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Quadrifocal Diffractive Lenses based on Aperiodically Structured Lucas Sequences

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ABSTRACT

In this work, we present a new family of aperiodic diffractive lenses based on the Lucas sequence, implemented in both binary-phase and kinoform configurations. These lenses generate four focal planes along the optical axis with a partially balanced energy distribution, offering enhanced control over axial light distribution. A comparative analysis reveals that the kinoform design achieves higher energy efficiency, while the axial positions between focal planes are directly related to the golden ratio, reflecting the intrinsic properties of the Lucas sequence and its close relationship with the Fibonacci sequence. The proposed lenses show strong potential for applications in optical trapping, where multiple particles can be confined simultaneously, and in multifocal imaging systems, where parallel acquisition can reduce scanning time and maintain uniform resolution across planes, as well as in quadrifocal intraocular lenses. This study demonstrates the versatility of Lucas-sequence-based diffractive lenses for advanced optical and laser-based technologies.

Introduction

Diffractive optical elements (DOEs) have become essential tools for light control and beam shaping in compact optical systems, enabling functionalities that are difficult to achieve with conventional refractive components^{1–4}. Their ability to modulate phase and amplitude of optical field distributions has facilitated their integration into a wide range of optical and laser-based applications, including axial beam shaping, optical trapping, and multifocal imaging systems^{5–7}. Alternative strategies for generating multiple focal regions have also been explored using structured light approaches, including orbital angular momentum beams, polarization-dependent modulation, and focal scanning techniques^{8–11}. As a result, these applications have driven the ongoing development of diffractive lens designs that provide enhanced focusing capabilities while preserving structural simplicity and design flexibility.

Traditional Fresnel zone plates rely on periodic radial arrangements designed to concentrate light into a primary focus^{12,13}. However, due to their diffractive nature, these structures also produce additional secondary foci associated with higher diffraction orders, which restrict independent control over the axial energy distribution^{12,14,15}. Although phase-only and kinoform implementations significantly improve energy efficiency by directing most of the incident power into the principal diffraction order^{16,17}, strictly periodic designs offer limited flexibility to precisely control the number, position, and relative intensity of the resulting foci.

Deterministic aperiodic structures have emerged as a powerful alternative for the development of multifocal diffractive lenses^{18–20}. Unlike periodic arrangements, aperiodic distributions provide additional degrees of freedom in the zone organization, allowing the design of specific axial irradiance profiles. Using mathematical sequences as design rules ensures reproducibility and deterministic behavior, avoiding the unpredictability inherent in random or pseudo-random approaches^{21,22}. Moreover, aperiodic sequences allow the definition of the number of focal planes, their relative intensities, and their axial positions, offering a level of control that is difficult to achieve with conventional periodic designs. Previous studies have demonstrated that aperiodic sequences can generate lenses with multiple focal planes, whose positions and relative intensities are directly related to the intrinsic properties of the sequence^{23,24}. Unlike conventional multifocal diffractive optical elements, which are often designed through numerical or iterative optimization procedures, sequence-based approaches provide deterministic analytical rules that directly determine the number and relative positions of the resulting focal planes.

Among the sequences explored in diffractive optics are the Fibonacci, Baum–Sweet, and Silver Mean sequences, which have produced bifocal and tetra focal lenses with characteristic axial irradiance profiles and well-defined focal-length ratios^{25–27}.

In particular, Fibonacci-based lenses exhibit focal-plane ratios that approach the golden ratio, reflecting the mathematical structure of the sequence²⁸. Closely related to the Fibonacci sequence but defined by different initial conditions, the Lucas sequence provides an alternative framework for generating unique multifocal behavior. While sharing several mathematical properties with Fibonacci, its distinct initial configuration enables the design of lenses with axial energy distributions and focal-plane ratios that are not accessible using the Fibonacci sequence alone.

In this work, we introduce a new family of aperiodic diffractive lenses whose zone distribution is governed by the Lucas sequence²⁹. The proposed Lucas diffractive lenses are implemented in both binary phase and kinoform configurations and generate four focal planes along the optical axis with a partially balanced energy distribution. The comparative analysis of both implementations demonstrates, as expected, the superior energy efficiency of the kinoform design. Moreover, the relationship between the focal planes is directly connected to the golden ratio, reflecting an intrinsic property of the Lucas sequence and its link to the Fibonacci sequence. These features position Lucas-sequence-based diffractive lenses as promising candidates for applications such as optical trapping, multifocal imaging systems, and quadrifocal intraocular lenses, where simultaneous and controlled multi-plane focusing is required.

Design of the Lucas diffractive lens

The design of the proposed family of diffractive lenses, termed *Lucas Diffractive Lenses* (LDLs), is based on the Lucas number sequence, which satisfies the same recurrence relation as the Fibonacci sequence but differs in its initial conditions²⁹⁻³¹. The Fibonacci sequence is defined by the seeds $F_0 = 0$ and $F_1 = 1$, obeying the recurrence rule $F_{k+1} = F_k + F_{k-1}$ and generating the sequence $\{F_k\}_{k=0}^{\infty} = \{0, 1, 1, 2, 3, 5, 8, 13, \dots\}$. The golden mean, also known as the golden ratio, is defined as the limit of two consecutive Fibonacci numbers: $\phi = \lim_{k \rightarrow \infty} (F_{k+1}/F_k) = (1 + \sqrt{5})/2$.

However, the Lucas sequence is defined by the initial values $L_0 = 2$ and $L_1 = 1$, while following the same recurrence relation $L_k = L_{k-1} + L_{k-2}$, which produces the sequence $\{L_k\}_{k=0}^{\infty} = \{2, 1, 3, 4, 7, 11, 18, \dots\}$. Owing to the well-established relationship between both sequences, the Lucas numbers can be expressed in terms of Fibonacci numbers as $L_k = F_{k-1} + F_{k+1} = F_k + 2F_{k-1}$ ³², and, conversely, the Fibonacci numbers can be recovered from the Lucas sequence through $F_k = (L_{k-1} + L_{k+1})/5 = (L_k + 2L_{k-1})/5$. Note that the golden ratio between two consecutive Lucas numbers also converges in the asymptotic limit, the golden ratio.

$$\lim_{k \rightarrow \infty} \frac{L_{k+1}}{L_k} = \lim_{k \rightarrow \infty} \frac{F_k + F_{k+2}}{F_{k-1} + F_{k+1}} = \frac{\phi^{-1} + \phi}{\phi^{-2} + 1} = \phi \quad (1)$$

This mathematical framework provides the basis for encoding the Lucas sequence into binary structures, which is exploited in the following to construct Lucas-based zone plates through the use of appropriate initial seeds and recursive generation rules.

For the design of a diffractive lens based on the Lucas sequence, two initial sequences, referred to as seeds, are considered: $S_0 = \{B, A\}$ and $S_1 = \{A\}$, thus obtaining binary sequences for each sequential order. Higher-order sequences are generated by following an equivalent recurrence rule as the Lucas number sequence, that is, S_n is obtained as the concatenation of the two previous sequences $S_n = S_{n-1} || S_{n-2}$, so that the sequences corresponding to the first seven orders are expressed as follows:

$$\begin{aligned} S_0 &= \{B, A\}, \\ S_1 &= \{A\}, \\ S_2 &= \{A, B, A\}, \\ S_3 &= \{A, B, A, A\}, \\ S_4 &= \{A, B, A, A, A, B, A\}, \\ S_5 &= \{A, B, A, A, A, B, A, A, B, A, A\}, \\ S_6 &= \{A, B, A, A, A, B, A, A, B, A, A, A, B, A, A, A, B, A\}. \end{aligned}$$

It is worth noting that each sequence of order n contains a number of elements of type B equal to the $(n-1)$ -th Fibonacci number, F_{n-1} , and a number of elements of type A equal to the $(n+1)$ -th Fibonacci number, F_{n+1} , as a direct consequence of the intrinsic relationship between the Fibonacci and Lucas sequences. Accordingly, the length ℓ_n of the Lucas sequence of order n coincides with the n -th term of the Lucas numbers, L_n , and can be written as $\ell_n = F_{n-1} + F_{n+1} = L_n$. As an illustrative example, in the case of the Lucas sequence S_6 , the total length is $\ell_6 = F_5 + F_7 = 5 + 13 = L_6$, indicating that it is composed of 13 elements of type A and 5 elements of type B . Consequently, the ratio between the lengths of sequences of consecutive orders $\frac{\ell_{n+1}}{\ell_n}$ converges to the golden ratio ϕ , as a consequence of the 1.

An Lucas Diffractive Lenses (LDL) of radius a , designed from a sequence S_n , is divided into ℓ_n rings of equal area. The phase in the plate is distributed along the quadratic radial coordinate $\zeta = (r/a)^2$ within the interval $[0, 1]$, divided into ℓ_n subintervals, each having a width of $d_n = \frac{1}{\ell_n}$. Based on this approach, two prototypes of Zone Plates with different phase profiles are proposed: the binary-phase profile and the kinoform phase profile.

For the construction of the binary-phase Lucas Zone Plate (LZP), the phase distribution in the lens depends on the elements $S_{n,j}$ of the corresponding sequence. If the element $S_{n,j} = A$, the corresponding phase, ϕ_n , for the j -th ring is $\phi_n = 0$, while for $S_{n,j} = B$, $\phi_n = \pi$. Accordingly, the phase distribution along the quadratic radial coordinate can be mathematically expressed as:

$$\phi_n(\zeta) = \pi \sum_{j=1}^{\ell_n} \text{rect}\left(\frac{\zeta - (j-1/2)d_n}{d_n}\right) \delta_{B,S_{n,j}}, \quad (2)$$

where $\delta_{B,S_{n,j}}$ is a Kronecker delta and rect denotes the rectangular function. Figure 1(a,b) show the phase distribution as a function of the quadratic radial coordinate and its corresponding representation on a cartesian plane for an LZP of order $n = 7$.

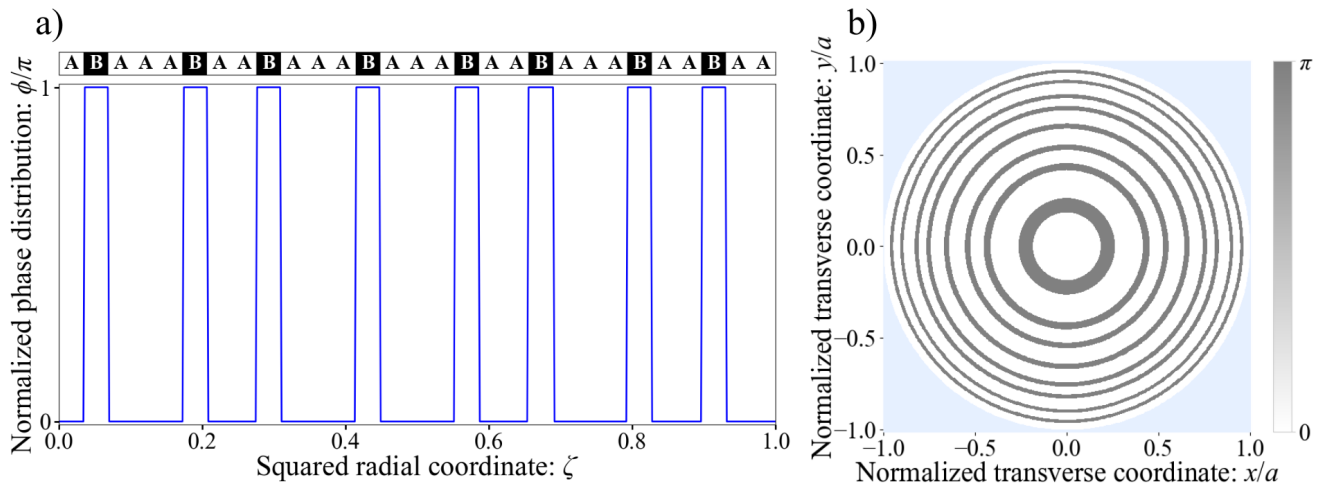


Figure 1. Normalized phase distribution of an LZP with order $n = 7$: a) as a function of the coordinate ζ and b) in a cartesian coordinate plane.

For the construction of the LDL with kinoform phase profile (KLZP), a linear phase distribution is considered in the interval $[0, 2\pi]$ in the consecutive zones where different phases appear. In this case, a phase "tooth" is constructed in the regions where the sequence presents the pair "AB", such that the slope of the tooth is $\frac{2\pi}{2d_n}$, while in the remaining regions $\phi_n = 0$. Mathematically, this phase distribution can be expressed as:

$$\phi_n(\zeta) = \frac{2\pi}{2d_n} \sum_{j=1}^{\ell_n-1} \text{rect}\left(\frac{\zeta - jd_n}{2d_n}\right) (\zeta - d_n(j-1)) \delta_{A,S_{n,j}} \delta_{B,S_{n,j+1}}. \quad (3)$$

Figure 2 shows the phase distribution of a KLZP of order $n = 7$ and its representation in a cartesian plane. The presence of constant-phase regions is a direct consequence of the sequence-based encoding rule and is consistent with the design strategy employed in other aperiodic kinoform diffractive lenses reported in the literature^{27,28}. For example, similar sequence-based kinoform implementations have been used in modified Fibonacci kinoform lenses for extended depth imaging applications³³.

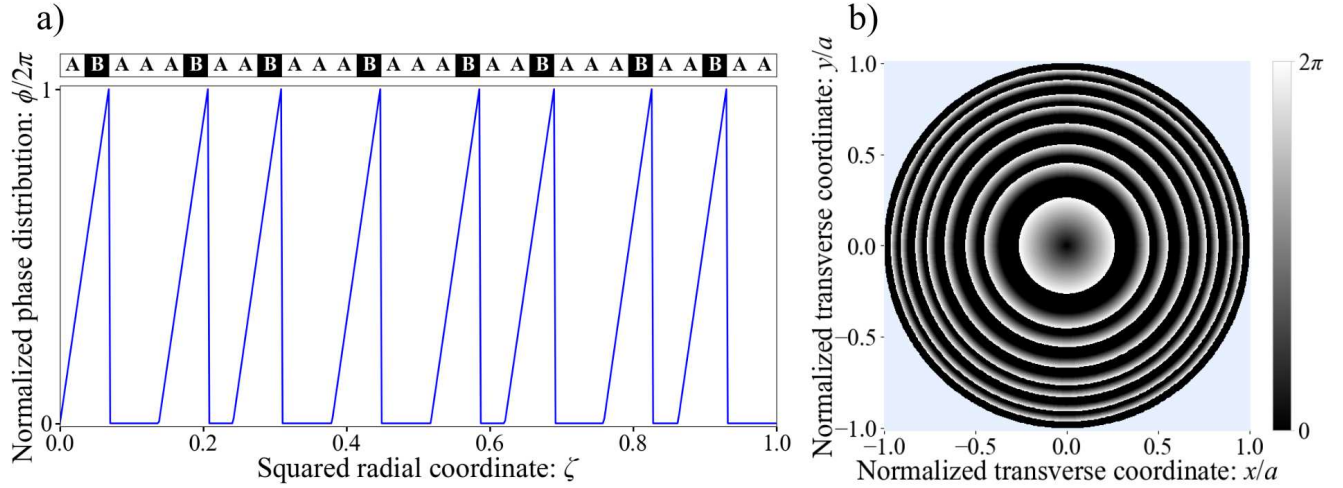


Figure 2. Normalized phase distribution of a KLZP with order $n = 7$: a) as a function of the coordinate ζ and b) in a cartesian coordinate plane.

Focusing Properties

To determine the focusing properties of LDLs, both binary-phase LZPs and KLZPs, the irradiance distribution of the lenses was computed according to the following expression³⁴:

$$I(u) = (2\pi u)^2 \left| \int_0^1 e^{i\phi_n(\zeta)} e^{-2i\pi u \zeta} d\zeta \right|^2, \quad (4)$$

where $u = \frac{a^2}{2\lambda z}$ is the reduced axial coordinate, λ is the wavelength of the employed light, and z is the axial distance at which the light beam propagates after passing through the lens.

Figure 3 shows the irradiance profiles of three LZPs of orders $n = 7$, $n = 8$, and $n = 9$, respectively. Four focal planes are observed, two outer ones with approximately equal intensities, and two inner ones with similar intensities but lower than those of the outer planes, reaching approximately 70% of the intensity of the outer foci. This suggests the existence of four focal planes that distribute the energy in a partially balanced manner. This multifocal behavior originates from the diffraction properties of the aperiodic zone distribution defined by the Lucas sequence. The corresponding phase structure introduces dominant diffraction components in the propagated field that produce constructive interference at specific axial positions, giving rise to the four main focal peaks.

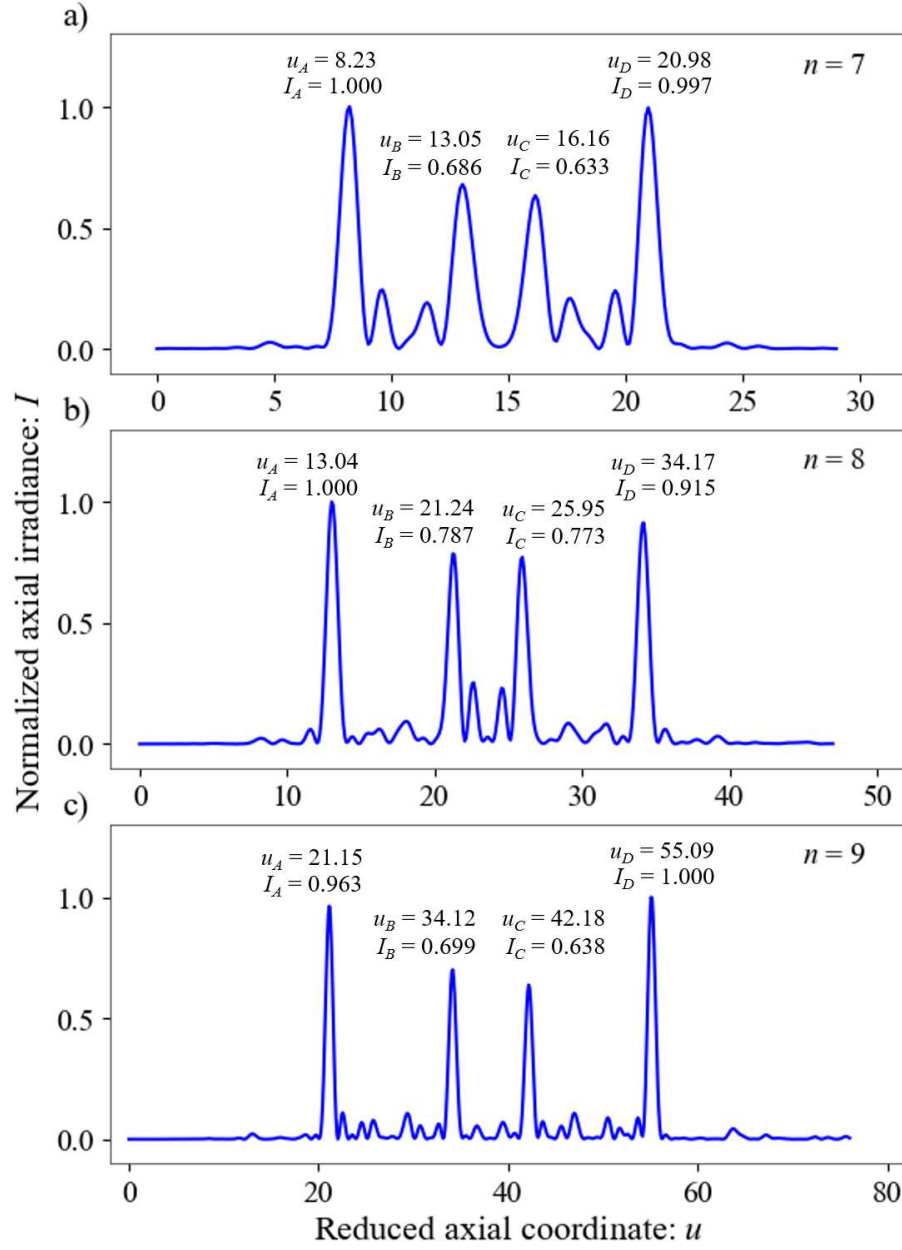


Figure 3. Normalized axial irradiance profiles of binary-phase Lucas Zone Plates (LZP) for orders $n = 7$, 8 , and 9 , exhibiting four well-defined focal peaks with a partially balanced irradiance distribution.

Table 1 lists the focal positions in the normalized coordinate u for three consecutive orders, namely $n = 7$, $n = 8$, and $n = 9$. An analysis of these data reveals that the focal positions u_{A_n} , u_{B_n} , and u_{D_n} converge toward the Fibonacci numbers of orders F_{n-1} , F_n , and F_{n+1} , respectively. In contrast, u_{C_n} is approximately equal to $2F_{n-1}$, that is, twice the value of u_{A_n} .

Table 1. Focal positions of the LZP for three consecutive orders and their relation to Fibonacci numbers.

Order n	Focus	Numerical value	Relation to F_n	Relative error (%)
7	u_A	8.23	$F_6 = 8$	2.88
	u_B	13.05	$F_7 = 13$	0.38
	u_C	16.16	$2F_6 = 16$	1.00
	u_D	20.98	$F_8 = 21$	0.10
8	u_A	13.04	$F_7 = 13$	0.31
	u_B	21.24	$F_8 = 21$	1.14
	u_C	25.95	$2F_7 = 26$	0.19
	u_D	34.17	$F_9 = 34$	0.50
9	u_A	21.15	$F_8 = 21$	0.71
	u_B	34.12	$F_9 = 34$	0.35
	u_C	42.18	$2F_8 = 42$	0.43
	u_D	55.09	$F_{10} = 55$	0.16

An additional relationship emerges when expressing the focal positions in terms of the Lucas sequence, namely

$$u_{A_n} = \frac{L_{n-2} + L_n}{5}, \quad u_{B_n} = \frac{L_{n-1} + L_{n+1}}{5}, \quad u_{C_n} = \frac{2(L_{n-2} + L_n)}{5}, \quad u_{D_n} = \frac{L_n + L_{n+2}}{5}.$$

Consequently, for a given order, the ratios between successive focal positions and the first focus can be approximated as

$$\frac{u_{B_n}}{u_{A_n}} \approx \varphi, \quad \frac{u_{C_n}}{u_{A_n}} \approx 2, \quad \frac{u_{D_n}}{u_{A_n}} \approx \varphi^2.$$

The analysis of the relative percentage error between the focal plane and its relation with Fibonacci numbers reveals consistent results. It is evident that, as the order increases, the error decreases significantly, with relative error values remaining below 1%, which supports the previously established relationships. For lower orders, the four focal peaks are broader and the agreement with the asymptotic Fibonacci based relations is less accurate, whereas for higher orders the peaks become narrower and better defined, and their axial positions approach more closely the predicted values.

The KLZP irradiance distribution for the same three consecutive orders ($n = 7$, $n = 8$, and $n = 9$) (see Figure 4) reveals focus properties similar to those of the LZP profile, exhibiting four focal planes located at the same focal positions as those corresponding to the binary-phase profile.

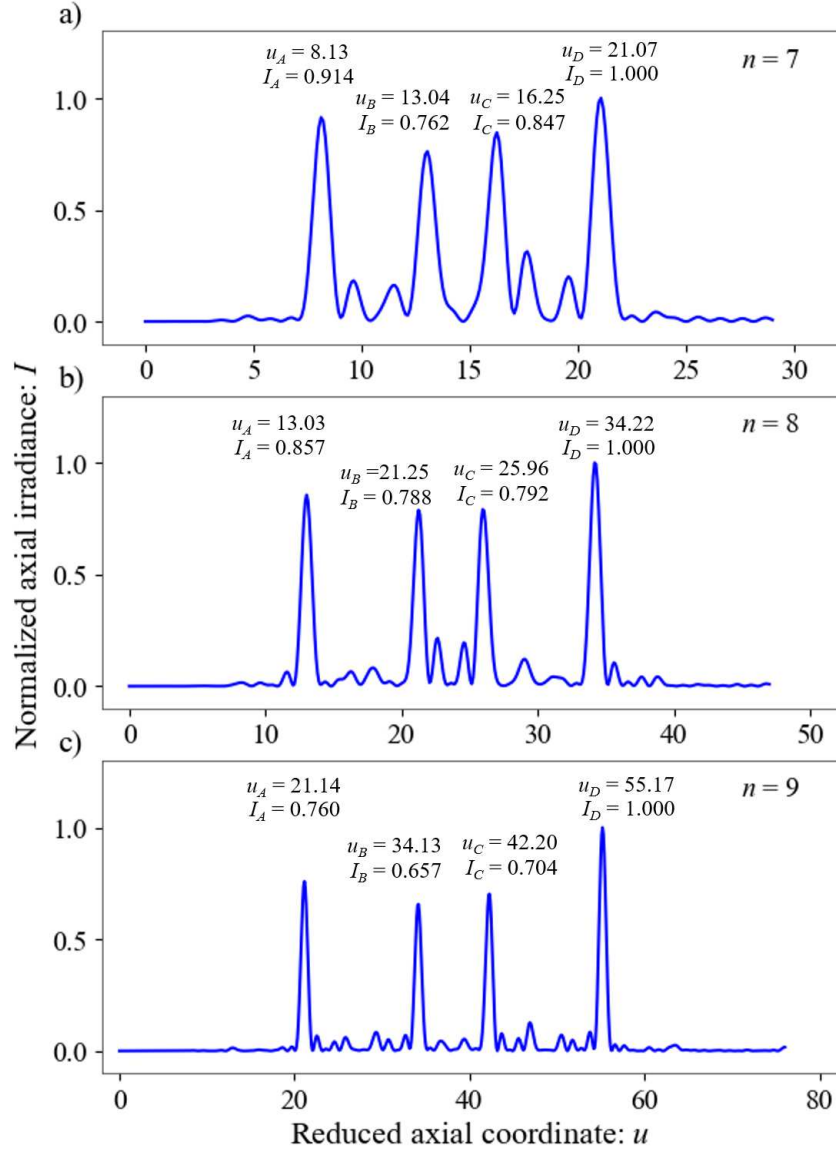


Figure 4. Normalized axial irradiance of an KLZP for three different orders: a) $n = 7$, b) $n = 8$, and c) $n = 9$.

A comparison between the irradiance distributions of the binary and kinoform phase profiles for a lens constructed from a sequence of order $n = 7$ reveals a significant energy improvement in the kinoform profile relative to the binary one. This is due to the fact that the kinoform profile apodizes the higher diffraction orders and concentrates most of the light into the first diffraction order (see Figure 5), whereas the binary profile distributes the energy more evenly among the various diffraction orders. A quantitative comparison of the four principal focal peaks shows that the normalized peak intensities obtained with the binary-phase design are approximately 51%, 42%, 36%, and 47% lower than those obtained with the kinoform profile for the first, second, third, and fourth focal planes, respectively, corresponding to an average reduction of about 44%. This result confirms the higher effective focusing efficiency achieved by the kinoform implementation.

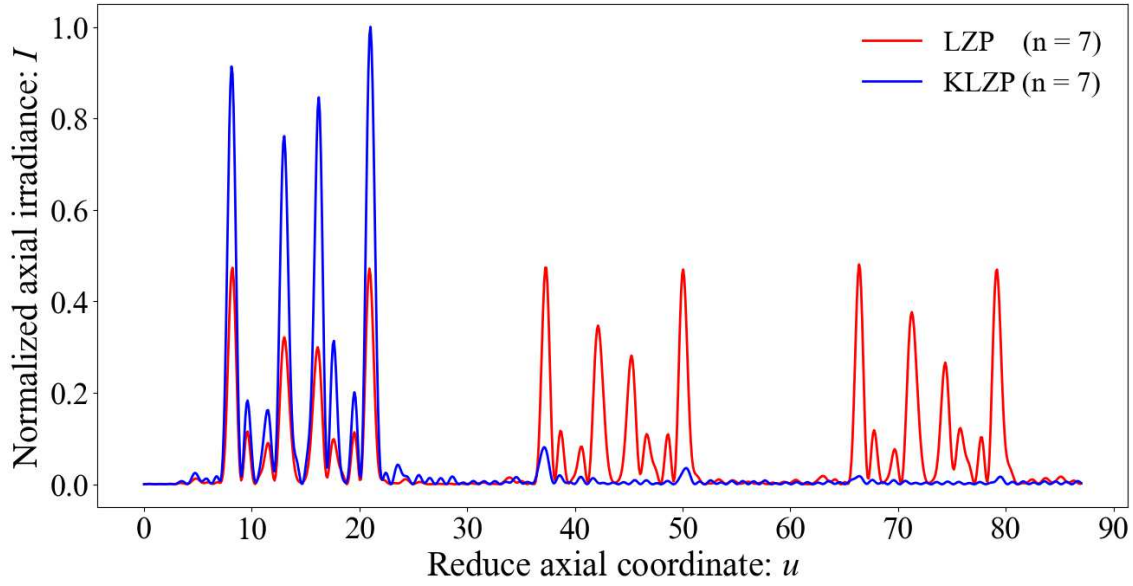


Figure 5. Comparison of normalized axial irradiance between a binary-phase LZP (red) and a kinoform KLZP (blue) for order $n = 7$.

Experimental results

To verify the focusing properties of the KLZP, the experimental setup shown in Figure 6 was used. In this arrangement, a He–Ne laser beam ($\lambda = 632.8\text{ nm}$, $P = 10\text{ mW}$), collimated by lens L_a (focal length $f_a = 160\text{ mm}$) and linearly polarized, is coupled into an optical fiber. The collimated beam is directed to a beam splitter (BS), which reflects part of the light toward a liquid crystal spatial light modulator (SLM) (Holoeye PLUTO, 1920×1080 pixels, pixel pitch of $8\text{ }\mu\text{m}$, 8-bit grayscale), operating in phase-only mode to digitally implement the KLZP. The wavefront modulated by the SLM passes again through the BS and is transmitted to a telescopic system composed of lenses L_b ($f_b = 200\text{ mm}$) and L_c ($f_c = 100\text{ mm}$), projecting the beam information generated by the lens onto the pupil plane.

The KLZP includes a one-dimensional diffraction grating that acts as a linear phase carrier, directing the diffracted light toward the first diffraction order in the Fourier plane of lens L_3 . To isolate this order, a diaphragm (PH) is added to the system, located at the focal plane of lens L_3 , acting as a spatial filter that transmits only the first diffracted order. To ensure precise alignment, the SLM is slightly tilted so that the linear phase carrier aligns the first diffraction order with the optical axis and the diaphragm aperture. This filtering stage not only isolates the desired signal but also reduces noise from the zero and higher diffraction orders generated by the pixelated structure of the SLM.

Finally, the irradiance profile produced by the KLZP lens along the optical axis is recorded using a camera sensor (8-bit grayscale, pixel pitch of $3.75\text{ }\mu\text{m}$, 1280×960 pixels) mounted on a motorized translation stage (Thorlabs LTS 300; travel range of 300 mm ; precision of $5\text{ }\mu\text{m}$), allowing the experimental determination of the focusing properties of the lens.

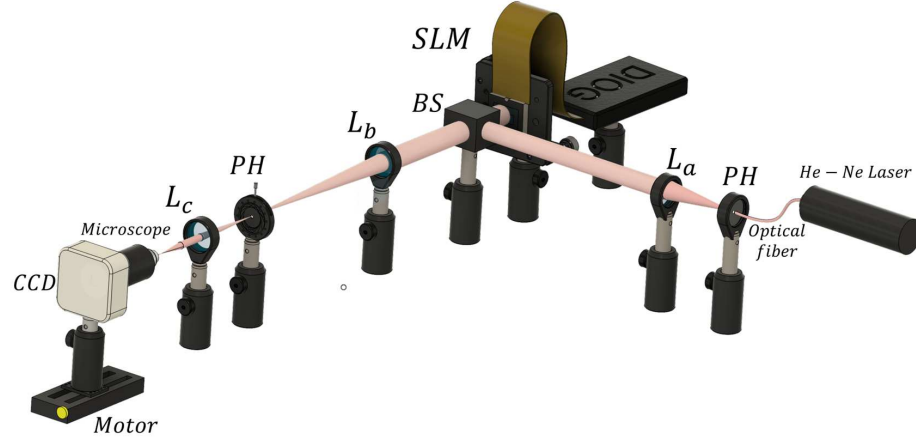


Figure 6. Experimental setup used to characterize the focusing properties of the KLZP, including He-Ne laser ($\lambda = 632.8$ nm), collimating lens L_a , beam splitter BS, phase-only SLM (Holoeye PLUTO), telescopic system (L_b , L_c), spatial filter PH, and motorized CCD camera.

To evaluate the focusing properties of the KLZP, a lens designed from a Lucas sequence of order $n = 7$ and radius $a = 1.5$ mm was employed. Figure 7 presents a comparison of the irradiance profiles in the yz plane obtained from numerical calculations (see Figure 7a) and from experimental irradiance measurements (see Figure 7b).

The experimental results confirm the generation of four focal planes by the KLZP lens. The experimentally determined focal positions, together with the numerically predicted ones, are summarized in Table 2. In all cases, the discrepancy between both approaches is below 1%, demonstrating strong agreement between theoretical predictions and experimental observations.

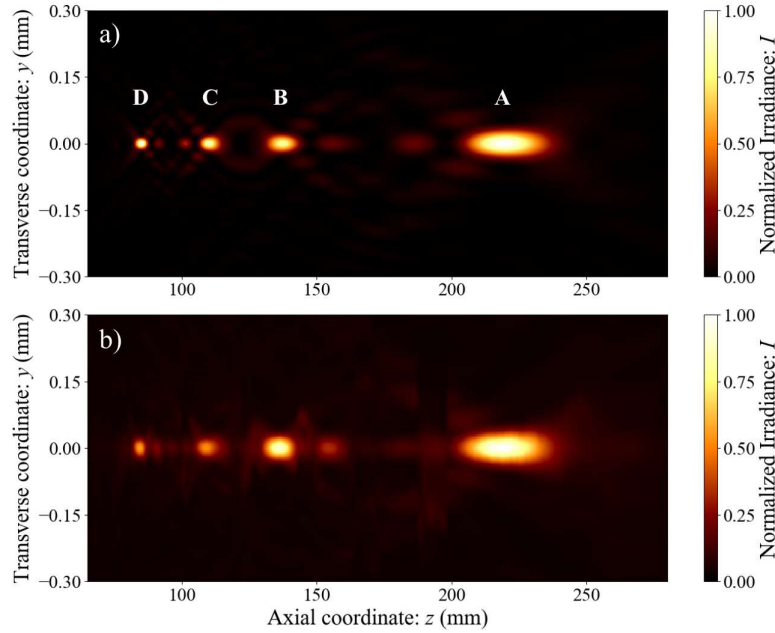


Figure 7. Comparison of the irradiance profiles in the yz plane for a KLZP lens of order $n = 7$ and radius $a = 1.5$ mm, obtained from numerical calculations (a) and experimental measurements (b).

Moreover, the ratios between the position of focus A and those of the remaining foci closely match the numerically predicted values, yielding $z_A/z_B = 1.61 \approx \phi$, $z_A/z_C = 2.01$, and $z_A/z_D = 2.59 \approx \phi^2$. These ratios highlight the intrinsic connection to the Lucas sequence, which underlies the KLZP lens design.

Table 2. Experimental and numerical focal positions of a KLZP lens of order $n = 7$ and radius $a = 1.5$ mm.

Focus	Experimental values (mm)	Numerical values (mm)	Discrepancy (%)
z_D	84.39	84.36	0.04
z_C	108.38	109.36	0.90
z_B	135.53	136.28	0.55
z_A	218.29	218.69	0.18

Discussion

The multifocal behavior observed in Lucas Diffractive Lenses (LDLs) originates directly from the deterministic aperiodic structure defined by the Lucas sequence. Unlike conventional Fresnel zone plates, where secondary foci arise from higher diffraction orders, the proposed lenses produce four dominant focal planes whose axial positions are intrinsically linked to the mathematical properties of the sequence. The convergence of focal locations toward Fibonacci numbers, together with the emergence of the golden ratio (ϕ) in the spacing between planes, establishes a clear and predictable correspondence between the generating sequence and the resulting optical response.

A particularly relevant finding is that a simple modification of the initial conditions of the recurrence relation ($L_0 = 2$, $L_1 = 1$), while retaining the same recursive rule as the Fibonacci sequence, leads to a qualitatively different focusing behavior. This change doubles the number of principal focal planes compared to previously reported Fibonacci-based diffractive lenses, which are typically bifocal. These results underscore the decisive role of the sequence seeds in controlling both the number and the relative distribution of foci along the optical axis. The comparison between binary-phase and kinoform implementations further clarifies the origin of the observed performance. Both designs produce identical focal positions, demonstrating that the quadrifocal response is determined primarily by the sequence encoding rather than by the specific phase profile. Nevertheless, the kinoform configuration significantly enhances energy efficiency by directing a larger fraction of the incident light into the desired foci and suppressing higher diffraction orders, achieving an average intensity increase of approximately 44% in the normalized peak intensities of the four principal focal planes relative to the binary-phase design. Experimental validation using a phase-only spatial light modulator confirmed the theoretical predictions, with discrepancies below 1% in all focal positions.

This excellent agreement validates the robustness of the Lucas-sequence approach as a deterministic framework for the design of multifocal diffractive elements. Current limitations include the moderate sequence orders tested ($n = 7-9$) and the monochromatic nature of the characterization. Higher orders are expected to yield sharper foci, while polychromatic performance and fabrication tolerances should be investigated in future studies, especially for ophthalmic applications.

Conclusion and Perspectives

This study introduces a new family of aperiodic diffractive lenses whose zone distribution is governed by the Lucas sequence, providing a deterministic approach to multifocal beam control. In the first diffraction order, these lenses generate four focal planes along the optical axis with a partially balanced energy distribution. Notably, the arrangement of the focal planes follows the golden ratio, corroborating the intrinsic connection between the Lucas and Fibonacci sequences^{29,32}. Moreover, we advance the understanding of the properties of this aperiodic sequence by showing that, although it is generated by the same linear recurrence relation as the Fibonacci sequence, differences in its initial generation conditions lead to zone plates that produce twice the number of foci compared to the Fibonacci case. Similarly, the intrinsic link between focal-plane positions and the golden ratio ensures robustness and scalability. In addition, the nature of the proposed design enables precise control over the axial position and relative spacing of the focal planes without relying on iterative optimization procedures. The kinoform implementation further enhances practical performance by increasing energy efficiency and suppressing undesired high diffraction orders. These distinctive features make Lucas-sequence-based diffractive lenses highly promising for advanced optical applications. Although the present work focuses on demonstrating the multifocal properties of Lucas-sequence-based diffractive lenses, further studies could explore the possibility of tailoring the relative energy distribution among the focal planes by modifying the sequence seeds, the encoding strategy, or the lens order, enabling additional flexibility in the design of multifocal optical elements.

When combined with a refractive base lens, the proposed diffractive structures could enable the development of novel multifocal intraocular lenses for cataract and presbyopia surgery. Compared to current bifocal and trifocal designs, such lenses

could provide an extended depth-of-focus concept in which the focal volume extension is achieved through the superposition of multiple foci under polychromatic illumination³⁵. Although increasing the number of focal planes may raise the risk of visual artifacts such as halos or reduced contrast, the partially balanced energy distribution demonstrated here constitutes a favorable starting point for optimized ophthalmic designs.

The multifocal character of these lenses is also attractive for artificial vision and robotic systems, where simultaneous imaging of different axial planes can be achieved without mechanical scanning. This capability enables fast and robust inspection of objects with features distributed across multiple depths, reducing system complexity and improving acquisition speed. In optical tomographic imaging, the simultaneous generation of multiple focal planes allows discrete axial sampling of a volume within a single exposure, reducing the need for axial scanning and minimizing motion-induced artifacts in dynamic samples. This approach is particularly relevant for biological imaging, microfluidics, and particle-based systems, where temporal stability is critical.

Additionally, the quadrifocal focusing capability can be exploited in optical trapping systems for the simultaneous confinement of multiple micro- or nanoparticles along the optical axis, enabling parallel manipulation in biophysical and soft-matter studies. Finally, due to the intrinsic advantages of diffractive optics at long wavelengths, the proposed design may be extended to the terahertz regime, where multifocal elements could support depth-resolved imaging and non-destructive testing applications.

Data availability

All data generated or analysed during this study are included in this published article.

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Author contributions statement

A.P., A.G., F.M., W.F., V.F., and J.M. conceived the experiments, A.P., A.G., F.M., W.F. and V.F. conducted the experiments, and A.P., F.M., V.F., and J.M. analyzed the results. All authors reviewed the manuscript.

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