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Development of Mathematical Reasoning through Artificial Intelligence: A Design-Based Learning Framework

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Abstract

This study investigated cognitive progression in mathematical reasoning following the implementation of an artificial intelligence (AI)-supported learning framework. A longitudinal pretest–posttest design was used with 500 secondary school students. Mathematical reasoning was assessed across eight domains, and cognitive development was examined via latent profile and transition analyses supported by predictive and Bayesian modeling. The findings revealed consistent improvements across all the reasoning domains and identified four distinct reasoning profiles: novice, developing, strategic, and advanced reasoning. Transition analyses indicated substantial movement toward higher reasoning profiles, particularly along the paths from Novice to Developing, Developing to Strategic, and Strategic to Advanced. Critical evaluation and reflective engagement emerged as the strongest predictors of cognitive progression, whereas the frequency of AI use demonstrated comparatively weaker effects. The use of Bayesian evidence strongly supported the effectiveness of the framework. The findings suggest that AI contributes most effectively to mathematical reasoning when learners engage critically and reflectively with AI-generated content rather than using AI primarily as an answer-producing tool.

Introduction

Generative artificial intelligence can now solve complex mathematical problems, generate proofs, and provide step-by-step explanations within seconds. However, a fundamental educational question remains unresolved: does access to increasingly intelligent systems strengthen students' mathematical reasoning, or does it reduce opportunities for independent cognitive development? Educational institutions increasingly integrate AI-powered tools into teaching and learning processes, creating new opportunities for personalized instruction, adaptive feedback, and independent learning. Despite these advances, a fundamental challenge remains unresolved. Access to accurate solutions does not necessarily translate into the development of mathematical reasoning because learners must still interpret, evaluate, justify, and connect mathematical ideas independently. This tension raises an important educational question regarding how AI can be utilized not merely as a source of answers but also as a catalyst for deeper mathematical thinking and cognitive development.

Mathematical reasoning occupies a central position in mathematics education because it enables learners to identify patterns, formulate conjectures, construct arguments, evaluate evidence, and develop generalizable solutions (Hanna, 2020; Jeannotte & Kieran, 2017; Nickerson, 2011). Contemporary educational frameworks increasingly recognize reasoning as a core competency required for success in both academic and professional settings. Students with strong reasoning abilities are better equipped to solve unfamiliar problems, transfer knowledge across contexts, and engage in higher-order thinking. Persistent difficulties in developing these competencies continue to be reported across educational systems, suggesting that innovative instructional approaches are needed to support meaningful reasoning development.

Recent research has highlighted the growing potential of AI-supported learning environments to enhance mathematics education. Studies have reported positive effects of AI-assisted instruction on problem-solving performance, learner engagement, personalized feedback, and academic achievement (Ellikkal & Rajamohan, 2025; Wan et al., 2025). Generative AI systems provide immediate explanations, alternative solution pathways, and interactive support that enrich learning experiences. Recent evidence shows that AI fosters reflective thinking and self-regulated learning by encouraging learners to evaluate information critically, compare alternative solutions, and monitor their own reasoning processes during problem solving (Lee et al., 2024; Zhou et al., 2024). These findings indicate that AI possesses considerable educational potential beyond its role as an information delivery tool.

Existing AI studies largely evaluate achievement gains, whereas little evidence exists regarding developmental transitions in mathematical reasoning profiles. Most studies evaluate AI effectiveness through achievement scores (Liao et al., 2024; Zheng et al., 2023), task completion rates, or student perceptions (Bhadila et al., 2026; Er et al., 2025; Nazaretsky et al., 2024), providing limited insight into how reasoning develops over time (Garbuio & Lin, 2021). Research examining cognitive progression remains comparatively scarce, particularly in mathematics education. Little is known about whether students move through distinct developmental stages of reasoning when learning with AI, how such transitions occur, and which dimensions of AI engagement contribute most strongly to cognitive advancement. Consequently, the mechanisms through which AI supports mathematical reasoning development remain insufficiently understood.

The present study addresses these limitations by conceptualizing mathematical reasoning as a developmental process rather than a static outcome. A design-based learning framework was developed to integrate AI into activities emphasizing reflection, evaluation, justification, and strategic problem solving. The study extends the current literature by combining latent profile analysis, cognitive transition analysis, multinomial modeling, and Bayesian inference to examine how students progress across reasoning profiles and how different forms of AI engagement influence that progression. This approach provides a more comprehensive understanding of the relationship between AI-supported learning and mathematical reasoning development.

This study aims to develop and evaluate an AI-supported learning framework designed to enhance mathematical reasoning and promote cognitive progression. Specifically, the study addresses the following research questions: (1) What latent profiles of mathematical reasoning emerge before and after participation in the AI-supported learning framework? (2) How do students transition across reasoning profiles during the intervention? (3) Which dimensions of AI engagement predict progression toward higher reasoning profiles? (4) To what extent does the framework contribute to improvements in mathematical reasoning? By addressing these questions, this study seeks to provide both theoretical and practical insights into the design of AI-supported learning environments that foster deeper mathematical thinking.

Theoretical Background

2.1 Mathematical reasoning

Mathematical reasoning is increasingly recognized as a critical educational outcome in the era of artificial intelligence because students are expected not only to obtain solutions but also to evaluate, justify, and refine mathematical ideas. Reasoning theory conceptualizes mathematics as a process of constructing logical relationships among concepts, evidence, and conclusions rather than merely applying procedural rules. Learners engage in reasoning when they formulate conjectures, test assumptions, and develop arguments that connect mathematical representations to underlying principles. Such processes are particularly important in AI-supported environments, where learners must assess the validity and appropriateness of AI-generated explanations rather than accept them uncritically (Alfaleh, 2026).

Mathematical reasoning is closely associated with higher-order thinking because it requires learners to analyze information, evaluate competing strategies, and generate defensible solutions for unfamiliar problems. Contemporary educational research increasingly emphasizes that the effective use of AI depends on learners' ability to engage in these higher-order cognitive processes. Recent studies have shown that AI-supported learning environments are most effective when students actively interrogate generated responses, compare alternative explanations, and reflect on their reasoning processes rather than relying on AI as a source of immediate answers (Hao et al., 2026). These findings position reasoning as a central mechanism through which AI can contribute to meaningful learning.

From the perspective of mathematical cognition, reasoning develops through the progressive organization and reorganization of conceptual structures that support problem solving and knowledge transfer. Cognitive development occurs when learners connect abstract mathematical concepts with evidence, representations, and strategic decision-making processes. Emerging scholarship on generative AI in education further suggests that productive interactions with AI can stimulate reflection, cognitive conflict, and conceptual refinement, thereby supporting deeper forms of mathematical understanding (Canonigo, 2024; Zeng, 2025). Mathematical reasoning therefore represents not only an outcome of learning but also a cognitive process through which learners construct, evaluate, and extend mathematical knowledge in technology-rich environments.

2.2 Artificial Intelligence in Mathematics Education

Generative artificial intelligence has emerged as one of the most transformative developments in contemporary education. Unlike conventional educational technologies that primarily deliver predefined content, generative AI systems can produce explanations, examples, feedback, and solution pathways in response to learners' needs. Recent advances in large language models have enabled students to engage in interactive mathematical dialog, explore alternative problem-solving approaches, and obtain immediate support during learning activities. Such capabilities have expanded the role of AI from an information-delivery tool to an active participant in the learning process, creating new opportunities for personalized and adaptive mathematics instruction (J. Liu et al., 2025).

The growing presence of generative AI has accelerated the development of AI-assisted learning environments that support students throughout different stages of mathematical problem solving. AI-assisted learning refers to instructional processes in which AI systems provide guidance, feedback, scaffolding, and adaptive support to facilitate learning. Research has shown that AI-assisted environments can improve learner engagement, promote self-regulated learning, and provide individualized assistance that is often difficult to achieve in traditional classroom settings (Lima et al., 2026; Yang, 2026). Educational benefits become particularly evident when AI is integrated into pedagogical designs that encourage students to interact actively with mathematical concepts rather than passively receiving solutions.

Recent scholarship has increasingly emphasized the concept of AI-mediated reasoning, which positions artificial intelligence as a cognitive partner that supports the development of higher-order thinking. AI-mediated reasoning occurs when learners use AI-generated explanations, feedback, and alternative perspectives as resources for evaluating arguments, testing conjectures, and refining mathematical understanding. Educational value therefore arises not from the answers produced by AI but from the reasoning processes stimulated through interaction with those answers. Studies have demonstrated that learners develop deeper conceptual understanding when they critically examine AI-generated outputs, compare competing solution strategies, and reflect on the validity of mathematical arguments (Cevikbas & Kaiser, 2026; Kaya & Yavuz, 2025). This perspective highlights the importance of designing AI-supported learning environments that promote reflection, critical evaluation, and meaningful engagement with mathematical ideas.

2.3 Constructivism and metacognition

Constructivist theory provides a foundational perspective for understanding how students develop mathematical reasoning in AI-supported learning environments. Piaget (1970) argued that learning occurs through the active construction and reorganization of cognitive structures as learners interact with new information and experiences. Vygotsky & Cole (1978) extended this perspective by emphasizing the role of social and cultural mediation in cognitive development, proposing that higher-order thinking emerges through interaction with tools, language, and more knowledgeable others. Artificial intelligence can function as such a mediating tool by providing explanations, alternative solution pathways, and feedback that encourage learners to examine, compare, and refine mathematical ideas. Learning therefore becomes a process of knowledge construction in which students actively engage with AI-generated information rather than passively receiving answers.

Metacognitive theory further explains why some learners benefit more from AI-supported learning than others do. Flavell (1979) defined metacognition as the capacity to monitor, regulate, and evaluate one's own cognitive processes during learning and problem solving. Metacognitive monitoring involves awareness of understanding and recognition of potential errors, whereas metacognitive regulation refers to the selection and adjustment of strategies used to achieve learning goals. Metacognitive evaluation occurs when learners assess the quality, validity, and effectiveness of their reasoning processes and

solutions. These mechanisms enable students to move beyond procedural task completion and engage more deeply with conceptual understanding.

Reflection and critical evaluation represent particularly important forms of metacognitive regulation in AI-supported learning environments. Reflection encourages learners to reconsider assumptions, compare alternative explanations, and integrate new information into existing knowledge structures. Critical evaluation requires learners to assess the accuracy, consistency, and appropriateness of AI-generated outputs before accepting them as valid. Recent studies have demonstrated that the educational benefits associated with artificial intelligence are strongest when learners actively monitor, regulate, and evaluate their interactions with AI rather than relying on generated responses uncritically (Papaneophytou & Nicolaou, 2025). Constructivism and metacognition therefore provide complementary theoretical foundations for explaining how AI-supported learning promotes mathematical reasoning through knowledge construction, reflective engagement, and evaluative thinking.

2.4 Conceptual Framework

The conceptual framework of this study is grounded in constructivist and metacognitive perspectives, which posit that meaningful learning emerges through active knowledge construction and deliberate regulation of cognitive processes. Although artificial intelligence provides learners with explanations, feedback, and alternative solution pathways, educational benefits do not arise automatically from exposure to AI-generated content. Rather, learning outcomes depend on how learners engage with, interpret, and evaluate the information produced through human-AI interactions. Consequently, the framework conceptualizes artificial intelligence as a cognitive catalyst whose educational value is mediated by metacognitive processes.

The framework proposes that AI interaction initiates a sequence of cognitive processes that contribute to mathematical reasoning development. Interaction with AI-generated explanations and solutions encourages learners to engage in reflection, whereby they compare alternative perspectives, reconsider assumptions, and connect new information with existing knowledge structures. Reflective engagement subsequently promotes critical evaluation, requiring learners to assess the validity, consistency, and appropriateness of AI-generated outputs. Through these processes, learners become increasingly capable of monitoring understanding, identifying errors, and refining problem-solving strategies. Reflection and critical evaluation are therefore positioned as central mechanisms linking AI interaction to cognitive development.

Cognitive progression is conceptualized as the developmental pathway through which metacognitive engagement contributes to increasingly sophisticated forms of mathematical reasoning. Students who engage more deeply in reflection and critical evaluation are expected to demonstrate greater movement toward higher reasoning profiles characterized by stronger capabilities in pattern recognition, conjecturing, logical reasoning, argumentation, generalization, verification, mathematical modeling, and strategic thinking. The framework therefore proposes that the educational impact of artificial intelligence

is indirect, operating through metacognitive processes that facilitate cognitive progression and ultimately enhance mathematical reasoning.

On the basis of this framework, the study hypothesizes that AI-supported learning promotes mathematical reasoning through cognitive progression mechanisms facilitated by reflective and evaluative engagement with AI-generated content. Students who engage more deeply in reflection and critical evaluation are expected to demonstrate greater movement toward higher reasoning profiles and stronger development across multiple dimensions of mathematical reasoning. This framework therefore integrates artificial intelligence, cognitive development, and mathematical reasoning into a unified explanatory model that guides the empirical analyses presented in this study.

Method

3.1 Research Design

This study employed a design-based learning approach to develop and evaluate an artificial intelligence (AI)-supported instructional framework aimed at enhancing students' mathematical reasoning. Design-based learning was selected because it allows instructional innovations to be implemented and examined within authentic learning environments while simultaneously generating theoretical insights into learning processes. The study adopted a longitudinal pretest—posttest design in which students' mathematical reasoning was assessed before and after their participation in the AI-supported learning intervention. The design enabled the investigation of cognitive development, latent reasoning profiles, and progression pathways associated with AI-mediated learning experiences.

3.2 Participants

The study involved 500 secondary school students enrolled in mathematics courses. The participants were selected through stratified random sampling to ensure representation across different academic ability levels. All the participants completed the intervention and both measurement occasions. The sample size was considered sufficient for latent profile modeling, transition analysis, and Bayesian inference because it provided stable estimation of profile membership probabilities and transition pathways. Participation was voluntary, and informed consent was obtained in accordance with institutional research ethics guidelines.

3.3 AI-Supported Learning Framework

The instructional framework was implemented over an eight-week intervention period consisting of sixteen learning sessions. Students were encouraged to use generative artificial intelligence systems as cognitive partners during mathematics learning activities. Several AI platforms, including ChatGPT, Gemini, Claude, and Microsoft Copilot, were available throughout the intervention. ChatGPT emerged as the dominant platform because of its accessibility, conversational capabilities, and widespread familiarity among students. Both free and premium versions of ChatGPT were utilized during classroom and independent learning activities. Rather than focusing on technology adoption itself, the framework

emphasized how students interacted with AI-generated explanations, feedback, and alternative solution pathways to support mathematical reasoning development.

Constructivist and metacognitive principles guided the design of the instructional framework. Learning activities were structured to encourage students to actively construct mathematical understanding through exploration, comparison, justification, and reflection. Artificial intelligence functions as a cognitive mediator that supports learners in generating ideas, examining multiple solution pathways, identifying errors, and evaluating mathematical arguments. Teachers acted as facilitators who monitored AI-supported interactions and encouraged students to justify conclusions via mathematical evidence. Students were explicitly instructed not to accept AI-generated answers without evaluation and were required to provide written explanations, critiques, and reflections regarding the validity of AI outputs.

The intervention progressively addressed eight dimensions of mathematical reasoning: pattern recognition, conjecturing, logical reasoning, mathematical argumentation, generalization, verification, mathematical modeling, and strategic thinking. Early activities focused on identifying relationships and generating conjectures, whereas later activities emphasized evaluation, verification, modeling, and strategic problem solving. Reflection and critical evaluation were embedded throughout the instructional process to strengthen metacognitive engagement. As shown in Table 1, each stage of the framework aligns specific reasoning objectives with targeted AI-supported learning activities. This progression ensured that artificial intelligence served as a catalyst for reasoning development rather than a mechanism for answer production.

Table 1
AI-supported learning activities and cognitive objectives

Week	Mathematical Reasoning Focus	Learning Activity	AI Function	Student Task
1	Pattern Recognition	Explore numerical and visual patterns	Generate explanations and examples	Compare patterns and identify relationships
2	Conjecturing	Develop mathematical conjectures	Suggest alternative hypotheses	Evaluate plausibility of conjectures
3	Logical Reasoning	Analyze logical structures	Generate step-by-step reasoning	Identify logical consistency and errors
4	Mathematical Argumentation	Construct and critique arguments	Provide alternative arguments	Justify and defend mathematical claims
5	Generalization	Extend solutions to broader situations	Generate generalized rules	Evaluate applicability of generalizations
6	Verification	Check solution validity	Provide verification procedures	Confirm correctness of solutions
7	Mathematical Modeling	Solve contextual mathematical problems	Generate modeling alternatives	Compare and refine mathematical models
8	Strategic Thinking	Solve complex nonroutine problems	Propose multiple solution strategies	Select and justify optimal strategies

Students interacted with AI systems through carefully designed prompts intended to stimulate reasoning rather than answer retrieval. Prompting activities encouraged learners to compare alternative explanations, evaluate mathematical validity, examine assumptions, and reflect on solution strategies. Such interactions were designed to cultivate deeper cognitive engagement and support progression across reasoning profiles. Representative examples of prompts used throughout the intervention are presented in Table 2.

Table 2
Representative AI Prompts Used During Learning Activities

Reasoning Domain	Example Prompt
Pattern Recognition	"Identify the pattern in this sequence and explain the mathematical relationship."
Conjecturing	"Generate three possible conjectures based on this numerical pattern and explain the reasoning behind each one."
Logical Reasoning	"Explain the logical steps required to solve this problem and identify possible errors."
Argumentation	"Provide an alternative mathematical argument and compare it with my solution."
Generalization	"How can this solution be generalized for any value of n?"
Verification	"Verify whether this solution is mathematically valid and explain why."
Mathematical Modeling	"Develop two mathematical models for this real-world problem and compare their assumptions."
Strategic Thinking	"Suggest multiple solution strategies and explain the advantages and limitations of each strategy."

To minimize answer-oriented use of artificial intelligence, the students were explicitly instructed not to copy AI-generated solutions without evaluation. Every interaction required written justification, comparison of alternative responses, identification of potential errors, and reflective commentary regarding the validity of AI-generated explanations. These procedures were implemented to ensure that AI functioned as a catalyst for reasoning development rather than a substitute for mathematical thinking.

Prior to implementation, the participating mathematics teachers attended a two-day workshop focused on AI-supported pedagogy, prompting strategies and procedures for facilitating reflective and evaluative learning activities.

3.4 Instruments

Prior to the main analyses, the reliability and internal consistency of the instruments were evaluated. Mathematical reasoning was measured via 24 items representing eight domains: Pattern Recognition, Conjecturing, Logical Reasoning, Argumentation, Generalization, Verification, Mathematical Modeling, and Strategic Thinking. AI engagement was measured via five indicators: frequency of AI use, AI-assisted problem solving, reflection, feedback utilization, and critical evaluation.

Internal consistency reliability was assessed via Cronbach's alpha coefficients. The results indicated high reliability across all mathematical reasoning domains, with alpha coefficients ranging from .886 to .905. The overall mathematical reasoning instrument demonstrated excellent reliability ($\alpha = .977$), indicating strong internal consistency among the 24 items. The AI engagement scale also demonstrated

acceptable reliability (α = .790), exceeding the commonly recommended threshold of .70 for educational research (Nunnally & Bernstein, 1994). These results suggest that the instruments provided stable and consistent measurements suitable for subsequent latent profile and transition analyses.

Table 3
Internal consistency reliability of the study instruments

Construct	Number of Items	Cronbach's Alpha
Pattern Recognition	3	0.905
Conjecturing	3	0.893
Logical Reasoning	3	0.899
Argumentation	3	0.894
Generalization	3	0.894
Verification	3	0.892
Mathematical Modeling	3	0.895
Strategic Thinking	3	0.886
Mathematical Reasoning (Overall)	**24**	**0.977**
AI Engagement	**5**	**0.790**

3.5 Data analysis

Data analysis was conducted in several sequential stages. First, descriptive statistics were calculated to examine changes in mathematical reasoning before and after the intervention. Second, latent profile analysis was performed to identify distinct reasoning profiles on the basis of the eight reasoning domains. Model selection was guided by information criteria and profile interpretability. Third, profile membership distributions were compared across measurement occasions to examine changes in cognitive composition.

Fourth, transition analysis was conducted to investigate movement between reasoning profiles from pretest to posttest. Transition probabilities were estimated and visualized via heatmaps and Sankey diagrams to reveal dominant cognitive progression pathways. Fifth, multinomial logistic regression was employed to examine the effects of AI engagement variables on profile transitions, allowing the identification of factors associated with progression toward higher reasoning profiles. Odds ratios were calculated to facilitate interpretation of predictor importance.

Finally, Bayesian inference was used to evaluate the overall effectiveness of the intervention. A Bayesian paired comparison of pretest and posttest reasoning scores was conducted via the Jeffreys–Zellner–Siow prior specification. Bayes factors were estimated to quantify the strength of evidence supporting changes in mathematical reasoning following participation in the AI-supported learning framework. This

combination of descriptive, latent profile, transition, predictive, and Bayesian analyses provided a comprehensive evaluation of cognitive development and the mechanisms underlying mathematical reasoning progression.

Results

4.1 Changes in Mathematical Reasoning Following AI-Supported Learning

Initial analyses revealed a consistent upward shift across all dimensions of mathematical reasoning following the implementation of the AI-supported learning framework. Although the magnitude of improvement varied across domains, the overall pattern suggested that cognitive gains were not restricted to a single aspect of reasoning. A detailed comparison of the pretest and posttest performance results is presented in Table 4.

Table 4
Descriptive Statistics of Mathematical Reasoning Domains Before and After AI-supported Learning

Domain	Min	Q1	Median	Mean	Q3	Max	
Pretest							
Pattern Recognition	1.000	2.333	3.000	2.968	3.667	5.000	
Conjecturing	1.000	2.333	3.000	2.983	3.667	5.000	
Logical Reasoning	1.000	2.333	3.000	2.979	3.667	5.000	
Mathematical Argumentation	1.000	2.333	3.000	2.975	3.667	5.000	
Generalization	1.000	2.333	3.000	3.025	3.667	5.000	
Verification	1.000	2.333	3.000	2.971	3.667	5.000	
Mathematical Modeling	1.000	2.333	3.000	2.981	3.667	5.000	
Strategic Thinking	1.000	2.333	3.000	2.943	3.667	5.000	
Domain	**Min**	**Q1**	**Median**	**Mean**	**Q3**	**Max**	**Δ Mean**
Posttest							
Pattern Recognition	1.000	3.000	3.667	3.531	4.333	5.000	0.563
Conjecturing	1.333	3.000	3.667	3.509	4.000	5.000	0.526
Logical Reasoning	1.000	3.000	3.667	3.523	4.000	5.000	0.544
Mathematical Argumentation	1.000	3.000	3.333	3.482	4.000	5.000	0.507
Generalization	1.333	3.000	3.667	3.512	4.000	5.000	0.487
Verification	1.000	3.000	3.667	3.528	4.000	5.000	0.557
Mathematical Modeling	1.333	3.000	3.667	3.528	4.333	5.000	0.547
Strategic Thinking	1.333	3.000	3.667	3.519	4.333	5.000	0.576
Note.							

All scores were measured on a five-point scale.

where higher scores indicate stronger mathematical reasoning.

The results demonstrate a uniform pattern of improvement across all eight reasoning domains. Gains were observed not only in the mean scores but also in the upward movement of medians and upper quartiles, indicating that progress occurred throughout the distribution rather than among a limited subset of participants. This pattern suggests that the intervention supported broad-based cognitive development and facilitated growth across multiple dimensions of mathematical thinking.

The largest increase was observed in *strategic thinking*, whereas *generalization* exhibited the smallest, although still meaningful, improvement. The relatively balanced growth across domains indicates that the framework promoted a comprehensive development of reasoning rather than strengthening isolated skills. Students appeared to be more capable of selecting appropriate solution strategies, evaluating mathematical arguments, and connecting ideas across different problem contexts.

A closer examination of the domain-specific changes further reveals that higher-order reasoning processes benefit substantially from AI-supported learning. Improvements in *verification, mathematical modeling, and logical reasoning* suggest that students increasingly engage in analytical and evaluative forms of thinking. Such developments are particularly important because these domains represent essential components of advanced mathematical competence and are often associated with successful problem solving in unfamiliar situations.

4.2 Identification of Latent Mathematical Reasoning Profiles

Improvements in the mean scores provide initial evidence of learning gains following the intervention. These findings, however, do not reveal whether students progressed through similar developmental pathways or formed distinct patterns of reasoning. Model evaluation indicated that a four-class solution provided the most appropriate representation of heterogeneity in the data, as summarized in Table 5. The emergence of four latent profiles suggests that mathematical reasoning develops through empirically distinguishable stages rather than as a single continuous progression.

Table 5
Latent Class Model Selection Statistics

Time Point	Model	Classes	Log-Likelihood	BIC	ICL
Pretest (T1)	VII	4	-3199,223	-6640,815	-6651,134
Posttest (T2)	VII	4	-3159,481	-6561,331	-6582,008

The stability of the four-class structure across both measurement occasions strengthens confidence in the robustness of the identified profiles. Rather than reflecting statistical artifacts, the classes appear to capture meaningful differences in students' reasoning capabilities. A more detailed examination of profile characteristics is presented through the domain-level patterns displayed in Fig. 2.

Profile labels were assigned on the basis of domain-specific mean patterns and theoretical descriptions of reasoning development reported by Lithner (2008) and Schoenfeld (2016). Clear separation among profiles can be observed across all reasoning domains. Students in the highest-performing profile consistently demonstrated strong performance on every indicator, whereas those in the lowest profile exhibited comparatively limited reasoning capabilities throughout the domain structure. The substantial distance between profiles indicates that the observed differences represent qualitatively distinct levels

of cognitive development rather than minor variations in performance. This pattern supports the interpretation of mathematical reasoning as a multidimensional and developmental construct.

4.3 Distribution of Mathematical Reasoning Profiles Across Measurement Occasions

Changes in average performance do not necessarily imply shifts in the overall cognitive composition of the student population. Examining class membership distributions provides additional insight into how students were redistributed across reasoning profiles following the intervention.

Table 6
Class Membership Distribution

Latent Class	Label	T1_n	T1_%	T2_n	T2_%
1	Novice Reasoners	193	38,6	47	9,4
2	Developing Reasoners	157	31,4	169	33,8
3	Strategic Reasoners	96	19,2	173	34,6
4	Advanced Reasoners	54	10,8	111	22,2

One of the most notable findings is the substantial reduction in the proportion of *novice reasoners*. This decline was accompanied by marked increases in the *strategic* and *advanced reasoner* groups, a pattern clearly reflected in Table 6. The redistribution suggests that the intervention was associated not only with higher scores but also with movement toward more sophisticated forms of mathematical reasoning. The magnitude of these shifts indicates that cognitive development occurred across a broad segment of participants rather than being confined to a small subgroup of high achievers. The findings therefore suggest that the AI-supported learning framework facilitated meaningful progression toward higher-order reasoning profiles at the population level.

4.4 Cognitive Progression Pathways between Reasoning Profiles

The observed redistribution of reasoning profiles raises important questions regarding the pathways through which students advanced during the intervention period. Transition analysis provides a more detailed perspective on how movement occurs across different stages of reasoning development.

The transition probabilities were concentrated primarily along pathways, leading to more advanced reasoning profiles. Several cells displayed markedly higher probabilities than others did, indicating that progression followed systematic developmental trajectories rather than random movement between profiles. The evidence from Fig. 3 suggests that students were substantially more likely to advance to higher reasoning levels than to remain stagnant or regress.

A more intuitive representation of these transitions is provided through the flow-based visualization shown in Fig. 4.

The strongest flows occurred from *Novice* to *Developing*, followed by transitions from *Developing* to *Strategic* and from *Strategic* to *Advanced*. These patterns reveal a gradual and cumulative progression process whereby students move through increasingly sophisticated levels of reasoning over time. Only a relatively small proportion remained in their original profiles, whereas the dominant trend reflected upward cognitive mobility. Such evidence aligns with developmental perspectives that view reasoning growth as a staged process built upon prior cognitive foundations.

4.5 Role of artificial intelligence in cognitive progression

The substantial profile transitions observed throughout the intervention period prompted further investigation into the specific dimensions of AI engagement associated with cognitive advancement. Multinomial modeling was therefore employed to identify which forms of AI use were most strongly linked to movement toward higher reasoning profiles.

Table 7
Multinomial Effects of AI Variables on Cognitive Progression

AI Factor	Developing OR	Strategic OR	Advanced OR
AI1 Frequency	0,751	0,852	0,677
AI2 Problem Solving	0,851	0,706	0,638
AI3 Reflection	1,548	1,346	1,544
AI4 Feedback Use	0,892	0,73	0,663
AI5 Critical Evaluation	2,028	1,54	1,501

Critical evaluation of AI-generated outputs emerged as the most influential predictor of cognitive progression. Its effect exceeds those associated with simple usage frequency or task-oriented interactions with AI. The results summarized in Table 6 indicate that the educational value of AI depends less on exposure to technology itself and more on the quality of the cognitive engagement that accompanies its use.

Reflection on AI-generated responses also demonstrated a consistently positive association with profile advancement. Together, these findings suggest that students benefit most when AI functions as a catalyst for reflection, evaluation, and reasoning rather than as a mechanism for obtaining immediate answers. The results highlight the importance of metacognitive engagement in transforming AI use into meaningful learning opportunities.

The relative importance of AI-related factors is further illustrated through odds ratio visualization. Both *critical evaluation and reflection* occupy the most prominent positions in Fig. 5, consistently outperforming other dimensions of AI engagement. Their dominance reinforces the argument that

higher-order cognitive interaction with AI constitutes the primary mechanism through which mathematical reasoning develops within the proposed framework.

4.6 Bayesian evaluation of AI-supported learning effectiveness

The collective findings point toward a substantial impact of the intervention on students' mathematical reasoning. To evaluate the strength of the evidence supporting this conclusion, Bayesian inference was conducted via paired comparisons of the pretest and posttest reasoning scores.

Table 8
Bayesian Pretest–Posttest Comparison

Outcome Variable	BF_{10}	Prior (r)	Evidence Category
Mathematical Reasoning	2.13×10^{48}	0.707	Extreme Evidence for H_1

The resulting Bayes factor provided overwhelming support for the alternative hypothesis. This exceptionally large value indicates that the observed improvement is far more consistent with the presence of a genuine intervention effect than with random variation alone. The evidence reported in Table 8 therefore offers strong inferential support for the effectiveness of the AI-supported learning framework in enhancing mathematical reasoning. Additional confirmation is provided through the distributional patterns visualized in the final figure.

The surface distribution reveals a noticeable shift in score density toward higher levels of mathematical reasoning following the intervention. A greater concentration of observations in the upper score range suggests that improvement occurred across the broader distribution rather than being driven by a limited number of participants. This visual pattern complements the Bayesian findings and the transition analyses, collectively indicating substantial cognitive gains associated with the implementation of the AI-supported learning framework.

Discussion

The principal finding of this study is that participation in the AI-supported learning framework was associated with substantial progression toward higher mathematical reasoning profiles. The transition patterns revealed that a considerable proportion of the students moved from novice and developing profiles to strategic and advanced profiles following the intervention. This pattern suggests that the framework influences the organization of mathematical thinking rather than merely improving task performance. Constructivist theory proposes that learning occurs through the active restructuring of cognitive schemas as learners encounter new experiences and resolve conceptual conflicts (Piaget, 1970; Vygotsky & Cole, 1978). AI-supported interactions appear to have created such opportunities by exposing students to alternative explanations, multiple solution pathways, and competing mathematical arguments. Similar findings have been reported by Y.-L. E. Liu et al. (2026), who reported that AI-

enhanced environments support higher-order thinking when learners actively engage with generated content. The present study extends this literature by demonstrating that AI-supported learning contributes to developmental movement across latent reasoning profiles, providing evidence that mathematical reasoning evolves through identifiable stages of cognitive sophistication rather than through uniform gains in achievement alone.

Improvement was observed across all eight dimensions of mathematical reasoning, including pattern recognition, conjecturing, logical reasoning, argumentation, generalization, verification, modeling, and strategic thinking. This consistency indicates that reasoning development occurred as an integrated cognitive process rather than as isolated skill acquisition. Contemporary theories of mathematical cognition emphasize that reasoning emerges from coordinated interactions among multiple cognitive processes that support problem representation, justification, and strategic decision making (Silver, 2013). Previous studies have frequently reported improvements in specific reasoning skills following technology-supported instruction (Hughes et al., 2020); however, evidence regarding simultaneous growth across multiple reasoning domains remains limited. The present findings suggest that AI-supported learning environments are most effective when they encourage learners to connect diverse forms of mathematical thinking. Mathematics instruction therefore benefits from AI designs that promote interconnected reasoning activities rather than narrowly focused procedural practice.

Another important finding concerns the identification of four distinct reasoning profiles that remained theoretically coherent across measurement occasions. The existence of these profiles indicates that learners differ not only in achievement level but also in the structure and sophistication of their reasoning processes. Developmental theories of cognition propose that learners progress through increasingly complex forms of reasoning as they integrate new knowledge with existing conceptual structures (Vosniadou, 2014). Earlier research has documented heterogeneous learning trajectories in mathematics education, although these studies have typically relied on achievement scores rather than latent cognitive profiles (Clements & Sarama, 2025). By examining transitions between reasoning profiles, the present study introduces a developmental perspective to AI-supported mathematics learning. This contribution provides a richer account of cognitive change and highlights the value of profile-based approaches for understanding how learners evolve over time.

Critical evaluation emerged as the strongest predictor of progression toward advanced reasoning profiles. Students who systematically questioned, verified, and scrutinized AI-generated responses demonstrated greater cognitive advancement than those who relied primarily on AI-generated solutions. This finding challenges the common assumption that educational benefits increase automatically with greater AI exposure. Instead, cognitive advancement appears to depend on how learners interrogate AI-generated information rather than how frequently they access it. One possible explanation is that critical evaluation requires learners to compare AI-generated information against existing conceptual knowledge, creating productive cognitive conflict that stimulates deeper reasoning development.

This finding suggests that the educational value of artificial intelligence lies not in answer generation but in the evaluative processes stimulated by interactions with those answers. Metacognitive theory identifies monitoring and evaluation as central mechanisms through which learners regulate their understanding and refine their cognitive strategies (Flavell, 1979). Similar conclusions were reported by Pickering & Swinnerton (2019), who argued that educational outcomes depend more heavily on learner engagement than on technological sophistication. The present study advances this perspective by providing empirical evidence that critical evaluation functions as a key mechanism linking AI engagement to mathematical reasoning development. Educational activities that require learners to verify claims, identify inconsistencies, and critique AI-generated explanations should therefore occupy a central position within AI-supported mathematics instruction.

Reflective engagement also plays a substantial role in explaining cognitive progression. Students who reflected on AI-generated explanations, compared with alternative solution pathways, and reconsidered their own assumptions were more likely to transition into higher reasoning profiles. Reflection appears to transform AI interactions into opportunities for metacognitive monitoring and conceptual refinement. Rather than accepting AI outputs passively, learners engage in cycles of interpretation, evaluation, and revision that strengthen their mathematical understanding. This interpretation aligns with self-regulated learning theory, which views reflection as a mechanism for integrating new information with existing knowledge structures and improving adaptive expertise (Kuiper & Pesut, 2004). Previous studies have similarly identified reflection as a critical factor in successful AI-supported learning environments (Yun, 2024). A notable contribution of the present study is the demonstration that reflective engagement is associated not only with improved performance but also with developmental progression across reasoning profiles.

Bayesian evidence, latent profile transitions, and multinomial modeling converge to support the effectiveness of the proposed framework. The consistency of findings across analytical approaches strengthens confidence that the observed changes reflect genuine cognitive development rather than measurement artifacts. More importantly, the results illuminate the mechanism through which AI contributes to mathematical reasoning. AI-generated explanations and alternative solutions created opportunities for reflection and critical evaluation, which subsequently stimulated metacognitive monitoring and reasoning growth. Artificial intelligence therefore functions as a cognitive partner rather than an automated tutor. This interpretation aligns with emerging perspectives in AI in education that emphasize human-AI collaboration as a process of coconstructing understanding rather than outsourcing cognition (Edwards et al., 2025). Future AI-supported learning environments should consequently prioritize reflection, evaluation, and intellectual engagement as core design principles capable of fostering deeper mathematical reasoning.

6. Theoretical and Practical Contributions

The present study contributes to the literature on artificial intelligence in education in three important ways. First, it reconceptualizes mathematical reasoning as a developmental construct rather than a

static achievement outcome. Existing studies frequently evaluate AI effectiveness via performance indicators or test scores, whereas the present study demonstrates that reasoning development can be understood as movement across qualitatively different cognitive profiles. This perspective extends the current understanding of mathematical reasoning by emphasizing developmental progression rather than isolated performance gains.

Second, the study introduces cognitive progression pathways as a framework for investigating learning processes in AI-supported environments. Previous research has focused predominantly on whether AI improves learning outcomes, whereas limited attention has been given to understanding how learners transition between different stages of cognitive development. By integrating latent profile analysis and transition modeling, this study provides empirical evidence that AI-supported learning is associated with systematic movement toward more sophisticated forms of reasoning. This study offers a novel theoretical and methodological perspective for examining cognitive development in technology-enhanced learning environments.

Third, the findings identify reflection and critical evaluation as the primary mechanisms underlying mathematical reasoning development. Research on artificial intelligence in education often emphasizes technological capabilities, yet the present findings demonstrate that educational benefits emerge primarily through learners' active engagement with AI-generated content. Reflection and critical evaluation were more influential than the simple frequency of AI use. This finding advances contemporary theories of AI-supported learning by highlighting metacognitive engagement as the critical mechanism connecting artificial intelligence with higher-order reasoning development.

From a practical perspective, the findings indicate that effective AI-supported mathematics instruction should move beyond answer generation and emphasize activities requiring learners to evaluate, justify, compare, and refine mathematical ideas. Teachers should design learning experiences that position AI as a cognitive partner rather than an automated solution provider. Such an approach can foster deeper reasoning, strengthen metacognitive regulation, and support sustained cognitive development across diverse mathematical domains.

Unlike prior studies that conceptualize mathematical reasoning as an achievement outcome, the present study conceptualizes reasoning as a dynamic developmental process characterized by transitions across latent cognitive profiles. Methodologically, this study demonstrates the value of integrating latent profile analysis, transition modeling, multinomial prediction, and Bayesian inference to examine cognitive development in AI-supported learning environments.

7. Educational Implications

7.1 Implications for Teachers

These findings indicate that effective AI-supported mathematics instruction depends less on the frequency of AI use and more on the quality of learner engagement. Teachers should therefore design classroom activities that require students to interrogate, evaluate, and justify AI-generated responses rather than merely obtaining answers. Learning tasks should incorporate structured questioning, error detection, comparisons of alternative solutions, and reflective discussion. Such practices can encourage students to develop habits of critical inquiry and strengthen their capacity to construct mathematical arguments independently. AI should consequently be positioned as a tool for stimulating reasoning rather than as a substitute for mathematical thinking.

7.2 Implications for Curriculum Designers

Curriculum developers should integrate artificial intelligence into mathematics curricula through reasoning-oriented learning experiences rather than through technology-oriented activities. The present findings suggest that reflection, verification, and critical evaluation are essential components of effective AI-supported learning. Curriculum frameworks should therefore embed opportunities for students to examine assumptions, compare competing explanations, evaluate the validity of mathematical claims, and justify conclusions via evidence. Learning outcomes should explicitly address metacognitive competencies alongside mathematical content knowledge. This approach can support the development of higher-order reasoning skills while ensuring that AI use remains aligned with broader educational objectives.

7.3 Implications for AI developers

The results provide important guidance for the design of future educational AI systems. Current AI platforms often prioritize efficiency, answer generation, and rapid problem solving. The findings from this study suggest that educational value emerges when AI systems encourage learners to reflect, critique, and evaluate generated outputs. Developers should therefore incorporate features that prompt justification, request alternative interpretations, highlight potential errors, and encourage users to explain their reasoning processes. Compared with systems optimized solely for answer accuracy, AI systems designed around cognitive engagement are more likely to foster meaningful learning. Such design principles can strengthen the role of artificial intelligence as a cognitive partner in educational settings.

7.4 Implications for educational policymakers

Educational policies concerning artificial intelligence should move beyond concerns related solely to access and technological adoption. Policy frameworks should emphasize pedagogical quality, critical AI literacy, and the responsible educational use of generative AI technologies. Professional development programs should prepare teachers to integrate AI into learning activities that promote reasoning, reflection, and evaluation. Assessment systems should also recognize higher-order thinking skills that emerge through human-AI interactions rather than focusing exclusively on procedural performance. Investment in AI-supported education should therefore prioritize instructional models that cultivate cognitive development, metacognitive regulation, and intellectual independence among learners. Such

policies can help ensure that artificial intelligence contributes to educational improvement while preserving the central role of human reasoning in learning.

Conclusion

The findings demonstrate that the AI-supported learning framework was associated with meaningful improvements in mathematical reasoning and facilitated progression toward more advanced cognitive profiles. Evidence from latent profile transitions, multinomial modeling, and Bayesian inference consistently indicated that students developed stronger reasoning capabilities following the intervention. Critical evaluation and reflective engagement emerged as the most influential factors associated with cognitive progression, suggesting that the educational value of artificial intelligence depends primarily on the quality of learners' intellectual interaction with AI-generated content rather than the frequency of technology use alone.

Several limitations should be acknowledged. The study was conducted within a single instructional framework and focused exclusively on mathematical reasoning, limiting the generalizability of the findings to other subject domains and educational contexts. Cognitive progression was examined across two measurement occasions, which restricted the ability to capture longer-term developmental trajectories. Additional contextual variables, such as teacher facilitation, prior mathematical achievement, and individual learning characteristics, were not incorporated into the analytical model and could influence reasoning development.

Future research should investigate the long-term sustainability of cognitive progression within AI-supported learning environments through multiwave longitudinal designs. Greater attention should also be given to examining how different forms of AI interaction influence specific reasoning processes across diverse educational settings. Integration of process data, learning analytics, and real-time interaction records would provide deeper insight into the mechanisms through which reflection, evaluation, and human-AI collaboration contribute to the development of advanced mathematical reasoning.

Declarations

Ethics Statement This study was reviewed and approved by the Institute for Research and Community Service (Lembaga Penelitian dan Pengabdian kepada Masyarakat, LP2M), Universitas Pekalongan, Indonesia. All research procedures were conducted in accordance with institutional ethical guidelines for research involving human participants.

Participant Consent Statement Written informed consent was obtained from all participants and their parents or legal guardians prior to participation in the study.

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Figures

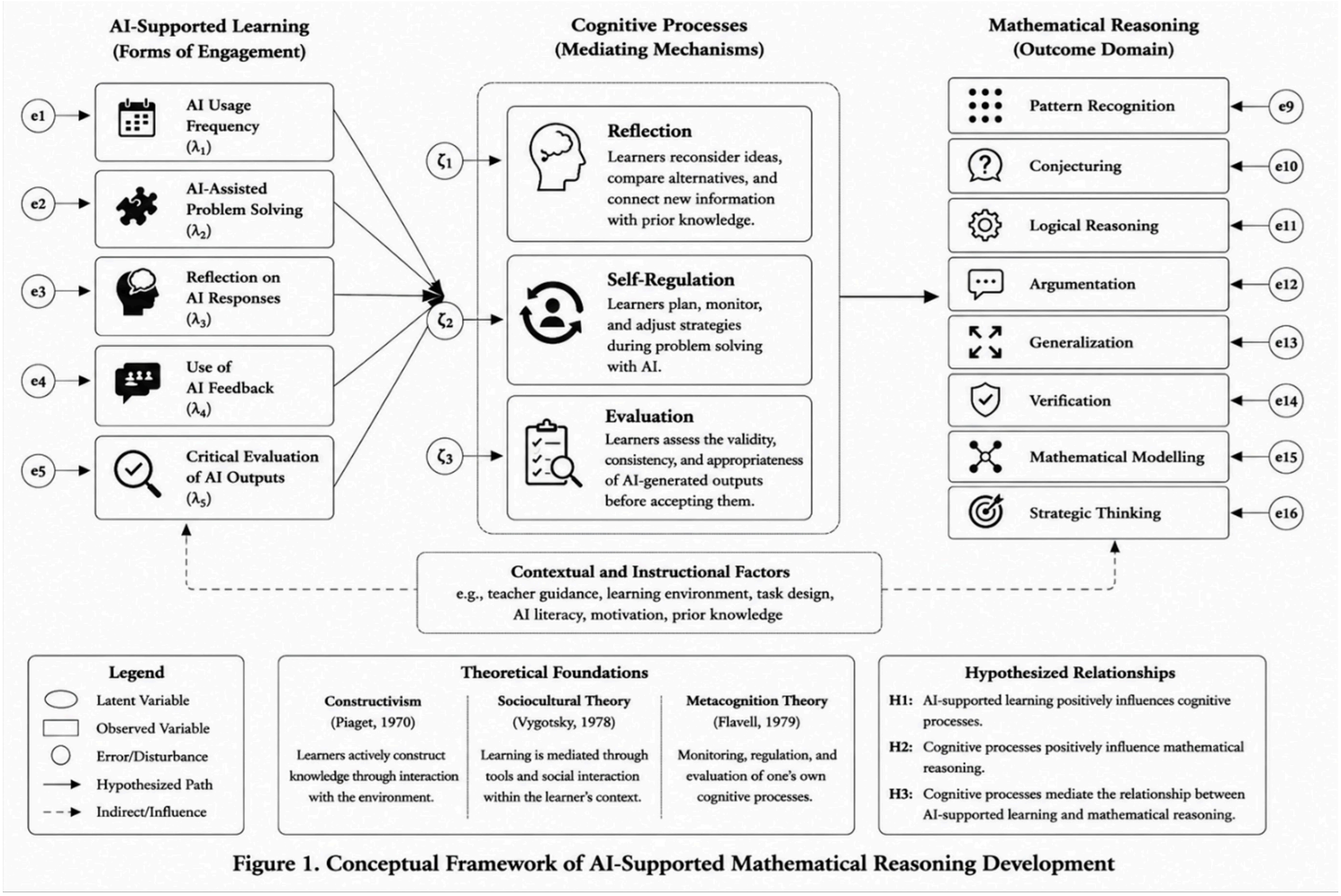


Figure 1. Conceptual Framework of AI-Supported Mathematical Reasoning Development

Figure 1

Conceptual framework illustrating the hypothesized relationships among AI-supported learning, metacognitive processes, and mathematical reasoning development. Reflection, self-regulation, and critical evaluation are proposed as mediating mechanisms linking AI engagement to higher-order mathematical reasoning.

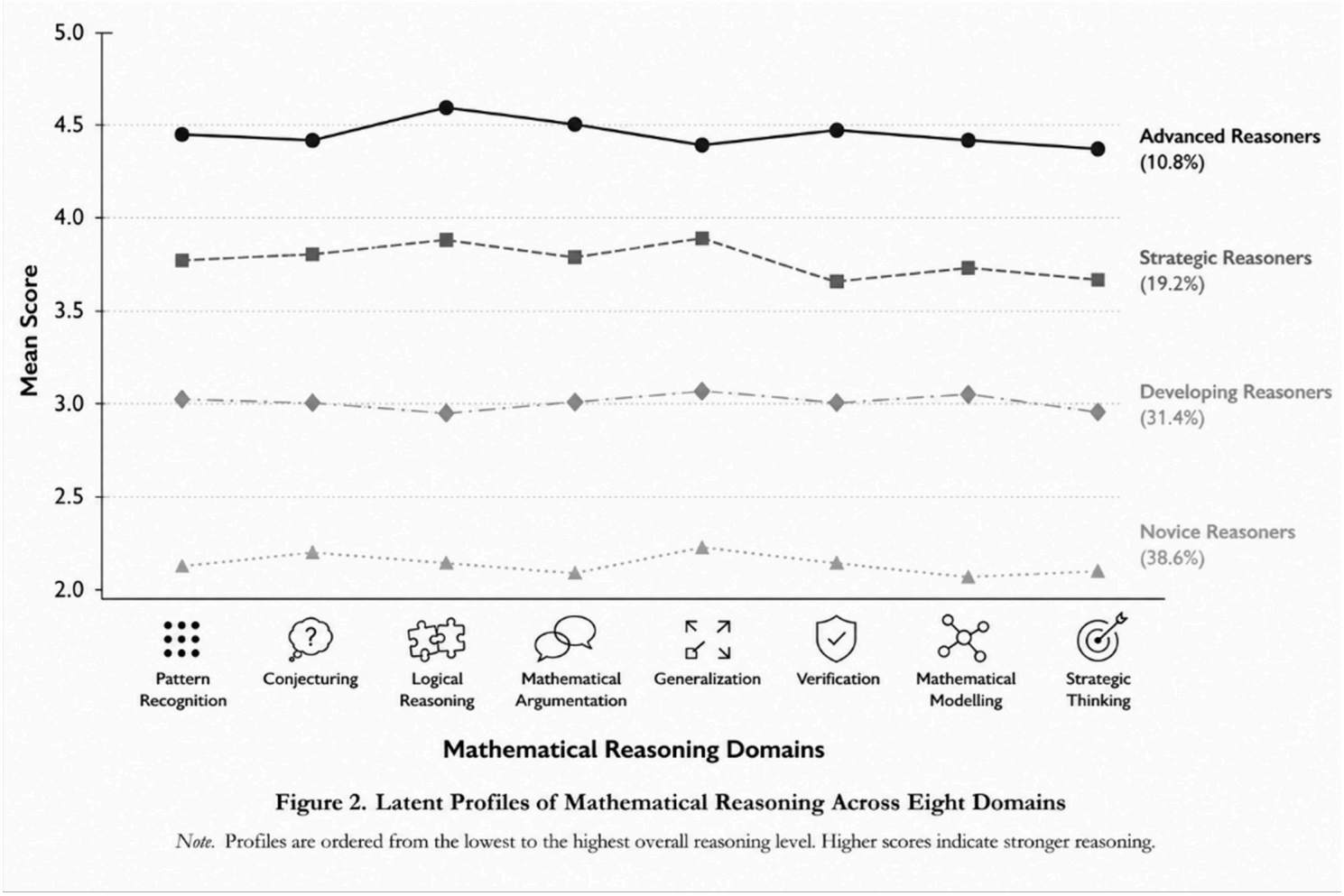


Figure 2. Latent Profiles of Mathematical Reasoning Across Eight Domains

Note. Profiles are ordered from the lowest to the highest overall reasoning level. Higher scores indicate stronger reasoning.

Figure 2

Latent mathematical reasoning profiles identified across eight reasoning domains at pretest. The profiles represent distinct levels of cognitive sophistication ranging from novice reasoners to advanced reasoners.

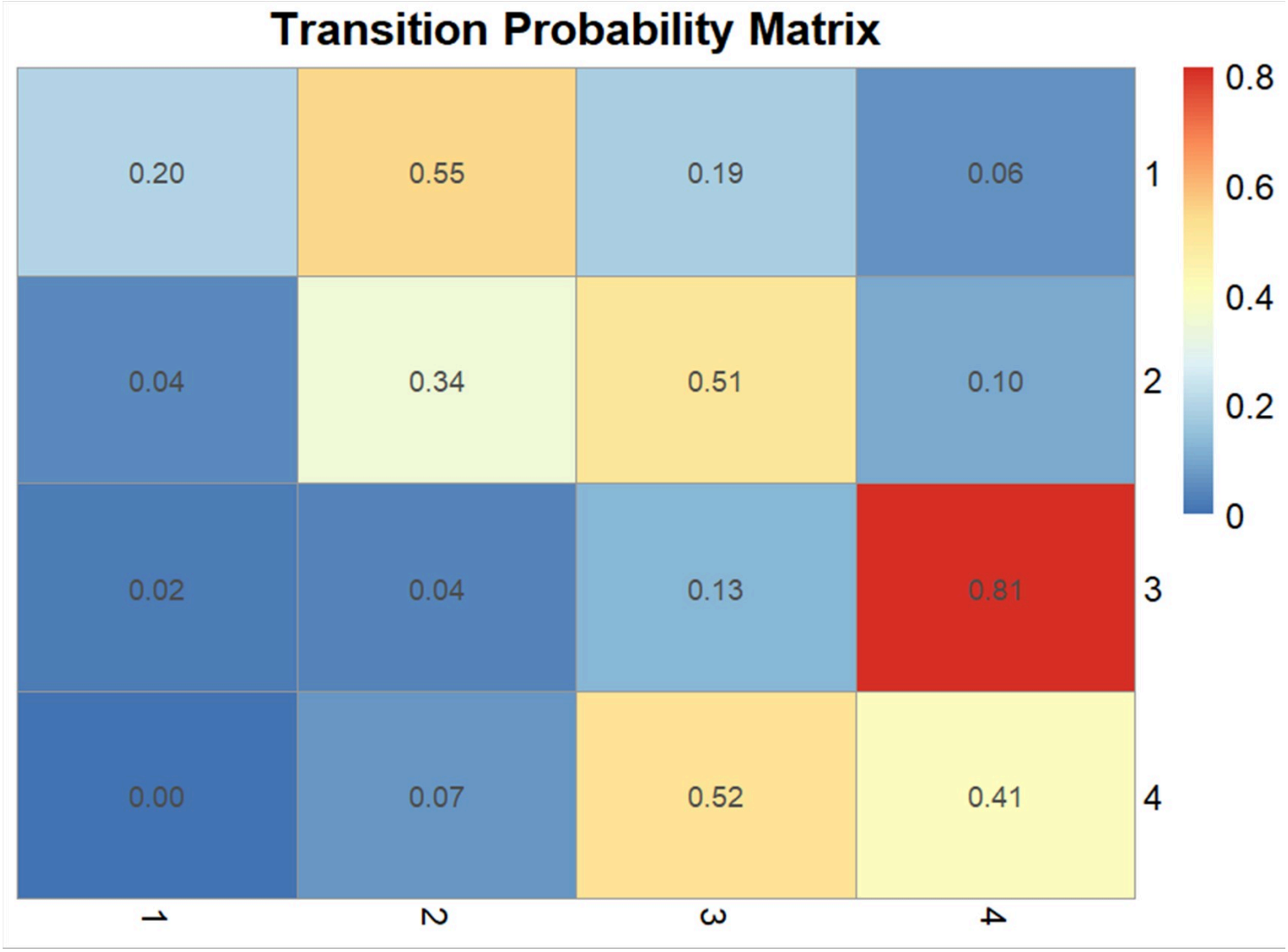


Figure 3

Heatmap of transition probabilities between latent mathematical reasoning profiles from pretest (T1) to posttest (T2). Warmer colors indicate higher probabilities of movement across reasoning profiles.

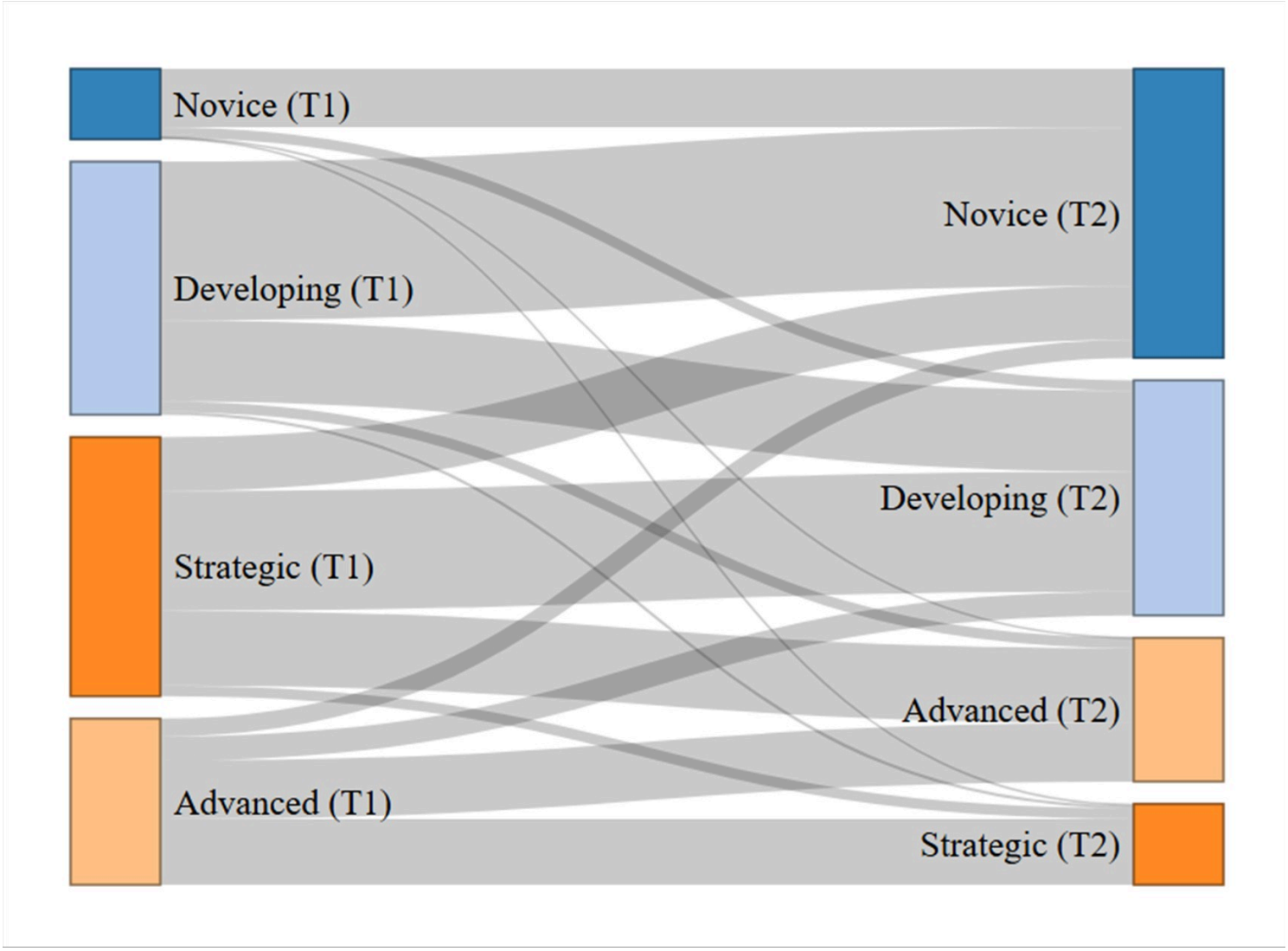


Figure 4

Cognitive progression pathways between latent mathematical reasoning profiles from pretest (T1) to posttest (T2). Flow widths represent the magnitude of student transitions across developmental reasoning stages.

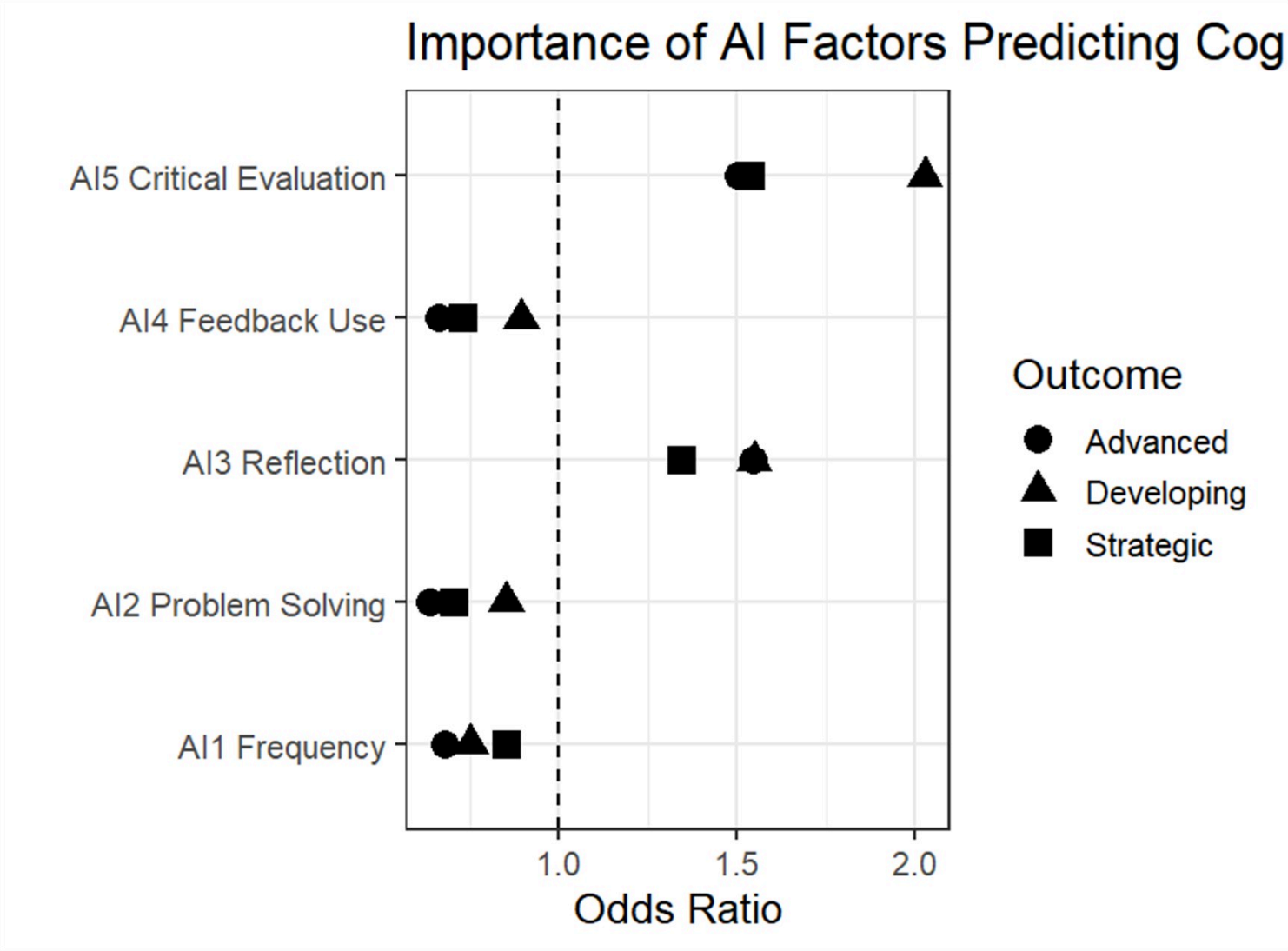


Figure 5

Relative importance of AI engagement factors in predicting progression toward higher mathematical reasoning profiles. Odds ratios are presented for developing, strategic, and advanced profile transitions.

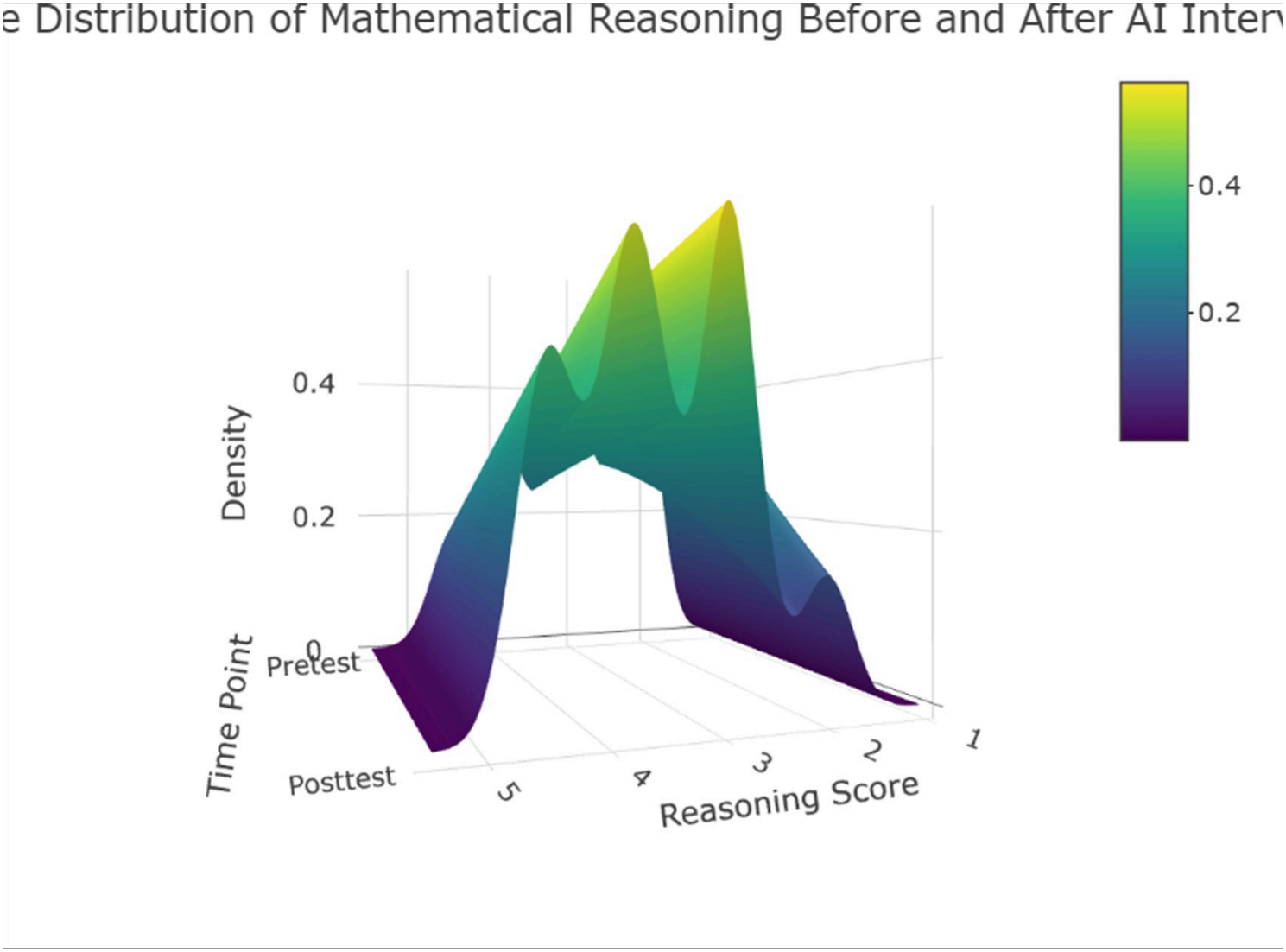


Figure 6

Three-dimensional surface distributions of mathematical reasoning scores before and after the AI-supported learning intervention. The surface illustrates shifts in score density and changes in the overall distribution of reasoning performance across measurement occasions.