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Rydberg Quasi-Contact Forces with Resonant Range Control via Fast Eigenstate Twisting

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Abstract – This article introduces a novel method to engineer sharply peaked, distance-selective interactions between neutral atoms by exploiting interaction-induced resonances within a resonantly driven Rydberg ladder system. By tuning laser parameters, a subsystem eigenstate twists rapidly and is brought into degeneracy with the atomic ground state at precisely defined interatomic separations, resulting in an effective fourth-order light-shift potential sharply peaked around this resonance distance. The linewidth of the resonant double excitation manifold defines the linewidth of the potential peak. Unlike previous off-resonant macrodimer-based schemes, our approach significantly enhances interaction strength, reaching MHz scales, and provides straightforward experimental tunability without requiring sub-wavelength positional control. Analytic expressions, validated through comprehensive master-equation simulations, detail the interaction profile’s amplitude, width, and resonant distance and robustness to laser fluctuations. This precise control facilitates parallel entangling gates crucial for measurement-based quantum computing and enables simulation of complex lattice Hamiltonians with customizable connectivity.

Introduction– Highly excited Rydberg atoms offer precisely controllable interactions that have substantial implications for advancing quantum technology. These interactions are inherently long-range, which can lead to problematic cross-talk between nonadjacent qubits, adversely affecting the fidelity of parallel quantum gates and complicating the implementation of quantum lattice models with customized connectivity. In conventional neutral-atom platforms, selective interactions require tightly focused beams, creating a trade-off between crosstalk, mode distortion, and defining a minimum lattice spacing. By contrast, encoding spatial selectivity directly in the interaction potential allows global or weakly focused fields to drive interactions only at the resonant distance, addressing the current bottleneck of scalability in atomic QPU. This approach calls for sharply peaked, distance-selective interactions that suppress unwanted long-range couplings while relaxing optical addressing constraints.

This property is particularly advantageous for performing parallel multi-qubit gates, especially those required for Measurement-based quantum computing (MBQC), which necessitates the rapid generation of large-scale, high-fidelity entanglement among a register of long-lived qubits. The distance-selective interaction allows parallel entangling operation with global pulses in the atomic lattice. The proposed interaction enables geometric entanglement filtering, expediting and parallelizing the syndrome extraction in surface error correction codes within a geometrically designed atomic lattice with global laser addressing. The proposed interaction enables the simulation of intricate lattice models, such as extended Hubbard, Ising, or Su–Schrieffer–Heeger (SSH) Hamiltonians, with customizable interaction graphs [1–3]. Consequently, such programmable interaction networks significantly enrich the exploration of quantum

phase transitions [4], topological states of matter [1, 2], quantum annealing processes [5], simulation of photosynthesis light harvesting [6], and Hopfield networks.

In the continuum limit, distance-selective interactions are analogous to zero-range potentials positioned at finite radii, known as *shell potentials*. Such δ -shell potentials exhibit distinctive scattering properties and are featured in exactly solvable scattering theory models [7–9]. Additionally, off-centered contact interactions have been theoretically explored for novel phenomena, such as controlled particle bunching and anti-bunching in a Harmonic trap [10], indicating promising opportunities for investigating unconventional scattering and bound-state physics at larger separations.

In this work, we introduce a novel method to realize distance-selective interactions by exploiting the fast twist of subsystem eigenstates in a resonantly driven Rydberg ladder configuration. Specifically, interaction-induced resonances form degeneracy between a manifold of doubly excited states and the ground state at precisely controlled interatomic distances, generating an effective potential sharply peaked at a target separation. Note that we are not using the anti-blockade mechanism, and even the two-Rydberg excitation at the interaction peak position may be absent if the peak lies within the blockade radius. More precisely, this peak corresponds to the potential experienced by the steady state of the driven, interacting, dissipative system, which appears as a Lorentzian potential that can even generate restoring forces, ensuring the persistence of trapping. We derive analytic expressions, validated by numerical simulations, that characterize the interaction amplitude, width, and resonant separation and demonstrate that these features are tunable through the driving laser parameters and robust against realistic laser fluctuations. Furthermore, we study three-body resonance peaks, illuminating the tech-

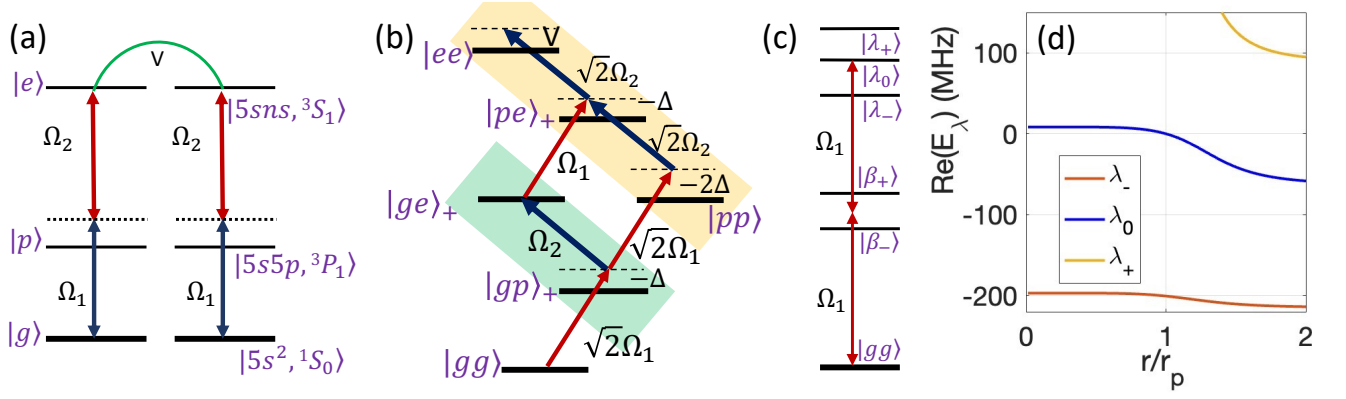


FIG. 1. **Level scheme for generating distance-selective, delta-function-like interactions.** (a) Two-photon resonant excitation of Rydberg atoms with strong interactions enables the engineering of tailored interaction profiles. Notably, in the regimes $\Omega_2/2\Delta > 1$ for positive detuning and $\Omega_2/2|\Delta| < 1$ for negative detuning, the resulting interaction acquires a sharply peaked, delta-function-like shape (see Fig. 2a). (b) This sharply localized interaction peak originates from an interaction-induced resonance. When $\Omega_1 \ll \Omega_2$, the two-atom Hilbert space naturally separates into three subspaces, i.e., the ground state, singly excited states (green boxes), and doubly excited states (yellow box), with transitions mediated by the weak Ω_1 field. (c) The interplay between strong coupling Ω_2 and interatomic interaction V becomes evident upon diagonalizing the singly and doubly excited subspaces. This yields the dressed eigenstates $|\beta_{\pm}\rangle$ and $|\lambda_{0,\pm}\rangle$, respectively. (d) The interaction shifts the $|\lambda_0\rangle$ state into resonance with the ground state at a specific interatomic distance r_p , see Eq. 5. This interaction-induced resonance sharply enhances the excitation probability (Fig. 2b), effectively producing an ultra-narrow, distance-selective interaction peak resembling a delta function (Fig. 2a).

nique that could be extended to higher-order many-body resonances. In particular, such genuine many-body resonances enable single-step, parallel stabilizer operations [15], direct implementation of multi-qubit gates [16], and the exploration of exotic many-body dynamics that cannot be reached through pairwise interactions alone.

Unlike previously studied soft-core potentials obtained through resonant Rydberg dressing [11–13], the current scheme operates in a distinct parameter regime to produce sharply localized, off-centered δ -function-like potentials, see discussion. Previous approach for making distance-selective interaction, applied off-resonant dressing of ground-state atoms into Rydberg macrodimer states [14], faced limitations including atom loss, continuum coupling, and demanding experimental controls. In contrast, our resonant scheme achieves orders-of-magnitude improvements in interaction sharpness and strength, simplifies experimental implementation, and enables precise, rapid tuning of the interaction profile without requiring stringent atomic position or orientation control, see discussion.

Scheme— We consider two strontium atoms, each driven by the resonant two-photon ladder scheme of Fig. 1a to the Rydberg state $|e\rangle$ via the intermediate state $|p\rangle$. In the rotating frame, the single-atom Hamiltonian reads

$$H_i = \frac{\Omega_1}{2} (\hat{\sigma}_{gp}^i + \hat{\sigma}_{pg}^i) + \frac{\Omega_2}{2} (\hat{\sigma}_{pe}^i + \hat{\sigma}_{ep}^i) - \Delta \sigma_{pp}^i, \quad (1)$$

with $\hat{\sigma}_{\alpha\beta} \equiv |\alpha\rangle\langle\beta|$, Rabi frequencies Ω_1 (ground– $|p\rangle$) and Ω_2 ($|p\rangle$ – $|e\rangle$), and detuning Δ from the intermediate level.

In the idealized, lossless limit, the system adiabatically follows the dark eigenstate $|d\rangle \propto \Omega_2|g\rangle - \Omega_1|e\rangle$, which acquires no light shift. For $\Omega_1 \ll \Omega_2$, atoms remain predominantly in $|g\rangle$ but are weakly dressed by the Rydberg state, with excited-state admixture $P_e \simeq (\Omega_1/\Omega_2)^2$. When two atoms are both in $|e\rangle$, they interact via the van der Waals potential $V_{ij} = \frac{C_6}{r_{ij}^6} \hat{\sigma}_{ee}^i \hat{\sigma}_{ee}^j$ where r_{ij} is their separation. To capture decoherence, we include spontaneous emission as the Lindblad operator $L_p = \sqrt{\gamma}|g\rangle\langle p|$ with $\gamma/2\pi = 7.6$ kHz for the $|p\rangle \rightarrow |g\rangle$ decay in ^{87}Sr .

At each interatomic distance, we solve the master equation to obtain the steady-state ρ_{ij} (the zero-eigenvalue right eigenvector of the Liouvillian superoperator) and define the light-shifted energy

$$U(r_{ij}) = \text{Tr}[\rho_{ij}(H_i + H_j + V_{ij})], \quad (2)$$

where $H_{\{i,j\}}$ are the single-atom Hamiltonians of Eq. 1, see the two-photon ladder scheme of Fig. 1(a), and V_{ij} is the van-der Waals interaction between Rydberg states. This $U(r)$ is the steady-state (driven–dissipative) internal energy and is the physically relevant observable under continuous driving and dissipation.

In the regime $\Omega_1 \ll \Omega_2$, the two-atom Hilbert space splits into three manifolds weakly coupled by Ω_1 . These manifolds are the ground state $|gg\rangle$, the single-excitation manifold spanned by $|gp\rangle_+ = (|gp\rangle + |pg\rangle)/\sqrt{2}$ and $|ge\rangle_+$ (highlighted by green in Fig. 1b), and the double-excitation manifold $\{|pp\rangle, |pe\rangle_+, |ee\rangle\}$ (highlighted by yellow). The strong coupling Ω_2 mixes the states in each subspace. Pre-diagonalizing the subsystems quantifies

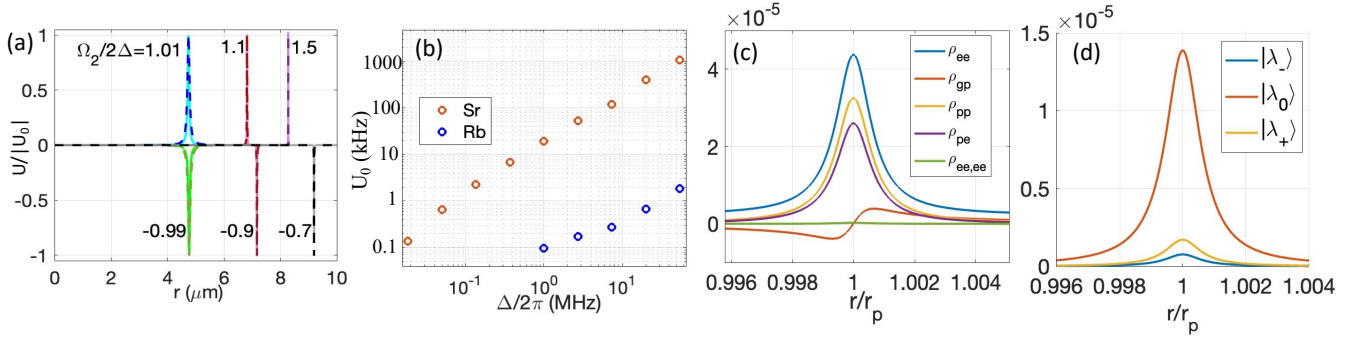


FIG. 2. **Interaction profile** (a) The analytic (dashed line) and numeric (solid line) interaction profile are presented based on Eq. 6 and 2, respectively. With Ω_1 and Δ held constant, varying Ω_2 tunes the amplitude, sign, and resonant separation r_p of the effective interaction $U(r)$. As explained in Eq. 8, deviation of $\Omega_2/2\Delta$ from 1 makes the spatial peak profile sharper and increases the contact interaction range r_p . This tunability allows designing different interaction graphs in a lattice. (b) Maximum interaction strength at a decoherence rate of $100s^{-1}$ achievable for Sr (upper circles) and Rb (lower circles). Here Ω_1 is tuned to adjust $100s^{-1}$ decoherence rate at the peak position. For Struntium, the applied parameter is in the $35\text{kHz} < \Omega_1/2\pi < 210\text{kHz}$ range. Interaction to loss ratio scales by Δ/γ . See below for applied parameters. (c) Distance-dependent steady-state populations (density-matrix elements) as a function of interatomic spacing r , see methods. (d) The Lorentzian peak emerges from the interaction-induced two-body resonance in the doubly-excited manifold (yellow subspace in Fig. 1b), which gets close to resonance with $|\lambda_0\rangle$ and hence the system energy at $r = r_p$ peaks due to fourth-order light-shift. Applied Parameters in (a,c,d): $\Delta/2\pi = 10$ MHz, $\Omega_1 = 200$ kHz, principal quantum number $n = 100$; panel (b,c,d) uses $\Omega_2/(2\Delta) = 1.1$.

the light-shifts experienced by the eigen-states at different interatomic separations, see Fig. 1c. In the single-excitation block, the coupling Hamiltonian in $\{|gp\rangle_+, |ge\rangle_+\}$ basis has eigenvalues $\beta_{\pm}/\hbar = -\frac{\Delta}{2} \pm \frac{1}{2}\sqrt{\Delta^2 + \Omega_2^2}$, which remain off-resonant with $|gg\rangle$.

By contrast, in the yellow shaded manifold of double-excitation subspace, the 3×3 Hamiltonian in $\{|pp\rangle, |pe\rangle_+, |ee\rangle\}$ basis

$$\frac{H_{\text{yellow}}}{\hbar} = \begin{pmatrix} -2\Delta - i\gamma & \sqrt{2}\Omega_2/2 & 0 \\ \sqrt{2}\Omega_2/2 & -\Delta - i\gamma/2 & \sqrt{2}\Omega_2/2 \\ 0 & \sqrt{2}\Omega_2/2 & V(r) \end{pmatrix} \quad (3)$$

supports $|\lambda_{\pm,0}\rangle$ eigenstates. The λ_0 eigenvalue is given by

$$\begin{aligned} E_{\lambda_0}(r) &= \mathcal{E}_{\lambda_0} + i\Gamma_0 \\ \mathcal{E}_{\lambda_0}(r) &= -\Delta + \frac{\Omega_2^2}{2} \left(\frac{1}{V(r) + \Delta} - \frac{\Delta}{\Delta^2 + \gamma^2/4} \right) \\ \Gamma_0 &= \frac{\Omega_2^2}{2} \frac{\gamma}{\Delta^2 + \gamma^2/4} \end{aligned} \quad (4)$$

Considering the space-dependent interaction $V = C_6/r^6$ and in the regime of $\Delta \gg \gamma$, Fig. 1d shows how $\text{Re}(E_{\lambda_0})$ crosses zero at the resonance condition

$$V(r_p) = \frac{2\Delta\Omega_2^2}{(\Omega_2^2 - 4\Delta^2 + \gamma^2)}. \quad (5)$$

At this separation $r = r_p$, the corresponding eigenstate $|\lambda_0(r_p)\rangle$ gets close to resonance with $|gg\rangle$, enhancing the fourth order lightshift to the ground state via $|gg\rangle \rightarrow |\beta_{\pm}\rangle \rightarrow |\lambda_0\rangle \rightarrow |\beta_{\pm}\rangle \rightarrow |gg\rangle$. The relevant steady-state

density-matrix elements are shown in Fig. 2(c), while the populations of the two-excitation subspace eigenstates are plotted in Fig. 2(d). These panels clearly demonstrate an enhanced population of the dominant two-excitation eigenstate at the resonance peak. Importantly, at the resonance point ($\mathcal{E}_{\lambda_0} = 0$), the broad linewidth of the resonant eigenstate ($\Omega_1 \ll \Gamma_0$) limits the population of $|\lambda_0\rangle$ and prevents a divergence of the fourth-order light-shift potential. Overall, this interaction-induced avoided crossing produces a distance-selective peak in the light shift, with a width set by the linewidth of $|\lambda_0\rangle$.

The solitary sharp peak happens in two regimes of parameters ($\Omega_2 > 2\Delta$ with $\Delta > 0$) and ($\Omega_2 < 2|\Delta|$ with $\Delta < 0$), see Fig. 2a. It forms a Lorentzian peak

$$U(r) = \frac{U_0}{1 + (r - r_p)^2/w^2}. \quad (6)$$

As the peak is formed by getting into resonance with the $|\lambda_0\rangle$ state, the width of the lightshift peak w is defined by the line width of this eigenstate (Γ_0). Considering the interatomic deviation of w from the resonance point r_p , the corresponding interaction induced levelshift of $\lambda_0(r_p \pm w) - \lambda_0(r_p) = \Gamma_0$, breaks the resonance. Hence, the potential width w is defined by

$$\begin{aligned} \left. \frac{d\lambda_0}{dr} \right|_{r_p} w = \Gamma_0 &\rightarrow w = \Gamma_0/\kappa \\ \kappa &= \frac{\Omega_2^2}{2(V + \Delta)^2} \left. \frac{dV}{dr} \right|_{r_p} = \frac{-3\Omega_2^2 V(r_p)}{r_p [V(r_p) + \Delta]^2}, \end{aligned} \quad (7)$$

which presents the HWHM of the Lorentzian interaction $U(r)$. Substituting $V(r_p)$ from Eq. 5, and considering

$$\gamma \ll \Delta$$

$$\frac{w}{r_p} = \frac{\gamma (3\Omega_2^2 - 4\Delta^2)^2}{12 \Delta \Omega_2^2 (\Omega_2^2 - 4\Delta^2 + \gamma^2)} \quad (8)$$

which shows that deviating from $\Omega_2^2 = 4\Delta^2$ reduces the w/r_p and makes the interaction more δ -function-like, see Fig. 2a for examples. Selecting a long-lived clock state, with a correspondingly small decay rate γ as the intermediate level, yields a much sharper resonance peak. The depth of interaction at the peak position is obtained perturbatively by deriving the steady state in the regime $\Omega_1/\Omega_2 \ll 1$ and $\Omega_1 \ll \gamma \ll |\Delta|$ as

$$U_0 = \frac{2 \Delta \Omega_1^4 [4 \Delta^2 (4 \Delta^2 + \gamma^2 + 3 \Omega_2^2) + \gamma^2 \Omega_2^2]}{\gamma^2 \Omega_2^2 [4 \Delta^2 \gamma^2 + (4 \Delta^2 + \Omega_2^2)^2]}, \quad (9)$$

with details explained in the Method section. Equations 5, 6, 8, and 9 together capture both the line shape and the maximum interaction strength in these two detuning regimes.

Having more than two atoms, the *many-body interaction* could also be engineered in this system. Fig. 3, studies an example of three-body interaction in an equilateral triangle geometry. The three-body interaction resonances emerge alongside the familiar two-body peaks in Fig. 3a. Under the collective-level scheme depicted in Fig. 3b, choosing $\Omega_1 \ll \Omega_2$ effectively divides the system into four decoupled subspaces. In this regime, the three-body interaction emerges under sixth-order light shift at a specific interatomic distance r_{p3} , where an eigenstate in the three excitation manifold marked by blue becomes resonant with the collective ground state, defined by

$$V(r_{p3}) = \frac{2\Delta^3 - 5\Delta\Omega^2 \pm \sqrt{4\Delta^6 - 4\Delta^4\Omega^2 + 9\Delta^2\Omega^4 + 4\Omega^6}}{2(2\Delta^2 - \Omega^2)}. \quad (10)$$

The fast twist of the resonant eigenstate in this manifold generates the sharp three-body interaction peak. By tuning parameters to access a specific many-body resonance, one can engineer tailored many-body connectivity within the lattice, ideal for the implementation of stabilizing operations.

Robustness to Experimental Uncertainties

We now analyze the robustness of the distance-selective interaction against fluctuations in the controlling laser parameters, specifically the detuning, Δ , and the Rabi frequencies, Ω_1 and Ω_2 . Our analysis quantifies how fluctuations, denoted as $\delta\Delta$, $\delta\Omega_1$, and $\delta\Omega_2$, propagate into the key figures of merit that characterize the interaction potential i.e., its peak position r_p , width w , and depth U_0 .

We first consider the *sensitivity of the peak interaction distance* r_p . This distance is determined by the resonance

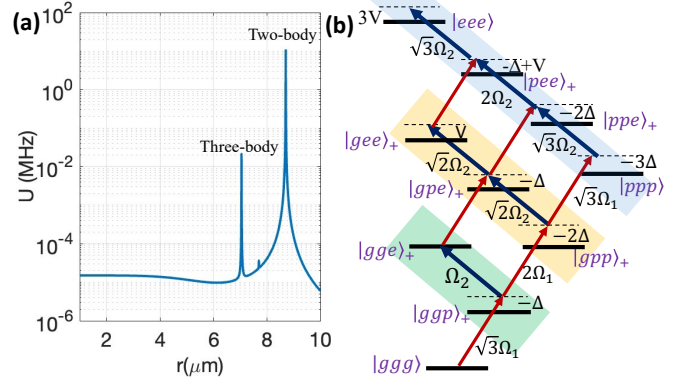


FIG. 3. **Three-Body Interaction** (a) Three- and two-body resonance peaks in the interaction profile of three atoms in an equilateral triangle geometry. (b) Level scheme in a three-atom collective basis. Here the Rydberg state $n=100$ is targeted by the driving parameters $\Omega_1/2\pi = 318\text{kHz}$, $\Delta/2\pi = 10\text{MHz}$, $\Omega_2 = 4\Delta$.

condition in Eq. 5 and is given by

$$r_p = \left[\frac{C_6(\Omega_2^2 - 4\Delta^2 + \gamma^2)}{2\Delta\Omega_2^2} \right]^{1/6} \quad (11)$$

By applying logarithmic differentiation of Eq. 5 and using the relation $\delta r_p/r_p = -(1/6) \delta V/V$ that is derived from $V(r_p) = C_6/r_p^6$, we obtain the fractional change in r_p with respect to fluctuations in Δ (at fixed Ω_2) and Ω_2 (at fixed Δ):

$$\begin{aligned} \frac{\delta r_p}{r_p} = & -\frac{1}{6} \left(1 + \frac{8\Delta^2}{\Omega_2^2 - 4\Delta^2 + \gamma^2} \right) \frac{\delta\Delta}{\Delta} \\ & -\frac{1}{3} \left(1 - \frac{\Omega_2^2}{\Omega_2^2 - 4\Delta^2 + \gamma^2} \right) \frac{\delta\Omega_2}{\Omega_2} \end{aligned} \quad (12)$$

To simplify the analysis, we introduce the dimensionless parameters $\epsilon \equiv \Omega_2^2/(2\Delta)^2 - 1$, which quantifies the deviation from the two-photon resonance condition, $\Omega_2 = 2\Delta$, at $\epsilon = 0$. In the desired operation regime $\gamma/\Delta \ll 1$, we define $\eta \equiv \gamma^2/\Delta^2 \approx 0$, and the sensitivity coefficients in Eq. (12) simplify to:

$$\frac{\delta r_p}{r_p} \approx -\frac{1}{6} \left(1 + \frac{2}{\epsilon + \eta} \right) \frac{\delta\Delta}{\Delta} - \frac{1}{3} \left(1 - \frac{1 + \epsilon}{\epsilon + \eta} \right) \frac{\delta\Omega_2}{\Omega_2} \quad (13)$$

Equation (12) reveals a critical divergence as $\epsilon \rightarrow -\eta \approx 0$, corresponding to the resonance condition. The sensitivity of the peak position is therefore minimized by operating away from this resonance.

Next, we analyze the *sensitivity of the interaction width* w . The relative interaction width, w/r_p , is given by Eq. 8 as:

$$\frac{w}{r_p} = \frac{\sqrt{\eta}(3\epsilon + 2)^2}{3(1 + \epsilon)(4\epsilon + \eta)} \quad (14)$$

At resonance ($\epsilon \approx 0$), the relative width reaches its maximum value, $(w/r_p)_{\max} \approx 4/(3\sqrt{\eta}) \approx 4\Delta/(3\gamma)$. As ϵ deviates from 0, w/r_p monotonically decreases, approaching the asymptotic value of $3\gamma/(4\Delta)$ for $\epsilon \gg 1$. Consequently, a desired relative width can be engineered by selecting the appropriate value of ϵ .

The fractional sensitivity of the width itself can be derived as

$$\frac{\delta w}{w} = \left[1 + \frac{8}{4\epsilon + \eta} - \frac{2(4\epsilon + \eta)}{4(1 + 3\epsilon) + \eta} \right] \frac{\delta\Delta}{\Delta} - \frac{4(1 + 3\epsilon) + \eta}{4\epsilon + \eta} \frac{\delta\Omega_2}{\Omega_2} \quad (15)$$

Finally, we examine the *sensitivity of the interaction depth* U_0 . The fractional change in U_0 , derived from Eq. 9 can be expressed as

$$\frac{\delta U_0}{U_0} = 4 \frac{\delta\Omega_1}{\Omega_1} + S_{\Delta}^{(U)} \frac{\delta\Delta}{\Delta} + S_{\Omega_2}^{(U)} \frac{\delta\Omega_2}{\Omega_2}. \quad (16)$$

Here, the sensitivity coefficients $S_{\Delta}^{(U)}$ and $S_{\Omega_2}^{(U)}$ are given by

$$S_{\Delta}^{(U)} = 1 + \frac{8\Delta^2(12\Delta^2 + \gamma^2 + 3\Omega_2^2)}{4\Delta^2(4\Delta^2 + \gamma^2 + 3\Omega_2^2) + \gamma^2\Omega_2^2} - \frac{8\Delta^2[\gamma^2 + 2(4\Delta^2 + \Omega_2^2)]}{4\Delta^2\gamma^2 + (4\Delta^2 + \Omega_2^2)^2}$$

$$S_{\Omega_2}^{(U)} = -2 + \frac{2\Omega_2^2(12\Delta^2 + \gamma^2)}{4\Delta^2(4\Delta^2 + \gamma^2 + 3\Omega_2^2) + \gamma^2\Omega_2^2} - \frac{4\Omega_2^2(4\Delta^2 + \Omega_2^2)}{4\Delta^2\gamma^2 + (4\Delta^2 + \Omega_2^2)^2}.$$

The sensitivity of the interaction position δr_p , width δw , and depth δU_0 diverges as the system approaches resonance, $\epsilon \approx 0$. This behavior reveals a fundamental experimental trade-off, where the choice of ϵ must simultaneously satisfy competing requirements. On one hand, the interaction width w should exceed the atomic localization length; on the other hand, the relative sensitivities $\delta w/w$ and $\delta r_p/r_p$ must remain sufficiently small to ensure high-fidelity operation. Beyond the choice of ϵ , the overall robustness is further limited by the intrinsic stability of the driving lasers. Taken together, these constraints, along with the target interatomic distance r_p , define the viable parameter regime of the system, including the principal quantum number n of the Rydberg state.

To illustrate the robustness argument in a distance-selective interaction in an optical lattice, we consider a typical lattice constant of $3 \mu\text{m}$, cooled to the motional ground state, with a Gaussian wave packet width that could be compressed up to $\sigma = 15\text{nm}$ in a normal optical lattice and beyond in Rydberg lattices [17]. To enable effective site-selective interactions, we fix the potential width to $w = 250\text{nm}$. This value is sufficiently narrow to prevent crosstalk between adjacent sites, yet sufficiently wide to ensure a uniform interaction over the atomic wave

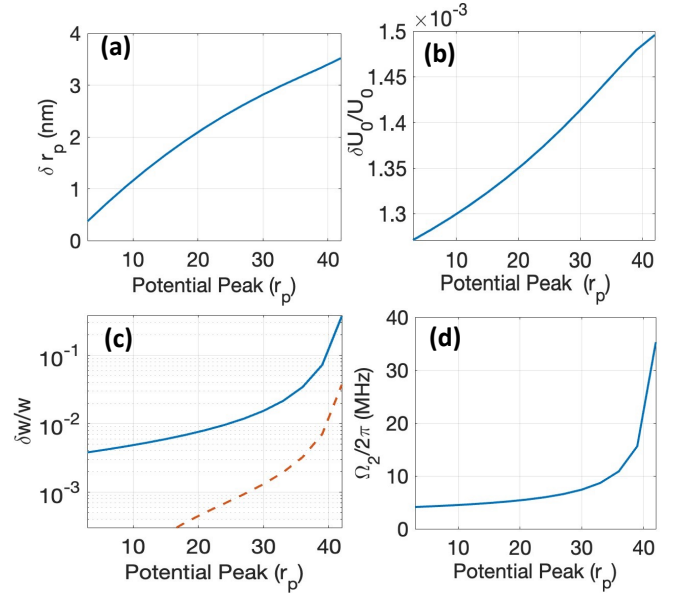


FIG. 4. Robustness of the interaction potential against laser parameter fluctuations. The plots show the calculated fluctuations in (a) the peak position, δr_p , (b) the relative interaction depth, $\delta U_0/U_0$, and (c) the relative interaction width, $\delta w/w$, as a function of the target peak distance r_p . The calculations assume fractional Rabi frequency fluctuations of $\delta\Omega/\Omega = 10^{-4}$ (10^{-5} for the red dashed line) and an absolute detuning fluctuation of $\delta\Delta = 2\pi \times 500\text{Hz}$. The interaction width is held constant at $w = 250 \text{ nm}$ to ensure uniform coupling across a delocalized atomic wave packet ($\sigma = 15 \text{ nm}$) while avoiding crosstalk, considering a lattice constant of $3\mu\text{m}$. (d) For each point, the required value of Ω_2 is determined via Eq. (14) to satisfy the fixed-width condition.

packet, thereby mitigating the effects of finite atomic localization. We assume experimentally realistic laser stability parameters, with fractional Rabi frequency fluctuations of $\delta\Omega/\Omega \approx 10^{-4}$ and detuning fluctuations of $\delta\Delta \approx 2\pi \times 500\text{Hz}$. Figure 4 shows the resulting variations in the potential characteristics (r_p , U_0 , w) as a function of the target interaction distance r_p . As r_p increases, the sensitivity of the potential parameters to laser fluctuations becomes more pronounced. Nevertheless, even at large separations such as $r_p = 42\mu\text{m}$, the system remains remarkably robust, e.g., a positional uncertainty of $\delta r_p = 3\text{nm}$ leads to only a 0.01% variation in the interaction strength experienced by the atom. Similarly, a substantial width uncertainty of $\delta w/w = 0.4$ results in only a 0.1% change in the on-site interaction, while the average crosstalk to neighboring atoms remains on the order of $10^{-3}U_0$. These results demonstrate that the proposed scheme enables robust, long-range, and spatially selective interactions, even in the presence of realistic laser noise, and is therefore well-suited for implementation with current state-of-the-art laser systems.

Discussion— Comparing the current approach with the previous realization of distance-selective interactions

based on off-resonant coupling to Rydberg macrodimers [14], several distinct advantages emerge. The *potential width* in our scheme is controlled by the laser parameters and the intermediate state linewidth. In contrast, the macrodimer-based scheme exhibits a finite-width peak, fundamentally limited by the macrodimer vibrational wavefunction and atomic motional states, thus requiring sub-wavelength control of atom positions.

Due to direct resonance-enhanced coupling at the target distance r_p , our method achieves substantially higher *potential depth*, potentially reaching the MHz scale. This enhancement represents orders-of-magnitude improvement over the macrodimer-based dressing approach, which yielded moderate couplings on the order of a few hundred hertz, inherently limited by its second-order, off-resonant nature.

Our technique offers flexible and straightforward *resonant distance tunability* r_p via adjustments of laser detunings or the selection of different Rydberg pair resonances. We provide analytical expressions explicitly relating system parameters to r_p , enabling precise targeting of nearest-neighbor or next-nearest-neighbor interactions. The macro-dimer scheme, on the other hand, determines r_p by the fixed macrodimer bond length of the chosen vibrational state, making adjustments more cumbersome and necessitating the selection of different molecular states or atomic species.

Our resonant ladder scheme allows *high-precision control* over the interaction via laser intensity and detuning, enabling rapid switching and fine-tuning of the interaction peak position, width, and strength. In comparison, the macrodimer scheme's spatial sharpness was constrained by atomic motional spread and required intricate experimental control, including ground-state cooling, careful alignment of atomic orientations, and precise management of multiple laser frequencies and polarizations.

On a different aspect, we should compare this work with the earlier works on resonant Rydberg dressing [11–13], where they achieved soft-core interaction profiles. Those softcore potentials feature flat-topped, non-divergent profiles that saturate at short range. They could be observed when $\Omega_2 < 2|\Delta|$ for positive detuning and $\Omega_2 > 2|\Delta|$ for negative detuning. They explore the case that interaction is so large that it decouples $|ee\rangle$ within the soft core. They found the maximum soft-core interaction strength for a specific laser intensity of $\Omega_2 = 2|\Delta|$ where λ_- in Fig. 1c gets in resonance with $|gg\rangle$. This resonance has been deployed for making an optical lattice using a standing wave driving field [17]. The current work is distinct in generating off-centered δ -like peak potential away from zero separation rather than a soft-core profile. Here we are working in a different regime with opposite detunings, i.e. $\Omega_2 < 2|\Delta|$ for negative detuning and $\Omega_2 > 2|\Delta|$ for positive detuning, the interaction profile forms a distance-selective delta func-

tion peak. Here, we discuss that the finite and specific value of interaction makes the other eigenstate λ_0 (see Fig. 1) in resonance with $|gg\rangle$ at a specific interatomic distance, causing a peak potential.

Outlook—Rydberg interactions have already proven indispensable across a wide range of quantum-technology applications [18–20]. By introducing a tunable, sharply peaked interaction profile centered at a chosen separation, our work adds a powerful new tool to this experimental and theoretical toolbox. In addition to the applications discussed in the Introduction, the introduced distance-selective interaction scheme opens a new avenue for creating diatomic molecules with micrometer-scale bond lengths [12]. This offers a unique window into molecular physics, e.g., to observe vibrational motion of this giant molecule in real time, or use local fields to perturb one atom and see the effect on the bond. Such studies are normally impossible for conventional molecules since bond lengths are too small and internal dynamics are too fast.

METHODS

Perturbative Master Equation Analysis

The effective interaction potential is derived analytically by solving the steady-state master equation perturbatively in the small parameter $\epsilon = \Omega_1/\Omega_2 \ll 1$. The system consists of two three-level atoms in a ladder configuration ($|g\rangle \rightarrow |p\rangle \rightarrow |e\rangle$), where a weak probe field Ω_1 couples the ground-to-intermediate transition and a strong dressing field Ω_2 couples the intermediate-to-Rydberg transition.

The steady-state density matrix ρ satisfies

$$\mathcal{L}[\rho] = -i[H, \rho] + \sum_{j=1,2} \Gamma \mathcal{D}[\sigma_{gp}^{(j)}][\rho] = 0$$

where $H/\Omega_2 = H_0 + \epsilon H_1$ with

$$H_0 = \sum_{j=1,2} \left[\frac{1}{2}(\sigma_{pe}^{(j)} + \text{h.c.}) - \frac{\Delta}{\Omega_2} \sigma_{pp}^{(j)} \right] + \frac{V(r)}{\Omega_2} \sigma_{ee}^{(1)} \sigma_{ee}^{(2)}$$

$$H_1 = \sum_{j=1,2} \frac{1}{2}(\sigma_{gp}^{(j)} + \text{h.c.})$$

and $\epsilon = \Omega_1/\Omega_2$. The Lindblad dissipator is $\mathcal{D}[L][\rho] = L\rho L^\dagger - \frac{1}{2}\{L^\dagger L, \rho\}$, accounting for spontaneous decay from $|p\rangle$ to $|g\rangle$ at rate Γ .

The master equation $\mathcal{L}/\Omega_2[\rho] = 0$ is vectorized into superoperator form:

$$\mathcal{M}_0 \vec{\rho} + \epsilon \mathcal{M}_1 \vec{\rho} = 0$$

where $\mathcal{M}_0$ and $\mathcal{M}_1$ are 81×81 superoperators acting on the vectorized density matrix in the two-atom Hilbert

space (dimension $3^2 = 9$). Here, $\mathcal{M}_1$ corresponds to H_1/Ω_2 part of dynamics and the rest of the dynamics which are zero order in ϵ all included in $\mathcal{M}_0$. The density matrix is expanded as a power series in $\epsilon = \Omega_1/\Omega_2$

$$\rho = \rho^{(0)} + \epsilon \rho^{(1)} + \epsilon^2 \rho^{(2)} + \epsilon^3 \rho^{(3)} + \epsilon^4 \rho^{(4)} + \mathcal{O}(\epsilon^5)$$

Substituting into the master equation and collecting terms at each order yields

$$\mathcal{M}_0 \tilde{\rho}^{(0)} = 0$$

$$\mathcal{M}_0 \tilde{\rho}^{(n)} = -\mathcal{M}_1 \tilde{\rho}^{(n-1)}, \quad n \geq 1$$

with $\rho_{gg,gg}^{(0)} = 1$ and the normalization constraint $\text{Tr}[\rho] = 1$ enforced at each order, the density matrix terms to the fourth order in ϵ are calculated.

The effective interaction potential is extracted from the expectation value of the total Hamiltonian:

$$U_{\text{eff}}(r)/\Omega_2 = \langle H_{(0)} + \epsilon H_{(1)} \rangle$$

To encounter the effects of van-der Waals interaction, we expand to the fourth order in ϵ and find the potential with non-zero elements of the density matrix as

$$U_{\text{eff}}(r) = -\Omega_1 \text{Re}[\rho_{gp}^{(1)} + \rho_{gp}^{(3)}] - \Omega_2 \text{Re}[\rho_{pe}^{(2)} + \rho_{pe}^{(4)}] - 2\Delta[\rho_{pp}^{(2)} + \rho_{pp}^{(4)}] + V(r)\rho_{ee,ee}^{(4)} \quad (17)$$

where $\rho_{\alpha,\beta} = \sum_j \rho_{\alpha,\beta,jj} = \sum_j \rho_{jj,\alpha,\beta}$. This procedure yields an analytic form of interaction used in deriving U_0 (Eq. 9) for the regime of $\Omega_1 \ll \gamma_p \ll |\Delta|$ and $\Omega_1 \ll \Omega_2$.

Forming bound states

The distance-selective interaction introduced above naturally gives rise to bound states at finite interatomic separation. The Lorentzian peak in $U(r)$, centered at r_p , acts as an effective off-centered potential well whose position, width, and depth are independently tunable via external driving fields. Such a potential departs qualitatively from conventional short-range molecular interactions and enables binding at distances determined by controllable resonance conditions rather than electronic structure. This opens a route to engineering-bound states with prescribed spatial characteristics.

For the Lorentzian trap of Eq. 6, to ensure that the potential well is deep enough to support at least one bound state, the depth U_0 and width w must be related by $U_0 w^2 > \frac{\hbar^2}{2m}$. Considering $U_0/2\pi = 1\text{MHz}$ with controlling parameters expressed in Fig. 2c, the potential width must be larger than 6nm.

The ground motional state of the mentioned trap is numerically obtained by solving the Schrodinger equation

$$-\frac{\hbar^2}{2m} \frac{d^2 \psi(x)}{dx^2} + \frac{U_0}{1 + (\frac{x}{w})^2} \psi(x) = E \psi(x), \quad (18)$$

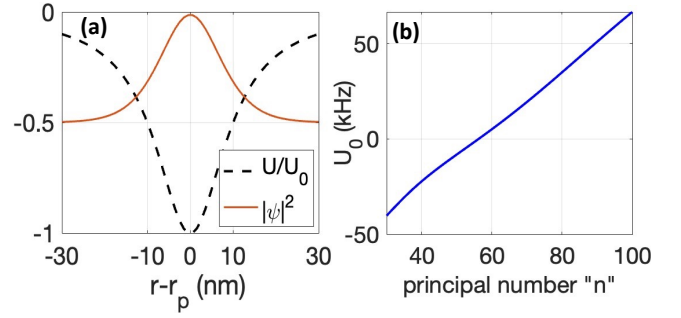


FIG. 5. (a) Bound state wavefunction for the trapping potential given by Eq. 6, with $U_0/2\pi = 1\text{MHz}$ and $w = 10\text{nm}$. (b) Dependence of the Lorentzian potential depth U_0 on the Rydberg principal quantum number n , arising from the n -dependence of the spontaneous decay rate γ_r . See text for details.

by discretizing the spatial coordinate, using the finite difference method, and solving for the eigenvalues E and eigenfunctions $\psi(x)$. The only trapped eigenstate is plotted in Fig. 5c for a case with $U_0/2\pi = 1\text{MHz}$ and $w = 10\text{nm}$. The atom is confined to a HWHM of 8nm.

Laser Noise Induced Heating of Bound States

We now analyze the effect of laser intensity fluctuations on the motional stability of atoms forming a bound state under the distance-selective interaction potential. To estimate the heating rate induced by laser intensity noise, we employ a harmonic approximation of the Lorentzian interaction potential near the resonance position, defining the relative coordinate $x = r - r_p$. The effective Hamiltonian governing the relative motion reads

$$H = \frac{P^2}{2m} + \frac{1}{2} m \omega_b^2 x^2, \quad (19)$$

where ω_b is the effective oscillation frequency of the bound state, given by

$$\omega_b = \sqrt{\frac{2|U_0|}{mw^2}}, \quad (20)$$

where w and U_0 are the width and depth of the Lorentzian peak defined in Eqs. 8, 9. In the regime $\Omega_1 \ll \Omega_2$ and $\gamma \ll \Delta$, this leads to the scaling $\omega_b \propto \Omega_1^2/\Omega_2^{2/3}$, or equivalently $\omega_b \propto I_1/I_2^{1/3}$.

Fluctuations in the laser intensities, denoted by $\xi_j = (I_j - I_{j0})/I_{j0}$, induce fluctuations in the effective oscillation frequency. To leading order, this results in the transformation

$$\omega_b^2 \rightarrow (1 + \xi_1(t) - \frac{1}{3}\xi_2(t))\omega_b^2, \quad (21)$$

where we have linearized the dependence on intensity fluctuations.

The resulting time-dependent perturbation to the Hamiltonian in Eq. 19 is then

$$H'(t) = \frac{1}{2} \left(\xi_1(t) - \frac{1}{3} \xi_2(t) \right) m\omega_b^2 x^2. \quad (22)$$

The heating rate can be obtained by evaluating transitions between motional eigenstates of the bound state using first-order time-dependent perturbation theory. For an atom initially in the motional state $|n\rangle$, the dominant transitions occur to $|n \pm 2\rangle$ with rates given by [21]

$$R_{n \pm 2 \leftarrow n} = \frac{\pi\omega_b^2}{16} S_\xi(2\omega_b)(n+1 \pm 1)(n \pm 1), \quad (23)$$

where $S_\xi(\omega)$ is the one-sided power spectral density of the effective fractional intensity noise,

$$S_\xi(2\omega_b) \equiv \frac{2}{\pi} \int_0^\infty d\tau \cos(2\omega_b\tau) \langle \xi(t)\xi(t+\tau) \rangle, \quad (24)$$

with $\xi(t) = \xi_1(t) - \frac{1}{3}\xi_2(t)$.

Assuming a population distribution $P(n, t)$ over motional states, the average energy of the bound state is $\langle E(t) \rangle = \sum_n P(n, t)(n + \frac{1}{2})\hbar\omega_b$. The energy increases as

$$\langle \dot{E} \rangle = \Gamma_\xi \langle E \rangle, \quad \Gamma_\xi = \pi^2 \nu_{tr}^2 S_\xi(2\nu_{tr}), \quad (25)$$

where $\nu_{tr} = \omega_b/2\pi$ is the oscillation frequency.

Experimental measurements of laser intensity noise provide realistic estimates for the heating rate. For the Ω_1 laser, measurements in strontium systems report $S_\xi(2\nu_{tr}) \approx 2.3 \times 10^{-13} \text{ Hz}^{-1}$ under servo stabilization, and $4 \times 10^{-12} \text{ Hz}^{-1}$ in the free-running case [22], corresponding to heating rates of $\Gamma_{\text{servo}} \approx 0.3 \text{ s}^{-1}$ and $\Gamma_{\text{free}} \approx 5 \text{ s}^{-1}$. Similarly, ultraviolet lattice systems driven by Ω_2 exhibit heating rates corresponding to lifetimes of several seconds [23].

At higher frequencies ($\nu_{tr} \gtrsim 25 \text{ kHz}$), the noise spectrum typically decreases as $S_\xi(\nu) \propto \nu^{-2}$ [21, 24]. In the present bound-state configuration, increasing ω_b shifts the system to higher frequencies where the noise is suppressed. As a result, although the parametric coupling strength increases with ω_b , this is compensated by the reduction in noise power, leading to a nearly constant heating rate.

In conclusion, laser intensity noise induces only weak parametric heating of the bound states, with rates comparable to or smaller than those observed in standard optical trapping experiments. This ensures that the bound states remain stable over experimentally relevant timescales.

More Realistic Decay Model

In the main text, including γ_p had a key role in generating the delta function-like interaction. It creates a driven-dissipative system that leads to steady-state formation.

It also regularizes the divergence of the fourth-order light shift to the ground state and defines the width of the Lorentzian peak. A fully realistic Lindblad model should account for Rydberg decoherence. However, this would render the perturbative analytical treatment intractable. Therefore, we use the simplified model in the main text to present and analyze the core physics, while discussing the effects of Rydberg decoherence numerically here.

We employ a simplified decay model for $|100s^3S_1\rangle$. The total spontaneous decay rate $\gamma_r = 2.7 \text{ ms}^{-1}$ is calculated following Ref. [25]. For high- n Rydberg S -states in the strontium triplet series, decay is dominated by transitions to low-lying P -states, particularly $|5p^3P_J\rangle$, since the Einstein A coefficient scales as the cube of the transition energy.

Decay rates to all m_j substates within a given $|np^3P\rangle$ manifold are equal. Starting from $|e\rangle = |100s^3S_1, m_j = 0\rangle$ and applying dipole selection rules ($\Delta m_j = 0, \pm 1$), we find that 1/7 of the total decay to the $|5p^3P\rangle$ manifold goes to the target state $|p\rangle = |5p^3P_1, m_j = 1\rangle$. In total, a direct decay channel with rate $\alpha\gamma_r = 0.05\gamma_r$, represented by the Lindblad term $\alpha\gamma_r|p\rangle\langle e|$.

The remaining decay channels, targeting other states, subsequently cascade to the ground state $|g\rangle = |5s^2^1S_0\rangle$. We model this indirect pathway as an effective two-step process with a rate: $\gamma_{\text{indirect}} = \frac{(1-\alpha)\gamma_r\gamma_p}{(1-\alpha)\gamma_r + \gamma_p}$ corresponding to the Lindblad term $\gamma_{\text{indirect}}|g\rangle\langle e|$.

We numerically compute the steady state, including both Rydberg γ_r and intermediate state γ_p decay channels, and extract the peak potential depth U_0 . As shown in Fig. 4b, U_0 depends on the Rydberg principal quantum number n solely through the n -dependence of γ_r . This is distinct from the interaction strength, which determines the peak position r_p and width w of the Lorentzian feature.

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Data availability All the data that support the findings of this study are included in this article. Any additional data is available from the corresponding author upon reasonable request.

Code availability All relevant code supporting the document is available upon request.

Author contributions The author, Mohammad-sadegh Khazali, performed the research, analyzed the results, and wrote the manuscript.

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