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Artificial Intelligence Modelling of Non-Fourier Heat Transport in Magnetite-Graphene Hybrid Nanofluid for Sensor Applications

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Abstract: Magnetite (Iron Oxide) -Graphene hybrid nanoparticles, in particular, are found to be very beneficial for sensor surface applications because of their superior thermal conductivity, magnetic responsiveness, and enhanced heat transfer capability, which enhances the thermal stability, sensitivity, and operational efficiency of contemporary biosensors, microelectronic sensing devices, and smart sensors. This work delves into the dynamics of a magnetite-graphene/water-ethylene glycol hybrid nanofluid's unsteady MHD squeezed flow across a porous sensor surface under the conditions of viscous dissipation, nonlinear thermal radiation, and the non-Fourier C-C heat flux model. Applying the right transformations reduces the system of governing partial DEs to a set of ordinary DEs. These are then resolved by use of the MATLAB BVP5C solvers. After that, we'll use the numerical dataset to train two artificial neural networks: LMBP. This study's results show that the LMBP network replicates the BVP5C solution at epoch 77 with a gradient of $7.17.10^{-8}$, a best validation MSE of $1.77.10^{-10}$, and a correlation value $R = 1$. Furthermore, As R and Ec parameter values rise, the temperature profile $\theta(\eta)$ also increases. Radiative heating and viscous dissipation are improved thermal conductivity.

Keywords: MHD, sensor surface, CC heat flux, Magnetite-Graphene Hybrid Nanofluid, LMBP ANN.

Nomenclature

u and v	velocity components along the x and y axes	C_f	skin friction coefficient
B_0	magnetic field	M	magnetic parameter
p	pressure	Nu_x	Nusselt number
q_r	radiative heat flux	τ_w	shear stress
T	temperature	Re_x	Reynolds number
t	time	Pr	Prandtl number
$U(x, t)$	free stream axial velocity	γ^*	relaxation time parameter
k^*	absorption coefficient	S	permeability parameter
σ^*	Stefan-Boltzmann constant	θ_w	temperature ration parameter
$q_w(x)$	heat flux	f_0	permeable velocity parameter

T_{∞}	free stream temperature	R	radiation parameter
a	squeeze flow strength	$q(x)$	free stream temperature
s and h_0	constant	f	fluid
b	squeeze flow index	nf	nanoparticles
v_0	surface permeable velocity	hnf	hybrid nanoparticles
q_0	heat flux	CCM	Cattaneo-Christov model
ϕ_1 and ϕ_2	Nanoparticle volume fraction	NF	nanofluid
Ec	Eckert number	HNF	hybrid nanofluid

Introduction

Nanofluids have been developed as effective heat transfer media chiefly because of their far greater thermal conductivity and improved energy transportation abilities over the ordinary fluids. Such fluids find extensive use in applications including microchannel cooling, solar collector panels, nuclear reactors, biomedical apparatus, and cooling of electronic devices. Abbas et al. [1] investigated the flow of nanofluids over a nonlinear sheet subjected to varying physical effects, and they found that reactive flow conditions improved mass and heat transfer rates. Ferdows et al. [2] worked on three-dimensional hybrid nanofluid stagnation-point flow and documented the enhanced thermal performance appropriate for industrial heat exchangers. Ullah et al. [3] shown the great promise of AI-based prediction approaches in thermal systems by modelling nanofluid flow across an exponentially extending surface using artificial neural networks. Elfatah et al. [4] conducted a review on use of nanofluids in microchannels and brought out their great role in the development of compact cooling technologies and MEMS devices. Lone et al. [5] conducted experiments with Cu, Al_2O_3 , and TiO_2 nanofluids over curved surfaces, and they observed that, because of mixing of nanoparticles they achieved better heat transfer. Ali et al. [6] used the CC heat flux model in MHD nanofluid flow with thermal radiation, thus stressing the significance of non-Fourier heat conduction effects.

Hybrid nanofluids exhibit enhanced thermal diffusivity, improved energy transport capability, and greater suspension durability than conventional monofluids due to the presence of multiple nanoparticle species. Because of their remarkable thermophysical properties, hybrid nanofluids are extensively utilized in automotive cooling systems, nuclear reactors, refrigeration and air-conditioning systems, energy storage devices, and aerospace thermal management applications. Maranna et al. [7] studied the performance of heat transmission in porous media by studying the behaviour of MHD hybrid nanofluid flow across porous stretching and shrinking sheets. Thamizhselvi and Narayana [8] used RSM-BBD to improve the heat transfer properties of MHD HNF flow across non-uniform sheets. Ramana et al. [9]

examined the flow of rotating HNF in 3D through a porous material while considering radiation absorption and Forchheimer effects across a stretched surface. The study highlighted the significance of this work for thermal engineering and rotating device systems. Jeelani et al. [10] focused on Maxwell hybrid nanofluid flow solar radiation and chemical reactions, stressing the use of renewable energy sources. Farooq et al. [11] considered the bioconvection phenomenon of MHD hybrid nanofluid flow and showed that microorganism movement enhances thermal transport. Gul et al. [12] investigated how radiative MHD HNF flows with Brownian motion and thermophoresis. Galal et al. [13] performed their investigation on 3D MHD nanofluid flow in the presence of inclined magnetization and found a strong dependence on the direction of the magnetic field. Imran et al. [14] developed artificial neural network-based thermal prediction models for MHD hybrid nanofluid flow and demonstrated their strong predictive accuracy through validation analyses. Gupta et al. [15] investigated the MHD Couette flow in porous ducts at thermal non-equilibrium conditions (a relevant technology to nuclear cooling systems). Hussein et al. [16] found that under certain flow conditions, the heat transfer efficiency is greatly improved when MHD ternary NF fluid pass across permeable cylinders. Shree et al. [17] used Fibonacci wavelet methods to solve Casson fluid MHD flow problems over sheets. The unstable MHD flow of NF containing copper and water between parallel plates that were impacted by suction and injection was studied by Abbas et al. [18].

The CC heat flux formulation predicts finite-speed thermal wave propagation, whereas Fourier's model is more realistic for high-temperature and fast heat transfer processes. Sidahmed et al. [19] examined viscoelastic nanofluid flow under constant heat flux conditions while incorporating the effects of chemical reactions. Alfannakh et al. [20] made a review of silver oxide nanoparticle flow incorporating CC heat flux and found that thermal performance was enhanced. Johari et al. [21] combined MHD nanofluids and CC heat flux to develop an efficient thermal management system for an X-ray system. Ramya et al. [22] investigated micropolar nanofluid flow with CC heat flux and chemical reactions. Shah et al. [23] studied fourth-grade nanofluid flow with CC double diffusion effects.

Sensor surfaces have attracted considerable attention in modern fluid flow and heat transfer studies because of their wide-ranging applications in biomedical instruments, multiphase electronic medical systems (MEMS), lubrication technologies, thermal sensing devices, industrial automation processes, and aerospace engineering systems. Khaled and Vafai [24] studied fluid flow and heat transfer over a sensor surface and proved that cooling and sensing systems may be more thermally efficient. Using a sensor surface, Hayat et al. [25] investigated the heat transfer properties and squeezed flow behaviour of a second-grade fluid.

In their study, Rashidi et al. [26] found analytical solutions for axisymmetric squeezing flows in two dimensions that are unstable when passed between parallel plates. MHD flow facilitated by water-functionalized metallic NF was studied by Haq et al. [27] and they demonstrated improved heat transfer properties that are suitable for use in nano-cooling and electronic thermal management systems. Siddiqui et al. [28] investigated the unstable squeezing flow of viscous MHD fluids between parallel plates using the homotopy perturbation technique; their results show that this method may be used in hydrodynamic sensor technologies and industrial stinging operations.

Because of their excellent thermal conductivity, superior magnetic behavior, and enhanced physicochemical stability, Fe_3O_4 and graphene nanoparticles play a vital role in applications like solar energy conversion systems, catalytic reactors, magnetic separation processes, smart coatings, and high-performance heat exchange equipment. In light of the above, this research develops a formula for the unstable MHD squeezed flow of a hybrid nanofluid consisting of Fe_3O_4 –Graphene/ H_2O – EG over a porous sensor surface. The present configuration is examined under the combined influences of the constant heat flux (CHF) model, nonlinear thermal radiation, and viscous dissipation effects. In order to numerically solve the system of nonlinear ODEs, the governing PDEs are first converted into them using appropriate similarity transformations. Then, the BVP5C solver in MATLAB is used for the numerical solution. Graphical analysis and presentation of the parameter's effect on temperature and velocity profiles, Nusselt number and friction factor are provided.

Mathematical Formulation

Consider the flow of a horizontal Magnetite–Graphene nanofluid over a surface of length L . Figure 1 shows the chosen coordinate system with the x -axis running parallel to the surface from its leading edge and the y -axis taken perpendicular to the surface. A higher value for $h(t)$ for the compressed channel containing the surface is much more than the thickness of the boundary. The length L sensor surface is positioned in a compressed channel at a height significantly above the layer, where the height $h(t) = h_0 e^{-st}$ when $b = 0$ and $h(t) = h_0/(s + bt)^{(1/b)}$ for $b > 0$.

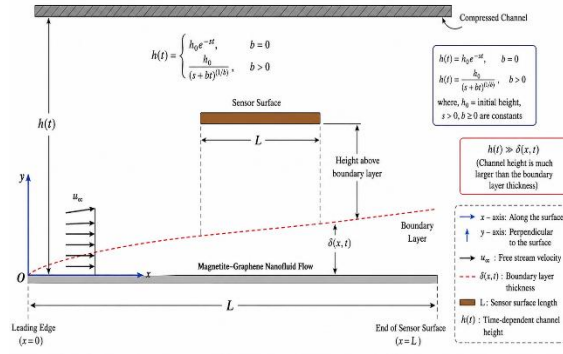


Figure 1: Physical model of squeezing sensor surface.

The current investigation is based on the premise that the free-stream compressive force starts at the surface's tip, where the moving upper plate approaches the still lower plate. Moreover, when the flow reaches the other end of the sensor surface, $v_w(t)$ represents the permeability velocity at the surface of the sensor.

To illustrate the governing equations that are under control, consider the following:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \quad (1)$$

$$u \frac{\partial u}{\partial x} + v \frac{\partial v}{\partial y} = \frac{\mu_{hnf}}{\rho_{hnf}} \frac{\partial^2 u}{\partial y^2} - \frac{1}{\rho_{hnf}} \frac{\partial p}{\partial x} - \frac{\sigma_{hnf} B_0^2}{\rho_{hnf}} u, \quad (2)$$

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} = - \frac{\sigma_{hnf} B_0^2}{\rho_{hnf}} U - \frac{1}{\rho_{hnf}} \frac{\partial p}{\partial x}, \quad (3)$$

$$u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + \frac{\partial T}{\partial t} + \Gamma \left(u \frac{\partial u}{\partial x} \frac{\partial T}{\partial x} + u \frac{\partial v}{\partial x} \frac{\partial T}{\partial y} + u^2 \frac{\partial^2 T}{\partial x^2} + v \frac{\partial v}{\partial y} \frac{\partial T}{\partial y} + v \frac{\partial u}{\partial y} \frac{\partial T}{\partial x} + v^2 \frac{\partial^2 T}{\partial y^2} + 2uv \frac{\partial^2 u}{\partial x \partial y} \right) = \frac{k_{hnf}}{(\rho C_p)_{hnf}} \frac{\partial^2 T}{\partial y^2} - \frac{1}{(\rho C_p)_{hnf}} \frac{\partial q_r}{\partial y} + \frac{\mu_{hnf}}{(\rho C_p)_{hnf}} \left(\frac{\partial u}{\partial y} \right)^2. \quad (4)$$

Equation (2) may be rewritten as follows after adding the pressure gradient factor from equation (3):

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} + \frac{\partial u}{\partial t} = \frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} + \frac{\mu_{hnf}}{\rho_{hnf}} \frac{\partial^2 u}{\partial y^2} + \frac{\sigma_{hnf} B_0^2}{\rho_{hnf}} (U - u). \quad (5)$$

The radiative heat transfer is denoted as:

$$q_r = - \frac{4\sigma^*}{3k^*} \frac{\partial T^4}{\partial y}. \quad (6)$$

Therefore, Eq's (4) is;

$$\frac{\partial T}{\partial t} + u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} = \frac{1}{(\rho C_p)_{hnf}} \left[k_{hnf} + \frac{16\sigma^* T_\infty^3}{3k^*} \right] \frac{\partial^2 T}{\partial y^2} + \frac{\mu_{hnf}}{(\rho C_p)_{hnf}} \left(\frac{\partial u}{\partial y} \right)^2, \quad (7)$$

subject to:

$$u(x, 0, t) = 0, \quad -k_{hnf} \frac{\partial T(x, 0, t)}{\partial y} = q(x), \quad v(x, 0, t) = v_w(t),$$

$$T(x, \infty, t) = T_{\infty}, \quad u(x, \infty, t) = U(x, t). \quad (8)$$

Table 1. Thermophysical properties of nanofluid and hybrid nanofluid.

Thermophysical Properties	Hybrid Nanofluid
Density	$\rho_{hnf} = (1 - \varphi_2)[(1 - \varphi_1)\rho_f + \varphi_1\rho_{n1}] + \varphi_2\rho_{n2}$
Heat capacity	$(\rho C_p)_{hnf} = (1 - \varphi_2)[(1 - \varphi_1)(\rho C_p)_f + \varphi_1(\rho C_p)_{n1}] + \varphi_2(\rho C_p)_{n2}$
Dynamic viscosity	$\mu_{hnf} = \frac{\mu_f}{(1 - \varphi_1)^{2.5}(1 - \varphi_2)^{2.5}}$
Thermal conductivity	$k_{hnf} = \frac{k_{n2} + 2k_{nf} - 2\varphi_2(k_{nf} - k_{n2})}{k_{n2} + 2k_{nf} + \varphi_2(k_{nf} - k_{n2})} \times (k_{nf})$ where $k_{nf} = \frac{k_{n1} + 2k_f - 2\varphi_1(k_f - k_{n1})}{k_{n1} + 2k_f + \varphi_1(k_f - k_{n1})} \times (k_f)$

Table 2. Thermophysical properties of Magnetite-Graphene and Water-Ethylene Glycol.

Thermophysical Properties	Magnetite (iron Oxide)	Graphene	$H_2O - EG$
Density $\rho(kg/m^3)$	5180	2200	1050
Heat capacity	670	700	0.40
Thermal conductivity	6.5	2000	3300
Prandtl number			6-7

The hybrid nanofluid's thermophysical characteristics are detailed in Table 1. Water, Ethylene glycol, magnetite and Graphene all have their physical characteristics detailed in Table 2.

Forms taken into account in this work for $U(x, t)$, $v_w(t)$ and $q(x)$ are as follows:

$$U(x, t) = ax, \quad v_w(t) = v_0\sqrt{a}, \quad q(x) = q_0x \quad (9)$$

Implementing an appropriate transformation are:

$$\psi = \sqrt{av_f}xf(\eta), \quad \eta = y\sqrt{a/v_f}, \quad \theta(\eta) = \frac{T - T_{\infty}}{(q_0x/k_f)\sqrt{v_f/a}}, \quad a = \frac{1}{s+bt}, \quad \theta_w = \frac{q(x)}{T_{\infty}}$$

$$T = T_{\infty}(1 + (\theta_w - 1)\theta). \quad (10)$$

Equation (1) is perfectly satisfied when ψ is the stream function with $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$ transform equations (5) and (7) into:

$$\frac{\mu_{hnf}/\mu_f}{\rho_{hnf}/\rho_f} f'''' + \left(f + \frac{b\eta}{2}\right) f''' - f'^2 + b(f' - 1) + \frac{1}{(1 - \varphi_2)[(1 - \varphi_1)\rho_f + \varphi_1\rho_{n1}] + \varphi_2\rho_{n2}} M(1 - f') = 0, \quad (11)$$

$$\frac{1}{Pr} \frac{1}{(\rho C_p)_{hnf}/(\rho C_p)_f} \left(\frac{k_{hnf}}{k_f} \theta'' + R[(1 + (\theta_w - 1)\theta)^3 + 3(\theta_w - 1)\theta'^2(1 + (\theta_w - 1)\theta)^2] \right) + \left(f + \frac{b\eta}{2}\right) \theta' - \left(f' + \frac{b}{2}\right) \theta + Ec f''^2 - \gamma^*[ff'\theta' + f^2\theta''] = 0. \quad (12)$$

subject to:

$$\begin{aligned} f(0) &= -f_0, \theta(0) = -\frac{k_f}{k_{hnf}}, f'(0) = 0, \\ f'(\infty) &= 1, \quad \theta(\infty) = 0, \end{aligned} \quad (13)$$

where;

$$R = \frac{4\sigma^* T_\infty^3}{k^* k_f}, Pr = \frac{(\mu C_p)_f}{k_f}, f_0 = \frac{v_0}{\sqrt{v_f}}, M = \frac{\sigma B_0^2}{\rho_f a}, \gamma^* = \Gamma b \text{ and } Ec = \frac{U^2}{C_p(T_w - T_\infty)}$$

Here are the definitions of C_f and the local Nu_x are:

$$C_f = \frac{\tau_w}{\rho_f U^2} \text{ and } Nu_x = \frac{x q_w}{k_f (T_w - T_\infty)}, \quad (14)$$

where τ_w and q_w are defined as;

$$\tau_w = \mu_{hnf} \left(\frac{\partial u}{\partial y} \right)_{y=0}, \quad q_w = -k_{hnf} \left(\frac{\partial T}{\partial y} \right)_{y=0} + (q_r)_{y=0}. \quad (15)$$

One obtains by applying Eq's (15) and Eq's (14);

$$Re_x^{1/2} C_f = \frac{\mu_{hnf}}{\mu_f} f''(0), \quad Re_x^{-1/2} Nu_x = - \left(\frac{k_f}{k_{hnf}} (1 + R\theta_w) \right) \frac{1}{\theta(0)}, \quad (16)$$

where $Re_x = Ux/v_f$.

Numerical Method

Nonlinearity generation is acknowledged as a crucial feature of advanced research in many technical, scientific, technological, and industrial problems. A system of non-linear partial or ordinary DEs is often the outcome of mathematical modelling of these challenges (11–13). We use the numerical method BVP 5C to solve the conventional DEs in order to get the solution to the current flow problem. Error estimates of orders 4 and 5 are used in this method, which is a built-in MATLAB algorithm. In the reduced similarity domain $\eta \in [0, \eta_\infty]$, the $\theta(\eta)$ and $f'(\eta)$ profiles are calculated for typical values of the governing physical parameters. The approach, which also automatically computes error estimates for various step sizes, may be able to control high-order error. By introducing the following variable to make system in first-order form:

$$\begin{aligned} y_1 &= f, y_2 = f', y_3 = f'', y_4 = \theta, y_5 = \theta. y_1' = y_2, y_2' = y_3, y_4' = y_5. \\ y_3' &= \frac{\frac{\rho_{hnf}}{\mu_f}}{\frac{\mu_{hnf}}{\mu_f}} \left[y_2^2 - \left(y_1 + \frac{b\eta}{2} \right) y_3 - b(y_2 - 1) - \frac{M(1-y_2)}{(1-\varphi_2)[(1-\varphi_1)\rho_f + \varphi_1\rho_{n1}] + \varphi_2\rho_{n2}} \right], \end{aligned} \quad (17)$$

$$\begin{aligned} y_5' &= \frac{-(y_1 + \frac{b\eta}{2})y_5 + (y_2 + \frac{b}{2})y_4 - Ec y_3^2 + \gamma^* y_1 y_2 y_5}{\frac{k_{hnf}}{k_f} + R[(1+(\theta_w-1)y_4)^3 + 3(\theta_w-1)y_5^2(1+(\theta_w-1)y_4)^2]} \\ &\quad - \frac{Pr \frac{(\rho c_p)_{hnf}}{(\rho c_p)_f}}{\gamma^* y_1^2}. \end{aligned} \quad (18)$$

It is translated as follows by the boundary conditions (13):

$$y_1(0) = -f_0, y_2(0) = 0, y_2(\infty) = 1, y_4(0) = -\frac{k_f}{k_{hnf}}, y_4(\infty) = 0. \quad (19)$$

Results and Discussions:

The coupled nonlinear ordinary DEs drive the flow of the Iron Oxide-Graphene/Water-Ethylene Glycol hybrid nanofluid across the sensor surface. These equations are numerically solved using the BVP 5C technique. ANN-LMBP is trained on the same dataset to provide a fast surrogate model. The default setting for the parametric study unless otherwise stated is $\phi_1 = 10\%$, $\phi_2 = 20\%$, $M = 0.5$, $R = 0.5$, $Ec = 0.2$, $Pr = 6.5$, $b = 0.2$, $\gamma^* = 0.1$, $\theta_w = 1.4$, and $f_0 = -0.2$. The discussion is structured figure by figure and table by table.

Levenberg-Marquardt Backpropagation Artificial Neural Networks (LMBP):

Over the past decade, LMBP-ANN has gained popularity as a powerful tool for computational fluid dynamics (CFD), notably in turbulence modelling, flow parameter optimisation, and nanofluid transport phenomena prediction. The LMBP-ANN surpasses traditional iterative methods, which may be expensive and numerically unstable, in terms of speed and stability, all while maintaining a high level of physical correctness. The model excels when dealing with nonlinear problems that defy conventional numerical solutions, such as hybrid multiphase systems, magnetohydrodynamics, and others. A trained artificial neural network (ANN) can improve data-driven predictions and parametric research, as well as act as a rapid stand-in for more complex simulations.

To solve a physical heat transfer problem with specified boundary conditions, the first step is to convert it to a system of ODEs. The LMBP approach uses a Mean Square Error (MSE) loss function minimisation strategy to train its algorithms to solve these specific equations. Layer size, number of layers, and activation functions used in network architecture design have a significant influence on the solution's efficiency and accuracy. If you're having difficulties tackling complex fluid flow problems with more traditional mathematical methodologies, LMBP-ANN provides a quick and dependable solution.

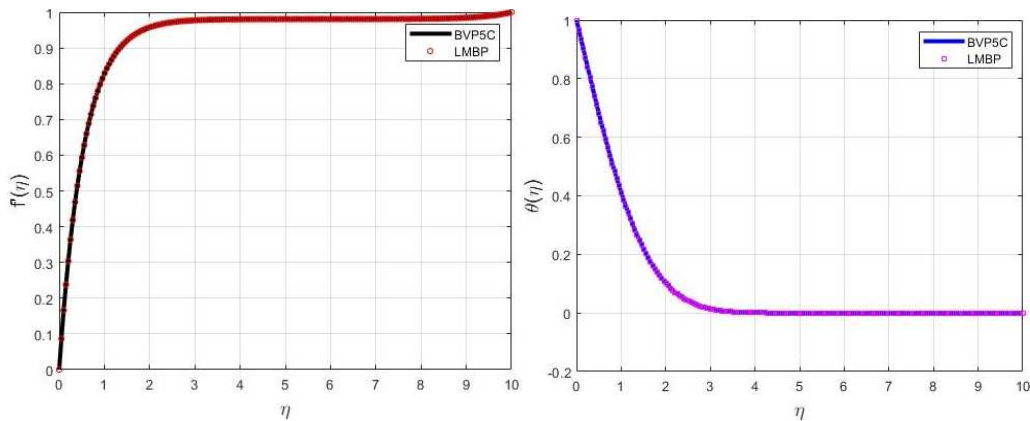


Figure 2 and 3: Comparison result of LMBP and BVP 5C outcomes for $f'(\eta)$ and $\theta(\eta)$

Figure 3(a, b) displays the BVP5C and trained neural network's for $f'(\eta)$ and $\theta(\eta)$ profile. The two curves coincide over the whole computational domain, indicating that the surrogate has learnt the underlying solution operator rather than merely fitting specific training points.

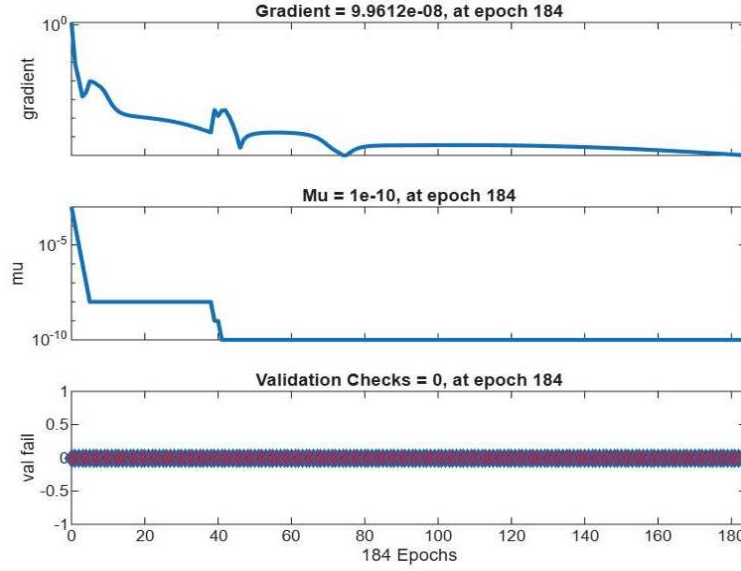


Figure 4: Assessment of the built neural network models during the training phase.

Figure 4 displays the training diagnostics of the ANN-LMBP model. The cost function gradient decreases monotonically and converges to 9.96×10^{-8} at the epoch 184. The Levenberg–Marquardt damping parameter μ converges to its lower bound 10^{-10} . The number of validation failures remains zero during the training. The simultaneous occurrence of a small gradient, a small damping parameter and no validation failures means that the optimiser found a well-conditioned minimum without overfitting.

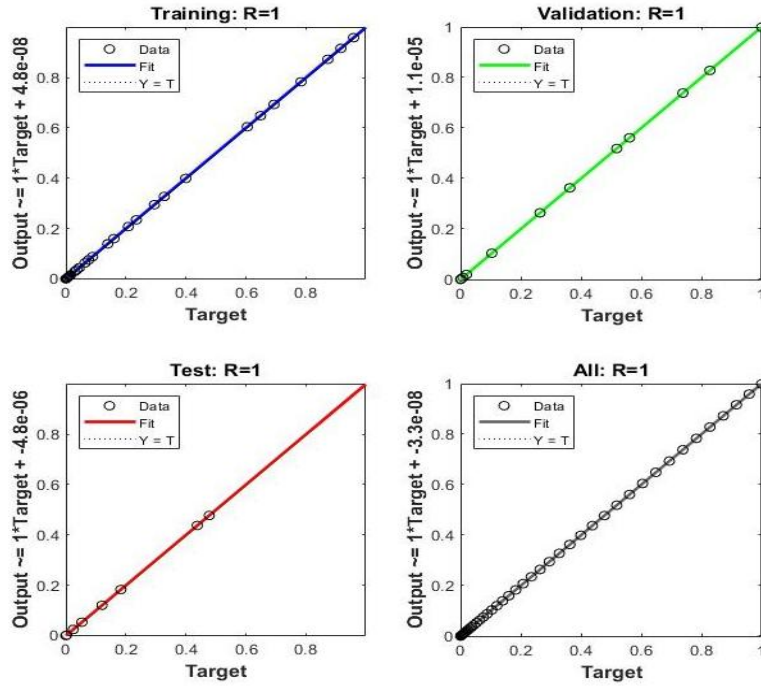


Figure 5: Visual representation of the regression results for several scenarios.

Figure 5 depicts the regression results for the training, validation, test and combined data partitions. The correlation coefficient is $R = 1$ at the precision shown and the bias of the linear fit is lower than 10^{-5} for each partition, indicating that the network reproduces the target values for the entire range of the parameter space without systematic deviation.

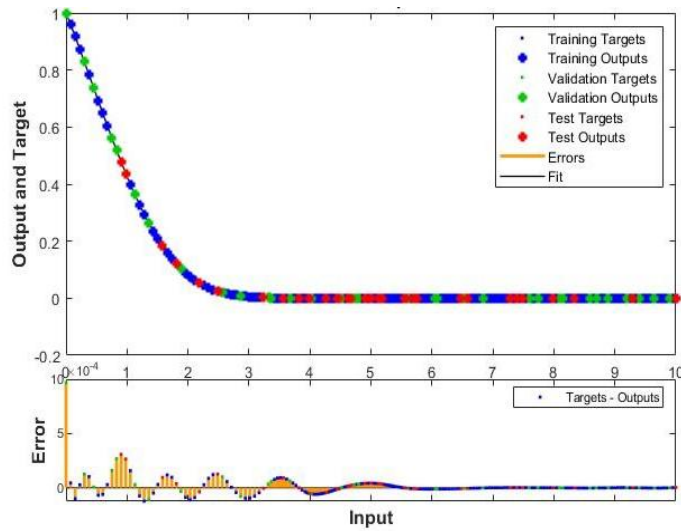


Figure 6: Function fitting performance of the ANN-LMBP model.

Figure 6 compares the network output to the target values given by the bvp5c reference. The training, validation and test points are indistinguishable from the reference curve within the line thickness of the plot with absolute errors limited to the interval. The errors decay with η at the same rate as the magnitude of the target, which implies that the network is able to reproduce the near-wall structure as well as the far-field decay.

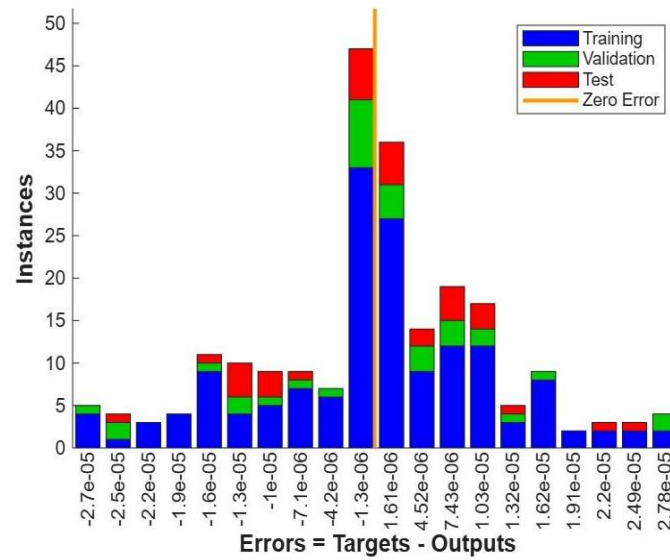


Figure 7: Error histogram ANN-LMBP.

Figure 7 displays the 20-bin error histogram of the ANN-LMBP model. The zero-error reference line forms a tiny peak in the distribution, with the most populated bin being about -1.3×10^{-6} . There seems to be no systematic bias in the predictions, since the histogram is almost symmetrical and there are no long tails. Additionally, the residuals are substantially within the engineering tolerances typically acceptable for boundary-layer calculations.

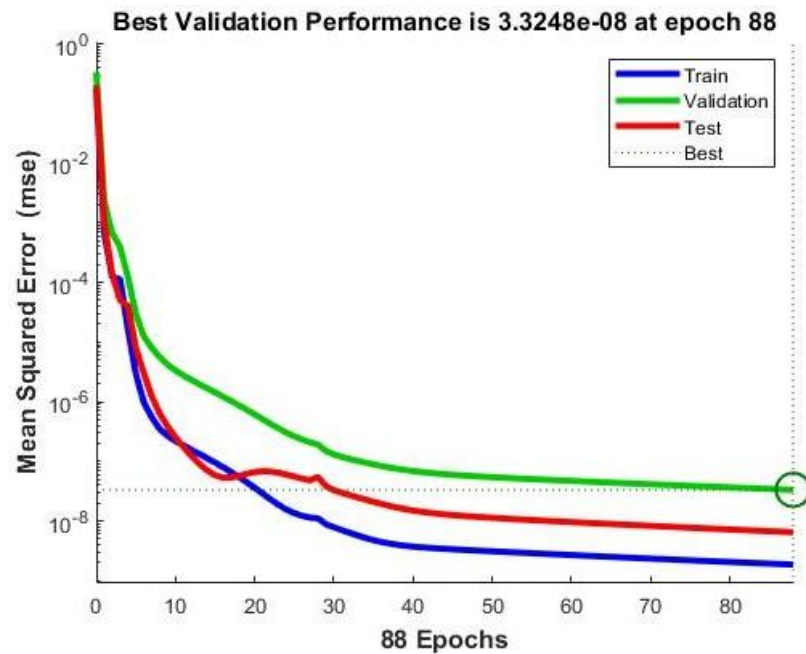


Figure 8: MSE results of the LMBP NN training.

In Figure 7, we can see the three data divisions' mean squared errors (MSEs) plotted against the training period. At epoch 88, the validation performance reaches its peak at 3.32×10^{-8} . The validation curve flattens and the training curve remains softly declining beyond this point, showing the convergence of the model to a generalising minimum. As observed, the test,

validation, and training curves were so close together during optimization that ANN-LMBP is a good surrogate model and that overfitting is not happening.

Table 3. Comparison of numerical results for $f''(\eta)$ and $\theta'(\eta)$ obtained by BVP4C, BVP5C and LMBP methods.

	BVP 4C		BVP 5C		LMBP	
η	$f''(0)$	$\theta'(0)$	$f''(0)$	$\theta'(0)$	$f''(0)$	$\theta'(0)$
0	0.61878	1	0.61878	1	0.618779	1
2	0.066216	0.0815471	0.066156	0.08149910	0.066121	0.08153900
4	0.017174	0.0001080	0.017170	0.00010812	0.017176	0.00010824
6	0.042383	3.6612e-05	0.042386	3.6617e-05	0.042386	3.6614e-05
8	0.083259	3.392e-05	0.083257	3.3922e-05	0.083262	3.3988e-05
10	0.141740	0	0.141739	0	0.141739	0

The numerical values of $f''(\eta)$ and $\theta'(\eta)$ as determined by BVP 4C, BVP 5C, and LMBP methods are compared in Table 3. All three approaches agree quite well on the findings, with very little discrepancies at the higher decimal places. The temperature gradient $\theta'(\eta)$ meets the far-field boundary requirements and approaches zero gradually as $\eta \rightarrow \infty$. As a result, the effectiveness, consistency, and precision of the suggested LMBP approach to nonlinear flow and heat transfer analysis are confirmed by the comparison.

Influence of the regulating physical variables:

Nanoparticle volume fractions:

The effect of the parameter ϕ_1 and ϕ_2 on $f'(\eta)$ is shown in Figure 9 and 10. From figure 9 it is noticed that, as the iron-oxide nanoparticle phase's volume fraction ϕ_1 goes up from 10% to 40%, the momentum is much thinner. Furthermore, as ϕ_1 increases it reduces the velocity. Similarly, Graphene nanoparticle phase's volume fraction ϕ_2 scale back both $f'(\eta)$ and corresponding thickness of the momentum layer. The underlying mechanism is that higher particle concentration elevates the effective dynamic viscosity of the hybrid suspension, thereby increasing internal resistance and thinning the momentum boundary layer. This dual character of nanoparticle loading simultaneously enhancing thermal transport while decelerating the flow through viscous effects is clearly evident. Figures 11 and 12 examine the effect of volume fractions on the $\theta(\eta)$. It can be observed from the figure 11 that an increasing ϕ_1 from 10% to 40% leads to an enhances in the $\theta(\eta)$ distribution. Additionally, thickness of the thermal layer also enhances for larger ϕ_1 parameter. Form the figure 12, it can be observed

that the temperature distribution becomes more pronounced when ϕ_2 increases from 10% to 40%. Apart from that, as ϕ_2 increases, reducing the thermal layer thickness. To put it physically, when the particle concentration is high, the hybrid mixture has a greater effective thermal conductivity, which means that heat diffuses more efficiently inside the boundary layer. In addition, the greater solid fraction augments volumetric heat capacity and internal heat generation, which collectively elevate the fluid temperature.

Magnetic parameter (M):

Pictured in Figure 13 is the $f'(\eta)$ curve throughout the different range of M . With the increase in M , reducing the $f'(\eta)$ distribution and thinning the momentum layer. Lorentz force, which occurs when an applied magnetic field interacts with an electrically conducting hybrid nanofluid, resulting in reduction of fluid flow. Figure 14 shows that the $\theta(\eta)$ profile when the parameter M increased between 0.2 to 0.8. The wall heat-transfer rate enhances as M intensifies because the imposed field's Lorentz force is opposed to the fluid's motion. Due to this, the greater M increases the $\theta(\eta)$ fields and the corresponding thermal layers thickness.

Squeeze flow index parameter (b):

Figure 15 shows how the squeeze parameter b ($= 0.2, 0.4, 0.6, 0.8$) on the $\theta(\eta)$ distribution. It is noticed that the $\theta(\eta)$ profile has decreased when the parameter b has been increased from 0.2 to 0.8. Furthermore, increasing b , resulting in reduction of thermal layer thickness. The reason for this is because the heat is being transferred away from the surface more effectively due to an increase in the squeezing effect, which improves the fluid motion between the plates. Because of this, the $\theta(\eta)$ distribution is lowered and the thermal layer is reduced as well.

Permeable velocity parameter (f_0):

Figures 16 and 17 show the impact of f_0 on $f'(\eta)$ and $\theta(\eta)$ correspondingly. By increasing the values of f_0 , the $f'(\eta)$ profile is depressed across the layer, as seen in figure 16. Also, when f_0 increases, both $\theta(\eta)$ and the thermal layer thickness decreases, as can be seen in figure 17. Physically, surface suction action is represented by f_0 . The fluid particles are ejected by a greater suction when f_0 rises from -2 to -0.5. Resulting in the reduction of $f'(\eta)$ and the momentum layer. A thinner thermal layer is another result of the rapid surface-to-subsurface evacuation of the hot fluid.

Temperature ratio (θ_w) and Radiation parameter R :

Figure 18 examine the influence of θ_w on the $\theta(\eta)$ profile. It can be observed that an increasing θ_w leads to an enhances in the $\theta(\eta)$ distribution. Additionally, thickness of the thermal layer also enhances for larger θ_w parameter. From a physical perspective, θ_w heightens the disparity between surface heat and fluid heat. Over the boundary layer, thermal diffusion is improved as the temperature ratio parameter is set to a bigger value, since it gives more heat to the fluid particles. Figure 19 illustrates the relationship between $\theta(\eta)$ profile and the radiation parameter (R). As R grows, the $\theta(\eta)$ also increases. Additionally, higher levels of R result in an improved thermal layer. Physically, Rise in R results in increase of temperature, which is linked to the spatial variation in the fluid's thermal conductivity. Enhanced thermal conductivity improves heat transfer across the fluid, resulting in elevated fluid temperatures due to more efficient thermal energy distribution.

Prandtl and Eckert Number:

Figure 20 depicts the changing $\theta(\eta)$ as a function of the Prandtl number (Pr). The temperature gradient across the boundary layer becomes narrower as Pr rises. Consequently, the thermal layer contracts and the $\theta(\eta)$ field becomes weaker. Thus, a more compact thermal boundary layer and poorer heat diffusion are associated with bigger Pr . The $\theta(\eta)$ and thermal layer is considerably raised by an increase in Ec as shown in Figure 21. The process of viscous dissipation is responsible for this result because it raises the fluid's temperature by transforming kinetic energy into thermal energy. A thicker thermal layer, indicative of improved energy conversion, is a result of an increase in Ec . The heat transport properties are enhanced and the temperature fluid is raised by increased viscous dissipation.

Figure 22 illustrates the effect of the thermal relaxation parameter (γ^*) on the $\theta(\eta)$. As γ^* increases, the temperature distribution decreases. Physically, a larger γ^* corresponds to a longer thermal relaxation time, meaning the fluid responds more slowly to thermal disturbances at the stretching surface. This non-Fourier effect reduces the transfer of heat from the surface into the fluid, producing a thinner and cooler thermal boundary layer.

Friction factor and Nusselt Number ($C_f Re_x^{1/2}$ and $Nu_x/Re_x^{1/2}$):

Figures 23 and 24 show the variation of the coefficient $C_f Re_x^{1/2}$ coefficient with different physical parameters. From these figures, it shows that, the higher values of M higher the $C_f Re_x^{1/2}$. In contrast, raising ϕ_1 and b decreases the $C_f Re_x^{1/2}$ coefficient. Figure 25 depict the variation of $Nu_x/Re_x^{-1/2}$ with θ_w and Pr . The local $Nu_x/Re_x^{-1/2}$ drops with the rise of

θ_w , but it reverses in higher Pr . In the other hand for higher values of R and γ^* enhances the $Nu_x/Re_x^{-1/2}$ as seen in Figure 26.

Table 4: Numerical values of $C_f Re_x^{1/2}$ and $Nu_x Re_x^{-1/2}$ for different physical parameter

ϕ_1	ϕ_2	M	R	Ec	Pr	b	γ^*	θ_w	f_0	$C_f Re_x^{1/2}$	$Nu_x Re_x^{-1/2}$
10%										1.114318	0.663919
20%										0.971222	0.731822
30%										0.832214	0.790688
	10%									0.982741	0.728219
	20%									0.971222	0.731822
	30%									0.950458	0.739855
		0.5								0.332038	0.919093
		1								0.703496	0.842170
		1.5								0.971222	0.731822
			0.5							0.971222	0.536823
			1							0.971222	0.731822
			1.5							0.971222	0.763925
				0.2						0.971222	1.106359
				0.4						0.971222	0.967289
				0.6						0.971222	0.731822
					5					0.971222	0.488618
					6					0.971222	0.657678
					7					0.971222	0.731822
						0.2				1.052016	0.449273
						0.4				0.971222	0.731822
						0.6				0.885010	0.983399
							0.2			0.971222	0.614623
							0.4			0.971222	0.672023
							0.6			0.971222	0.731822
								1.2		0.971222	0.771595
								1.4		0.971222	0.748383
								1.6		0.971222	0.731822
									-0.2	1.008386	0.762986
									-0.4	0.989647	0.748128
									-0.6	0.971222	0.731822

Table 4 shows the physical factors affect $C_f Re_x^{1/2}$ and $Nu_x Re_x^{-1/2}$. The increase in ϕ_1 and ϕ_2 leads to reduced $C_f Re_x^{1/2}$ and increased $Nu_x Re_x^{-1/2}$ due to better thermal conductivity of hybrid nanofluid. Because of the Lorentz force, the magnetic parameter M increases $C_f Re_x^{1/2}$ and decreases $Nu_x Re_x^{-1/2}$. As parameter R rises, $Nu_x Re_x^{-1/2}$ increases because radiation transports thermal energy. Ec reduces $Nu_x Re_x^{-1/2}$ when viscous dissipation raises fluid temperature. Due to reduced thermal diffusivity and thermal boundary layer thickness, Pr

promotes $Nu_x Re_x^{-1/2}$. Furthermore, parameter b raises $Nu_x Re_x^{-1/2}$. Other hand, increasing the γ^* , θ_w , R and θ_w delaying $Nu_x Re_x^{-1/2}$.

Conclusion

The study discusses the steady flow of an iron oxide–graphene hybrid nanofluid suspended in a $H_2O - EG$ base fluid over a sensor surface in the presence of a MHD, thermal radiation and C-C heat flux. The governing equations are solved using the BVP5C technique. The obtained results are further validated through an independent implementation of the Levenberg–Marquardt back-propagation algorithm, which demonstrates excellent agreement with the numerical solutions. Here are the key takeaways from the investigation:

- The ANN-LMBP surrogate can recreate the verified numerical solution to within the line width of the reference plot, and it may be used instead of the complete numerical solver for quick parametric exploration of the current setup.
- A combination of electromagnetic drag and higher effective viscosity on the wall shear stress, increases $Re_x^{1/2} C_{fx}$ along with the M , ϕ_1 and ϕ_2 parameter.
- γ^* decreases both the temperature gradient and the Nu_x . This non-Fourier effect highlights the need of using the C-C model in fast or high-frequency thermal dynamics and the inapplicability of the traditional Fourier law of thermal conduction in fluids with limited thermal relaxation time.
- Viscous dissipation, defined by the Ec , is always increasing the temperature of the fluid and reducing the Nu_x , which becomes more pronounced at an increased level of stretching.
- Because high- Pr fluids have a lower thermal diffusivity and thus thinner thermal boundary layers, a lower Pr is associated with a lower $\theta(\eta)$.
- As R and Ec parameter values rise, the temperature profile $\theta(\eta)$ also increases. Radiative heating and viscous dissipation are improved thermal conductivity.

Future work: The present investigation is limited to steady two-dimensional flow under prescribed free-surface boundary conditions. Future studies may extend the analysis to unsteady flow regimes, 3D configurations, and broader categories of non-Newtonian base fluids.

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Clinical trial: not applicable.

Data availability: The data that supports the findings of this study are available within the article.

Credit Author Statement: Ganesh Kumar - Writing original draft and Formal analysis, Chidananda- software and Calculation;

Declaration of Conflict: The authors declare that there have no competing financial interest or personal relationship there could have appear to influence the work reported in this work.

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Figures:

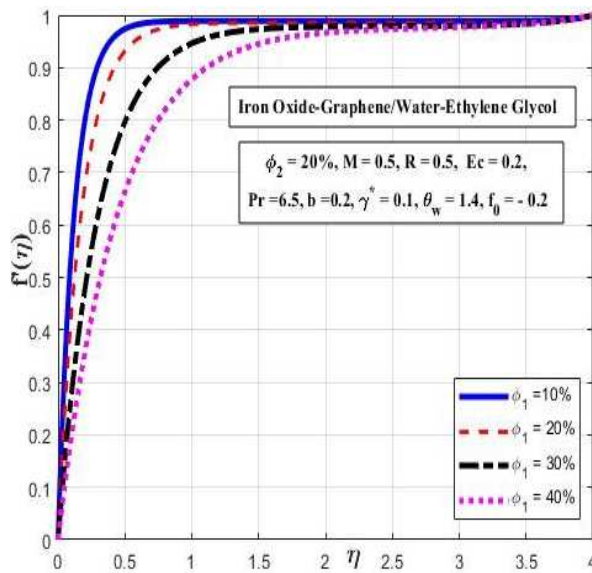


Figure 9: behaviour of ϕ_1 on $f'(\eta)$.

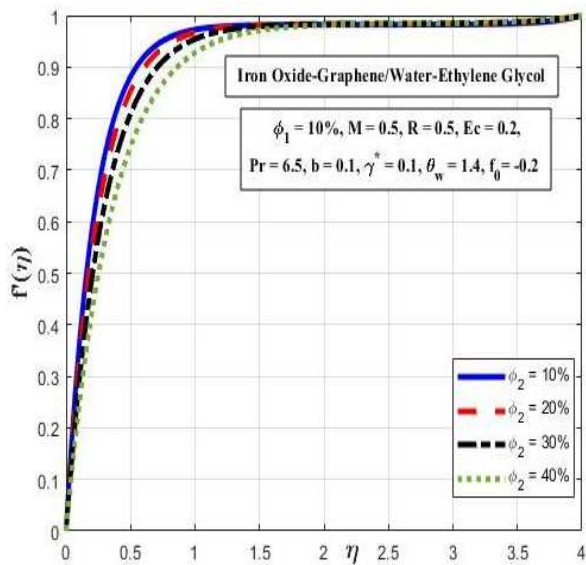


Figure 10: behaviour of ϕ_2 on $f'(\eta)$.

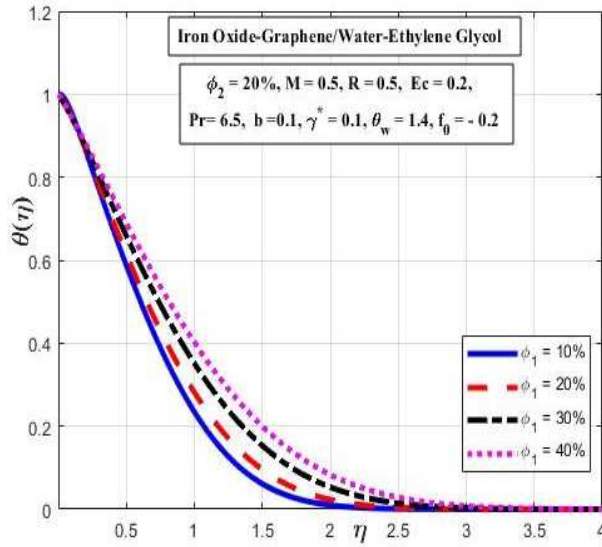


Figure 11: behaviour of ϕ_1 on $\theta(\eta)$.

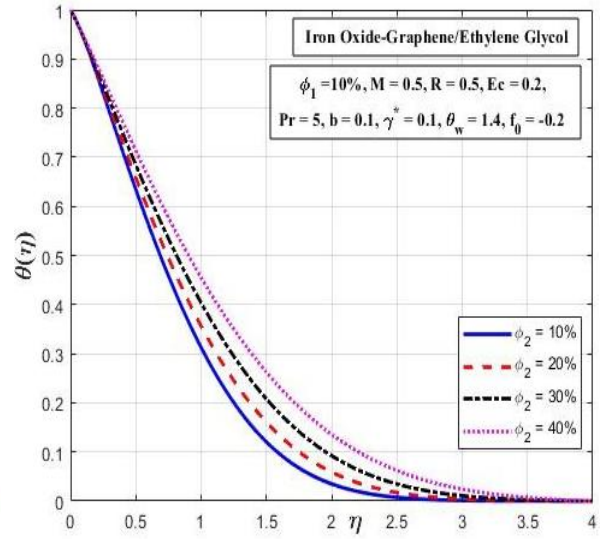


Figure 12: behaviour of ϕ_2 on $\theta(\eta)$.

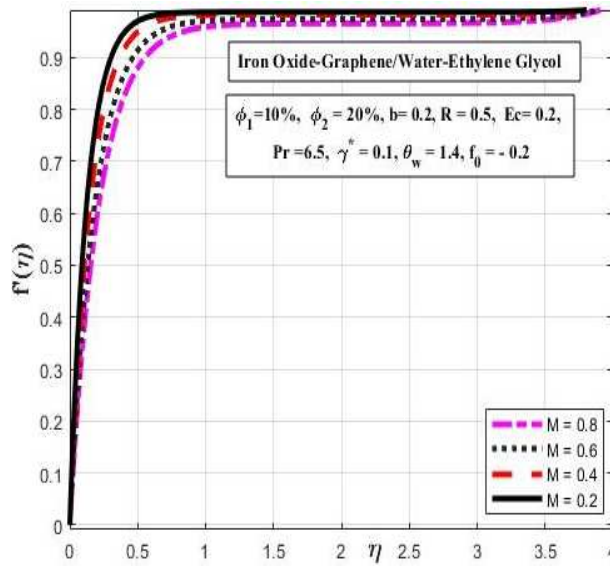


Figure 13: behaviour of M on $f'(\eta)$.

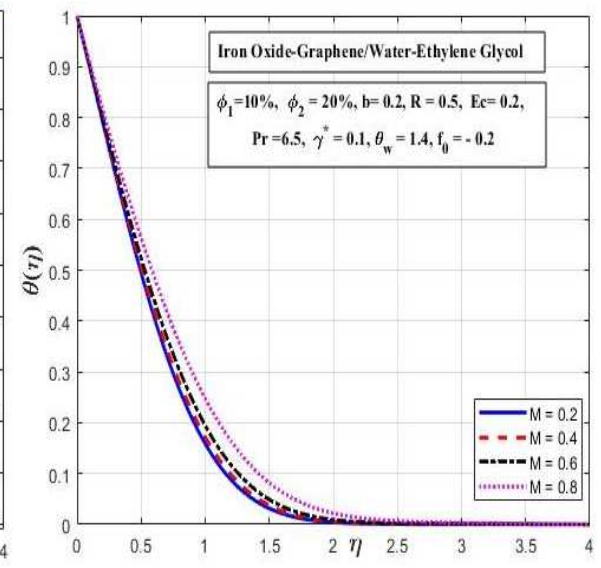


Figure 14: behaviour of M on $\theta(\eta)$.

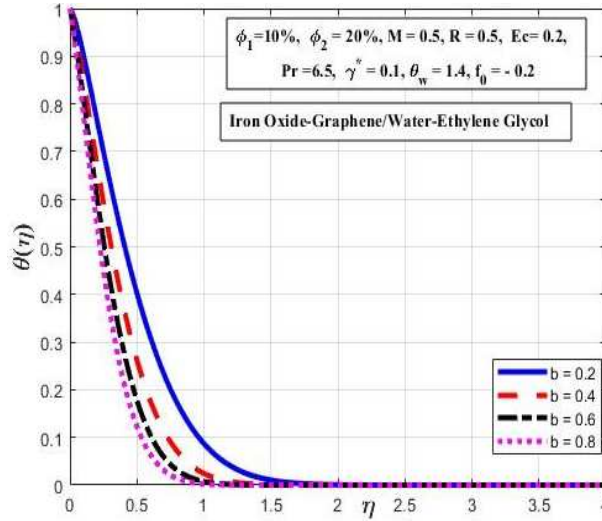


Figure 15: behaviour of b on $\theta(\eta)$.

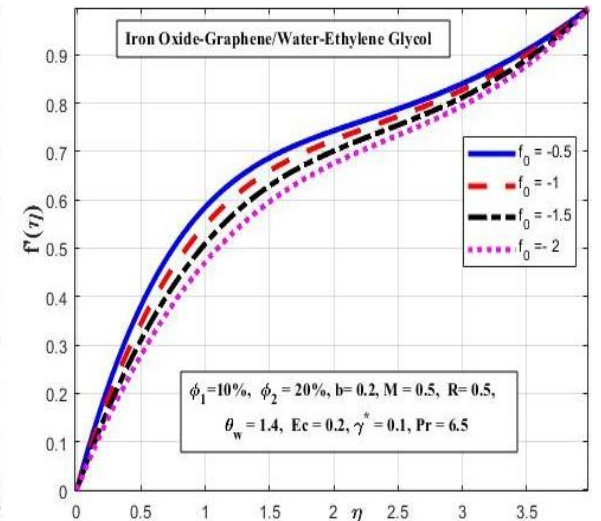


Figure 16: behaviour of f_0 on $f'(\eta)$.

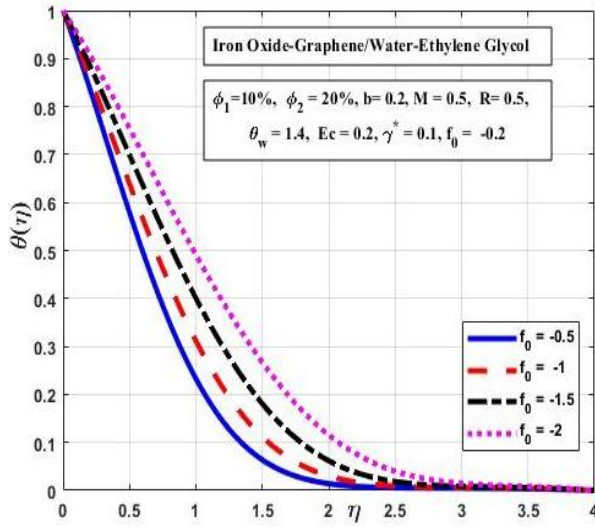


Figure 17: behaviour of f_0 on $\theta(\eta)$.

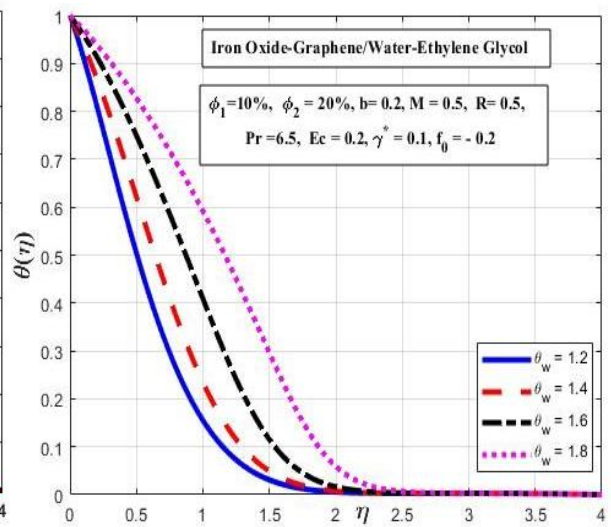


Figure 18: behaviour of θ_w on $\theta(\eta)$.

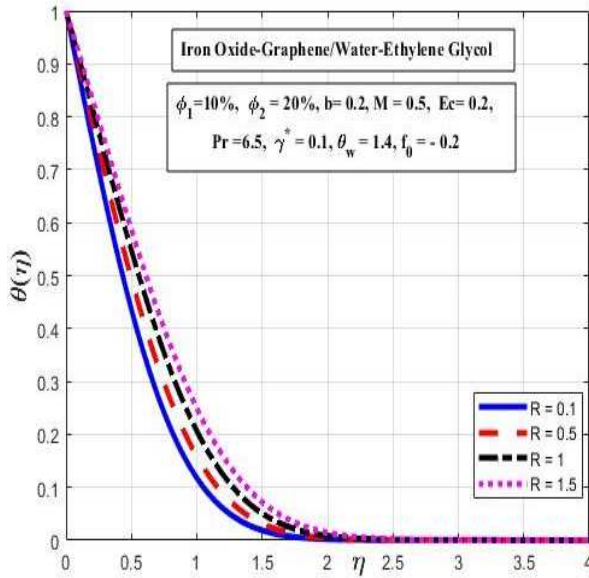


Figure 19: behaviour of R on $\theta(\eta)$.

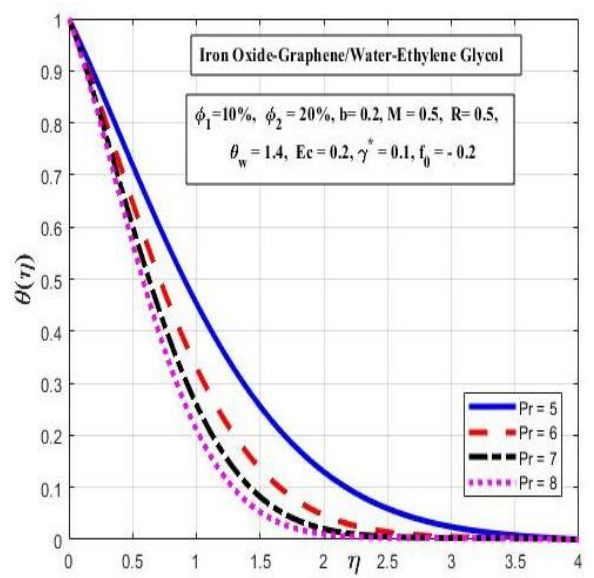


Figure 20: behaviour of Pr on $\theta(\eta)$.

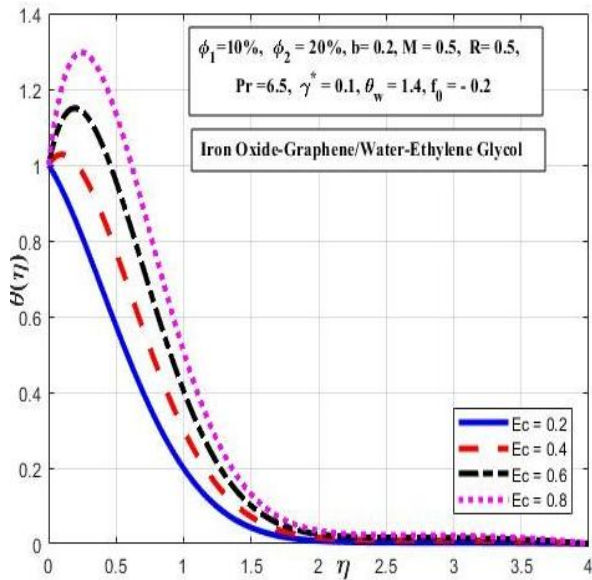


Figure 21: behaviour of Ec on $\theta(\eta)$.

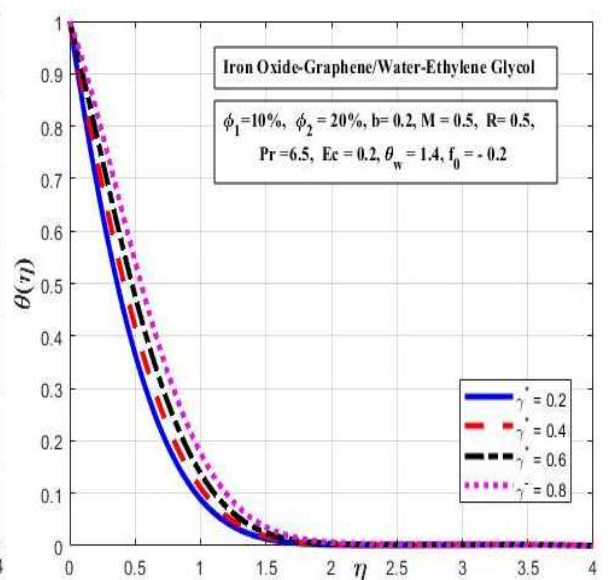


Figure 22: behaviour of γ^* on $\theta(\eta)$.

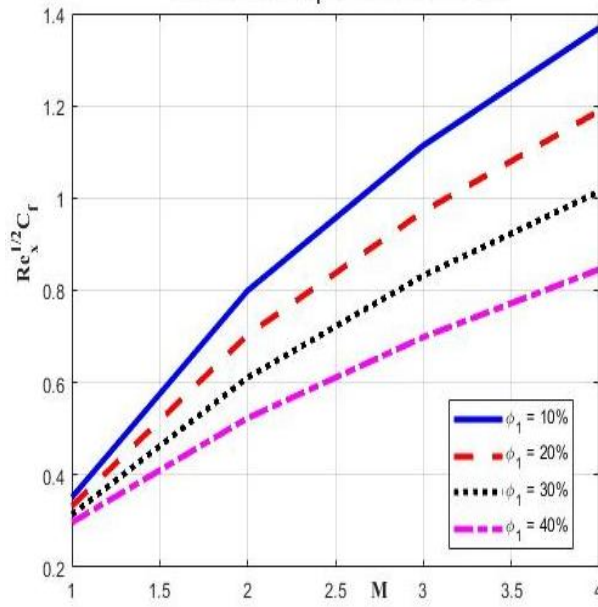


Figure 23

Figure 23: behaviour of ϕ_1 and M on $Re_x^{1/2} C_f$.

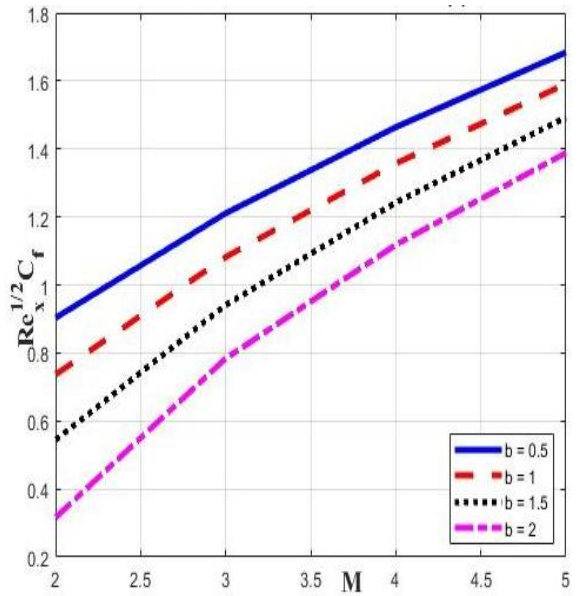


Figure 24

Figure 24: behaviour of ϕ_1 and M on $Re_x^{1/2} C_f$.

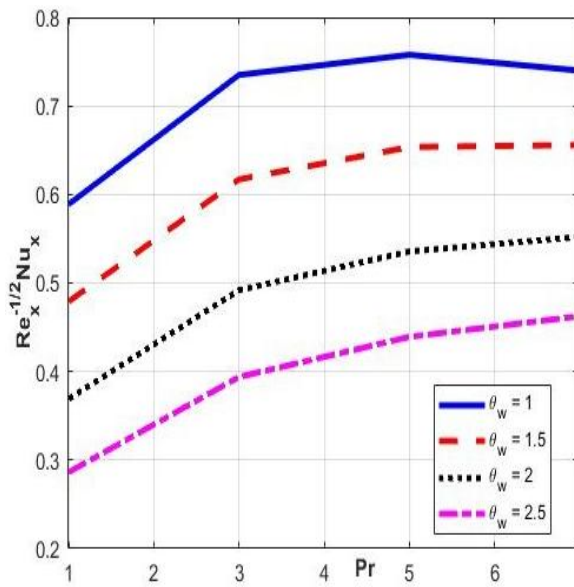


Figure 25

Figure 25: behaviour of θ_w and Pr on $Re_x^{-1/2} Nu_x$.

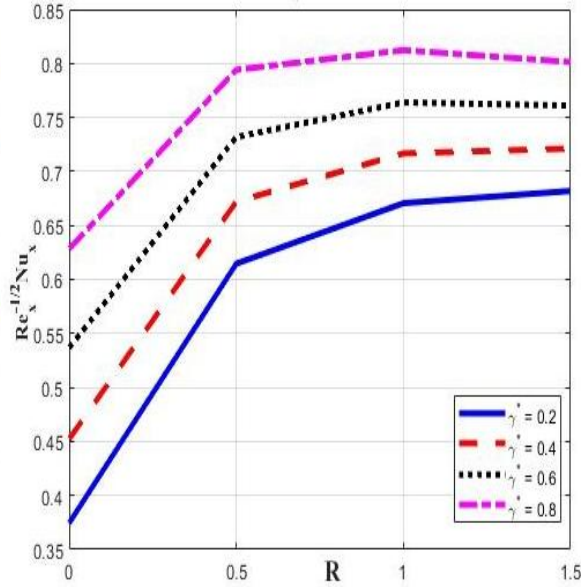


Figure 26

Figure 26: behaviour of γ^* and R on $Re_x^{-1/2} Nu_x$.