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# Seismic analytical reliability analysis of triangular rock wedge slope stability

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# Seismic analytical reliability analysis of triangular rock wedge slope stability


## Abstract

The reliability analysis of rock slope stability serves as an effective method for assessing prevalent uncertainties in geotechnical and geometrical parameters. This study investigates an analytical solution known as Jointly Distributed Random Variables (JDRV) for the reliability analysis of rock slope stability under planar sliding conditions, accounting for uncertainties across all stochastic variables involved in the analysis, including rock parameters, seismicity, and geometry. In this context, the stochastic parameters selected include shear strength, unit weight, and geometric parameters, which are modeled using truncated normal distributions. Conversely, the horizontal seismic coefficient is represented using an exponentially truncated distribution. The statistical results from the JDRV method reveal that it occupies a position statistically similar to that of the First Order Second Moment (FOSM) method, while also demonstrating superior performance compared to the Monte Carlo Simulation (MCS) method. Notably, the JDRV method offers significant reductions in computational time and costs associated with determining safety factor probability density functions (PDFs) in comparison to MCS. Furthermore, this study emphasizes the importance of considering all variables as stochastic parameters. The findings indicate that when all variables are treated as stochastic, a lower reliability index or a higher probability of failure is observed. Additionally, the sensitivity analysis reveals that the friction angle of sliding surfaces is the most influential parameter in rock slope stability analysis.



**Keywords**: Analytical reliability analysis, Jointly distributed random variables, Monte Carlo simulation, Rock wedge slope stability, Seismic stability analysis


## 1. Introduction

In mountainous areas, earthquakes are one of the most common causes of slope instability. Therefore, rock slope stability must be assessed under seismic conditions, which has prompted researchers to develop several approaches to estimating its stability. It has been observed that rock slope stability is stochastic rather than deterministic due to uncertainty regarding many parameters affecting its stability. This renders the reliability analysis of rock slopes an effective method for quantifying the uncertainties associated with slope engineering. In other words, the stability of a rock slope can be assessed based on the distribution of the controlling parameters.

Regarding rock slope stability, two main stochastic approaches are often developed. Two common approaches account for uncertainties associated with joint/fracture geometrical properties and uncertainty associated with rock material characteristics. Due to uncertainty regarding fracture geometry within the rock mass, stochastic fracture networks have been developed, which are well documented and widely discussed [1-4]. A second approach frequently employed in the analysis of rock slope stability involves estimating the probability of slope failure while incorporating uncertain parameters into the stability assessment [5, 6].

Stochastic analyses of the slope were performed by six major methods: MCS [7], Response Surface Method (RSM) [8], Random Finite Element Method (RFEM) [9], Geostatistics [10], approximate methods [11-13], and analytical methods [14]. Analytical methods employ mathematical expressions to represent the PDFs of input variables. In this study, the JDRV method allows for the derivation of a mathematical expression for the safety factor density function by analytically integrating the PDFs within a slope stability analysis model. [5, 15].

In quantitative analysis, the MCS method involves randomly generating points to represent uncertainties associated with probability computation. It can also be applied to solving problems

that cannot be addressed analytically, such as slope stability problems on rock slopes. Tamimi et al. [16] developed a correlation-based method for assessing the reliability of rock slopes through the use of analytical and simulation techniques. Park et al. [17] implemented the probabilistic method to analyze rock slope stability in a practical setting. Jimenez-Rodriguez et al. [18] presented an approach based on a general systems methodology for analyzing rock slope stability. By integrating discrete rigid finite element theory, plastic limit analysis, stochastic programming, and MCS, Li et al. [19] formulated a new method to assess rock slopes with multiple failure modes. However, the randomness of seismic loading is not considered. Johari and Lari [20] analyzed the stability of rock wedges using a system reliability approach and taking into account related failure modes. Tiwari and Latha [21] proposed a method for evaluating the reliability of rock slopes, taking into account the uncertainty in peak parameters and residual resistance. Their approach employs a quantitative risk assessment system based on geological strength index values.

As a substitute for the MCS method, the RSM method employs an approximate model in place of the numerical model, enabling the system to be estimated and the uncertainty propagated to be analyzed. Li et al. [22] introduced a stochastic RSM for reliability assessment that utilizes non-normal variables associated with rock slope stability concerning planar failures.

Within the framework of RFEM, elasto-plastic finite element analysis is integrated with random fields produced using the local average subdivision method. Significant efforts to implement this approach have been made by researchers such as Griffiths et al. [23], Gravanis et al. [24], and Liu et al. [25]. Griffiths et al. [23] investigated the probability of slope failure by employing finite element methods combined with the first-order reliability method, without considering spatial variations, as well as utilizing the more sophisticated probabilistic analysis tools available in RFEM. Gravanis et al. [24] proposed an analytical method for calculating the failure probability of rock slopes subjected to planar sliding, taking into account the influence of spatial variability on slope stability. Liu et al. [25] utilized RFEM to assess a three-dimensional slope, incorporating spatially random soil variability. To ensure that the two-dimensional analysis results, derived from the most conservative section, remain adequately conservative, a new methodology was introduced for identifying the most critical section.

Based on a spatially correlated random variation model, geostatistics is an estimation technique. As a result of geostatistical modeling and block analysis, Young [26] developed a method for estimating the local probability of failure in open pit slopes. Mayer and Stead [27] demonstrated that conventional probabilistic design principles cannot adequately simulate spatial heterogeneity using statistical upscaling techniques. In geostatistical methods, spatial dependencies are assessed through variograms, while probabilistic approaches tend to overlook these inherent correlations. To assess the impact of correlation among shear strength parameters on the reliability of rock slopes, Aladejare and Wang [28] developed a Bayesian approach to describe rock properties along with a reliability-based design approach for the probabilistic analysis of rock slope stability. Meanwhile, Liu et al. [29] employed the geostatistical method to represent the spatial variability of the mechanical parameters of rock masses.

Approximate methods for calculating the safety factor generally necessitate the variance, mean, and performance function of the input variables. Utilizing the widely used spreadsheet platform, Low [30] conducted a reliability analysis and developed a reliability-based design for a two-dimensional joint rock slope employing the first-order reliability method. Additionally, Park et al. [31] derived an equation for wedge failure through the point estimation method.

The most important methods for assessing the stability of rock slopes during earthquakes include the Newmark sliding block analysis [32], the pseudo-static method [33, 34], the Makdisi-

Seed analysis [34, 35], and the Nonlinear analysis approach [34]. Due to simplicity and utility, the pseudo-static method has been commonly employed in the seismic stability analysis of rock slopes.

According to Johari et al. [36], the JDRV method is applicable for assessing the reliability of pseudo-static stability in rock slopes experiencing planar sliding. Zhao et al. [37] conducted a dependable analysis of rock slopes using MCS, applying the nonlinear Barton-Bandis failure criteria along with a modified pseudo-dynamic method. Zhao et al. [38] studied the nonlinear Barton-Bandis failure criterion in the context of a three-dimensional wedge system subjected to pseudo-static earthquake forces. Meanwhile, Zhang et al. [39] utilized the pseudo-static approach to model seismic loads, employing the simplified Bishop method to assess the stability factors of rock slopes containing weak layers.

In spite of several different types of research on stochastic rock slope stability, utilizing various methods and taking into account the uncertainty in parameters, it has not been possible to simultaneously investigate the uncertainties in geometrical, seismic, and rock parameters, particularly using an analytical approach. By applying the JDRV method in this study, computational costs, time, and accuracy of reliability evaluations of the stability problems of wedges with plane sliding are reduced significantly, while by combining fundamental relations in the JDRV method with Jacobian equations, the PDF equation of the rock slope safety factor is determined using fundamental relations in the JDRV method.

To evaluate the accuracy and efficiency of the proposed method, the MCS and FOSM results have been compared. Comparisons have been made based on the reliability indices of these approaches. Two different approaches have been used to conduct sensitivity analysis on rock slope stability using the proposed method. A sensitivity analysis based on the JDRV method was applied to the analysis of rock slope stability as a function of parameter changes. The first approach involves increasing the coefficient of variation (COV) of six stochastic input parameters with the assistance of the standard deviations and means of the parameters. (e.g., new COV = old COV + 0.1×COV). As for the second approach, the mean friction angle, as well as parameters related to geometry, such as the triangular wedge coordinates (y), are taken into account as variables. Alternatively, other input parameters are maintained at a constant value.

## 2. Rock slope triangular wedge with plane sliding

As part of the stochastic stability analysis in the present study, the triangular wedge with plane sliding of the rock slope is considered. Fig. 1 illustrates a typical figure of this rock slope with its parameters. In this figure, N is the normal force applied at the base of the slice, and T is the rock shear strength.

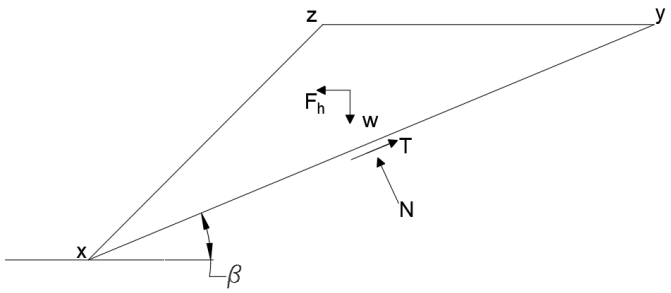



Fig. 1. Triangular wedge with planar failure surface in pseudo-static slope analysis

For this slope, the equation for determining the safety factor based on the limit equilibrium method can be expressed as follows [34]:

$$FS = \frac{cL_{ab} + [W\cos(\beta) - F_h \sin(\beta)]\tan(\varphi)}{W\sin(\beta) + F_h \cos(\beta)} \quad (1)$$

where:

$$W = \gamma A \quad (2)$$

$$F_h = k_h W \quad (3)$$

FS: Factor of safety against sliding
A: Wedge area
W: Weight of the rock wedge resting on the failure surface
$F_h$: Horizontal earthquake inertial forces
$k_h$: Dimensionless horizontal pseudo-static coefficients
c: Cohesive strength along the sliding surface
φ: Friction angle of the sliding surface
β: Angle of the failure surface, measured from horizontal
γ: Unit weight of the rock

This study considers the triangular wedge coordinate [x] to be the origin coordinate from a geometric standpoint, whereas c, γ, φ, and $k_h$ are assumed to be stochastic. For this purpose, all the mentioned parameters are randomly generated. Furthermore, the controlling geometry parameters, like coordinates of the triangular wedge ([y], [z]), are randomly generated to compute the effective parameters within the equation. The angle of the failure surface (β) and also the length of the failure plane ($L_{ab}$) can be calculated by the coordinates of the triangular slope, while Heron's formula can be used to calculate the area (A) from the coordinates of the triangular slope. According to this statement, the area of a triangle whose sides have lengths a, b, and c is [41]:

$$s = \frac{a+b+c}{2} \quad (4)$$

$$A = \sqrt{s(s-a)(s-b)(s-c)} \quad (5)$$

where:
A: Area of wedge
s: Semi-perimeter
a: Length of side xy
b: Length of side yz
c: Length of side xz

## 3. Jointly distribution random variables method

JDRV is a stochastic analysis method that offers several advantages over MCS, such as:

- Stability of Results: In JDRV, the mean and standard deviation of the results remain constant regardless of the number of simulations performed. As a result of this stability, results are consistent and reliable, making it easier to interpret and compare them.
- Efficiency: In contrast to MCS, where calculations must be repeated for each simulation, JDRV requires fewer iterations, allowing it to save computational time and resources.

- Convergence: The convergence rate of JDRV is typically faster than that of MCS. As a result, fewer simulations are required to reach convergence with JDRV, reducing the overall analysis time.
- Accuracy: JDRV typically delivers more precise estimates of the mean and standard deviation than MCS, thanks to its stability and convergence characteristics. This enhanced accuracy is especially advantageous in addressing complex or high-dimensional issues.

Overall, the JDRV method offers improved stability, efficiency, convergence, and accuracy compared to Monte Carlo Simulation, making it a valuable analytical tool in various fields of research and analysis.

In this approach, the probability density functions of the input variables and their statistical interrelationships are formulated mathematically. Theorems 1-4 provide statistical and probabilistic relationships between parameters [6, 14, 42, 43].

**Theorem 1**

$Z_1, Z_2, \ldots, Z_n$ are right continuous random variables with marginal $f_{Z1}(z_1)$, $f_{Z2}(z_2)$, …, $f_{Zn}(z_n)$ densities. The joint PDF distribution of $Z_1, Z_2, \ldots, Z_n$ are independent if and only if all $Z_1, Z_2, \ldots, Z_n$, $f_{Z1, Z2, \ldots, Zn}(Z_1, Z_2, \ldots, Z_n)$ are defined as follows:

$$f_{Z1,Z2,\ldots,Zn}(z_1,z_2,\ldots,z_n)=f_{z_1}(z_1)f_{z_2}(z_2)\ldots f_{z_n}(z_n) \tag{6}$$

**Theorem 2**

If $Z_1, Z_2, \ldots, Z_n$ are continuous random variables in which the joint probability density are specified by $f_{Z1, Z2, \ldots, Zn}(Z_1, Z_2, \ldots, Z_n)$ and let $T_1=g_1(Z_1, Z_2, \ldots, Z_n)$, $T_2=g_2(Z_1, Z_2, \ldots, Z_n)$,…, $T_n=g_n(Z_1, Z_2, \ldots, Z_n)$ define a one-to-one transformation between $(Z_1, Z_2, \ldots, Z_n)$ and $(t_1, t_2, \ldots, t_n)$. Let $t_1=g_1(Z_1, Z_2, \ldots, Z_n)$, …, $t_n=g_n(Z_1, Z_2, \ldots, Z_n)$ can only be solved for $(Z_1, Z_2, \ldots, Z_n)$ in terms of $(t_1, t_2, \ldots, t_n)$ as $Z_1=h_1(t_1, t_2, \ldots, t_n)$, $Z_2=h_2(t_1, t_2, \ldots, t_n)$,…, $Z_n=h_n(t_1, t_2, \ldots, t_n)$. Then the joint probability of $t_1, t_2, \ldots, t_n$ is:

$$f_{T_1,T_2,\ldots,T_n}(t_1,t_2,\ldots,t_n)=\left|J(t_1,t_2,\ldots,t_n)\right|f_{Z_1,Z_2,\ldots,Z_n}(h_1(t_1,t_2,\ldots,t_n),\ldots,h_n(t_1,t_2,\ldots,t_n)) \tag{7}$$

$$J(t_1,t_2,\ldots,t_n)=\begin{bmatrix} \frac{\delta z_1}{\delta t_1} & \frac{\delta z_1}{\delta t_2} & \cdots & \frac{\delta z_1}{\delta t_n} \\ \frac{\delta z_2}{\delta t_1} & \frac{\delta z_2}{\delta t_2} & \cdots & \frac{\delta z_2}{\delta t_n} \\ \vdots & \vdots & \ddots & \vdots \\ \frac{\delta z_n}{\delta t_1} & \frac{\delta z_n}{\delta t_2} & \cdots & \frac{\delta z_n}{\delta t_n} \end{bmatrix} \tag{8}$$

where:

In general. $|J(t_1, t_2, \ldots, t_n)|$represents the absolute of $J(t_1, t_2, \ldots, t_n)$

**Theorem 3**

Suppose the joint PDF of continuous random variables $Z_1, Z_2, \ldots, Z_n$ is $f_{Z1, Z2, \ldots, Zn}(Z_1, Z_2, \ldots, Z_n)$, then the marginal PDF of $Z_i$ will be as follows:

$$f_{Z_i}(Z_i) = \iint_{R_{z_i}} \dots \int f_{z_1,z_2,\dots,z_n}(z_1, z_2, \dots, z_n)\, dz_1 dz_2 \dots\, dz_{i-1} dz_{i+1} \dots\, dz_n \tag{9}$$

**Theorem 4**

In the case of Z, a random variable characterized by its probability density function of $f_z(z)$, and T, a function of z with the form of T = g(z), T's probability density can be derived as follows:

$$f_T(t) = f_z(g^{-1}(t)) \times \left| \frac{dg^{-1}(t)}{dt} \right| \tag{10}$$

Generally, this relationship is applicable to functions g(z) that are monotonically increasing or decreasing.

## 4. First order second moment method

FOSM is an approximate technique that employs the Taylor series expansion of the function under consideration. The expansion is truncated at the linear expression (first order). By utilizing this modified expansion along with the first two moments of the random variables, the first two moments of the dependent variable (specifically the second moment) can be derived. This method focuses solely on the random variables and does not take into account the shape of the probability density function (PDF). The mean and standard deviation of a function are computed using the means and standard deviations of the input variables, as illustrated in the following equation. [44]:

$$E_{Function} \cong \sum_{j=1}^{n} g(E(x_j)) \tag{11}$$

$$\sigma^2_{Function} \cong \sum_{j=1}^{n} \left(\frac{\partial g}{\partial x_j}\right)^2 \sigma^2(x_j) \tag{12}$$

where:

$x_j$: Variables to be input

$g(x_j)$: The Function of the input variables, $x_j$

n: Numbers of variables

$E_{Function}$: Mean

$\sigma^2_{Function}$: Variance

## 5. Assessment of the reliability of rock slope stability based on plane sliding

A safety factor relationship was examined in this study using the equation Eq. (13), while the probability density function was independently derived using Eqs. (6) through (10). Appendix A contains the methods used to derive these equations.

$$FS = \frac{k_1 k_5 + [k_2 k_6 \cos(k_7) - k_4 k_2 k_6 \sin(k_7)] k_3}{k_2 k_6 \sin(k_7) + k_4 k_2 k_6 \cos(k_7)} \tag{13}$$

where:

$$\begin{cases} k_1 = c \\ k_2 = \gamma \\ k_3 = \tan(\varphi\pi / 180) \\ k_4 = k_h \\ k_5 = L_{ab} \\ k_6 = A \\ k_7 = \beta \end{cases} \quad (14)$$

To determine the safety factor of triangular wedges in rock slopes experiencing plane sliding, a MATLAB program was created to compute the PDF and Cumulative Density Function (CDF) based on the derived mathematical functions $k_1$ to $k_7$, $f_{K1}(k_1)$ to $f_{K7}$ ($k_7$), and Eqs. (A.1) to (A.15). While Eq. (A.14) lacks analytical solutions, it must be addressed numerically. Furthermore, the safety factor for rock slopes can also be calculated using MCS along with Equations (B.1) to (B.9) for comparison purposes. As the input data is a combination of normal and exponential distributions, the results of (B.1) to (B.9) were coded in the same computer program. The difference between JDRV and MCS is that skewness is taken into account in the equation obtained as a result of the inherent analytical analysis. However, in MCS, the appropriate function must be fitted to the output of the problem. Therefore, the MCS results should be analyzed in terms of an epsilon-skew-normal distribution [45].

## 5.1. Stochastic parameters

Six input parameters are defined as random variables to calculate the uncertainty in rock slope stability. The selected parameters are the controlling geometrical parameters such as triangular wedge coordinates (y, z), horizontal pseudo-static coefficients ($k_h$), apparent cohesion (c), sliding surface friction angle (φ), and rock unit weight (γ). The friction angle, apparent cohesion, unit weight, and triangular wedge coordinates mentioned earlier are modeled using a normal truncated distribution. In addition, the horizontal pseudo-static coefficient is modeled with an exponential truncated distribution. An important point to note is that cohesion and friction angle are negatively correlated [46]. However, for mathematical convenience, they are considered uncorrelated variables. The described stochastic processes are listed in Tables 1 and 2.

Table 1. Norm-truncated stochastic parameters

| Parameters | Mean | Standard deviation | COV |
|---|---|---|---|
| c(kPa) | 10.0 | 2.0 | 0.200 |
| φ(Deg.) | 35.0 | 5.0 | 0.143 |
| γ(kN/m³) | 26.0 | 1.0 | 0.038 |
| y(m) | 45.0 | 2.0 | 0.045 |
| z(m) | 15.0 | 2.0 | 0.134 |

Table 2. Truncated exponential parameters for stochastic processes

| Parameters | λ | Mean=1/λ | Standard deviation | Minimum | Maximum |
|---|---|---|---|---|---|
| $k_h$ | 10.00 | 0.10 | 0.10 | 0.01 | 0.16 |

## 6. Illustrative example

To illustrate the effectiveness and precision of the proposed method, an illustrative example based on plane sliding as the output variable is presented. For verification of the results of the present method with the MCS, the final PDF for the safety factor is calculated utilizing the same data for both methods. For this purpose, $10^8$ simulations are conducted for MCS. A summary of the results is presented in Figs. 2 to 7.

### 6.1. Comparing the safety factor stability distribution by the JDRV and MCS

This sub-section compares the PDF and CDF of safety factor distribution obtained by the JDRV and MCS in three approaches: (1) rock slope parameters, (2) rock slope parameters and seismic effect, and (3) rock slope, geometric parameters, and seismic effect (all stochastic variables).

Based on the approach that considers rock slope parameters, Figs. 2 and 3 display the PDF and CDF of the safety factor, respectively. Fig. 2 indicates that the results of the proposed method are very close to MCS. Although the results show a low difference in the peak values. However, Fig. 3 illustrates a perfect match between the CDF of the proposed method and MCS. Fig. 4 and 5 demonstrate PDF and CDF of the safety factor of rock slope stability with plane sliding considering the rock slope parameters and seismic effect, respectively. Fig. 4 shows the relevant results of the proposed method according to MCS and also indicates that the mean result of the JDRV method is higher than MCS. However, Fig. 5 indicates a perfect match between the proposed method and MCS, like the approach to rock slope parameters. Figs. 6 and 7 illustrate the PDF and CDF of the stability rock slope safety factor with triangular plane sliding surface based on the rock slope, geometric parameters, and seismic effect (all stochastic variables), respectively. Fig. 6 shows the relevant results of the JDRV method concerning MCS. Although the results show a slight difference in the peak values, and indicate that the mean result of the JDRV method is higher than MCS. Fig. 7 demonstrates an ideal match between the proposed method and MCS. However, it shows a slight difference in high CDF values. The evaluation of the safety factor stability distribution using the proposed method reveals an excellent agreement with the results from MCS, demonstrating that the proposed method possesses both commendable accuracy and efficiency.

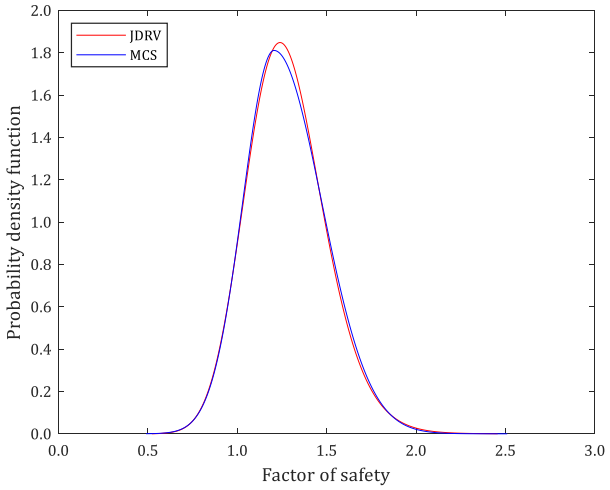



Fig. 2. The PDF of the safety factor accounts for the uncertainties associated with the parameters of the rock slope

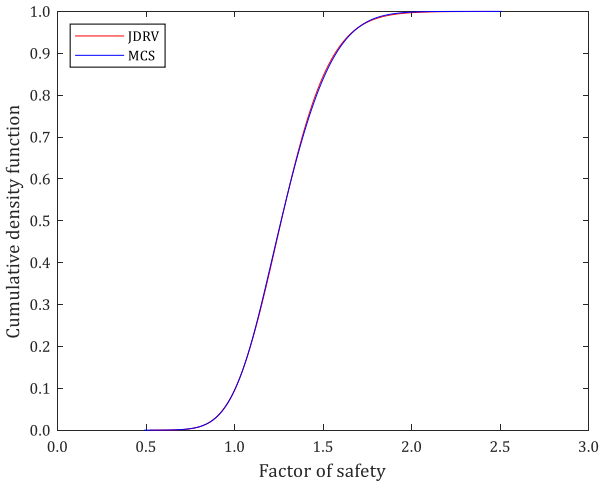



Fig. 3. The CDF of the safety factor considering uncertainties of rock slope parameters

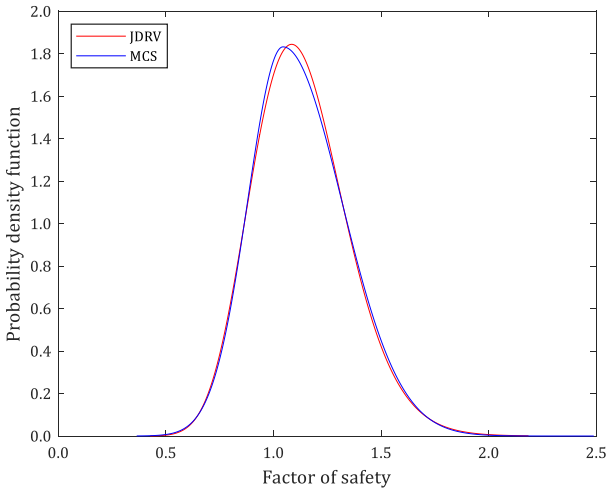



Fig. 4. The PDF of the safety factor takes into account the uncertainties in the rock slope parameters as well as the seismic effects

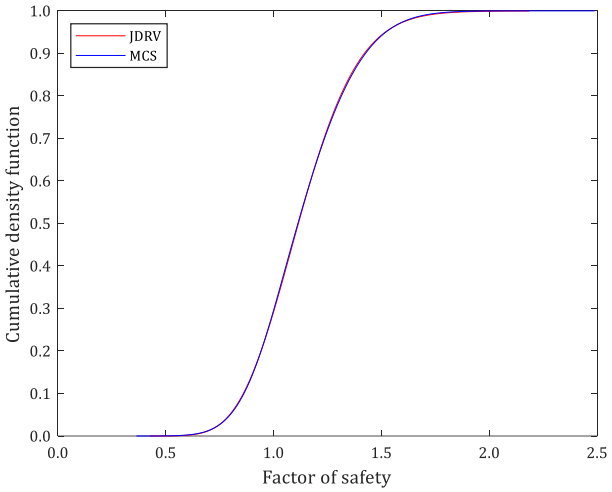



Fig. 5. The CDF of the safety factor considering the uncertainties of rock slope parameters and seismic effect

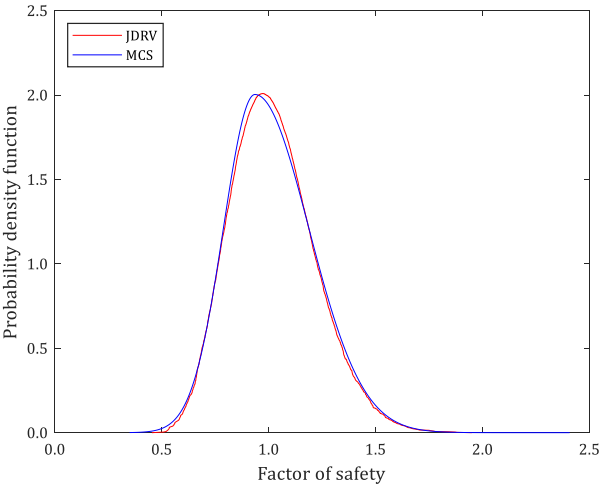



Fig. 6. The PDF of the safety factor is derived from the uncertainties associated with rock slope parameters, geometric factors, and seismic effects

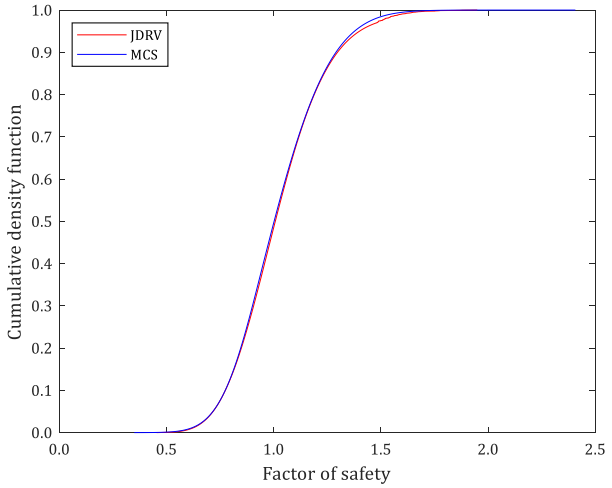



Fig. 7. The CDF of the safety factor based on the uncertainties of rock slope, geometric parameters, and seismic effect

## 6.2. Determination of the safety factor stability probabilistic specification by FOSM

The derivation of mathematical functions required for the FOSM method is provided in Appendix B. Table 3 summarizes a step-wise procedure of the FOSM method for evaluating the safety factor of rock slope stability considering all stochastic parameters, including rock slope, geometric parameters, and seismic effect, based on the Eqs. (11) and (12). The first column of this table lists the desired parameters, while the second and third columns present the corresponding mean values and standard deviations of the parameters as detailed in Tables 1 and 2. Columns 4 and 5 are derived based on the mentioned FOSM Equations and the obtained relations in Appendix B. The mean and variance values, accounting for uncertainties in the rock slope, geometric parameters, and seismic effect analysis, are 1.0010 and 0.0298, respectively.

Table 3. Results from the FOSM method for the slope safety factor

| Parameters ($x_i$) | Mean ($\mu_{xi}$) | Standard deviation ($\sigma_{x_i}$) | $\partial FS/\partial x_i\big\|_{\mu_{x_i}}$ | $(\partial FS/\partial x_i)^2 \times \sigma_{x_i}^2$ |
|---|---|---|---|---|
| c(kPa) | 10.00 | 2.0000 | 0.0076 | 2.3363e-4 |
| $\gamma$(kN/m$^3$) | 26.00 | 1.0000 | -0.0029 | 8.6343e-6 |
| tan($\varphi$)(Rad.) | 0.01 | 0.0015 | 1.3205 | 0.0133 |
| $k_h$ | 0.10 | 0.1000 | -2.0192 | 0.0145 |
| $L_{ab}$(m) | 54.09 | 1.6600 | 0.0014 | 5.5151e-6 |
| A(m$^2$) | 450.00 | 42.4000 | -1.6986e-04 | 5.1870e-5 |
| $\beta$(Rad.) | 0.59 | 0.0200 | -2.0219 | 0.0017 |
| | Mean of FS = 1.0010 | | | Sum = Variance of FS = 0.0298 |

## 7. Sensitivity analysis

To assess the impact of parameter changes on rock slope stability, a sensitivity analysis using the JDRV method was performed. This analysis considers the uncertainties associated with the rock slope, geometric parameters, and seismic effects, all of which are treated as stochastic variables. Two distinct approaches are presented for the analysis.

In the first approach, the COV of six stochastic input parameters is increased based on the standard deviation and mean values (e.g., the new COV = old COV + 0.1×COV). The influence of changes in each parameter is examined by varying one parameter while keeping the other stochastic input parameters within their defined ranges.

In Fig. 8, the variations in the factor of safety distributions are illustrated. The findings of this study suggest that the internal friction angle of the sliding surface, along with geometric parameters that govern rock slope stability—specifically the coordinates of the triangular wedge (y)—exert the most significant influence on the stability of rock slopes. According to Fig. 9, the internal friction angle of the sliding surface, as well as the controlling geometrical parameters (the triangular wedge coordinates (y)), predicted a lower probability of failure than other factors. Also, Table 4 summarizes the effect of controlling parameters considering an increased COV of each parameter to the amount of 10% and being stochastic, while the other controlling parameters were considered deterministic. The first column of the table delineates the desired parameters, while the second column presents the COV of the safety factor, which accounts for the effects of individual parameter changes. In this analysis, one input parameter is incrementally increased, with all other parameters held constant. The third column illustrates the ratio between the COV of the safety factor derived from each specific parameter change and the COV of the original model, wherein the COV of all stochastic parameters is uniformly increased.

In conclusion, considering the internal friction angle, horizontal pseudo-static coefficients, and the coordinates of the triangular wedge (y) as stochastic variables alone can cover the effect of the uncertainty of all parameters to an acceptable extent. Furthermore, acceptable results with less computational cost and time can be obtained.

Table 4. Sensitivity analysis of the ratio of COV variations of the safety factor, considering being stochastic of each of the parameters

| Parameters | COV | $COV/COV_{orginal}(\%)$ |
|---|---|---|
| φ | 0.1869 | 88.04% |
| c | 0.0171 | 8.05% |
| γ | 0.0028 | 1.32% |
| $k_h$ | 0.0861 | 40.56% |
| y (Coordinate) | 0.0377 | 17.75% |
| z (Coordinate) | 0.0065 | 3.06% |
| Original | 0.2123 | - |

The second approach considers the mean friction angle as a variable, along with parameters governing the geometry, such as the triangular wedge coordinates (y). To achieve this objective, the means of the other stochastic input parameters are maintained constant, as detailed in Table 1. Specifically, the means of parameters "φ" and "b" are varied within their respective minimum and maximum values. As expected, Fig. 10 demonstrates that increasing the friction angle decreases the probability of failure. Furthermore, Fig. 11 illustrates that with increasing the controlling geometry parameters like coordinates of the triangular wedge (y), the probability of failure has a downward trend.

The reliability index of the safety factor for the stability of a rock slope exhibiting planar sliding is calculated using Eq. (15). This calculation is valid under the assumption that the distribution of the FS follows a normal distribution [45].

$$\beta = \frac{|1 - E(FS)|}{\sigma_{FS}} \tag{15}$$

where:
E(FS): The mean value of the safety factor
$\sigma_{FS}$: The standard deviation of the safety factor

The probability of failure for the safety factor in the second approach is calculated using Eq. (16) [45].

$$P_f = \phi(-\beta) = 1 - \varphi(\beta) \tag{16}$$

where:
$\phi$ : The normal cumulative density function

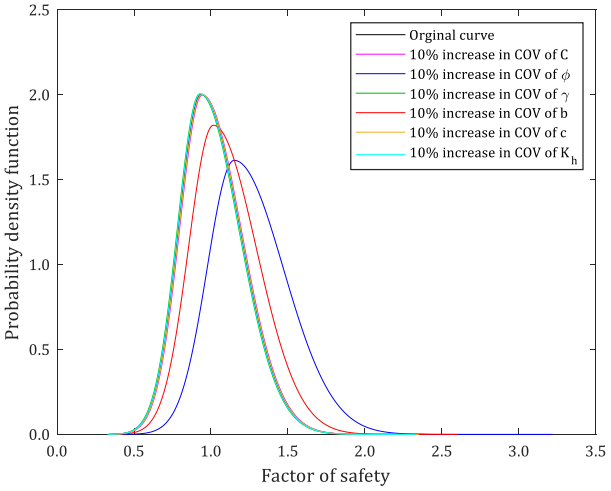



Fig. 8. Variation of the PDF of the FS in the sensitive analysis

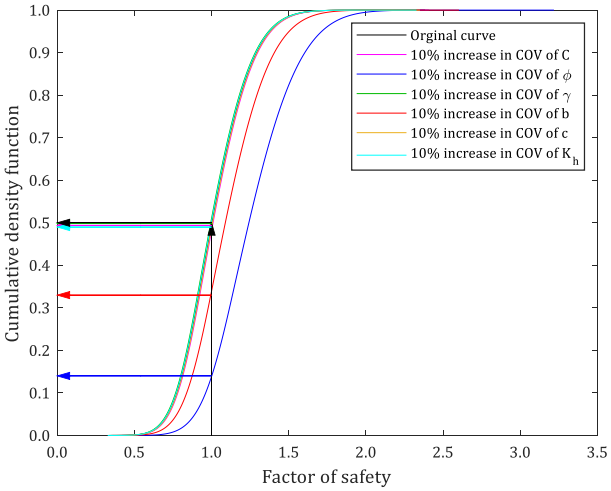



Fig. 9. Variation of the CDF of the FS in the sensitive analysis

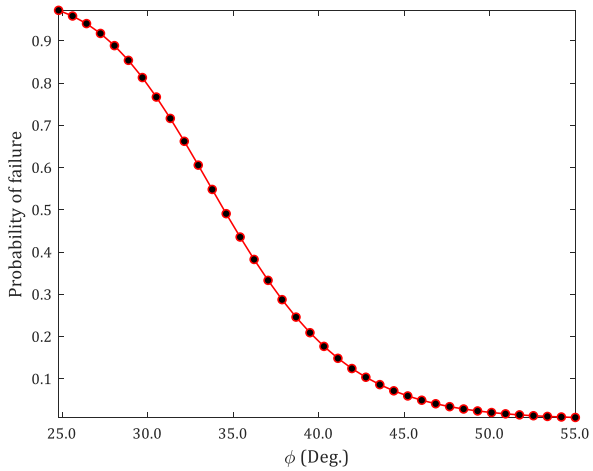



Fig. 10. Sensitivity analysis of φ versus probability of failure

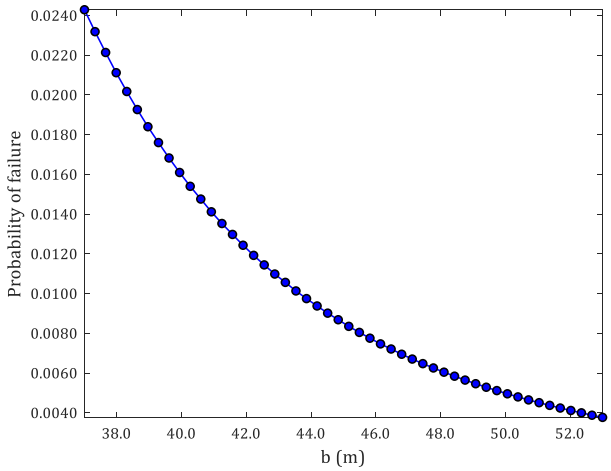



Fig. 11. Sensitivity analysis of coordinates of the triangular wedge (y) versus probability of failure

## 8. Results and Discussion

This section provides a comparative analysis of the MCS, JDRV, and FOSM methods to evaluate the efficiency and accuracy of the proposed method in determining the PDF of the safety factor for rock slope triangular wedges experiencing planar sliding. Furthermore, it examines the results derived from various approaches to analyzing rock slope stability under conditions of planar sliding, including (1) rock slope parameters, (2) rock slope parameters and seismic effect, and (3) rock slope, geometric parameters, and seismic effect (considering all stochastic variables), simultaneously has been done. Also, the reliability assessment between the mentioned approaches has been investigated. The results are shown in Figs. 13 to 16. Information regarding the comparison between methods is summarized in Table 5. Furthermore, the probability of failure and reliability indices for the various approaches are presented and compared in Tables 6 and 7, respectively.

## 8.1. Comparison of the statistical results of the JDRV, FOSM methods, and MCS

This sub-section compares the statistical results obtained from the JDRV, FOSM, and MCS methods. The proposed method has been evaluated for its efficiency and accuracy in relation to the results produced by the two other methods.

Figs. 13 and 14 illustrate the PDF and CDF of the FS based on the three methods when the rock slope, geometric parameters, and seismic effect (all stochastic variables) are considered stochastic variables. As shown in the presented figures, the precision of the JDRV method is observed compared to the FOSM method. Fig. 13 demonstrated that the relevant results of the JDRV method are closer to MCS with respect to the results of the FOSM method. Also, it indicates a significant difference in the peak values between the proposed and FOSM methods. The results show that FOSM is an approximate method, and the obtained equation from this method does not consider skewness. The comparison between the methods shows the effectiveness of the proposed method. Fig. 14 indicates an ideal match between the proposed method and MCS, too. However, this figure shows a slight difference in the high values of the CDF of the JDRV and FOSM methods.

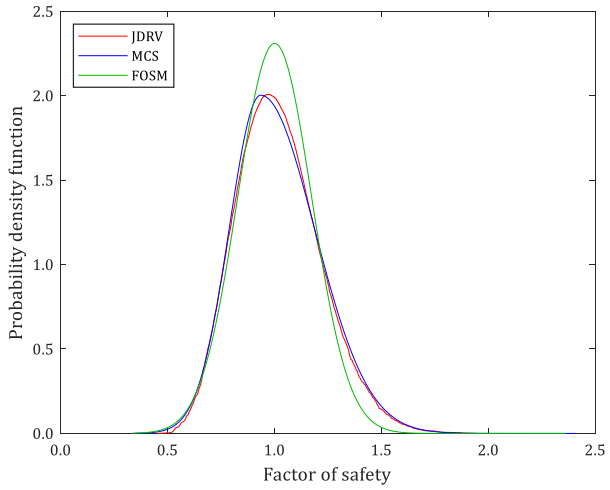



Fig. 13. PDFs of the safety factors of all stochastic variables by three methods

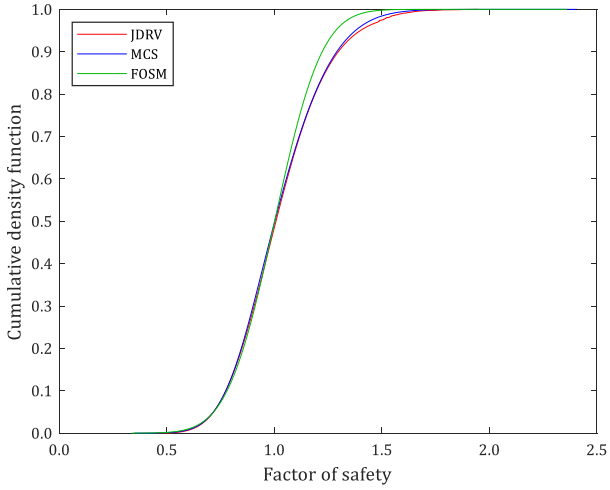



Fig. 14. CDFs of the safety factors of all stochastic variables by three methods

The mean and variance of the FS based on the three methods in the geometric and seismic analysis modes are listed in Table 5. It is evident that the results obtained from the JDRV method closely align with those produced by the MCS method. Also, one of the advantages of the JDRV method concerning MCS is the computational time, as compared in this table. The time required to cover the domain for MCS is longer than the JDRV method.

Table 5. Comparison of the statistical results by the methods

| Methods | Mean | Standard deviation | Time |
|---|---|---|---|
| JDRV (Analytical) | 1.0189 | 0.2016 | 3 Days |
| MCS (Simulation) | 0.9354 | 0.1992 | 11 Days |
| FOSM (Approximate) | 1.0010 | 0.1726 | A few minutes after developing equations |

## 8.2. Comparison of the approaches for considering uncertainty variables

This sub-section is assigned to the discussion about the determined reliability indices of the investigated approaches (1) rock slope parameters, (2) rock slope parameters and seismic effect, and (3) rock slope, geometric parameters, and seismic effect (all stochastic variables). This

comparison shows the importance of considering all variables as stochastic parameters. The results are illustrated in Figs 15 and 16. In addition, information regarding the comparison between results and reliability indices of different approaches is summarized in Tables 6 and 7, respectively.

As a result of the comparison of the PDF and CDF of the safety factor across the three stability analysis approaches, the predictions made by the proposed method are illustrated in Figs. 15 and 16, respectively. It can be seen that the rock slope, geometric parameters, and seismic effect predicted the upper probability of failure concerning the analysis. The probability related to the critical safety factor (Fs = 1) for all three approaches is shown in Fig. 16 and listed in Table 6.

Table 6. Comparison of the probability of failure related to the critical safety factor (Fs = 1)

| Approaches | Probability of failure |
|---|---|
| Rock slope parameters | 0.0959 |
| Rock slope parameters and seismic effect | 0.2923 |
| Rock slope, geometric parameters, and seismic effect | 0.4855 |

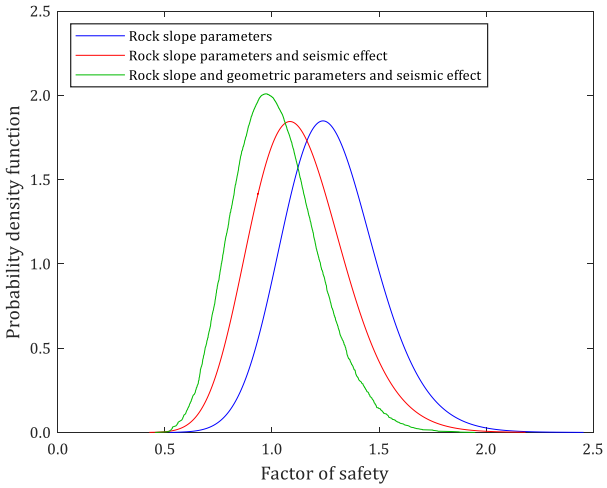



Fig. 15. PDFs of the safety factor of the approaches

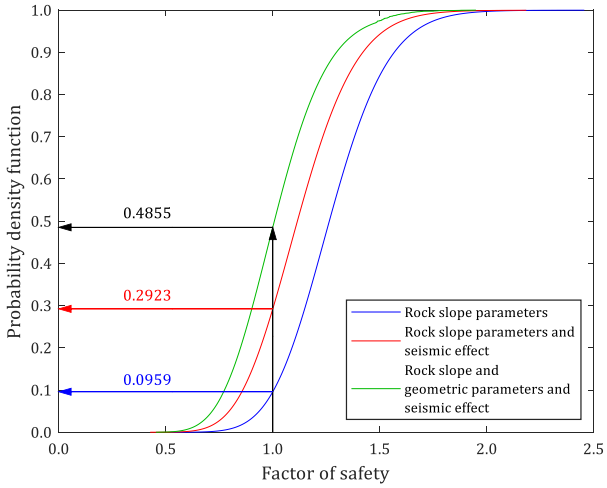



Fig. 16. CDFs of the safety factor of the approaches

The reliability indices, mean value, and standard deviation of the three approaches are listed in Table 7. As a result of considering all variables as stochastic parameters, a lower reliability index or greater probability of failure can be achieved regarding stability analysis.

Table 7. Comparison of the reliability index of the slope safety factor by three approaches

| Approaches | Reliability Index (β) | Mean (μ) | Standard deviation (σ) |
|---|---|---|---|
| Rock slope parameters | 1.2478 | 1.2775 | 0.2224 |
| Rock slope parameters and seismic effect | 0.5895 | 1.1298 | 0.2201 |
| Rock slope, geometric parameters, and seismic effect | 0.0938 | 1.0189 | 0.2016 |

## 9. Conclusion

Considering the inherent uncertainties associated with seismic, geometrical, and geotechnical properties, the stability analysis of rock slopes presents significant challenges. This study employs the JDRV method as an analytical solution for the reliability analysis of rock slope stability under planar sliding conditions, treating all variables as stochastic parameters, including rock slope characteristics, geometric parameters, and seismic effects. To achieve this, the PDF

equation of the rock slope safety factor is derived using the Jacobian approach based on fundamental relations inherent to the JDRV method. The PDFs of the safety factor are established under three scenarios: (1) considering rock slope parameters alone, (2) incorporating both rock slope parameters and seismic effects, and (3) integrating rock slope parameters, geometric parameters, and seismic effects, thereby accounting for all stochastic variables. The comparison of results obtained from the JDRV method with those from the MCS and FOSM methods demonstrates the accuracy and robustness of the JDRV method. The findings of this study can be summarized as follows:

I) The statistical results of JDRV are close to those of the MCS with respect to the results of the FOSM method.
II) The stochastic analysis by the JDRV method reduces the computational time and costs in determining the PDFs of safety factors compared to MCS.
III) It can be concluded from this study that considering all variables as stochastic parameters leads to lower reliability indices or a greater probability of failure when considering stability analysis.
IV) The sensitivity analysis reveals that the friction angle of the sliding surface and the horizontal pseudo-static coefficients are the most influential parameters affecting rock slope stability.

## Appendix A

The following appendix is a description of the mathematical functions $k_1$ to $k_7$ and FS, along with their domains:

$$F_{k_1}(k_1)=\frac{1}{\sigma_c\sqrt{2\pi}}\exp(-0.5(\frac{k_1-c_{mean}}{\sigma_c})^2) \qquad c_{min}\le k_1\le c_{max} \tag{A.1}$$

$$F_{k_2}(k_2)=\frac{1}{\sigma_\gamma\sqrt{2\pi}}\exp(-0.5(\frac{k_2-\gamma_{mean}}{\sigma_\gamma})^2) \qquad \gamma_{min}\le k_2\le \gamma_{max} \tag{A.2}$$

$$F_{k_3}(k_3)=F_\varphi(k_3)\times\left|\frac{d}{dk_3}(\frac{180}{\pi}\tan^{-1}(k_3))\right|=\frac{180}{\pi(1+k_3^{\,2})\sigma_\varphi\sqrt{2\pi}}\exp(-0.5(\frac{\frac{180}{\pi}\tan^{-1}(k_3)-\varphi_{mean}}{\sigma_\varphi})^2) \tag{A.3}$$

$$\tan(\varphi_{min})\le k_3\le\tan(\varphi_{max})$$

$$F_{k_4}(k_4)=\frac{\lambda_{k_4}\exp(-\lambda_{k_4}k_4)}{\exp(-\lambda_{k_4}k_{4min})-\exp(-\lambda_{k_4}k_{4max})} \qquad 0.01\le k_4\le k_{4max} \tag{A.4}$$

$$F_{k_5}(k_5)=\frac{1}{\sigma_{L_{ab}}\sqrt{2\pi}}\exp(-0.5(\frac{k_5-L_{ab_{mean}}}{\sigma_{L_{ab}}})^2) \qquad L_{ab_{min}}\le k_5\le L_{ab_{max}} \tag{A.5}$$

$$F_{k_6}(k_6)=\frac{1}{\sigma_A\sqrt{2\pi}}\exp(-0.5(\frac{k_6-A_{mean}}{\sigma_A})^2) \qquad A_{min}\le k_6\le A_{max} \tag{A.6}$$

$$F_{k_7}(k_7)=\frac{1}{\sigma_\beta\sqrt{2\pi}}\exp(-0.5(\frac{k_7-\beta_{mean}}{\sigma_\beta})^2) \qquad \beta_{min}\le k_7\le\beta_{max} \tag{A.7}$$

The following equations can be written using Theorem 2:

$$\begin{cases} u=FS=g_1(k_1,k_2,k_3,k_4,k_5,k_6,k_7)=\dfrac{k_1k_5+[k_2k_6\cos(k_7)-k_4k_2k_6\sin(k_7)]k_3}{k_2k_6\sin(k_7)+k_4k_2k_6\cos(k_7)} \\ v=g_2(k_1,k_2,k_3,k_4,k_5,k_6,k_7)=k_2 \\ w=g_3(k_1,k_2,k_3,k_4,k_5,k_6,k_7)=k_3 \\ z=g_4(k_1,k_2,k_3,k_4,k_5,k_6,k_7)=k_4 \\ x=g_5(k_1,k_2,k_3,k_4,k_5,k_6,k_7)=k_5 \\ y=g_6(k_1,k_2,k_3,k_4,k_5,k_6,k_7)=k_6 \\ j=g_7(k_1,k_2,k_3,k_4,k_5,k_6,k_7)=k_7 \end{cases} \tag{A.8}$$

$$\begin{cases} k_1=h_1(u,v,w,z,x,y,j)=\dfrac{u[(wy\sin(j))+(wzy\cos(j))]-[(wy\cos(j))-(wzy\sin(j))]v}{x} \\ k_2=h_2(u,v,w,z,x,y,j)=v \\ k_3=h_3(u,v,w,z,x,y,j)=w \\ k_4=h_4(u,v,w,z,x,y,j)=z \\ k_5=h_5(u,v,w,z,x,y,j)=x \\ k_6=h_6(u,v,w,z,x,y,j)=y \\ k_7=h_7(u,v,w,z,x,y,j)=j \end{cases} \tag{A.9}$$

As a result of Theorem 1, the following equations can be written:

$$f_{k_1,k_2,k_3,k_4,k_5,k_6,k_7}(k_1,k_2,k_3,k_4,k_5,k_6,k_7)=f_{k_1}(k_1)f_{k_2}(k_2)f_{k_3}(k_3)f_{k_4}(k_4)f_{k_5}(k_5)f_{k_6}(k_6)f_{k_7}(k_7) \tag{A.10}$$

According to Theorem 2, the following equations can be written:

$$\begin{aligned} F_{u,v,w,z,x,y,j}(u,v,w,z,x,y,j)=&\left|J(u,v,w,z,x,y,j)\right|f_{k_1,k_2,k_3,k_4,k_5,k_6,k_7}(h_1(u,v,w,z,x,y,j),\\ &h_2(u,v,w,z,x,y,j),h_3(u,v,w,z,x,y,j),h_4(u,v,w,z,x,y,j),h_5(u,v,w,z,x,y,j),\\ &h_6(u,v,w,z,x,y,j),h_7(u,v,w,z,x,y,j)) \end{aligned} \tag{A.11}$$

where:

$$J(u,v,w,z,x,y,j)=\begin{vmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} & J_{16} & J_{17} \\ J_{21} & J_{22} & J_{23} & J_{24} & J_{25} & J_{26} & J_{27} \\ J_{31} & J_{32} & J_{33} & J_{34} & J_{35} & J_{36} & J_{37} \\ J_{41} & J_{42} & J_{43} & J_{44} & J_{45} & J_{46} & J_{47} \\ J_{51} & J_{52} & J_{53} & J_{54} & J_{55} & J_{56} & J_{57} \\ J_{61} & J_{62} & J_{63} & J_{64} & J_{65} & J_{66} & J_{67} \\ J_{71} & J_{72} & J_{73} & J_{74} & J_{75} & J_{76} & J_{77} \end{vmatrix} =$$

$$\begin{vmatrix} \frac{\partial k_1}{\partial u} & \frac{\partial k_1}{\partial v} & \frac{\partial k_1}{\partial w} & \frac{\partial k_1}{\partial z} & \frac{\partial k_1}{\partial x} & \frac{\partial k_1}{\partial y} & \frac{\partial k_1}{\partial j} \\ \frac{\partial k_2}{\partial u} & \frac{\partial k_2}{\partial v} & \frac{\partial k_2}{\partial w} & \frac{\partial k_2}{\partial z} & \frac{\partial k_2}{\partial x} & \frac{\partial k_2}{\partial y} & \frac{\partial k_2}{\partial j} \\ \frac{\partial k_3}{\partial u} & \frac{\partial k_3}{\partial v} & \frac{\partial k_3}{\partial w} & \frac{\partial k_3}{\partial z} & \frac{\partial k_3}{\partial x} & \frac{\partial k_3}{\partial y} & \frac{\partial k_3}{\partial j} \\ \frac{\partial k_4}{\partial u} & \frac{\partial k_4}{\partial v} & \frac{\partial k_4}{\partial w} & \frac{\partial k_4}{\partial z} & \frac{\partial k_4}{\partial x} & \frac{\partial k_4}{\partial y} & \frac{\partial k_4}{\partial j} \\ \frac{\partial k_5}{\partial u} & \frac{\partial k_5}{\partial v} & \frac{\partial k_5}{\partial w} & \frac{\partial k_5}{\partial z} & \frac{\partial k_5}{\partial x} & \frac{\partial k_5}{\partial y} & \frac{\partial k_5}{\partial j} \\ \frac{\partial k_6}{\partial u} & \frac{\partial k_6}{\partial v} & \frac{\partial k_6}{\partial w} & \frac{\partial k_6}{\partial z} & \frac{\partial k_6}{\partial x} & \frac{\partial k_6}{\partial y} & \frac{\partial k_6}{\partial j} \\ \frac{\partial k_7}{\partial u} & \frac{\partial k_7}{\partial v} & \frac{\partial k_7}{\partial w} & \frac{\partial k_7}{\partial z} & \frac{\partial k_7}{\partial x} & \frac{\partial k_7}{\partial y} & \frac{\partial k_7}{\partial j} \end{vmatrix} = \frac{(yv\sin(j)+vzy\cos(j))}{x} \tag{A.12}$$

$$\begin{cases} J_{11}=\dfrac{(yv\sin(j)+vzy\cos(j))}{x} \\ J_{12}=\dfrac{-(vy\cos(j)-vzy\sin(j))}{x} \\ J_{13}=\dfrac{(u(y\sin(j)+zy\cos(j))-w(y\cos(j)-zy\sin(j)))}{x} \\ J_{14}=\dfrac{(uvy\cos(j)+wvy\sin(j))}{x} \\ J_{15}=\dfrac{(u(v\sin(j)+vz\cos(j))-w(v\cos(j)-vz\sin(j)))}{x} \\ J_{16}=\dfrac{(u(vy\cos(j)-vzy\sin(j))+w(vz\sin(j)+vzy\cos(j)))}{x} \\ J_{17}=\dfrac{-(u(vy\sin(j)+vzy\cos(j))-w(vy\cos(j)-vzy\sin(j)))}{x^2} \\ J_{it}=1 \quad \text{if } i=t \quad (i\geq 2 \text{ and } t\geq 1) \\ J_{it}=0 \quad \text{if } i\neq t \quad (i\geq 2 \text{ and } t\geq 1) \end{cases} \tag{A.13}$$

Based on Theorem 3, the following equations can be written:

$$f_U(u)=\int_{\alpha_6}^{\beta_6}\int_{\alpha_5}^{\beta_5}\int_{\alpha_4}^{\beta_4}\int_{\alpha_3}^{\beta_3}\int_{\alpha_2}^{\beta_2}\int_{\alpha_1}^{\beta_1} f_{U,V,W,Z,x,y,j}(u,v,w,z,x,y,j)\,dv\,dw\,dz\,dx\,dy\,dj \tag{A.14}$$

The integral bounds of the aforementioned equation are established in the following paragraphs. Since this integral involves differentials dv, dw, dz, dx, dy, and dj the variables v, w z, x, y, and j correspond to the functions $k_2$, $k_3$, $k_4$, $k_5$, $k_6$, and $k_7$. Consequently, the bounds of the integral can also be expressed as functions of $k_2$, $k_3$, $k_4$, $k_5$, $k_6$, and $k_7$. To achieve this, $k_{2min}$, $k_{3min}$,

…, $k_{7min}$ are related to the bounds $\alpha_1, \alpha_2, \ldots, \alpha_6$ respectively, and $k_{2max}$, $k_{3max}$, …, $k_{7max}$ are related to $\beta_1, \beta_2, \ldots, \beta_6$ respectively.

$$g_1(k_{1min}, k_{2min}, k_{3min}, k_{4min}, k_{5min}, k_{6min}, k_{7min}) \le u = FS \le g_2(k_{1max}, k_{2max}, k_{3max}, k_{4max}, k_{5max}, k_{6max}, k_{7max}) \quad \text{(A.15)}$$

## Appendix B

The derivation of the mathematical functions required for the FOSM method is presented in this Appendix:

$$FS = \frac{ux + [vy\cos(k) - zvy\sin(k)]w}{vy\sin(k) + zvy\cos(k)} \quad \text{(B.1)}$$

where:

$$\begin{cases} u = c \\ v = \gamma \\ w = \tan(\pi\varphi / 180) \\ z = k_h \\ x = L_{ab} \\ y = A \\ k = \beta \end{cases} \quad \text{(B.2)}$$

The derivative of a function of input variables, $x_j$:

$$\frac{\partial FS}{\partial u} = \frac{x}{vy(\sin(k) + z\cos(k))} \quad \text{(B.3)}$$

$$\frac{\partial FS}{\partial v} = \frac{-ux}{v^2 y(\sin(k) + z\cos(k))} \quad \text{(B.4)}$$

$$\frac{\partial FS}{\partial w} = \frac{\cos(k) - z\sin(k)}{\sin(k) + z\cos(k)} \quad \text{(B.5)}$$

$$\frac{\partial FS}{\partial z} = \frac{-(ux\cos(k) + vwy)}{(vy\sin(k) + z\cos(k))^2} \quad \text{(B.6)}$$

$$\frac{\partial FS}{\partial x} = \frac{u}{vy(\sin(k) + z\cos(k))} \quad \text{(B.7)}$$

$$\frac{\partial FS}{\partial y} = \frac{-ux}{vy^2(\sin(k) + z\cos(k))} \quad \text{(B.8)}$$

$$\frac{\partial FS}{\partial k} = w - \frac{-((\cos(k) - z\sin(k))(ux + vwy\cos(k) - vwyz\sin(k)))}{(vy\sin(k) + z\cos(k))^2} \quad \text{(B.9)}$$

## References

[1] P. Starzec and J. Andersson, "Application of two-level factorial design to sensitivity analysis of key block statistics from fracture geometry," *International Journal of Rock Mechanics and Mining Sciences,* vol. 39, no. 2, pp. 243-255, 2002.

[2] P. H. Kulatilake, J.-g. Um, M. Wang, R. F. Escandon, and J. Narvaiz, "Stochastic fracture geometry modeling in 3-D including validations for a part of Arrowhead East Tunnel, California, USA," *Engineering geology,* vol. 70, no. 1-2, pp. 131-155, 2003.

[3] F. Tonon and S. Chen, "Closed-form and numerical solutions for the probability distribution function of fracture diameters," *International Journal of Rock Mechanics and Mining Sciences,* vol. 44, no. 3, pp. 332-350, 2007.

[4] R. Jimenez-Rodriguez and N. Sitar, "Influence of stochastic discontinuity network parameters on the formation of removable blocks in rock slopes," *Rock mechanics and rock engineering,* vol. 41, no. 4, pp. 563-585, 2008.

[5] A. Johari, S. Mousavi, and A. Hooshmand Nejad, "A seismic slope stability probabilistic model based on Bishop's method using analytical approach," *Scientia Iranica,* vol. 22, no. 3, pp. 728-741, 2015.

[6] A. Johari, A. Javadi, M. Elmi, and S. Raei, "An analytical approach to reliability assessment of shear wave velocity relationship," *Scientia Iranica,* vol. 20, no. 6, pp. 1685-1694, 2013.

[7] N. Metropolis and S. Ulam, "The Monte Carlo method," *Journal of the American Statistical Association,* vol. 44, no. 247, pp. 335-341, 1949.

[8] L. Faravelli, "Response-surface approach for reliability analysis," *Journal of Engineering Mechanics,* vol. 115, no. 12, pp. 2763-2781, 1989.

[9] R. G. Ghanem and P. D. Spanos, *Stochastic finite elements: a spectral approach*. Courier Corporation, 2003.

[10] M. Lee, "DESRA-2: Dynamic effective stress response analysis of soil deposits with energy transmitting boundary including assessment of liquefaction potential," *Research Report, University of British Columbia,* 1978.

[11] E. Rosenblueth, "Point estimates for probability moments," *Proceedings of the National Academy of Sciences,* vol. 72, no. 10, pp. 3812-3814, 1975.

[12] J.-P. Wang and D. Huang, "RosenPoint: A Microsoft Excel-based program for the Rosenblueth point estimate method and an application in slope stability analysis," *Computers & Geosciences,* vol. 48, pp. 239-243, 2012.

[13] A. H. Ang and W. H. Tang, *Probability concepts in engineering planning and design, basic principles*. John Wiley & Sons Incorporated, 1975.

[14] P. G. Hoel, "Introduction to statistical theory/Paul G. Hoel, Sidney C. Port, Charles J. Stone," *The Houghton-Mifflin series in statistics,* 1971.

[15] A. Johari and A. Khodaparast, "Analytical stochastic analysis of seismic stability of infinite slope," *Soil Dynamics and Earthquake Engineering,* vol. 79, pp. 17-21, 2015.

[16] S. Tamimi, B. Amadei, and D. M. Frangopol, "Monte Carlo simulation of rock slope reliability," *Computers & structures,* vol. 33, no. 6, pp. 1495-1505, 1989.

[17] H.-J. Park, T. R. West, and I. Woo, "Probabilistic analysis of rock slope stability and random properties of discontinuity parameters, Interstate Highway 40, Western North Carolina, USA," *Engineering Geology,* vol. 79, no. 3-4, pp. 230-250, 2005.

[18] R. Jimenez-Rodriguez, N. Sitar, and J. Chacón, "System reliability approach to rock slope stability," *International Journal of Rock Mechanics and Mining Sciences,* vol. 43, no. 6, pp. 847-859, 2006.

[19] Z. Li, Z. Hu, X. Zhang, S. Du, Y. Guo, and J. Wang, "Reliability analysis of a rock slope based on plastic limit analysis theory with multiple failure modes," *Computers and Geotechnics,* vol. 110, pp. 132-147, 2019.

[20] A. Johari and A. M. Lari, "System reliability analysis of rock wedge stability considering correlated failure modes using sequential compounding method," *International Journal of Rock Mechanics and Mining Sciences,* vol. 82, pp. 61-70, 2016.

[21] G. Tiwari and G. M. Latha, "Reliability analysis of jointed rock slope considering uncertainty in peak and residual strength parameters," *Bulletin of Engineering Geology and the Environment,* vol. 78, no. 2, pp. 913-930, 2019.

[22] D. Li, Y. Chen, W. Lu, and C. Zhou, "Stochastic response surface method for reliability analysis of rock slopes involving correlated non-normal variables," *Computers and Geotechnics,* vol. 38, no. 1, pp. 58-68, 2011.

[23] D. Griffiths, J. Huang, and G. A. Fenton, "Influence of spatial variability on slope reliability using 2-D random fields," *Journal of Geotechnical and Geoenvironmental Engineering,* vol. 135, no. 10, pp. 1367-1378, 2009.

[24] E. Gravanis, L. Pantelidis, and D. Griffiths, "An analytical solution in probabilistic rock slope stability assessment based on random fields," *International Journal of Rock Mechanics and Mining Sciences,* vol. 71, pp. 19-24, 2014.

[25] Y. Liu, W. Zhang, L. Zhang, Z. Zhu, J. Hu, and H. Wei, "Probabilistic stability analyses of undrained slopes by 3D random fields and finite element methods," *Geoscience Frontiers,* vol. 9, no. 6, pp. 1657-1664, 2018.

[26] D. Young, "Probabilistic slope analysis for structural failure," in *International Journal of Rock Mechanics and Mining Sciences & Geomechanics Abstracts*, 1993, vol. 30, no. 7: Elsevier, pp. 1623-1629.

[27] J. Mayer and D. Stead, "A comparison of traditional, step-path, and geostatistical techniques in the stability analysis of a large open pit," *Rock Mechanics and Rock Engineering,* vol. 50, no. 4, pp. 927-949, 2017.

[28] A. E. Aladejare and Y. Wang, "Influence of rock property correlation on reliability analysis of rock slope stability: from property characterization to reliability analysis," *Geoscience Frontiers,* vol. 9, no. 6, pp. 1639-1648, 2018.

[29] F. Liu *et al.*, "Spatial variability and time decay of rock mass mechanical parameters: A landslide study in the Dagushan open-pit mine," *Rock Mechanics and Rock Engineering,* vol. 53, no. 7, pp. 3031-3053, 2020.

[30] B. Low, "Reliability analysis of rock slopes involving correlated non-normals," *International Journal of Rock Mechanics and Mining Sciences,* vol. 44, no. 6, pp. 922-935, 2007.

[31] H. J. Park, J.-G. Um, I. Woo, and J. W. Kim, "The evaluation of the probability of rock wedge failure using the point estimate method," *Environmental Earth Sciences,* vol. 65, no. 1, pp. 353-361, 2012.

[32] N. M. Newmark, "Effects of earthquakes on dams and embankments," *Geotechnique,* vol. 15, no. 2, pp. 139-160, 1965.

[33] K. Terzaghi, "Mechanism of landslides. Application of geology to engineering practice (Berkey volume), *Paige, Chairman, Geological Society of America Bulletin,* vol. 83, 1950.

[34] S. L. Kramer, *Geotechnical earthquake engineering*. Pearson Education India, 1996.

[35] F. I. Makdisi and H. B. Seed, "Simplified procedure for estimating dam and embankment earthquake-induced deformations," *Journal of the Geotechnical Engineering Division,* vol. 104, no. 7, pp. 849-867, 1978.

[36] A. Johari, M. Momeni, and A. Javadi, "An analytical solution for reliability assessment of pseudo-static stability of rock slopes using jointly distributed random variables method," 2015.

[37] L. Zhao *et al.*, "Rock slope reliability analysis using Barton-Bandis failure criterion with modified pseudo-dynamic approach," *Soil Dynamics and Earthquake Engineering,* vol. 139, p. 106310, 2020.

[38] L.-h. Zhao, K.-f. Jiao, D.-j. Li, and S. Zuo, "System reliability analysis of seismic pseudo-static stability of rock wedge based on nonlinear Barton—Bandis criterion," *Journal of Central South University,* vol. 27, no. 11, pp. 3450-3463, 2020.

[39] W.-g. Zhang, F.-s. Meng, F.-y. Chen, and H.-l. Liu, "Effects of spatial variability of weak layer and seismic randomness on rock slope stability and reliability analysis," *Soil Dynamics and Earthquake Engineering,* vol. 146, p. 106735, 2021.

[40] K. Kendig, "Is a 2000-year-old formula still keeping some secrets? " *The American Mathematical Monthly,* vol. 107, no. 5, pp. 402-415, 2000.

[41] H. Tijms, *Understanding Probability*. Cambridge University Press, 2012.

[42] A. Johari and A. Khodaparast, "Modelling of probability liquefaction based on standard penetration tests using the jointly distributed random variables method," *Engineering geology,* vol. 158, pp. 1-14, 2013.

[43] H. Duzgun, T. Bozdag, and A. Pasamehmetoglu, "Probabilistic wedge stability analysis by advanced first order second moment (FOSM) method," in *International symposium on mine planning and equipment selection*, 1994, pp. 833-838.

[44] G. S. Mudholkar and A. D. Hutson, "The epsilon–skew–normal distribution for analyzing near-normal data," *Journal of Statistical Planning and Inference,* vol. 83, no. 2, pp. 291-309, 2000

[45] Wu, X. Z. (2013). Trivariate analysis of soil ranking-correlated characteristics and its application to probabilistic stability assessments in geotechnical engineering problems. Soils and Foundations, 53(4), 540–556.

[46] G. A. Fenton and D. V. Griffiths, *Risk assessment in geotechnical engineering*. John Wiley & Sons, New York, 2008.