lambda(4+\lambda)+2\alpha(4+(-2+\lambda)\lambda)\right)}{32(-2+2\alpha+\lambda^2)^2}$. It follows that, when $\alpha_{C1} < \alpha < 1-\lambda/2$, we have $CS^A - CS^N > 0$, where $\alpha_{C1} = \frac{m^2(2-\lambda)\left(24+(-2+\lambda)\lambda(12+7\lambda)\right)}{32(-2+\lambda^2)^2}$. Further, comparing social welfare yields $SW^A - SW^N = \frac{m^2(-2+2\alpha+\lambda)\left(-24+\lambda(24+(2-7\lambda)\lambda)+2\alpha(12+7(-2+\lambda)\lambda)\right)}{32(-2+2\alpha+\lambda^2)^2}$. Solving $SW^A - SW^N = 0$, we obtain $\alpha = 1-\frac{\lambda}{2}$ and $\alpha = \frac{24+(-2+\lambda)\lambda(12+7\lambda)}{24+14(-2+\lambda)\lambda}$. Since $\frac{24+(-2+\lambda)\lambda(12+7\lambda)}{24+14(-2+\lambda)\lambda} > 1-\frac{\lambda}{2}$, it follows that when $\alpha < 1-\lambda/2$, we have $SW^A - SW^N > 0$.

Proof of Proposition 13 We compare the impact of EW service on consumer surplus and social welfare under no encroachment. Under no encroachment with self-operated EW service (Model $\bar{N}$), consumer surplus and social welfare are given by

$$\begin{aligned} CS^{\bar{N}} = &\frac{m^2(v^2-4\varphi)^2(v^4-8v^2\varphi+20\varphi^2)}{32(v^4-7v^2\varphi+12\varphi^2)^2} \\ &+ \frac{4fm\rho(v^2-4\varphi)\varphi(v^4-16v^2\varphi+44\varphi^2)+4f^2\rho^2\varphi^2(17v^4-120v^2\varphi+212\varphi^2)}{32(v^4-7v^2\varphi+12\varphi^2)^2} \end{aligned} \tag{89}$$

$$\begin{aligned} SW^{\bar{N}} = &\frac{4f^2\rho^2\varphi(-8v^6+103v^4\varphi-426v^2\varphi^2+572\varphi^3)}{32(v^4-7v^2\varphi+12\varphi^2)^2} \\ &+ \frac{m^2(v^2-4\varphi)^2(7v^4-50v^2\varphi+92\varphi^2)+4fm\rho(v^2-4\varphi)\varphi(7v^4-58v^2\varphi+116\varphi^2)}{32(v^4-7v^2\varphi+12\varphi^2)^2} \end{aligned} \tag{90}$$

Setting $CS^{\bar{N}} - CS^{N} = 0$, we find that there is no real root. Moreover, when $f = 0$, we have $CS^{\bar{N}} - CS^{N} = \frac{(-m)^2(2v^2-11\varphi)(v^2-4\varphi)^2\varphi}{32(v^4-7v^2\varphi+12\varphi^2)^2} > 0$. Therefore, $CS^{\bar{N}} > CS^{N}$.

Setting $SW^{\bar{N}} - SW^{N} = 0$, we find that there is no real root. Moreover, when $f = 0$, we have $SW^{\bar{N}} - SW^{N} = \frac{m^2(v^2-4\varphi)^2(7v^4-50v^2\varphi+92\varphi^2)}{32(v^4-7v^2\varphi+12\varphi^2)^2} > 0$. Therefore, $SW^{\bar{N}} > SW^{N}$.

Derivation of Critical Thresholds of outsourced EW The comparison of the equilibrium EW decisions under different EW modes is as follows.

Table B.12: Equilibrium solutions of Model $\bar{N}T$

Variable	Equilibrium value
$w^{\bar{N}T}$	$\frac{7m\varphi_t - mv^2 - 2f\varphi_t\rho}{14\varphi_t - 2v^2}$
$t^{\bar{N}T}$	$\frac{3m\varphi_t(7\varphi_t - v^2) + 2f(45\varphi_t^2 - 20\varphi_t v^2 + 2v^4)\rho}{4(42\varphi_t^2 - 13\varphi_t v^2 + v^4)}$
$q_t^{\bar{N}T}$	$\frac{7m\varphi_t - mv^2 - 2f\rho\varphi_t}{24\varphi_t - 4v^2}$
$p_e^{\bar{N}T}$	$\frac{5m\varphi_t(7\varphi_t - v^2) + 2f(19\varphi_t^2 - 16v^2\varphi_t + 2v^4)\rho}{4(42\varphi_t^2 - 13v^2\varphi_t + v^4)}$
$T_e^{\bar{N}T}$	$\frac{-v(-7m\varphi_t + mv^2 + 26f\varphi_t\rho - 4fv^2\rho)}{4(42\varphi_t^2 - 13\varphi_t v^2 + v^4)}$
$p_t^{\bar{N}T}$	$\frac{17m\varphi_t - 3mv^2 + 2f\varphi_t\rho}{24\varphi_t - 4v^2}$
$D_e^{\bar{N}T}$	$\frac{\varphi_t(7m\varphi_t - mv^2 - 26f\varphi_t\rho + 4fv^2\rho)}{2(42\varphi_t^2 - 13\varphi_t v^2 + v^4)}$
$\pi_S^{\bar{N}T}$	$\frac{\big(m(-7\varphi_t + v^2) + 2f\varphi_t\rho\big)^2}{8(42\varphi_t^2 - 13\varphi_t v^2 + v^4)}$
$\pi_P^{\bar{N}T}$	$\frac{m^2(-7\varphi_t + v^2)^2(39\varphi_t^2 - 12\varphi_t v^2 + v^4)}{16(42\varphi_t^2 - 13\varphi_t v^2 + v^4)^2} - \frac{4fm\varphi_t(21\varphi_t^3 + 39\varphi_t^2 v^2 - 13\varphi_t v^4 + v^6)\rho - 4f^2\varphi_t^2(543\varphi_t^2 - 168\varphi_t v^2 + 13v^4)\rho^2}{16(42\varphi_t^2 - 13\varphi_t v^2 + v^4)^2}$
$\pi_T^{\bar{N}T}$	$\frac{-\varphi_t(-7m\varphi_t + mv^2 + 26f\varphi_t\rho - 4fv^2\rho)^2}{16(6\varphi_t - v^2)(-7\varphi_t + v^2)^2}$

Table B.13: Equilibrium solutions of Model $\bar{D}T$

Variable	Equilibrium value
$w^{\bar{D}T}$	$\frac{7m\varphi_t - mv^2 - 2f\varphi_t\rho}{14\varphi_t - 2v^2}$
$q_d^{\bar{D}T}$	$\frac{-mv^2(-2+\lambda) + m\varphi_t(-12+7\lambda) - 2f\varphi_t\lambda\rho}{-2v^2(-2+\lambda^2) + 2\varphi_t(-12+7\lambda^2)}$
$t^{\bar{D}T}$	$\frac{3m\varphi_t(7\varphi_t - v^2)(-1+\lambda) + 2f\big(\varphi_t v^2(20-11\lambda^2) + v^4(-2+\lambda^2) + \varphi_t^2(-45+28\lambda^2)\big)\rho}{2(7\varphi_t - v^2)\big(-v^2(-2+\lambda^2) + \varphi_t(-12+7\lambda^2)\big)}$
$q_t^{\bar{D}T}$	$\frac{m(7\varphi_t - v^2)(-1+\lambda) + 2f\varphi_t\rho}{-2v^2(-2+\lambda^2) + 2\varphi_t(-12+7\lambda^2)}$
$p_e^{\bar{D}T}$	$\frac{5m\varphi_t(7\varphi_t - v^2)(-1+\lambda) + 2f\big(\varphi_t v^2(16-9\lambda^2) + v^4(-2+\lambda^2) + \varphi_t^2(-19+14\lambda^2)\big)\rho}{2(7\varphi_t - v^2)\big(-v^2(-2+\lambda^2) + \varphi_t(-12+7\lambda^2)\big)}$
$T_e^{\bar{D}T}$	$\frac{v\big(m(7\varphi_t - v^2)(-1+\lambda) + 2f(\varphi_t(13-7\lambda^2) + v^2(-2+\lambda^2))\rho\big)}{2(7\varphi_t - v^2)\big(-v^2(-2+\lambda^2) + \varphi_t(-12+7\lambda^2)\big)}$
$p_t^{\bar{D}T}$	$\frac{-mv^2(-3+\lambda+\lambda^2) + m\varphi_t(-17+\lambda(5+7\lambda)) + 2f\varphi_t(-1+\lambda^2)\rho}{-2v^2(-2+\lambda^2) + 2\varphi_t(-12+7\lambda^2)}$
$p_d^{\bar{D}T}$	$\frac{m}{2}$
$D_e^{\bar{D}T}$	$\frac{\varphi_t\big(m(7\varphi_t - v^2)(-1+\lambda) + 2f(\varphi_t(13-7\lambda^2) + v^2(-2+\lambda^2))\rho\big)}{(7\varphi_t - v^2)\big(-v^2(-2+\lambda^2) + \varphi_t(-12+7\lambda^2)\big)}$
$\pi_S^{\bar{D}T}$	$\frac{m^2(-7\varphi_t + v^2)^2(39\varphi_t^2 - 12\varphi_t v^2 + v^4)(-1+\lambda)^2}{4(7\varphi_t - v^2)\big(-v^2(-2+\lambda^2) + \varphi_t(-12+7\lambda^2)\big)} + \frac{4fm\varphi_t(7\varphi_t - v^2)(-1+\lambda)\rho + 4f^2\varphi_t^2\rho^2}{4(7\varphi_t - v^2)\big(-v^2(-2+\lambda^2) + \varphi_t(-12+7\lambda^2)\big)} + \frac{4fm\varphi_t(7\varphi_t - v^2)(-1+\lambda)\big(2\varphi_t v^2(3-5\lambda^2) + v^4(-1+\lambda^2) + 3\varphi_t^2(1+7\lambda^2)\big)\rho}{4(7\varphi_t - v^2)\big(-v^2(-2+\lambda^2) + \varphi_t(-12+7\lambda^2)\big)}$
$\pi_P^{\bar{D}T}$	$\frac{4f^2\varphi_t^2\big(543\varphi_t^2 - 168\varphi_t v^2 + 13v^4 - 2(315\varphi_t^2 - 94\varphi_t v^2 + 7v^4)\lambda^2 + 4(-7\varphi_t + v^2)^2\lambda^4\big)\rho^2}{4(-7\varphi_t + v^2)^2\big(\varphi_t(12-7\lambda^2) + v^2(-2+\lambda^2)\big)^2} + \frac{4f^2\varphi_t^2\big(543\varphi_t^2 - 168\varphi_t v^2 + 13v^4 - 2(315\varphi_t^2 - 94\varphi_t v^2 + 7v^4)\lambda^2 + 4(-7\varphi_t + v^2)^2\lambda^4\big)\rho^2}{4(-7\varphi_t + v^2)^2\big(\varphi_t(12-7\lambda^2) + v^2(-2+\lambda^2)\big)^2}$
$\pi_T^{\bar{D}T}$	$\frac{\varphi_t(6\varphi_t - v^2)\big(m(7\varphi_t - v^2)(-1+\lambda) + 2f(\varphi_t(13-7\lambda^2) + v^2(-2+\lambda^2))\rho\big)^2}{4(-7\varphi_t + v^2)^2\big(\varphi_t(12-7\lambda^2) + v^2(-2+\lambda^2)\big)^2}$

The case of no encroachment with outsourced EW service. Treating $\pi_P^{\bar{N}T} - \pi_P^{N}$ as a quadratic function of f, solving $\pi_P^{\bar{N}T} - \pi_P^{N} = 0$ shows that it has no real root. Moreover, when $f = 0$, we have $\pi_P^{\bar{N}T} - \pi_P^{N} = \frac{3m^2\varphi_t^2(-7\varphi_t+v^2)^2}{16(42\varphi_t^2-13\varphi_t v^2+v^4)^2} > 0$. Thus, it follows that $\pi_P^{\bar{N}T} > \pi_P^{N}$. Solving $p_t^{\bar{N}T} - w^{\bar{N}T} = \frac{m(35\varphi_t^2-12\varphi_t v^2+v^4)+2f\varphi_t(19\varphi_t-3v^2)\rho}{4(42\varphi_t^2-13\varphi_t v^2+v^4)} > 0$ yields the threshold at which the unit profit of the reselling channel is zero:

$$\hat{f}_1^{TN} = \frac{35m\varphi_t^2 - 12m\varphi_t v^2 + mv^4}{6\varphi_t v^2\rho - 38\varphi_t^2\rho} \tag{91}$$

When $f > \hat{f}_1^{TN}$, we have $p_t^{\bar{N}T} - w^{\bar{N}T} > 0$. Since the denominator of $\hat{f}_1^{TN}$ is negative, while the sign of $35m\varphi_t^2 - 12m\varphi_t v^2 + mv^4$ is uncertain, $\hat{f}_1^{TN}$ is not necessarily always positive.

Solving $D_e^{\bar{N}T} = \frac{\varphi_t(7m\varphi_t - mv^2 - 26f\varphi_t\rho + 4fv^2\rho)}{2(42\varphi_t^2 - 13\varphi_t v^2 + v^4)} > 0$ yields the threshold for positive EW demand:

$$\hat{f}_2^{TN} = \frac{m(7\varphi_t - v^2)}{26\varphi_t\rho - 4v^2\rho} \tag{92}$$

When $f < \hat{f}_2^{TN}$, we have $D_e^{\bar{N}T} > 0$. When $v^2 < 6\varphi_t$, it follows that $\hat{f}_2^{TN} - \hat{f}_1^{TN} = \frac{-m(6\varphi_t - v^2)(-7\varphi_t + v^2)^2}{\varphi_t(247\varphi_t^2 - 77\varphi_t v^2 + 6v^4)\rho} > 0$.

The case of direct encroachment with outsourced EW service. Treating $\pi_P^{\bar{D}T} - \pi_P^D$ as a quadratic function of f, solving $\pi_P^{\bar{D}T} - \pi_P^D = 0$ shows that it has no real root. Moreover, when $f = 0$, we have $\pi_P^{\bar{D}T} - \pi_P^D = \frac{1}{4}m^2(-1+\lambda)^2\left(-\frac{1}{(-2+\lambda^2)^2} + \frac{39\varphi_t^2 - 12\varphi_t v^2 + v^4}{\left(\varphi_t(12-7\lambda^2) + v^2(-2+\lambda^2)\right)^2}\right) > 0$. Thus, it follows that $\pi_P^{\bar{D}T} > \pi_P^D$.

Solving $p_t^{\bar{D}T} - w^{\bar{D}T} = \frac{m(35\varphi_t^2 - 12\varphi_t v^2 + v^4)(-1+\lambda) + 2f\varphi_t\left(v^2(3-2\lambda^2) + \varphi_t(-19+14\lambda^2)\right)\rho}{2(7\varphi_t - v^2)\left(-v^2(-2+\lambda^2) + \varphi_t(-12+7\lambda^2)\right)} > 0$ yields the threshold at which the unit profit of the reselling channel is zero:

$$\hat{f}_1^{TD} = \frac{m(35\varphi_t^2 - 12\varphi_t v^2 + v^4)(1-\lambda)}{2\varphi_t\left(v^2(3-2\lambda^2) + \varphi_t(-19+14\lambda^2)\right)\rho} \tag{93}$$

When $f > \hat{f}_1^{TD}$, we have $p_t^{\bar{D}T} - w^{\bar{D}T} > 0$. Since the denominator of $\hat{f}_1^{TD}$ is negative, while the sign of $m(35\varphi_t^2 - 12\varphi_t v^2 + v^4)$ is uncertain, $\hat{f}_1^{TD}$ is not necessarily always positive.

Solving $D_e^{\bar{D}T} = \frac{\varphi_t\left(m(7\varphi_t - v^2)(-1+\lambda) + 2f(\varphi_t(13-7\lambda^2) + v^2(-2+\lambda^2))\rho\right)}{(7\varphi_t - v^2)\left(-v^2(-2+\lambda^2) + \varphi_t(-12+7\lambda^2)\right)} > 0$ yields the threshold for positive EW demand:

$$\hat{f}_2^{TD} = \frac{m(7\varphi_t - v^2)(-1+\lambda)}{2\left(-v^2(-2+\lambda^2) + \varphi_t(-13+7\lambda^2)\right)\rho} \tag{94}$$

When $f < \hat{f}_2^{TD}$, we have $D_e^{\bar{D}T} > 0$. When $v^2 < 6\varphi_t$ and $(12-7\lambda^2)\varphi_t + v^2(-2+\lambda^2) > 0$, it follows that $\hat{f}_2^{TD} - \hat{f}_1^{TD} = \frac{m(-7\varphi_t + v^2)^2(-1+\lambda)\left(-v^2(-2+\lambda^2) + \varphi_t(-12+7\lambda^2)\right)}{2\varphi_t\left(-v^2(-2+\lambda^2) + \varphi_t(-13+7\lambda^2)\right)\left(v^2(3-2\lambda^2) + \varphi_t(-19+14\lambda^2)\right)\rho} > 0$. The analysis under agency encroachment is analogous and is therefore omitted for brevity. In addition, it can be shown that $\hat{f}_2^N - \hat{f}_2^{TN} = \frac{m(v^2\varphi - \varphi_t(v^2+3\varphi))}{2(13\varphi_t - 2v^2)\rho(2v^2 - 7\varphi)} > 0$, $\hat{f}_1^N - \hat{f}_1^{TN} = \frac{35m\varphi_t^2 - 12m\varphi_t v^2 + mv^4}{38\varphi_t^2\rho - 6\varphi_t v^2\rho} + \frac{mv^4 - 6mv^2\varphi + 8m\varphi^2}{6v^2\rho\varphi - 20\rho\varphi^2} > 0$, $\hat{f}_2^D - \hat{f}_2^{TD} = \frac{m(-1+\lambda)(-v^2\varphi + \varphi_t(v^2+3\varphi))}{2\left(-v^2(-2+\lambda^2) + \varphi_t(-13+7\lambda^2)\right)\rho\left(v^2(-2+\lambda^2) + (7-4\lambda^2)\varphi\right)} > 0$, and $\hat{f}_1^D - \hat{f}_1^{TD} = \frac{m(1-\lambda)(v^2 - 4\varphi)(v^2 - 2\varphi)}{2\rho\varphi\left(v^2(3-2\lambda^2) + 2(-5+4\lambda^2)\varphi\right)} + \frac{m(35\varphi_t^2 - 12\varphi_t v^2 + v^4)(-1+\lambda)}{2\varphi_t\left(v^2(3-2\lambda^2) + \varphi_t(-19+14\lambda^2)\right)\rho} > 0$.

Proof of Proposition 14 The case without supplier encroachment. Comparing the equilibrium EW prices in the two models yields $p_e^{\bar{N}T} - p_e^{\bar{N}} = \frac{1}{4}\left(\frac{5m\varphi_t(7\varphi_t - v^2) + 2f(19\varphi_t^2 - 16\varphi_t v^2 + 2v^4)\rho}{42\varphi_t^2 - 13\varphi_t v^2 + v^4} + \frac{-3mv^2 + 8m\varphi + 2f\rho\varphi}{v^2 - 3\varphi}\right)$. When $f < \hat{f}_{l1}$, we have $p_e^{\bar{N}T} > p_e^{\bar{N}}$, where

$$\hat{f}_{l1} = \frac{m(7\varphi_t - v^2)(13\varphi_t v^2 - 3v^4 - 33\varphi_t\varphi + 8v^2\varphi)}{2\rho\left(19\varphi_t^2 v^2 - 16\varphi_t v^4 + 2v^6 - 5(3\varphi_t^2 - 7\varphi_t v^2 + v^4)\varphi\right)} \tag{95}$$

Comparing the equilibrium EW periods in the two models yields
$T_e^{\bar{N}T} - T_e^{\bar{N}} = \frac{1}{4}v\left(m\left(\frac{1}{6\varphi_t - v^2} + \frac{1}{v^2 - 3\varphi}\right) + 2f\rho\left(\frac{1}{-7\varphi_t + v^2} + \frac{1}{-6\varphi_t + v^2} + \frac{1}{-v^2 + 3\varphi} + \frac{1}{-v^2 + 4\varphi}\right)\right)$. When $f < \hat{f}_{l2}$, we have $T_e^{\bar{N}T} > T_e^{\bar{N}}$, where

$$\hat{f}_{l2} = \frac{3m(7\varphi_t - v^2)(v^2 - 4\varphi)(2\varphi_t - \varphi)}{2\rho\left(42\varphi_t^2(2v^2 - 7\varphi) + v^2(7v^2 - 24\varphi)\varphi - 13\varphi_t(v^4 - 12\varphi^2)\right)} \tag{96}$$

The case with supplier direct encroachment. Comparing the equilibrium EW prices in the two models yields

$$\begin{aligned} p_e^{\bar{D}T} - p_e^{\bar{D}} = &\frac{5m\varphi_t(7\varphi_t - v^2)(-1+\lambda) + 2f\left(\varphi_t v^2(16-9\lambda^2) + v^4(-2+\lambda^2) + \varphi_t^2(-19+14\lambda^2)\right)\rho}{2(7\varphi_t - v^2)\left(-v^2(-2+\lambda^2) + \varphi_t(-12+7\lambda^2)\right)} \\ &+ \frac{m(-1+\lambda)(v^2 - 4\varphi)\varphi + f\rho\left(-v^4(-2+\lambda^2) + 2v^2(-5+3\lambda^2)\varphi + 2(5-4\lambda^2)\varphi^2\right)}{(v^2 - 4\varphi)\left(v^2(-2+\lambda^2) + (6-4\lambda^2)\varphi\right)}. \end{aligned} \tag{97}$$

When $f < \hat{f}_{l3}$, we have $p_e^{\bar{D}T} > p_e^{\bar{D}}$, where

$$\hat{f}_{l3} = \frac{-m(7\varphi_t - v^2)(-1+\lambda)\big(5\varphi_t v^2(-2+\lambda^2) - 2v^2(-2+\lambda^2)\varphi - 6\varphi_t(-1+\lambda^2)\varphi\big)}{2\big(\varphi_t v^2(16-9\lambda^2) + v^4(-2+\lambda^2) + \varphi_t^2(-19+14\lambda^2)\big)\rho\big(v^2(-2+\lambda^2) + (6-4\lambda^2)\varphi\big)} \tag{98}$$

Comparing the equilibrium EW periods in the two models yields $T_e^{\bar{D}T} - T_e^{\bar{D}} = \frac{1}{2}v\left(\frac{m(7\varphi_t-v^2)(-1+\lambda)+2f\left(\varphi_t(13-7\lambda^2)+v^2(-2+\lambda^2)\right)\rho}{(7\varphi_t-v^2)\left(-v^2(-2+\lambda^2)+\varphi_t(-12+7\lambda^2)\right)} - \frac{f\rho}{v^2-4\varphi} + \frac{m(-1+\lambda)-f(-2+\lambda^2)\rho}{v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi}\right)$. When $f < \hat{f}_{l4}$, we have $T_e^{\bar{D}T} > T_e^{\bar{D}}$, where

$$\hat{f}_{l4} = \frac{m(7\varphi_t - v^2)(-1+\lambda)\big(\varphi_t(-12+7\lambda^2) + (6-4\lambda^2)\varphi\big)}{2\big(-v^2(-2+\lambda^2) + \varphi_t(-13+7\lambda^2)\big)\rho\big(v^2(-2+\lambda^2) + (6-4\lambda^2)\varphi\big)} \tag{99}$$

The case without supplier encroachment. We have $D_e^{\bar{N}T} - D_e^{\bar{N}} = \frac{1}{2}\left(\frac{\varphi_t(7m\varphi_t - mv^2 - 26f\varphi_t\rho + 4fv^2\rho)}{42\varphi_t^2 - 13\varphi_t v^2 + v^4} + \frac{\varphi\left(m(v^2-4\varphi)+2f\rho(-2v^2+7\varphi)\right)}{(v^2-4\varphi)(v^2-3\varphi)}\right)$. When $f < \hat{f}_{l5}$, we have $D_e^{\bar{N}T} > D_e^{\bar{N}}$. Since $\hat{f}_{l5} > \hat{f}_2^N$, it follows that $D_e^{\bar{N}T} > D_e^{\bar{N}}$ always holds, where

$$\hat{f}_{l5} = \frac{m(7\varphi_t - v^2)(v^2 - 4\varphi)}{2\rho(13\varphi_t v^2 - 2v^4 - 46\varphi_t\varphi + 7v^2\varphi)} \tag{100}$$

The case with supplier direct encroachment. We have $D_e^{\bar{D}T} - D_e^{\bar{D}} = \frac{\varphi_t\left(m(7\varphi_t-v^2)(-1+\lambda)+2f(\varphi_t(13-7\lambda^2)+v^2(-2+\lambda^2))\rho\right)}{(7\varphi_t-v^2)\left(-v^2(-2+\lambda^2)+\varphi_t(-12+7\lambda^2)\right)} + \frac{\varphi\left(m(-1+\lambda)(v^2-4\varphi)-2f\rho(v^2(-2+\lambda^2)+(7-4\lambda^2)\varphi)\right)}{(v^2-4\varphi)\left(v^2(-2+\lambda^2)+(6-4\lambda^2)\varphi\right)}$. When $f < \hat{f}_{l6}$, we have $D_e^{\bar{D}T} > D_e^{\bar{D}}$. Since $\hat{f}_{l6} > \hat{f}_2^D$, it follows that $D_e^{\bar{D}T} > D_e^{\bar{D}}$ always holds, where

$$\hat{f}_{l6} = \frac{m(7\varphi_t - v^2)(-1+\lambda)(-2+\lambda^2)(v^2-4\varphi)}{-2v^2(-2+\lambda^2)\rho\big(v^2(-2+\lambda^2) + (7-4\lambda^2)\varphi\big) + 2\varphi_t\rho\big(v^2(26-27\lambda^2+7\lambda^4) + (-92+101\lambda^2-28\lambda^4)\varphi\big)} \tag{101}$$

Proof of Proposition 15 These results can be obtained from Derivation of Critical Thresholds of outsourced EW.

Proof of Proposition 16 In the parameter region where both self-operated EW service and outsourced EW service improve the platform's profit, the platform's optimal EW mode is determined by comparing $\pi_P^{\bar{N}}$ with $\pi_P^{\bar{N}T}$ (or $\pi_P^{\bar{D}}$ with $\pi_P^{\bar{D}T}$).

The case in which the supplier does not encroach. Let $\pi_P^{\bar{N}} = \pi_P^{\bar{N}T}$. Then, when $\hat{\varphi}_{N1} < \varphi < \frac{v^2\varphi_t}{v^2-3\varphi_t}$, the threshold $\hat{f}_{R1}$ exists. In this case, we have $\hat{f}_1^{TN} < \hat{f}_1^N$ and $\hat{f}_2^{TN} < \hat{f}_2^N$.

When $f < \hat{f}_1^{TN}$ or $f > \hat{f}_2^N$, we have $\pi_P^{\bar{N}} < \pi_P^N$, $\quad \pi_P^{\bar{N}T} < \pi_P^N$.

When $\hat{f}_1^N < f < \hat{f}_2^N$, we have $\pi_P^{\bar{N}} > \pi_P^N$. Moreover, when $\varphi < \hat{\varphi}_{N1}$, we have $\pi_P^{\bar{N}} > \pi_P^{\bar{N}T}$, and when $\hat{\varphi}_{N1} < \varphi < \frac{v^2\varphi_t}{v^2-3\varphi_t}$, $\quad f > \hat{f}_{R1}$, we have $\pi_P^{\bar{N}T} < \pi_P^{\bar{N}}$.

When $\hat{f}_1^{TN} < f < \hat{f}_2^{TN}$, we have $\pi_P^{\bar{N}T} > \pi_P^N$. Further, when $\hat{\varphi}_{N1} < \varphi < \frac{v^2\varphi_t}{v^2-3\varphi_t}$, $\quad f < \hat{f}_{R1}$, or $\varphi > \frac{v^2\varphi_t}{v^2-3\varphi_t}$, we have $\pi_P^{\bar{N}T} > \pi_P^{\bar{N}}$. The expression of $\hat{\varphi}_{N1}$ is given by

$$\hat{\varphi}_{N1} = \frac{v^2(45 - v^4)}{-639 + 369v^2 - 61v^4 + 3v^6} \tag{102}$$

The case with supplier direct encroachment. Let $\pi_P^{\bar{D}} = \pi_P^{\bar{D}T}$. Then, when $\hat{\varphi}_{D1} < \varphi < \frac{v^2\varphi_t}{v^2-3\varphi_t}$, the threshold $\hat{f}_{R2}$ exists. In this case, we have $\hat{f}_1^{TD} < \hat{f}_1^D$ and $\hat{f}_2^{TD} < \hat{f}_2^D$.

When $f < \hat{f}_1^{TD}$ or $f > \hat{f}_2^D$, we have $\pi_P^{\bar{D}} < \pi_P^D$, $\quad \pi_P^{\bar{D}T} < \pi_P^D$.

When $\hat{f}_1^D < f < \hat{f}_2^D$, we have $\pi_P^{\bar{D}} > \pi_P^D$. In this case, when $\varphi < \hat{\varphi}_{D1}$, we have $\pi_P^{\bar{D}T} < \pi_P^{\bar{D}}$, and when $\hat{\varphi}_{D1} < \varphi < \frac{v^2\varphi_t}{v^2-3\varphi_t}$, $\quad f > \hat{f}_{R2}$, we have $\pi_P^{\bar{D}T} < \pi_P^{\bar{D}}$.

When $\hat{f}_1^{TD} < f < \hat{f}_2^{TD}$, we have $\pi_P^{\bar{D}T} > \pi_P^D$. Moreover, when $\hat{\varphi}_{D1} < \varphi < \frac{v^2\varphi_t}{v^2-3\varphi_t}$, $\quad f < \hat{f}_{R2}$, or $\varphi > \frac{v^2\varphi_t}{v^2-3\varphi_t}$, we have $\pi_P^{\bar{D}T} > \pi_P^{\bar{D}}$. The expression of $\hat{\varphi}_{D1}$ is given by

$$\hat{\varphi}_{D1} = \frac{v^2\left(504 - 369v^2 + 64v^4 - 3v^6 - 14(-7+v^2)(-3+v^2)\lambda^2 + (-7+v^2)^2(1+v^2)\lambda^4\right) - \sqrt{H_1}}{2(-3+v^2)\left(-639 + 156v^2 - 9v^4 - 24(-7+v^2)\lambda^2 + 4(-7+v^2)^2\lambda^4\right)} \tag{103}$$

where

$$H_1 = v^4\Big((774 - 369v^2 + 58v^4 - 3v^6)^2 \\ + 4(-7 + v^2)(-6 + v^2)(-3 + v^2)(129 - 40v^2 + 3v^4)\lambda^2 \\ - 2(-7 + v^2)^2(4392 - 2697v^2 + 645v^4 - 71v^6 + 3v^8)\lambda^4 \\ + 4(-7 + v^2)^4(-3 + v^2)\lambda^6 + (-7 + v^2)^6\lambda^8\Big). \tag{104}$$

The case with supplier agency encroachment. Let $\pi_P^{\bar{A}} = \pi_P^{\bar{A}T}$. Then, when $\hat{\varphi}_{A1} < \varphi < \frac{v^2\varphi_t}{v^2-3\varphi_t}$, the threshold $\hat{f}_{R2}$ exists. In this case, we have $\hat{f}_1^{TA} < \hat{f}_1^A$ and $\hat{f}_2^{TA} < \hat{f}_2^A$.

When $f < \hat{f}_1^{TA}$ or $f > \hat{f}_2^A$, we have $\pi_P^{\bar{A}} < \pi_P^A$, $\quad \pi_P^{\bar{A}T} < \pi_P^A$.

When $\hat{f}_1^A < f < \hat{f}_2^A$, we have $\pi_P^{\bar{A}} > \pi_P^A$. In this case, when $\varphi < \hat{\varphi}_{A1}$, we have $\pi_P^{\bar{A}T} < \pi_P^{\bar{A}}$, and when $\hat{\varphi}_{A1} < \varphi < \frac{v^2\varphi_t}{v^2-3\varphi_t}$, $\quad f > \hat{f}_{R2}$, we have $\pi_P^{\bar{A}T} < \pi_P^{\bar{A}}$.

When $\hat{f}_1^{TA} < f < \hat{f}_2^{TA}$, we have $\pi_P^{\bar{A}T} > \pi_P^A$. Moreover, when $\hat{\varphi}_{A1} < \varphi < \frac{v^2\varphi_t}{v^2-3\varphi_t}$, $\quad f < \hat{f}_{R3}$, or $\varphi > \frac{v^2\varphi_t}{v^2-3\varphi_t}$, we have $\pi_P^{\bar{A}T} > \pi_P^{\bar{A}}$.