```
function [r_ab,R,PLOT] =Circle_Para_LS(rou_j,theta_1_j,Plot_switch)
%
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% A least-square fit for determining parameters of the circle equation (x?a)^2 + (y?b)^2 = R^2
using observations r_j:=[x_j, y_j]:=[?_j*cos(?_1_j), ?_j*sin(?_1_j)].
%
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%------------------------
% Syntax
%------------------------
% [d,AOOp,Center,Circ,R] =Circle_Para_LS(rou_j,theta_1_j) returns the downward FAC density
%[d,AOOp,Center,Circ,R] =Circle_Para_LS(...,'OptionalVariableName',OptionalVariableValue,...)
% MFACE_v1(...,'OptionalVariableName',OptionalVariableValue,...)
%OptionalVariableName is
%OptionalVariableName list:
%------------------------
% Inputs
%------------------------
%rou_j: ?_j(t), denoting the distance between O and the ring tip
%theta_1_j:    ?_1_j, when length(rou_j)==length(theta_1_j),
%                              ?_{j-1}_j, when length(rou_j)==length(theta_1_j)+1,
%                              (Here, ?_i_j denotes the cross angle between the    i-th and j-th cores)
%         Plot_switch: a key to contrle plot,    1 for plotting the LS results and 0 for not
%------------------------
% Outputs
%------------------------
%r_ab: complex coordinate of the LS-fitted circle center.    :
%(the orientation of circle center:angle_r_ab=angle(r_ab)/pi*180;the distance between r_ab and O:
d=abs(r_ab):    )
%R: LS-fitted radius
% PLOT: a structure comprising the handles and data of the figure.
%------------------------
%Examples
%------------------------
%---% 1st example: LS fit to 3 points
%rou_j=rand(3,1);theta_1_j=rand(3,1)*180;[r_ab,R,PLOT] =Circle_Para_LS(rou_j,theta_1_j,1)
%---% 2nd example: LS fit to 4 points,
%rou_j=rand(4,1);theta_1_j=rand(3,1)*180;
%[r_ab,R,PLOT] =Circle_Para_LS(rou_j,theta_1_j,1)
%---% 3rd example
% rou_j=rand(3,1);theta_1_j=rand(3,1)*180;[r_ab,R,PLOT] =Circle_Para_LS(rou_j,theta_1_j,1);
% close all
```

```
% Inputs_complex=PLOT.outputCircData*5./(5+rand(size(PLOT.outputCircData)))+3;
% Inputs_complex=Inputs_complex'*Inputs_complex(1)
%
[r_ab,R,PLOT2] =Circle_Para_LS(abs(Inputs_complex),angle(Inputs_complex(2:end))/pi*180,1);
%
%%
```

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%%
% Cite As
%Fang et al., (2021). Variations in tree-ring geometric center: implications for wind speed change and basal area increment ...
%------------------------
%Communication to:
%------------------------
% Maosheng He
% hmq512@gmail.com
% 20-04-2021
%%
if nargin<3,Plot_switch=0;end
rou_j=rou_j(:);
theta_1_j=theta_1_j(:);
theta_1_j=mod(theta_1_j,360);
% transfer ?_j-1_j to ?_1_j
if length(theta_1_j)< length(rou_j)
    theta_1_j=cumsum([0;theta_1_j(1:length(rou_j)-1)/360*2*pi]);
end
rou_j_comp=rou_j(:).*exp(1i*theta_1_j);%?_j_complex:=?_j*exp(1i*?_1_j);
rou_j_comp=rou_j_comp(:).';
%% LS fit to x^2+y^2+A*x+ B*y+C=0 derived  from (x?a)^2 + (y?b)^2 = R^2
xy1 = [real(rou_j_comp); imag(rou_j_comp); ones(size(rou_j_comp))];
ABC = [xy1']\abs(rou_j_comp)'.^2; % coefficents A, B, C and
r_ab=ABC(1)/2+ABC(2)/2*1i; % coordinates of the circle center
R = sqrt(abs(r_ab)^2+ABC(3)); % radius

%% plot
```

```
if Plot_switch,
    theta_i=linspace(0,2*pi,1000);
    PLOT.outputCircData=r_ab+[R*exp(1i*theta_i)];
    PLOT.gcf=figure;
    PLOT.gca=subplot(1,1,1);
    if length(rou_j_comp(:))<5
    PLOT.input=compass(rou_j_comp,'b');
    else
    PLOT.input=scatter(real(rou_j_comp),imag(rou_j_comp),'bx');
    end
    hold on;
    PLOT.output_circle=plot(real(PLOT.outputCircData),imag(PLOT.outputCircData),'r-');
    hold on; PLOT.output_center=scatter(real(r_ab),imag(r_ab),'r+');
    axis equal
else
    PLOT=nan;
end
```