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Encoding-Aware Fisher Diagnostics in Open Quantum Systems: Separating Backflow from Process-Parameter Readability

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Abstract

Fisher information is a central diagnostic in quantum readout, sensing, and open-system dynamics, but a positive Fisher derivative is not by itself a non-Markovianity witness. This paper does not introduce a new universal monotonicity measure. Instead, it establishes an encoding-aware diagnostic rule for deciding when Fisher growth can be interpreted as information backflow and when it should be read as ordinary parameter imprinting or measurement readability. The rule separates three operational layers: input-encoded optimized Fisher geometry, channel-parameter Fisher information, and fixed-readout Fisher information. For input-encoded parameters propagated by parameter-independent CP-divisible dynamics, optimized quantum Fisher information has the expected Lyapunov contraction interpretation; a revival is therefore a genuine backflow candidate. For channel-encoded parameters, Fisher information can grow even under a Markovian semigroup because the dynamics progressively writes the process parameter into the output state. A calibrated binary readout makes the measurement layer explicit. Dephasing, amplitude-damping, and two-qubit X-state examples then show the quantum-information payoff: Fisher readability, coherence, concurrence, and complementarity coincide in special calibrated sectors but separate in general. In particular, amplitude-damped X-state slices contain regions with vanishing concurrence but nonzero phase Fisher readability. The resulting criterion is an interpretation rule, not a replacement for established non-Markovianity witnesses: Fisher growth becomes causally meaningful only after the encoding, dynamical map, and readout layer have been fixed.

Keywords: quantum Fisher information; quantum information processing; open quantum systems; non-Markovianity; quantum sensing; measurement diagnostics

1 Introduction

Fisher information is often treated as a universal indicator of information loss and recovery in open quantum dynamics. This intuition is correct in an important but narrower setting: an already encoded parameter is transported by a parameter-independent dynamical map and is monitored with an optimized, contractive Fisher geometry. Outside that setting, the same mathematical increase of Fisher information can express a different physical process. In particular, when the estimated parameter belongs to the channel itself, Fisher growth may simply record the increasing readability of a process parameter as the channel acts.

The central result of this paper is an encoding-aware separation of these cases. The question is not only whether a Fisher signal increases in time, but what causal role the parameter plays. Is it stored in the initial state and passively processed by the dynamics? Is it a parameter of the

dynamical map? Or is it accessed through a fixed measurement record whose Fisher information depends on analyzer geometry? These alternatives lead to distinct diagnostic meanings.

This point is deliberately framed as an interpretation rule rather than as a new universal witness. QFI-flow and related optimized metrological quantities already provide non-Markovianity witnesses under their own hypotheses. The present contribution is prior to that witness question: it specifies when the Fisher quantity under discussion belongs to the Lyapunov layer in which a backflow statement is meaningful, and when it belongs instead to a channel-estimation or fixed-readout layer.

This distinction is operationally important for quantum information processing and quantum sensing. A monotone or Lyapunov-type Fisher quantity requires a precise dynamical setting: an input-encoded parameter, a parameter-independent evolution, and an optimized or contractive Fisher metric. A fixed classical readout is a different object; it can be experimentally useful but it is not automatically a Lyapunov function. A channel-encoded Fisher quantity is different again; it quantifies process readability and can grow under a memoryless semigroup without implying non-Markovian backflow.

The purpose of this paper is to make this separation into a usable diagnostic rule for open quantum systems. We separate three layers:

$$\begin{aligned} &\text{Lyapunov-Fisher geometry,} \\ &\text{fixed-readout Fisher information,} \\ &\text{channel-parameter Fisher information.} \end{aligned} \tag{1}$$

The central question is therefore not merely whether

$$\frac{d}{dt}\mathcal{I}(\theta, t) > 0 \tag{2}$$

occurs, but which parameter is being estimated, which map processes it, and which readout produces the Fisher signal.

Main contributions

The paper makes four contributions.

- (i) It formulates an input/channel/readout separation for Fisher diagnostics in open quantum systems. This separation identifies the causal role of the estimated parameter before assigning a backflow interpretation to temporal Fisher growth.
- (ii) It states the Lyapunov-Fisher claim in its sharp operational form: for input-encoded parameters processed by parameter-independent CP-divisible dynamics, optimized QFI or a contractive monotone Fisher metric cannot increase.
- (iii) It gives a Markovian control example showing that channel-encoded Fisher information for a damping rate can grow under a memoryless amplitude-damping semigroup. This growth is process-parameter imprinting, not memory backflow.
- (iv) It extends the distinction to calibrated two-qubit readout layers. In the Bell-dephasing benchmark Fisher readability, coherence visibility, and concurrence are locked together; in amplitude-damped X-state slices they separate, including regions where concurrence vanishes while phase Fisher readability remains nonzero.

This framing complements established QFI-flow and optimized metrological witnesses of non-Markovianity. Its purpose is not to replace those witnesses, and the diagnostic rule below is not itself a new monotone. It is a layer-identification criterion: it specifies the preconditions under which a Fisher increase can be given the stronger causal meaning of information backflow.

The distinction made here is not between different formulas for Fisher information, but between different causal roles of the estimated parameter. Existing QFI-flow approaches ask whether a chosen distinguishability measure revives during open-system evolution. The present work asks a prior diagnostic question: was the parameter whose distinguishability revives encoded in the input state and passively processed by the dynamics, or is it a parameter of the dynamics itself that is progressively written into the output? This input/channel/readout separation is the basis for the interpretation rule developed below.

For quantum-information diagnostics, the payoff is practical. Fisher or visibility data may be used as a low-cost entrance layer for deciding whether richer quantum-information reconstructions are justified, but only after their causal status is known. In a calibrated Bell-dephasing sector, Fisher, coherence, and concurrence revivals coincide. In amplitude-damped X-state sectors, Fisher readability can remain positive after concurrence has vanished. Thus the same mathematical idea of Fisher sensitivity supports different physical statements in different layers.

2 Related work and positioning

QFI flow as a non-Markovianity indicator was introduced and studied by Lu, Wang, and Sun [3]. A closely related and important optimized state-based metrological witness is the quantum-interferometric-power approach of Dhar, Bera, and Adesso [4]. Their quantity is constructed from a minimum quantum Fisher information over local Hamiltonians; revivals of this optimized metrological figure of merit witness non-CP-divisible dynamics. In the terminology used here, this belongs to the input- or state-encoded optimized layer. The purpose of the present work is different: we separate such optimized witnesses from channel-encoded Fisher growth and from fixed-readout record Fisher information, for which temporal increase need not by itself carry the same causal backflow meaning. Non-Markovianity itself has several inequivalent formulations, including trace-distance backflow, CP-divisibility, and entanglement-based criteria [5, 6]. These approaches already show that a positive information-flow derivative is meaningful only relative to the information quantity being monitored.

The present paper adds a prior interpretation layer to these witnesses. It addresses a separation that is easy to overlook in applications but is decisive for causal interpretation: the monitored parameter can be input encoded or channel encoded. This is compatible with known contractivity results for monotone quantum metrics, including the Bures/Fisher metric under parameter-independent completely positive trace-preserving maps [1, 2]. Recent work has sharpened this point by characterizing Markovianity through Fisher-metric contraction and by reviewing the dynamical role of quantum Fisher metrics [7, 8, 9]. These results support the Lyapunov side of the present distinction.

It is also compatible with quantum channel parameter estimation, where the parameter is part of the map rather than part of the input state. This line includes one-parameter channel estimation, fiber-bundle and channel-statistical approaches, and general-dynamics formulations of quantum parameter estimation [10, 11, 12, 13]. In that setting Fisher growth may reflect improved readability of a process parameter rather than return of information that was stored in the initial state. Recent model studies continue to use QFI matrices as witnesses of non-Markovianity and other open-system features [14]. Our contribution is to turn the input/channel/readout distinction into an explicit quantum-information diagnostic for interpreting Fisher growth in open-system readout.

The two-qubit examples connect the rule to complementarity and measurement-cost diagnostics. In simple Bell-sector dephasing, Fisher readability, coherence visibility, and concurrence can coincide. In amplitude-damped and mixed X-state sectors they separate. Here measurement-cost triage means the operational use of low-cost observables, such as visibility or fixed-readout Fisher information, to decide whether more expensive reconstructions of entanglement, process structure, or complementarity budgets are justified. No claim in the present

work depends on a separate triage formalism; related complementarity and triage perspectives are cited only as context [18, 19, 20].

This framing is suited to a quantum-information diagnostic presentation because it states exactly what a Fisher signal can certify. The same formal Fisher expression can support different physical statements depending on whether it is attached to an initial codeword, a dynamical rate, or a fixed measurement channel. The manuscript therefore makes the causal content explicit: Fisher growth is a signal of changing distinguishability, while causal backflow is a stronger statement that requires an input-encoded parameter and a parameter-independent post-encoding dynamics.

For a quantum-information reader, this matters because the same experimental record can be used for different tasks: parameter sensing, memory diagnostics, entanglement monitoring, or measurement-cost triage. The present work supplies a rule for not confusing these tasks. It identifies when a Fisher signal can be used as an entrance diagnostic for backflow-sensitive reconstruction and when it should instead be interpreted as ordinary process-parameter readability.

3 Fisher readout and encoding layers

Let a measurement with outcomes y produce probabilities $p(y|\theta, t)$. The classical Fisher information of this readout is

$$\mathcal{I}_y(\theta, t) = \sum_y p(y|\theta, t) [\partial_\theta \ln p(y|\theta, t)]^2. \quad (3)$$

This is the standard classical Fisher information of the measured distribution. The quantum Fisher information $\mathcal{F}_Q[\rho_\theta(t)]$ is the corresponding optimized infinitesimal distinguishability over all measurements and is equivalent to the Bures/Fisher metric normalization used in quantum estimation theory [1, 2]. The causal interpretation depends on where the parameter enters and on which Fisher layer is used.

Definition 1 (Input-encoded Fisher layer). *A parameter is input encoded if it is stored in the initial system state before the open-system dynamics acts:*

$$\rho_\theta^{\text{in}}(t) = \Lambda_t[\rho_\theta(0)]. \quad (4)$$

The map Λ_t is independent of θ . Fisher decay and recovery can then be interpreted as loss and possible return of information about an initially stored degree of freedom, in the same spirit as contractivity-based non-Markovianity diagnostics [3, 5, 6].

Definition 2 (Channel-encoded Fisher layer). *A parameter is channel encoded if it belongs to the dynamical map:*

$$\rho_\theta^{\text{ch}}(t) = \Lambda_t^\theta[\rho_0]. \quad (5)$$

The Fisher signal quantifies readability of the process parameter. Its temporal growth can occur because the dynamics needs time to imprint the parameter into the state; this is the setting of quantum channel parameter estimation rather than passive processing of an initially encoded state family [10, 11, 12, 13].

Definition 3 (Fixed-readout Fisher layer). *A fixed-readout Fisher layer is the classical Fisher information of a specified measurement, analyzer setting, or coarse-grained observation. It can be operationally useful, but it need not coincide with optimized QFI and it is not automatically a Lyapunov quantity.*

Remark 1. *The three layers can coexist in a real experiment. A model may contain an initially encoded phase, an unknown damping rate, a nuisance visibility, and a fixed analyzer. A causal Fisher analysis should state which parameter is being tracked and which parameters are held fixed or treated as nuisance parameters.*

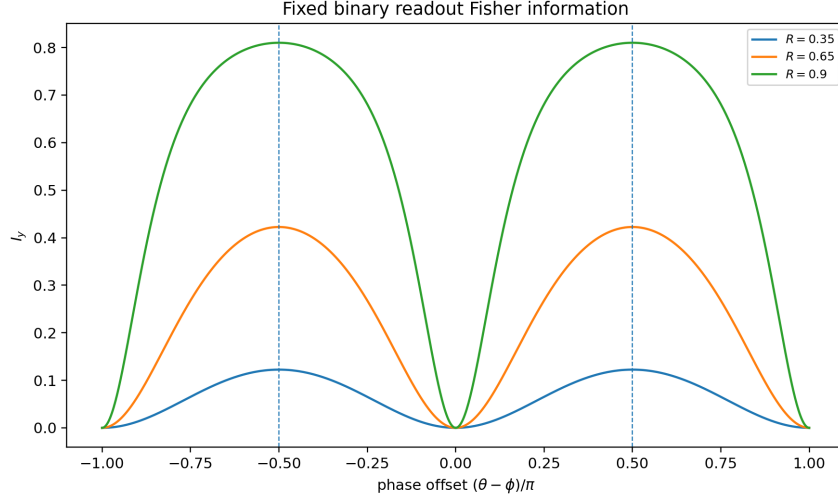


Figure 1: Fixed-readout Fisher information for a binary phase measurement. The optimized quadrature gives $\mathcal{I}_y^{\text{opt}} = R^2$, while a mismatched analyzer produces a lower readout Fisher signal. This is why a fixed classical Fisher readout should not be identified automatically with a Lyapunov Fisher quantity.

4 Explicit fixed-readout measurement model

A potential ambiguity in Fisher-flow diagnostics is the relation between state-space Fisher geometry and the Fisher information of a concrete measurement. Since optimized quantum Fisher information and classical readout Fisher information coincide only for suitable measurements, we make the fixed-readout layer explicit [1].

Consider a two-outcome phase readout with probabilities

$$p_{\pm}(\theta, t) = \frac{1}{2} [1 \pm R(t) \cos(\theta - \phi)], \quad (6)$$

where $R(t)$ is the measurable visibility amplitude and ϕ is the analyzer phase. The corresponding classical Fisher information is

$$\mathcal{I}_y(\theta, t) = \frac{R(t)^2 \sin^2(\theta - \phi)}{1 - R(t)^2 \cos^2(\theta - \phi)}. \quad (7)$$

The optimal quadrature setting satisfies $\cos(\theta - \phi) = 0$, giving

$$\mathcal{I}_y^{\text{opt}}(t) = R(t)^2. \quad (8)$$

Thus a fixed classical readout becomes a clean proxy only after the analyzer is calibrated to the encoded quadrature and after $R(t)$ is known to be the processed input-encoded visibility. Away from this matched setting, the same state can produce a smaller or distorted classical Fisher signal because of measurement geometry.

This example shows why fixed classical Fisher information should not be equated with optimized QFI without calibration. The same evolving state family can produce different Fisher traces under different analyzer phases. Therefore, a fixed-readout Fisher signal has a direct Lyapunov interpretation only when the readout is known to track the input-encoded tangent direction and when the underlying optimized Fisher geometry is contractive.

5 Lyapunov-Fisher layer and the clean dephasing benchmark

The Lyapunov intuition for Fisher information is correct when applied to the appropriate object. If the parameter is input encoded and the later map is parameter independent, CP-divisible

dynamics cannot increase the optimized QFI or any contractive monotone Fisher metric [2, 7, 8, 9]. This is the precise version of the Lyapunov intuition.

Lemma 1 (Input-encoded Fisher contractivity). *Let $\rho_\theta(t) = \Lambda_t[\rho_\theta(0)]$ and suppose $\Lambda_t = V_{t,s}\Lambda_s$ for $t \geq s$, with $V_{t,s}$ completely positive and trace preserving, as in the CP-divisibility formulation of quantum Markovianity [6]. Then*

$$\mathcal{F}_Q[\rho_\theta(t)] \leq \mathcal{F}_Q[\rho_\theta(s)]. \quad (9)$$

Thus an increase in input-encoded optimized QFI is incompatible with CP-divisible parameter-independent evolution.

Proof. Since $\rho_\theta(t) = V_{t,s}[\rho_\theta(s)]$ and $V_{t,s}$ is parameter independent and CPTP, monotonicity of the Bures metric implies monotonicity of the QFI:

$$\mathcal{F}_Q[V_{t,s}(\rho_\theta(s))] \leq \mathcal{F}_Q[\rho_\theta(s)]. \quad (10)$$

The argument fails if $V_{t,s}$ itself depends on θ , because then the map is no longer passive processing of the same parameterized state family. $\square$

A one-qubit dephasing model gives the simplest positive benchmark for coherence-loss and coherence-revival diagnostics in open quantum systems [5]. Let a phase θ be encoded in the equatorial coherence before the dephasing channel acts:

$$\rho_\theta(t) = \frac{1}{2} \begin{pmatrix} 1 & r_\perp \kappa(t) e^{-i\theta} \\ r_\perp \kappa^*(t) e^{i\theta} & 1 \end{pmatrix}. \quad (11)$$

The binary readout has visibility amplitude $R(t) = r_\perp |\kappa(t)|$. For the optimal quadrature readout,

$$\mathcal{I}_{\text{opt}}^{\text{in}}(t) = r_\perp^2 |\kappa(t)|^2. \quad (12)$$

Therefore,

$$\frac{d}{dt} \mathcal{I}_{\text{opt}}^{\text{in}}(t) > 0 \iff \frac{d}{dt} |\kappa(t)|^2 > 0. \quad (13)$$

This is the clean case in which Fisher growth, coherence revival, and a backflow reading coincide.

6 Channel-encoded Fisher growth under Markovian amplitude damping

The same inference fails when the parameter belongs to the channel. This section provides the decisive Markovian control case: a positive Fisher derivative can arise without memory. When the estimated parameter is the damping rate, the tangent vector is created by the channel during the evolution. Fisher growth then measures how well the process has imprinted its own parameter into the observed state. Consider a standard Markovian amplitude-damping semigroup with rate Γ and survival probability $e^{-\Gamma t}$ [15, 16]. Starting from the excited state and measuring ground/excited population gives

$$p_1(t) = e^{-\Gamma t}, \quad p_0(t) = 1 - e^{-\Gamma t}. \quad (14)$$

If the estimated parameter is Γ , the binary Fisher information is

$$\mathcal{I}_\Gamma(t) = \frac{(\partial_\Gamma p_0)^2}{p_0} + \frac{(\partial_\Gamma p_1)^2}{p_1} = \frac{t^2 e^{-\Gamma t}}{1 - e^{-\Gamma t}}. \quad (15)$$

This quantity grows at early times and decays at late times. There is no contradiction with Markovianity: the derivative $\partial_\Gamma e^{-\Gamma t} = -t e^{-\Gamma t}$ contains a factor t because the process rate becomes readable only as the process unfolds. The tangent vector $\partial_\Gamma \rho_\Gamma(t)$ is generated by the dynamics rather than being a contracted image of an initial tangent vector. Dissipation can erase information about initial-state features while simultaneously writing information about its own rate into the observed distribution.

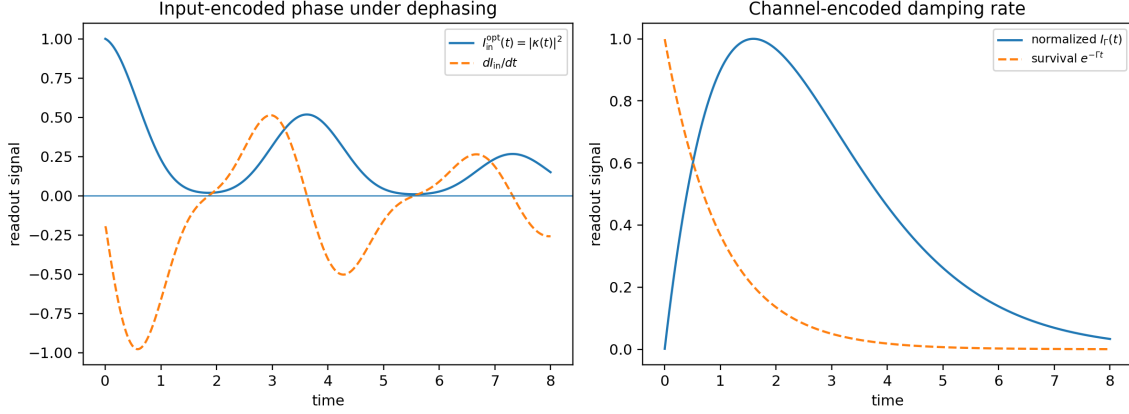


Figure 2: Central comparison. Left: input-encoded Fisher information under dephasing tracks the surviving coherence and can be interpreted as a backflow signal when it revives. Right: channel-encoded Fisher information for a Markovian damping rate can increase while the survival probability decays monotonically. This is ordinary process-parameter imprinting, not memory backflow.

7 Encoding-aware causal Fisher-readout rule

The previous examples motivate the following diagnostic rule.

Diagnostic Rule 1 (Encoding-aware Fisher-readout criterion). *A positive Fisher-flow signal*

$$\partial_t \mathcal{I}_y(\theta, t) > 0 \quad (16)$$

supports a causal backflow interpretation only after the following checks have been passed:

- (i) *the tracked parameter is input encoded, or channel-encoded contributions have been separated;*
- (ii) *the post-encoding evolution is parameter independent for the tracked parameter;*
- (iii) *the Fisher layer is either optimized QFI/monotone metric or an explicitly calibrated fixed readout;*
- (iv) *the signal is compared with a Markovian channel-parameter-imprinting control.*

If these checks fail, Fisher growth should be interpreted as increased readability or sensitivity, not as a backflow certificate.

Remark 2. *This rule is not proposed as a new non-Markovianity monotone. It translates known metric contractivity into a readout protocol and adds the practical warning that channel estimation and backflow detection are different tasks.*

8 Two-qubit Fisher layers

The two-qubit setting is where the diagnostic separation becomes most relevant for quantum information processing. Here a Fisher signal may be used as a low-cost readout of a phase, while the target quantum-information resource may be entanglement, concurrence revival, or a complementarity-budget redistribution. These are different layers. The purpose of the following examples is therefore not to define a new entanglement monotone, but to show precisely when a calibrated Fisher readout coincides with concurrence and when it does not.

Table 1: Diagnostic meaning of different Fisher layers. Only the first row has a direct Lyapunov/backflow reading under CP-divisible parameter-independent dynamics.

Fisher layer	Parameter location	Safe interpretation of growth
Optimized QFI or monotone metric	input state	Backflow candidate, provided the subsequent map is parameter independent.
Fixed classical readout	input state	Calibrated readability; not automatically a Lyapunov quantity.
Channel-parameter Fisher	dynamical map	Process-parameter imprinting rather than memory backflow.
Two-qubit layer Fisher	specified observable sector	Layer-specific readability; compare with concurrence or defect separately.

8.1 Bell-dephasing calibration sector

For a Bell-sector phase encoded before pure dephasing, the optimal Bell-quadrature Fisher information is

$$\mathcal{I}_{\text{Bell}}^{\text{in}}(t) = |\kappa(t)|^2. \quad (17)$$

For the symmetric Bell benchmark the concurrence and local complementarity defect satisfy, using Wootters concurrence and the local complementarity-budget notation,

$$C_{AB}^2(t) = |\kappa(t)|^2, \quad D_A(t) = 1 - |\kappa(t)|^2. \quad (18)$$

Here the concurrence layer follows the standard two-qubit concurrence construction, while D_A follows the complementarity-budget framework [17, 18]. Thus

$$\dot{\mathcal{I}}_{\text{Bell}}^{\text{in}} > 0 \iff \frac{d}{dt} C_{AB}^2 > 0 \iff \frac{d}{dt} D_A < 0. \quad (19)$$

This is the positive calibration sector: Fisher readability, pair entanglement, and complementarity-budget redistribution are locked together because the readout, encoding, and resource layer have been matched.

8.2 Amplitude-damped X-state separation

The locking is not generic. Consider amplitude-damped X-state slices parameterized by a damping parameter γ and a coherence scale λ . X states are useful here because their concurrence layer has a closed analytic form obtained from the standard two-qubit concurrence formula [17]. The readout is the calibrated input-phase measurement of Eq. (6), applied to the relevant Bell-sector quadrature after the damping map. The formulas below are therefore specified Fisher readouts of a chosen observable sector, not optimized entanglement monotones.

For the Φ -like slice the nonzero entries are

$$\rho_{11} = \frac{1 + \gamma^2}{2}, \quad \rho_{22} = \rho_{33} = \frac{\gamma(1 - \gamma)}{2}, \quad \rho_{44} = \frac{(1 - \gamma)^2}{2}, \quad \rho_{14} = \frac{\lambda(1 - \gamma)}{2} e^{-i\theta}. \quad (20)$$

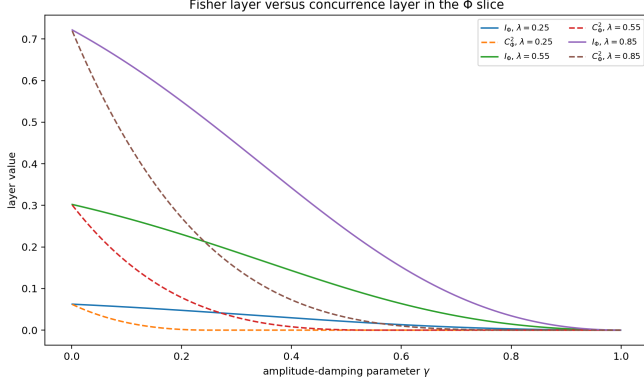
The occupied phase-readout sector has weight $w_\Phi = \rho_{11} + \rho_{44} = 1 - \gamma + \gamma^2$. The calibrated Fisher readout and concurrence are

$$\mathcal{I}_\Phi(\gamma, \lambda) = \frac{\lambda^2(1 - \gamma)^2}{1 - \gamma + \gamma^2}, \quad (21)$$

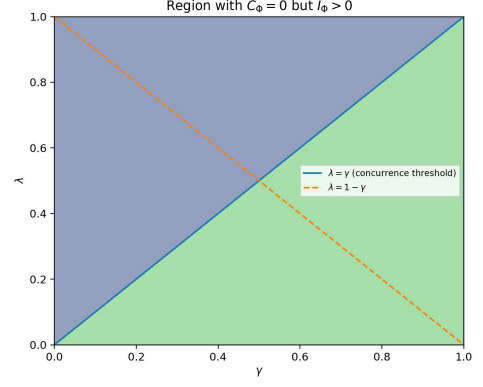
$$C_\Phi(\gamma, \lambda) = (1 - \gamma) \max\{\lambda - \gamma, 0\}. \quad (22)$$

Consequently, for $0 < \lambda \leq \gamma < 1$ one has

$$C_\Phi = 0, \quad \mathcal{I}_\Phi > 0. \quad (23)$$



(a) Fisher layer and concurrence layer.



(b) Region with $C_\Phi = 0$ but $\mathcal{I}_\Phi > 0$.

Figure 3: Two-qubit X-state layer separation. In the pure-dephasing Bell benchmark Fisher readability and concurrence coincide. In amplitude-damped X-state slices, Eqs. (21) and (22) show that phase Fisher readability can survive after concurrence vanishes.

The Fisher readout can still read phase coherence in a layer that no longer carries bipartite concurrence.

For the Ψ -like slice, with phase-carrying coherence in the $|01\rangle, |10\rangle$ sector, the corresponding quantities are

$$\mathcal{I}_\Psi(\gamma, \lambda) = \lambda^2(1 - \gamma), \quad C_\Psi^2(\gamma, \lambda) = \lambda^2(1 - \gamma)^2. \quad (24)$$

Again, Fisher readability and concurrence are related but not identical outside the pure-dephasing Bell calibration sector.

Proposition 1 (Two-qubit layer separation). *In the Bell-dephasing calibration sector, the input-phase Fisher layer and concurrence layer have identical revival windows. In the amplitude-damped Φ -like X-state slice, there is an open parameter region $0 < \lambda \leq \gamma < 1$ in which the concurrence is zero while the calibrated phase Fisher readout is strictly positive.*

Proof. The Bell-dephasing statement follows directly from $\mathcal{I}_{\text{Bell}}^{\text{in}} = C_{AB}^2 = |\kappa|^2$. For the Φ -like X-state slice, Eqs. (21) and (22) give $C_\Phi = 0$ whenever $\lambda \leq \gamma$, while $\mathcal{I}_\Phi > 0$ whenever $\lambda > 0$ and $\gamma < 1$. Therefore $0 < \lambda \leq \gamma < 1$ is a region of nonzero phase Fisher readability without bipartite concurrence. $\square$

This proposition is the main two-qubit message. A Fisher signal can be a useful quantum-information entrance diagnostic, but its certified content is layer dependent. In the calibrated Bell sector it identifies the same revival windows as concurrence. In the amplitude-damped X-state sector it certifies phase readability only; it does not certify entanglement recovery and does not by itself certify non-Markovian memory.

Quantum-information payoff. The X-state examples show why the encoding-aware distinction is more than terminology. The same experimental class of Fisher readout can either track concurrence revival in a calibrated Bell sector or remain positive after concurrence has vanished in an amplitude-damped sector. For QIP applications this is the useful distinction: Fisher data can guide measurement-cost triage, but it must be reported together with the encoded parameter, sector weight, and resource layer it is meant to diagnose.

9 Discussion

The central lesson is that Fisher information is a readout of sensitivity, and only some Fisher layers are Lyapunov layers. If the parameter is initially encoded and subsequently processed by

a parameter-independent CP-divisible map, optimized QFI growth contradicts contraction and supports a backflow interpretation. If the parameter belongs to the channel, Fisher growth can be the normal time required for the process to make its own rate readable.

The explicit binary measurement model separates optimized Lyapunov Fisher from fixed-readout Fisher. Coherence visibility, concurrence, complementarity defects, and Fisher readouts are not interchangeable. They may coincide in carefully chosen benchmark sectors, but those coincidences should be treated as model-dependent identities, not universal rules. In practice this suggests a triage workflow: first specify the encoding, then specify the measurement layer, and only then interpret temporal Fisher growth. This also gives a practical design principle: Fisher data may be used as a low-cost entrance layer, but the decision to reconstruct concurrence, complementarity budgets, or full process information should depend on which physical question the Fisher layer can actually certify.

The two-qubit X-state examples are controlled tests of this workflow. The Bell-dephasing sector is a positive calibration case: Fisher, coherence, and concurrence move together. The amplitude-damped X-state sectors are cautionary cases: a positive Fisher layer can remain after concurrence has vanished. This is not a paradox; it shows that the readout still sees a phase parameter in a state that no longer carries bipartite entanglement according to concurrence.

10 Conclusion

We have established an encoding-aware diagnostic rule for Fisher signals in open quantum systems. The rule is not a new universal non-Markovianity monotone; it is a practical criterion for deciding whether a positive Fisher derivative belongs to a Lyapunov/backflow layer or to a channel-estimation or fixed-readout layer. It separates three operational layers that are often conflated: input-encoded optimized Fisher geometry, fixed-readout Fisher information, and channel-parameter Fisher information. In the first layer, parameter-independent CP-divisible dynamics enforces Fisher contraction, so a revival of optimized QFI is a genuine backflow candidate. In the channel-encoded layer, Fisher growth can occur under a Markovian semigroup because the process parameter is being progressively imprinted into the output. In the fixed-readout layer, Fisher information is experimentally valuable but inherits a Lyapunov meaning only after the analyzer has been calibrated to the relevant tangent direction.

The dephasing, amplitude-damping, and two-qubit X-state examples show that this distinction is operational, not semantic. Fisher readability, coherence visibility, concurrence, and complementarity may coincide in calibrated sectors, but they separate in general. The amplitude-damped X-state region with $C_\Phi = 0$ and $\mathcal{I}_\Phi > 0$ makes this separation explicit. Thus Fisher or visibility data can serve as efficient triage signals for quantum-information experiments, provided that the encoding layer and readout layer are stated before causal claims are made.

A natural next step is to combine this readout rule with quantum-information protocols that use complementarity, concurrence, and Fisher observables as progressively richer diagnostic layers in structured two-qubit or photonic platforms. Such extensions should keep the same distinction explicit: Lyapunov-Fisher, fixed-readout Fisher, and channel-parameter Fisher answer different diagnostic questions.

A Derivation of the two-qubit X-state Fisher layers

This appendix records the elementary derivation of the X-state formulas used in the main text. The purpose is not to define a new entanglement measure, but to make explicit the measurement model behind Eqs. (21) and (22).

A.1 Amplitude-damping map and phase encoding

For each qubit we use the amplitude-damping Kraus operators

$$E_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\gamma} \end{pmatrix}, \quad E_1 = \begin{pmatrix} 0 & \sqrt{\gamma} \\ 0 & 0 \end{pmatrix}. \quad (25)$$

A phase θ is encoded in the relevant Bell-sector coherence before readout. The Fisher readout considered in Sec. 6 is the calibrated binary phase readout of Eq. (6), restricted to the occupied two-dimensional coherence sector after damping. For a coherence $ze^{-i\theta}$ supported in a sector of total weight w , the calibrated quadrature readout has visibility amplitude $R = 2|z|/w$ and contributes wR^2 to the classical Fisher information. Equivalently,

$$\mathcal{I}_{\text{sector}} = \frac{4|z|^2}{w}. \quad (26)$$

A.2 Φ -like slice

The Φ -like amplitude-damped X-state has diagonal entries

$$\rho_{11} = \frac{1+\gamma^2}{2}, \quad \rho_{22} = \rho_{33} = \frac{\gamma(1-\gamma)}{2}, \quad \rho_{44} = \frac{(1-\gamma)^2}{2}, \quad (27)$$

and phase-carrying coherence

$$\rho_{14} = \frac{\lambda(1-\gamma)}{2}e^{-i\theta}. \quad (28)$$

The occupied readout sector for this coherence has weight

$$w_\Phi = \rho_{11} + \rho_{44} = 1 - \gamma + \gamma^2. \quad (29)$$

Using Eq. (26) gives

$$\mathcal{I}_\Phi(\gamma, \lambda) = \frac{\lambda^2(1-\gamma)^2}{1-\gamma+\gamma^2}. \quad (30)$$

For a two-qubit X-state the concurrence is

$$C = 2 \max \{0, |\rho_{14}| - \sqrt{\rho_{22}\rho_{33}}, |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}\}. \quad (31)$$

Here $\rho_{23} = 0$, hence

$$C_\Phi(\gamma, \lambda) = (1-\gamma) \max\{\lambda - \gamma, 0\}. \quad (32)$$

It follows that $C_\Phi = 0$ but $\mathcal{I}_\Phi > 0$ whenever $0 < \lambda \leq \gamma < 1$.

A.3 Ψ -like slice

For the Ψ -like slice the phase-carrying coherence lies in the $|01\rangle, |10\rangle$ sector:

$$\rho_{23} = \frac{\lambda(1-\gamma)}{2}e^{-i\theta}, \quad \rho_{22} = \rho_{33} = \frac{1-\gamma}{2}, \quad \rho_{11} = \gamma, \quad \rho_{44} = 0. \quad (33)$$

The corresponding sector weight is $w_\Psi = 1 - \gamma$, and Eq. (26) yields

$$\mathcal{I}_\Psi(\gamma, \lambda) = \lambda^2(1-\gamma). \quad (34)$$

The X-state concurrence gives

$$C_\Psi(\gamma, \lambda) = \lambda(1-\gamma), \quad C_\Psi^2(\gamma, \lambda) = \lambda^2(1-\gamma)^2. \quad (35)$$

Thus the Fisher readout and concurrence are related but not identical outside the pure-dephasing Bell calibration sector.

Statements and declarations

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Data and code availability. The submission package contains the scripts used to generate the figures and the numerical layer checks, including the fixed-readout Fisher model and the two-qubit X-state layer figures. The source files are intended to accompany the manuscript submission.

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