Supplemental Information for "Universal RKKY-like Interactions Between Tilted Skyrmions in Chiral Magnets"

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SUPPLEMENTARY NOTE 1: SKYRMION-SKYRMION INTERACTION IN CHIRAL FILMS WITH TITLED SYMMETRIC MAGNETIC ANISOTROPY

In the absence of external field $H = 0$ and with a titled magnetic anisotropy of $E_{\rm an} = d\iint -[\frac{K}{2}(m_y \sin\theta + m_z \cos\theta)^2 + \frac{K_1}{2} m_z^2]{\rm dxdy}$, where $K_1 = 1.256\,{\rm MJ/m^3}$, $K = 0.365{\rm MJ/m^3}$, and the rest model parameters are the same as those used in the main text, we repeat what have been done in the main text. As mentioned in the main text, K_1 cancels the shape anisotropy such that K is the net symmetric magnetic anisotropy in the yz-plane with a titled angle of θ from the z-axis. These parameters correspond to $\kappa \equiv \pi^2 D^2/(16AK) = 0.9 < 1$ such that isolated skyrmions are metastable states [1, 2]. Different from H-induced asymmetric magnetic anisotropy where a spin prefers aligning along the field direction, this anisotropy equally prefers spins in both directions of the easy axis. We consider again the total energy $E(x, y)$ of two skyrmions centered on $(0, 0)$ and on (x, y), respectively, in the background of ferromagnetic state of $\mathbf{m} = (0, -\sin\theta, -\cos\theta)$. The spins at two skyrmion centers are fixed along $(0, \sin\theta, \cos\theta)$.

Figure S1(a) shows the total energy $E(x, y)$ for the case of $\theta = \pi/2$ (easy-axis along the y). The red, black, and blue curves are $E(x, y)$ for two skyrmions align along the y-axis, the x-axis, and along the line of $y = x$. Similar to the case in the main text, $E(x, y)$ monotonically decreases along the x-axis, or a repulsive interaction between two skyrmions in the direction. $E(x, y)$ is oscillatory along all other directions with a well-defined period $\lambda(x) = \lambda_0/\sin\phi$ where $\phi = \tan^{-1}(y/x)$, or demonstrating a RKKY-like interaction. Like that in the case of a titled field, $\lambda_0 \approx 136 - 44 \approx 90\,{\rm nm}$ is not sensitive to the value of K and depends only on D/A and θ.

We can also see that the oscillatory attraction-repulsion interaction relates to the periodic stripy structure of the in-plane skyrmion along the x-direction. Figure S1(b) shows how m_x (red) and m_z (blue) vary with y along lines of $x = 0$ (solid curves) and $x = 20$nm (dashed curves). One sees clearly an oscillatory behavior and the period change with the value of x according to nearly $\lambda_0/\sin\phi$ relation, as shown by almost linear line of $1/\lambda$ vs. $\sin\phi$ in the inset with ϕ defined above. This proves a nearly stripy structure of in-plane skyrmion along the x-direction, the same conclusion as those in the main text for H-induced in-plane skyrmion.

The hidden periodic stripy spin structure is also intrinsic for titled symmetric magnetic anisotropy and the period is not sensitive to value K as long as the film support isolated titled skyrmions. Again, Θ is defined as the angle between $\mathbf{m}$ and the easy-axis (defined as z'-axis with y'-axis in the yz-plane), and Φ the angle between the x'-axis (perpendicular to yz-plane) and the prjoection of $\mathbf{m}$ to the plane perpendicular to the easy-axis. Introducing $\kappa \equiv \pi^2 D^2/(16AK)$ and using the natural length unit $4A/\pi D$, the energy in function of Θ and Φ is:

$$E = \frac{\pi^2 D^2 d}{16A} \iint \Bigg\{ (\nabla\Theta)^2 + \sin^2\Theta\,(\nabla\Phi)^2 + \frac{4}{\pi}[\sin\Phi\partial_x\Theta + \sin\Theta\cos\Theta\cos\Phi\partial_x\Phi - \sin\theta\sin^2\Theta\partial_y\Phi + \cos\theta(\sin\Theta\cos\Theta\sin\Phi\partial_y\Phi - \cos\Phi\partial_y\Theta)] + \frac{1}{\kappa}\sin^2\Theta \Bigg\} d^2\mathbf{x} \tag{1}$$

The equations of Θ and Φ of a stable spin texture are,

$$2\nabla^2\Theta - \sin 2\Theta\,(\nabla\Phi)^2 + \frac{4}{\pi}[\sin\theta\sin 2\Theta\partial_y\Phi + 2\sin^2\Theta(\cos\Phi\partial_x\Phi + \cos\theta\sin\Phi\partial_y\Phi)] - \frac{2}{\kappa}\sin\Theta\cos\Theta = 0 \tag{2}$$

$$\sin 2\Theta\,(\nabla\Theta)\cdot(\nabla\Phi) + \sin^2\Theta\nabla^2\Phi - \frac{2}{\pi}[\sin\theta\sin 2\Theta\partial_y\Theta + 2\sin^2\Theta(\cos\Phi\partial_x\Theta + \cos\theta\sin\Phi\partial_y\Theta)] = 0, \tag{3}$$

For an in-plane skyrmion ($\theta = \pi/2$) centred at $(0, 0)$ with boundary conditions of $\Theta(0, 0) = 0$, $\Theta(|x| \to \infty, or|y| \to \infty) = \pi$ and far from the skyrmion center, where $\Theta' \equiv \pi - \Theta(x, y) \ll 1$, we can simplify Eqs. (2) and (3) by keeping only the linear terms of Θ' which become

$$\nabla^2\Theta' - \Theta'\left[(\nabla\Phi)^2 - \frac{4}{\pi}\partial_y\Phi + \frac{1}{\kappa}\right] = 0, \tag{4}$$

$$\nabla\Theta' \cdot \nabla\Phi - \frac{2}{\pi}\partial_y\Theta' + \frac{\Theta'}{2}\nabla^2\Phi = 0. \tag{5}$$

The same as H-controlled anisotropic case, Eqs. (4) and (5) admit following asymptotic solution,

$$\Theta(x, y) = \pi - \pi e^{-k_x|x| - k_y|y|},\ \Phi(y) = \frac{2}{\pi}y + \Phi_0, \tag{6}$$

with $k_x^2 + k_y^2 = 1/\kappa - 4/\pi^2$, and Φ_0 is a constant, exactly the same as that in the main text.

We can also show that skyrmion-skyrmion attraction comes from the match of the periodic stripy structures of two in-plane skyrmions in their overlapped region while skyrmion-skyrmion repulsion is due to mismatch of structures of them. Same as what was done in the main text, we examine Φ of two independent skyrmions: one centred at $(0, 0)$ and the other at (x_0, y_0). Figure S1(e) shows $\Phi(l)$ (the red for the first skyrmion and the black for the second skyrmion) in the overlapped region for $(x_0, y_0) = (0, 136\text{nm})$, $(-50\text{nm}, 136\text{nm})$, and $(70\text{nm}, 136\text{nm})$ when two akyrmions attract each other, where $l \equiv |\sqrt{x_0^2 + y_0^2}(|y|/|y_0| - 1/2)$ measures distance from the middle point between the two skyrmion along the line of $y = (y_0/x_0)x$. Indeed, $\Phi(l)$ for two skyrmions are almost identical in the overlapped region. They are also the same as the $\Phi(l)$ of the bi-skyrmion molecule.

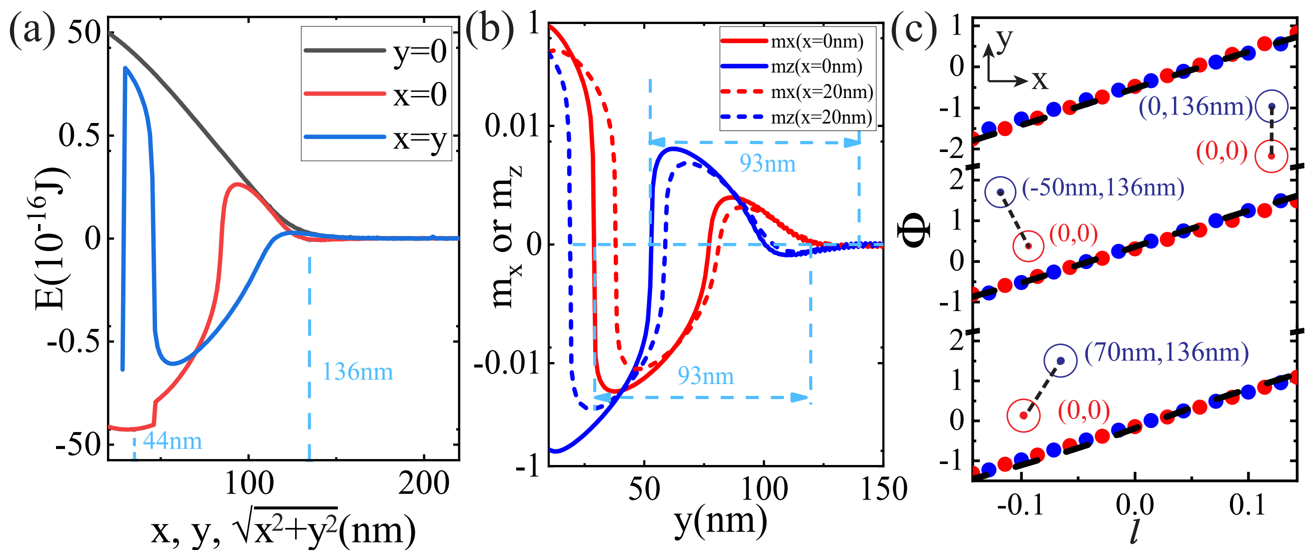

FIG. S1. (a) $E(x, y)$ as a function of y along $x = 0$ (black line), or x along $y = 0$ (red line), or $\sqrt{x^2 + y^2}$ along $y = x$ (blue line). The vertical blue dash-lines indicate local minimum. (b) m_x (red) and m_z (blue) as a function of y for $x = 0$nm (solid curve) and 20 nm (dashed lines). (c) $\Phi(l)$ of skyrmion centred at $(0, 0)$ (the black lines) and (x_0, y_0) (the red lines), and $\Phi(l)$ of the two skyrmions in their fully relaxed state (the blue dotted lines) for $(x_0, y_0) = (0, 136\text{nm})$, $(-50\text{nm}, 136\text{nm})$, and $(70\text{nm}, 136\text{nm})$

In summary, the RKKY-like skyrmion-skyrmion interaction is universal for titled skyrmions and comes from their anisotropic periodic stripe spin structures. These universal features does not depend on the origins of titled skyrmions, no matter whether it is due to a titled magnetic field which is asymmetric or due to the crystalline magnetic anisotropy in which both direction along the easy-axis are equivalent.

SUPPLEMENTARY NOTE 2: THE ACCURACY OF NUMERICAL SIMULATIONS

To ascertain teh accuracy of our numerical results, we simulate the same model in two different sample sizes of 600 nm × 600 nm × 0.5 nm and 1200 nm × 1200 nm × 0.5 nm with two mesh sizes of 1 nm × 1 nm × 0.5 nm and 0.5 nm × 0.5 nm × 0.5 nm. Fig. S2(a) shows the numerical $E(x = 0, y)$ (interaction energy between two skyrmions centered at $(0, 0)$ and $(0, y)$, respectively) as a function of y for sample sizes of 600 nm × 600 nm × 0.5 nm (black line) and 1200 nm × 1200 nm × 0.5 nm (red dashed line) and mesh size of 1 nm × 1 nm × 0.5 nm; as well as for sample sizes of 600 nm × 600 nm × 1 nm and mesh size of 0.5 nm × 0.5 nm × 0.5 nm (blue short dashed line). The perfect overlap of three sets of data demonstrates that the boundary effect is neglisgible for our sample size and mesh size is smaller enough to accurate numerical results. The green dots in the figure denote the energies minimums of skyrmion-skyrmion separations of the biskyrmion molecule.

Figure S2(b) shows the trajectories of two isolated in-plane skyrmions, initially one at $(0,0)$ and the other at $(0,-40\,\mathrm{nm})$ (red), $(120\,\mathrm{nm},0)$ (green), and $(40\,\mathrm{nm},-160\,\mathrm{nm})$ (blue), respectively, in a sample of $600\,\mathrm{nm}\times600\,\mathrm{nm}\times0.5\,\mathrm{nm}$. The trajectories show how two skyrmions bound together at various separations (66.5 nm and 156.5 nm), depending on the initial locations. The results of two skyrmion dynamics agree with the energy curves in Fig. S2(a).

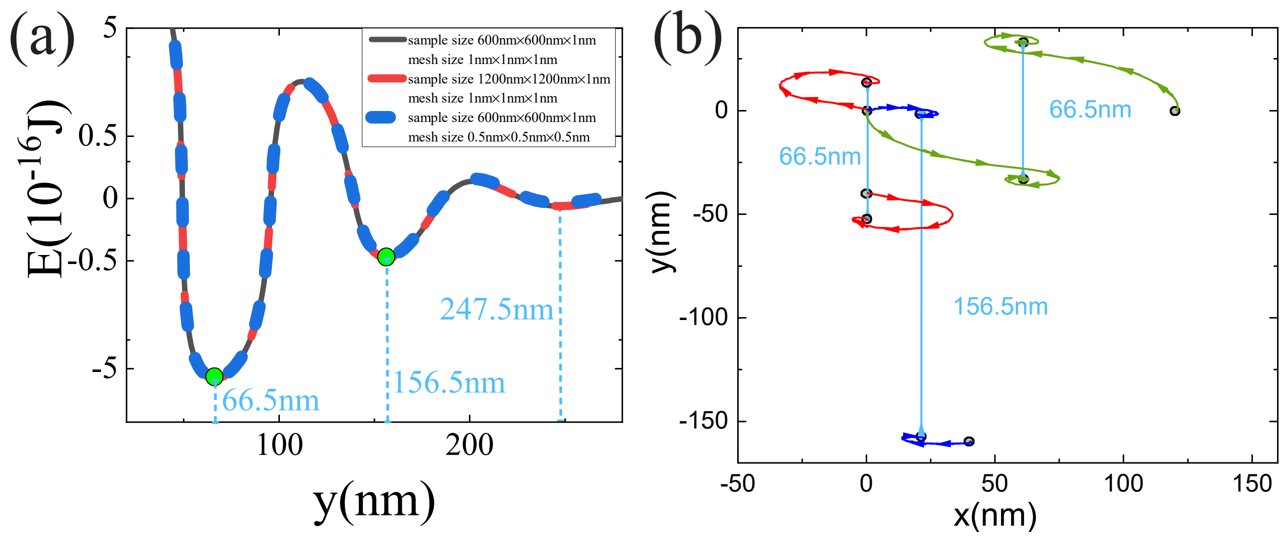

FIG. S2. (a) $E(x=0,y)$ vs. y for sample of $600\,\mathrm{nm}\times600\,\mathrm{nm}\times1\,\mathrm{nm}$ and meshsize of $1\,\mathrm{nm}\times1\,\mathrm{nm}\times1\,\mathrm{nm}$ (black line); for sample of $1200\,\mathrm{nm}\times1200\,\mathrm{nm}\times1\,\mathrm{nm}$ and meshsize of $1\,\mathrm{nm}\times1\,\mathrm{nm}\times1\,\mathrm{nm}$ (red dash line); and for sample of $600\,\mathrm{nm}\times600\,\mathrm{nm}\times1\,\mathrm{nm}$ and meshsize of $0.5\,\mathrm{nm}\times0.5\,\mathrm{nm}\times1\,\mathrm{nm}$ (blue short dashed line). (b) The trajectories of two isolated skyrmions. one is initially centered at $(0,0)$ and the other $(0,-40\,nm)$ (red lines), $(120\,nm,0)$ (green lines) and $(40\,nm,-160\,nm)$ (blue lines).

SUPPLEMENTARY NOTE 3: DERIVATION OF EQUATIONS AND SOLUTION FOR THE WAVY TAIL

Without losing generality, one can assume the easy axis lying in the yz-plane and the angle between easy-axis (parallel with the external field) and the z-axis is θ_H. The magnetization is described by Θ defined as the angle between $\mathbf{m}$ and the easy-axis (defined as z'-axis with y'-axis in the yz-plane) and Φ defined as the angle between the x'-axis (perpendicular to yz-plane) and the prjoection of $\mathbf{m}$ to the plane perpendicular to the easy-axis. The magnetization is then $\mathbf{m}=(\sin\Theta\sin\Phi,\sin\theta_H\cos\Theta+\cos\theta_H\sin\Theta\cos\Phi,\cos\theta_H\cos\Theta-\sin\theta_H\sin\Theta\cos\Phi)$. In terms of $\kappa'\equiv\pi^2D^2/(8\mu_0M_sAH)$ and using the natural length of $4A/\pi D$, the total magnetic energy of the film is

$$E=\frac{\pi^2D^2d}{16A}\iint\Big\{(\nabla\Theta)^2+\sin^2\Theta\,(\nabla\Phi)^2+\frac{4}{\pi}[\sin\Phi\partial_x\Theta+\sin\Theta\cos\Theta\cos\Phi\partial_x\Phi-\sin\theta\sin^2\Theta\partial_y\Phi \\ +\cos\theta(\sin\Theta\cos\Theta\sin\Phi\partial_y\Phi-\cos\Phi\partial_y\Theta)]+\frac{2}{\kappa'}(\cos\Theta+1)\Big\}d^2\mathbf{x}, \tag{7}$$

where d is the thickness of the sample. The equations for a (meta)stable spin texture is

$$2\nabla^2\Theta-\sin2\Theta\,(\nabla\Phi)^2+\frac{4}{\pi}[\sin\theta_H\sin2\Theta\partial_y\Phi+2\sin^2\Theta(\cos\Phi\partial_x\Phi+\cos\theta_H\sin\Phi\partial_y\Phi)]+\frac{2}{\kappa'}\sin\Theta=0 \tag{8}$$

$$\sin2\Theta\,(\nabla\Theta)\cdot(\nabla\Phi)+\sin^2\Theta\nabla^2\Phi-\frac{2}{\pi}[\sin\theta_H\sin2\Theta\partial_y\Theta+2\sin^2\Theta(\cos\Phi\partial_x\Theta+\cos\theta_H\sin\Phi\partial_y\Theta)]=0, \tag{9}$$

The boundary conditions for a in-plane skyrmion of $\theta_h=\pi/2$ are $\Theta(0)=0$ and $\Theta(\infty)=\pi$. Far from the skyrmion center, $\Theta'\equiv\pi-\Theta\ll1$. Eqs. (8) and (9), linear in Θ', become

$$\nabla^2\Theta'-\Theta'\left[(\nabla\Phi)^2-\frac{4}{\pi}\partial_y\Phi+\frac{1}{\kappa'}\right]=0, \tag{10}$$

$$\partial_x \Theta' \partial_x \Phi + \partial_y \Theta' \left(\partial_y \Phi - \frac{2}{\pi} \right) + \frac{\Theta'}{2} \nabla^2 \Phi = 0. \tag{11}$$

A particular solution of Eq. (11) is $\Phi(y) = \frac{2}{\pi} y$. Substituting this into Eq. (10), one has the Helmholtz equation, $\nabla^2 \Theta' - \Theta' \left(\frac{1}{\kappa'} - \frac{4}{\pi^2} \right) = 0$. One possible solution is $\Theta' = \exp(k_x x + k_y y + C)$, where k_x, k_y and C are constant when $1/\kappa' - 4/\pi^2 > 0$.

SUPPLEMENTARY NOTE 4: BOUNDARY BETWEEN THE RKKY-LIKE PHASE AND NON-ISOLATED SKYRMION PHASE

In the main text, we have shown that stripy structures of Eq. (6), which gives rise to the RKKY-like interaction, is valid only for $\kappa' > \pi^2/4$ in an in-plane field. To derive the boundary between the RKKY-like phase and non-existence of isolated skyrmion phase in $\kappa' - \theta_H$ parameter space, we go back to Eqs. (2) and (3) in the main text. Far from the skyrmion center, $\Theta' \equiv \pi - \Theta \ll 1$, and we have

$$\nabla^2 \Theta' - \Theta' \left[(\nabla \Phi)^2 - \frac{4}{\pi} \sin \theta_H \partial_y \Phi + \frac{1}{\kappa'} \right] = 0, \tag{12}$$

$$\nabla \Theta' \cdot \nabla \Phi - \frac{2}{\pi} \sin \theta_H \partial_y \Theta' + \frac{\Theta'}{2} \nabla^2 \Phi = 0. \tag{13}$$

Periodic stripy tail solution of an in-plane skyrmions is

$$\Theta'(x, y) = \pi - \pi e^{-k_x |x| - k_y |y|}, \ \Phi(y) = \frac{2}{\pi} \sin \theta_H y + \Phi_0, \tag{14}$$

where $k_x^2 + k_y^2 = 1/\kappa' - 4 \sin^2 \theta_H / \pi^2$, and Φ_0 is a constant. When $\kappa' < \pi^2/(4 \sin^2 \theta_H)$, $k_x^2 + k_y^2 > 0$ the skyrmion is metastable. When the inequality relation does not hold i.e. $\kappa' > \pi^2/(4 \sin^2 \theta_H)$, isolated skyrmions does not exist which was confirmed by Mumax3 simulations. However, a conical-like state with $\Theta = const$ exists as shown below. Equations (2) and (3) in the main text for $\Theta = const$ become,

$$\sin \Theta \left[\cos \Theta \, (\nabla \Phi)^2 - \frac{4}{\pi} \sin \theta_H \cos \Theta \partial_y \Phi + \frac{1}{\kappa'} \right] = 0, \tag{15}$$

$$\sin^2 \Theta \nabla^2 \Phi = 0. \tag{16}$$

The equations admit following conical solution,

$$\Theta = \arccos \left(\frac{\pi^2}{4 \kappa' \sin^2 \theta_H} \right), \ \Phi(y) = \frac{2}{\pi} \sin \theta_H y + \Phi_0, \tag{17}$$

The energy density of this state is: $-4 \sin^2 \theta_H [1 - \pi^2/(4\kappa' \sin^2 \theta_H)]^2 / \pi^2 < 0$. Thus, the necessary condition for a stable isolated skyrmion is $\kappa' < \pi^2/4 \sin^2 \theta_H > 4$ which should also satisfy $\kappa' < 4$ [2]. In summary, $\kappa' = \pi^2/(4 \sin^2 \theta_H)$ is the boundary between RKKY-like phase and non-existence isolated skyrmion phase. As shown in the main text, this analytic boundary agrees with Mumax3 simulations well.

SUPPLEMENTARY NOTE 5: RKKY-LIKE INTERACTION FOR INTERFACIAL DMI

There are two types of DMI interaction: bulk DMI and the interfacial one. Naturally, one may question whether the oscillatory skyrmion interaction and the periodic stripy skyrmion structure still exist if we replace the bulk DMI by an interfacial DMI. Here we show that the general features do not depend on the type of DMI. However, the attractive direction is now perpendicular to skyrmion centre spin direction, instead of parallel. Simultaneously, the stripy tail structures of an isolated skyrmion also rotate a 90° degree with respected to the bulk DMI case. The occurrences of the simultaneous changes in the anisotropic skyrmion interaction and stripy skyrmion tail structures further support our assertion of the origin of the RKKY-like interaction.

Below, we repeat the simulations presented in the main text for field-induced anisotropy by replacing the bulk DMI with interfacial DMI, $E_{DMI} = \iint m_z (\nabla \cdot \mathbf{m}) - (\mathbf{m} \cdot \nabla) m_z \mathrm{d}^2\mathbf{x}$ with the same DMI constant of $D = 2\,\mathrm{mJ/m^3}$. All other parameters remain the same as those for Figs. 1-3 in the main text. For $\mathbf{H} = 0.164\mathrm{T}$ parallel to the $-y$-direction and skyrmion centre spin along the y-direction, Figure S3(a) is the density plot of $m_z > 0$ (red) and $m_z < 0$ (blue) to reveal stripy structures along the x-direction, perpendicular to the skyrmion centre spin direction. The black dots denotes the skyrmion center. Figure S3(b) plots distributions of m_x and m_z along various direction. The oscillatory distributions reveal the periodic stripy structure of the skyrmion. Figure S3(c) shows oscillatory $E(x, y)$ with minima on the x-axis, instead of y-axis, at $x_{min} = 66.5, 156.5, 247.5\,\mathrm{nm}$. The separation between two neighbouring minimum points is $\lambda \approx 156.5 - 66.5 \approx 247.5 - 156.5 \approx 90 \sim 91\,\mathrm{nm}$, and not sensitive to H. Interestingly, this period is the same as that in the main text, implying bulk and interfacial DMIs with the same D-values are the same on the RKKY-like interaction, except the direction of attractive and repulsive forces. Thus, two such isolated skyrmions shall attract each other along the x-axis. $E(x, y)$ decreases monotonically with y (red line), indicating a repulsive force between two skyrmions along the y-axis. In between, the interaction along other radium, $E(x, y)$, thus interaction, oscillates with a well defined period as explained in the main text. As shown in the inset of Fig. S3(a), the periodic stripes are parallel to centre spin direction (the y-direction). Thus, two titled skyrmions attract each other when they align along the x-axis (perpendicular to the centre spin direction), and repel each other when they align along y-axis.

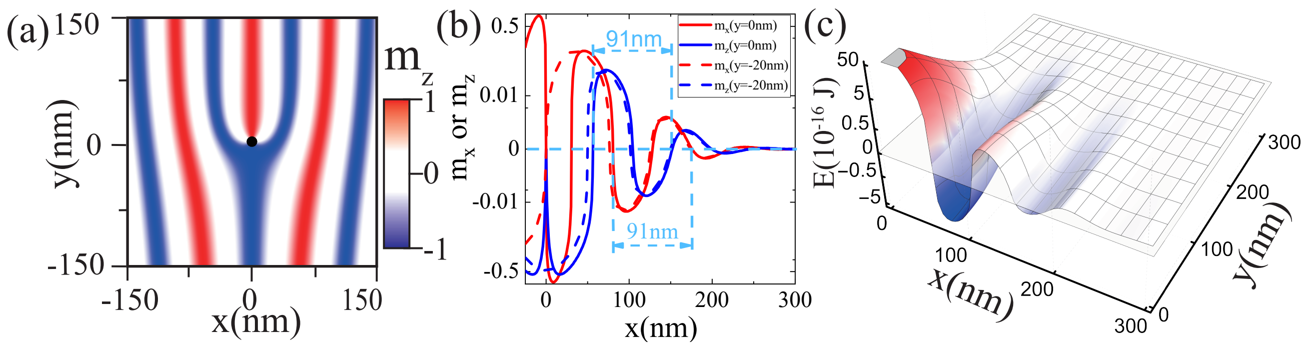

FIG. S3. (a) Density plots of an in-plane skyrmion with interfacial DMI: $m_z > 0$ (red) and $m_z < 0$ (blue). The skyrmion center is denoted by the black dots. (b) m_x (red) and m_z (blue) as a function of x for $y = 0\,\mathrm{nm}$ (solid curve) and $20\,\mathrm{nm}$ (dashed lines). (c) $E(x, y)$.

The change of attractive direction with the change of type of DMI interaction can be understand from the skyrmion equation, for interfacial DMI and $\mathbf{H}$ lies in $yz-$plane and has an angle of θ_H with the $z-$axis.

$$E = \frac{\pi^2 D^2 d}{16A} \iint (\nabla\Theta)^2 + \sin^2\Theta\,(\nabla\Phi)^2 + \frac{4}{\pi}[\sin\theta_H \sin^2\Theta\partial_x\Phi + \cos\theta_H\,(\cos\Phi\partial_x\Theta - \sin\Theta\cos\Theta\sin\Phi\partial_x\Phi) + \sin\Phi\partial_y\Theta + \sin\Theta\cos\Theta\cos\Phi\partial_y\Phi] + \frac{2}{\kappa'}(\cos\Theta + 1)d^2\mathbf{x} \tag{18}$$

Θ and Φ of a stable spin texture satisfy,

$$-2\nabla^2\Theta + \sin 2\Theta\,(\nabla\Phi)^2 + \frac{4}{\pi}[\sin\theta_H \sin 2\Theta\partial_x\Phi + 2\sin^2\Theta(\cos\theta_H \sin\Phi\partial_x\Phi - \cos\Phi\partial_y\Phi)] - \frac{2}{\kappa'}\sin\Theta = 0 \tag{19}$$

$$\sin 2\Theta\,(\nabla\Theta)\cdot(\nabla\Phi) + \sin^2\Theta\nabla^2\Phi + \frac{2}{\pi}[\sin\theta_H \sin 2\Theta\partial_x\Theta + 2\sin^2\Theta(\cos\theta_H \sin\Phi\partial_x\Theta - \cos\Phi\partial_y\Theta)] = 0. \tag{20}$$

For an in-plane skyrmion of $\theta_H = \pi/2$ are $\Theta(0) = 0$ and $\Theta(\infty) = \pi$ and far from the skyrmion center, $\Theta' \equiv \pi - \Theta \ll 1$. Then, Eqs. (19) and (20), linear in Θ', become

$$\nabla^2\Theta' - \Theta'\left[(\nabla\Phi)^2 + \frac{4}{\pi}\partial_x\Phi + \frac{1}{\kappa'}\right] = 0, \tag{21}$$

$$\partial_x\Theta'\left(\partial_x\Phi + \frac{2}{\pi}\right) + \partial_y\Theta'\partial_y\Phi + \frac{\Theta'}{2}\nabla^2\Phi = 0. \tag{22}$$

Periodic stripy tail solution of an in-plane skyrmions is

$$\Theta(x,y) = \pi - \pi e^{-k_x|x|-k_y|y|},\ \Phi(x) = -\frac{2}{\pi}x + \Phi_0,, \tag{23}$$

where $k_x^2 + k_y^2 = 1/\kappa' - 4/\pi^2$, and Φ_0 is a constant. Different from the bulk DMI where Φ is linear in y, Φ is linear in x, showing the stripes parallel to the y and leading to an attraction along the x-axis. Similar to the bulk DMI case, k_x^{-1} and k_y^{-1} depends on κ, period λ of the periodic stripy structure depends only on the length scale $4A/\pi D$ which is $\lambda = 2\pi\left(\frac{2}{\pi}\right)^{-1} \times \frac{4A}{\pi D} \approx 93\,\text{nm}$.

SUPPLEMENTARY NOTE 6: SPIN STRUCTURE OF BI-SKYRMION

Figure 2c in the main text is the spin structure of an in-plane isolated skyrmion. Here we plot spin tructure of a bi-skyrmion when the two skyrmions attract each other. As shown in Fig. S4(a), the spin structure of two in-plane skyrmions, one at the sample center $(0,0)$ and the other at $(0, 66.5\,\text{nm})$, the first potential minimum discussed in the main text, the in-plane skyrmions are anisotropic as shown by both the distribution of in-plane spin component (arrows) and the distribution of $m_z > 0.5$ (red) and $m_z < -0.5$ (blue). This anisotropic strucutre leads to a non-central force between two skyrmion as discussed in the main text. Figure S4(b) reveal periodically arranged stripes when we plot the map of $m_z > 0$ (red) and $m_z < 0$ (blue). Of course, the same conclusion can be made if one plots $m_x > 0$ and $m_x < 0$. Notably, two skyrmions (denoted by two dots) share the same blue region between them, in supporting our assertion that the attractive interaction originates from the matching of the wavy tails.

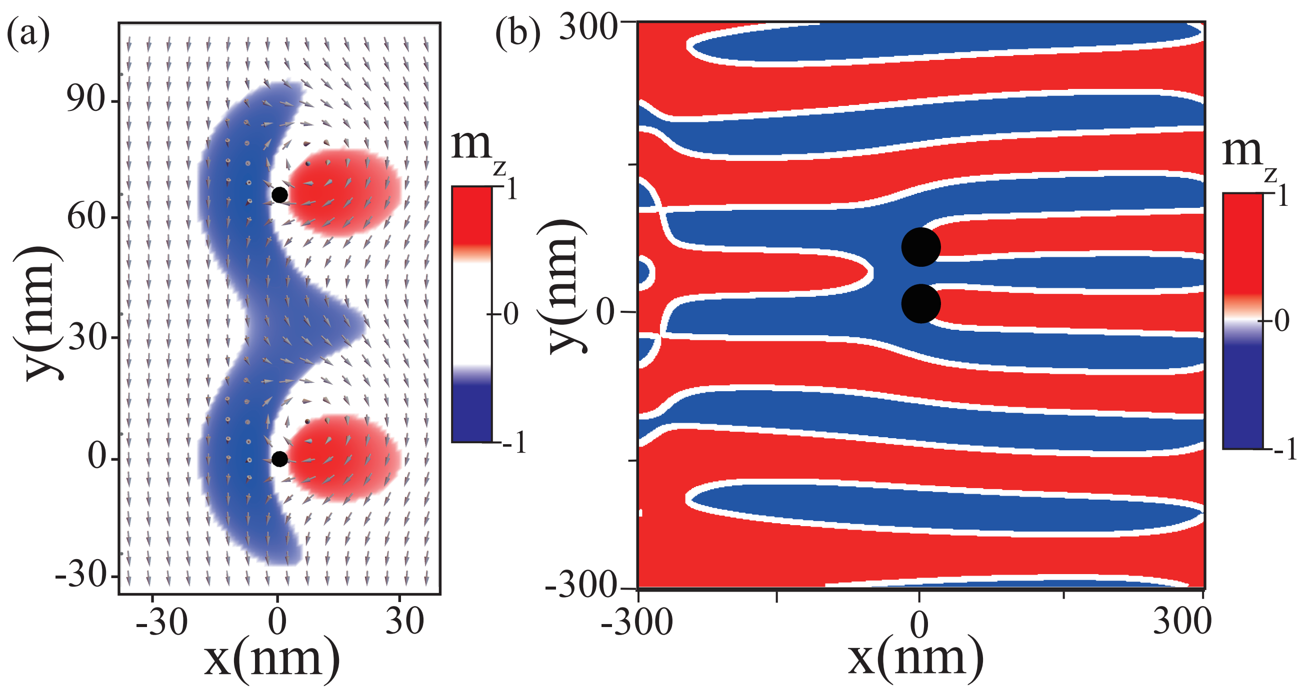

FIG. S4. Spin structure of two in-plane skyrmions, with one skyrmion located at the sample center while the other located at the first minimum of $(0, 66.5\,\text{nm})$. The spins at the centers (denoted by the black circles) point to $\hat{y}$ direction. (a) The arrows denote projections of $\mathbf{m}$ in the xy-plane. The m_z distribution is encoded by color: red for $m_z > 0.5$ and blue for $m_z < -0.5$. (b) Map of $m_z > 0$ (red) and $m_z < 0$ (blue). The periodically arranged red and blue stripes reveal the corrugated spin strucutre for the two in-plane skyrmions. Two skyrmion share the same blue region in between.

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