Appendix 2: Description of Method 2 with multiple day-level PA observations and multiple one-time measured covariates

Two different sources of variability are essential for variance adjustment: variability of PA across participants (between-individual variability) and day-to-day variability within a participant (within-individual variability). When we have multiple day-level PA observations (expressed as a p-dimensional vector $\boldsymbol{X}_{ij}$) and multiple one-time measured covariates (expressed as a q-dimensional vector $\boldsymbol{Z}_i$), these are represented by covariance matrices. For $\boldsymbol{X}_{ij}$, we use the following decomposition:

$$\boldsymbol{X}_{ij} = \boldsymbol{\xi}_i + \boldsymbol{\nu}_{ij},$$

where $\boldsymbol{\xi}_i \in \mathbb{R}^p$ captures the latent, person-specific deviation for participant i, and $\boldsymbol{\nu}_{ij} \in \mathbb{R}^p$ is the day-to-day fluctuation for day j of participant i. Here, $\boldsymbol{\Sigma}_{\xi\xi}$ is the between-individual covariance matrix and $\boldsymbol{\Sigma}_{\nu\nu}$ is the within-individual covariance matrix.

For each individual i, we construct two weight matrices that determine the contribution of the observed participant mean and the covariates to the adjusted estimate. Specifically, a $p \times p$ matrix $\boldsymbol{W}_{i1}$ and a $q \times p$ matrix $\boldsymbol{W}_{i2}$ are defined as:

$$\boldsymbol{W}_{i1} = (\boldsymbol{\Sigma}_{\xi\xi} + \boldsymbol{\Sigma}_{\nu\nu}/g_i - \boldsymbol{\Sigma}_{\xi z}\boldsymbol{\Sigma}_{zz}^{-1}\boldsymbol{\Sigma}_{z\xi})^{-1}(\boldsymbol{\Sigma}_{\xi\xi} - \boldsymbol{\Sigma}_{\xi z}\boldsymbol{\Sigma}_{zz}^{-1}\boldsymbol{\Sigma}_{z\xi})$$

and

$$\boldsymbol{W}_{i2} = \boldsymbol{\Sigma}_{zz}^{-1}\boldsymbol{\Sigma}_{z\xi}(\boldsymbol{I}_{p\times p} - \boldsymbol{W}_{i1}),$$

where $\boldsymbol{I}_{p\times p}$ denotes the $p \times p$ identity matrix. These weight matrices determine the extent to which the observed participant-level mean $\bar{\boldsymbol{X}}_i$ and the deviation of covariates $(\boldsymbol{Z}_i - \boldsymbol{\mu}_z)$ contribute to the adjusted estimate. The resulting variance-adjusted mean is given by

$$\tilde{\boldsymbol{X}}_i' = \boldsymbol{\mu}_\xi'(\boldsymbol{I}_{p\times p} - \boldsymbol{W}_{i1}) + \bar{\boldsymbol{X}}_i'\boldsymbol{W}_{i1} + (\boldsymbol{Z}_i - \boldsymbol{\mu}_z)'\boldsymbol{W}_{i2}.$$