Research Square

Preprints are preliminary reports that have not undergone peer review.
They should not be considered conclusive, used to inform clinical practice,
or referenced by the media as validated information.

Modal Frames as Resources: From formulation to simulation

Spyridon K. Chronopoulos

spychro@gmail.com

University of Ioannina https://orcid.org/0000-0003-0132-2338

Dimosthenis Michopoulos

University of Peloponnese

Konstantinos P. Peppas

University of Peloponnese https://orcid.org/0000-0002-8772-2502

Research Article

Keywords: Quantum resource theory, modal frames, measurement bases, quantum signal processing, quantum measurement channels, frame conversion, coherent modal reconstruction, quantum pulse gates, semantic quantum communication

Posted Date: June 2nd, 2026

DOI: https://doi.org/10.21203/rs.3.rs-9868980/v1

License: This work is licensed under a Creative Commons Attribution 4.0 International License.
Read Full License

Additional Declarations: The authors declare no competing interests.

Modal Frames as Resources: From formulation to simulation

Spyridon K. Chronopoulos [1*], Dimosthenis Michopoulos [2], and Konstantinos P. Peppas [2]

[11] Electronics-Telecommunications and Applications Laboratory, Physics Department, University of Ioannina, 45110 Ioannina, Greece; spychro@gmail.com (S.K.C.)
[2] Department of Informatics and Communications, University of Peloponnese, 22100 Tripoli, Greece; dimmihop@gmail.com (D.M.); peppas@uop.gr (K.P.P.)
* Correspondence: spychro@gmail.com (S.K.C.)

Highlights

- Proposition of Modal Frame Resource Theory (MFRT).
- The measurement basis is treated as a controllable quantum resource.
- Definition of modal frame information and frame-conversion cost.
- Representing modal frames through quantum measurement channels.
- Demonstrating the concept with a simple simulation of two modes.

Abstract

Quantum theories relevant to resources ordinarily study the potential gain from a quantum state once the measurement basis has already been chosen. Nevertheless, in modern photonic and matter/wave science, the measurement basis can be shaped and changed. The meaning is that this basis not only corresponds to a mathematical formulation but can be a physical resource (e.g., hardware).

In this work, we propose a new theory named Modal Frame Resource Theory (MFRT), which could serve as a simple framework for studying quantum measurement bases. A modal frame consists of a set of modes used by measurement apparatus like time-bin, frequency, spatial, or path modes. Specifically, modal frames are not only used for final detection, but can also be important for the distribution of quantum information across orthogonal modes. Then these can be recombined without measurement, such as in multicarrier signal processing. But stricter requirements exist, such as the preservation of quantum coherence.

For the purpose of explaining the proposed complementary concept of quantum theory, we define modal frame information, free-preserving operation, and the frame conversion distance. Regarding the free frame-preserving operation, an operation that can relabel or provoke phase-shifting in the modes, it does not mix them when transforming them into a new modal frame.

Each of the aforesaid is mentioned as clearly as possible using simple and comprehensible examples. The framework differentiates ideal measurement frames from noisy experimental implementations. A simple time-bin simulation (of two modes) is also presented as a proof of concept. The simulation compares a source frame {|early⟩, |late⟩} with a rotated receiver frame and shows how modal-frame mismatch changes the transition and the resulting MFRT compatibility score. So, the main point is that in programmable quantum systems, the measurement basis does not correspond to a fixed setting that has been chosen before starting the experiment. In more detail, the basis can be manipulated through a full processing cycle such as measuring, comparing, optimizing, converting, and engineering it as a quantum resource.

Keywords: Quantum resource theory, modal frames, measurement bases, quantum signal processing, quantum measurement channels, frame conversion, coherent modal reconstruction, quantum pulse gates, semantic quantum communication.

1. Introduction

In quantum theory, the observation is never independent of the way we choose to observe it. A quantum system does not appear in one "universal language". On the contrary, it answers the question that a measurement device asks of it. So, if the device's question is relevant to position, then the answer is relevant to position. Similarly, this applies elsewhere (e.g., energy, temporal and spatial modes, paths, etc.).

The "language of measurement" in quantum theory is mostly considered as being fixed in advance. The procedure includes choosing a basis, defining relative states, and then calculating probabilities. For example, in atomic physics, it is common to use the energy eigenbasis, that is, the set of states in which an atom has definite energy values. In quantum information, it is common to use the computational basis, which corresponds to the standard logical states used to encode qubits, such as $|0\rangle$ and $|1\rangle$. Existing quantum resource theories apply the reasoning that coherence is usually defined relative to a fixed basis, asymmetry relative to a fixed symmetry group, and reference-frame restrictions relative to a missing or shared frame. All the theory behind this reasoning have been extremely successful in showing that properties such as coherence, asymmetry, or lack of a shared reference frame can be quantified as physical resources [1]. However, modern quantum technologies recontextualize the whole logic. In photonic, atomic, and matter-wave platforms, the measurement basis is no longer relevant to a pure theoretical or and mathematical choice. It can be physically programmed. Modern laboratory apparatus can alter temporal modes, separate frequency modes, route spatial modes, interfere path modes, or implement mode-selective detection. Thereby, the basis becomes part of the apparatus, and the apparatus becomes part of the resource. The latter corresponds to a structure that can be controlled, optimized, and used for improving a quantum task. For example, a detector may initially distinguish the time-bin modes $|\text{early}\rangle$ and $|\text{late}\rangle$. If the device is configured so that it instead distinguishes the superposition modes $(|\text{early}\rangle+|\text{late}\rangle)/\sqrt{2}$ and $|\psi\rangle= (|\text{early}\rangle-|\text{late}\rangle)/\sqrt{2}$, then the modal frame has changed. This is not just a relabeling of the probable outcomes. It requires a physical modal transformation. In the proposed MFRT, this transformation is therefore treated as a non-free resource.

In the case of having different quantum analyzers, let's hypothesize that each is designed to read the same type of signal in a different way. One analyzer reads time-bin modes, another reads frequency modes, while another reads spatial modes. Every analyzer asks a different question, and the answer depends on the modal frame that is being used by the device.

When the source and the receiver use the same modal frame, the outcome is clear and informative. On the contrary, in the case of mismatched frames, the outcome becomes uncertain, and information from the source can even be lost. The ability to adjust the modal response of an analyzer so that it approaches a target modal frame is therefore not just a technical detail. It is a physical capability that, in the proposed MFRT scheme, can be quantified as a resource.

There is, however, an even stronger possibility. The analyzers should not be imagined only as destructive measurement boxes. In a programmable quantum device, they could act as coherent modal processors. In that case, the input quantum signal is not immediately measured. Instead, it

can be distributed across several orthogonal modal channels (for example, temporal, spectral, spatial, or path modes) and then to be recombined by a properly designed modal decoder. This is similar in intent to multicarrier communications (e.g., orthogonal frequency division multiplexing – OFDM), where information is distributed across orthogonal carriers and reconstructed by a receiver that knows the signal's structure. The quantum version is more delicate because the modal transformations must preserve coherence, and destructive measurement must be postponed until the final readout.

This observation leads to a broader view. A modal frame is not only a way of measuring a quantum system. It can also be part of an encoding, transmission, and reconstruction architecture. If the modal frame is chosen properly, it may preserve information, distribute it across suitable modal axes, and allow a decoder to reconstruct the intended signal. If the modal frame is inadequately matched to the device or channel, information may become inaccessible or require additional physical resources to be effectively recovered. Specifically, every frame can be considered as a measurement channel allowing us to pose significant questions relevant to (a) the quantity of the preserved information when there is a difference between preparation and measurement frames and (b) the cost to convert one measurement frame to another. So, the quantum Hilbert space refers to the "space" of possible states. In accordance, the modal frame is a specific set of modes within this space. In simple words, Hilbert space could be "the room", while the modal frame is the system of axes (i.e., the language / the alphabet) with the help of which we describe or measure what is inside the room. The theory also supports a broader type of processing view. In this starting point of Modal Frame Resource Theory (MFRT), the central idea is simple. In a programmable quantum system, the measurement basis is not just a setting. It is part of the device control set. Therefore, it can be treated as a resource. We call such a programmable basis a modal frame. A modal frame is a collection of distinguishable modes that a device can use to read, process, or reconstruct a quantum signal. These modes can be relevant to time, frequency, spatial, polarization, or path. So, a modal frame can determine what kind of information is extracted, preserved, and how well the signal can be reconstructed. MFRT provides a framework for treating this interface as a quantum resource. This perspective is strongly inspired by temporal-mode photonics. Temporal modes provide high-dimensional bases for photonic quantum information, and can be generated, manipulated, and detected with current technology. Brecht, Reddy, Silberhorn, and Raymer presented photon temporal modes as a complete framework intended for quantum information science, emphasizing that temporal modes can support high-dimensional encoding and optical communication compatibility [2]. Quantum pulse gates go further by enabling mode-selective operations and measurements. From this perspective, the apparatus does not only measure a signal. It specifies which modal component is accessible, selectable, and transformable. So, the device can select and act only on a specific mode, while the other modes are almost left unaffected. Experimental work has demonstrated temporal-mode measurement tomography and high-dimensional temporal-mode state tomography using such devices [3].

The specific developments pose a new question. Standard resource theories often wonder about the quantity of useful quantum resources that a state contains, given the fact of a fixed basis. In comparison to the previous, the proposed theory raises a different issue. Taking for granted the possibility of programming, comparing, converting, and using the basis itself for coherent reconstruction, then the relevant question would be what the resource value of the basis.

Therefore, MFRT should not be addressed as a new interpretation of quantum mechanics. There is no intention of replacing standard quantum theory, and there is no claim that measurement disturbance can be ignored. On the contrary, the proposed theory and experiment offer a theoretical resource framework for programmable quantum measurement bases and coherent modal architectures. The goal is to quantify two important facts. First, modal frame information quantifies the part of the encoded signal which remains accessible when the source and the receiver (measurement device) operate in different modal frames. Simply, it implies the performance of the two kinds of modal frames toward interpreting the same quantum signal. The frame information measures how much of the encoded quantum signal still remains accessible when the source and the receiver operate with different modal frames. Second, the distance of frame-conversion measures the difficulty of converting one measurement frame into another using only free operations. The term of free operations (such as relabeling modes or adding phase shifts) means simple changes that do not create a genuinely new modal structure. For example, the swapping of the labels named as "early" and "late" is a free operation (in the way that no physical change is applied), while creating a new superposition of early and late is not. A photon can reach the detector early or late. These two possible arrival times can be considered as quantum states (i.e., $|\text{early}\rangle$ and $|\text{late}\rangle$ respectively), and this is called time-bin encoding. In this sort of encoding, "early" and "late" denote two possible arrival-time modes of a photon. They function as two distinguishable modal states, similar to $|0\rangle$ and $|1\rangle$ in a qubit (but still, in quantum mechanics, the photon can also be in a superposition: $|\psi\rangle = (|\text{early}\rangle + |\text{late}\rangle)/\sqrt{2}$). Therefore, if free operations are not enough for a specific transformation, then additional physical control is required. This could be the use of a pulse shaper, an interferometer, a mode converter, or a quantum pulse gate to convert one modal frame into another. So, a non-free transformation modifies the modal response of the measurement device so that the receiver's modal frame becomes aligned, or approximately aligned, with a desired target modal frame. Frame-conversion distance reports the volume of the needed control turning one measurement language into another.

The new modal encoder–decoder viewpoint adds a third motivation relevant to a suitable modal frame that may not only support measurement but can also help toward carrying, processing, and reconstructing quantum information. In this context, MFRT can support a principle such as not measuring the modal components too early, instead of transforming them coherently, preserving their phase relations, and decoding them before final detection.

Two levels are defined for keeping the theory structure clear. The first level is called MFRT-pure, which deals with ideal complete rank-one (measurement outcome) frames. This means that the measurement frame includes all the modes needed to describe the system, and each measurement outcome corresponds to one single, perfectly distinguishable mode. The second level, called MFRT-noisy, includes realistic experimental effects such as detector noise, loss, imperfect mode selectivity, and classical post-processing. This separation allows us to build the theory, first in the ideal case and then in the experimental case. This distinction is important because not every loss of information has the same generative cause. Taking into consideration the ideal case, information may be lost because two modal frames are genuinely mismatched. In the noisy theory, information may also be lost due to the fact that the detector is imperfect, modes overlap, or the experimental device introduces noise. By differentiating these two cases, MFRT can recognize fundamental frame mismatch from practical experimental imperfections.

To support the theoretical development, this work also includes a useful two-mode simulation based on the time-bin modes |early⟩ and |late⟩. The simulation compares a source modal frame with a rotated receiver modal frame and shows the alteration in transition probabilities, in accordance to the frame mismatch angle variation. An MFRT score is then introduced to visualize how the receiver outcomes remain correlated with the original source modes. So, this simulation is intended as a first proof of concept showing how modal-frame compatibility can be quantified and visualized.

The structure of the paper, apart from the introduction, is split into six remaining sections. The second section presents related scientific work by appropriate bibliographic references. In the third section, the MFRT for quantum signal processing is analyzed with various examples. Then, section four introduces information about the innovation of the proposed MFRT scheme while giving the physical interpretation and potential impact. Section five presents simulation properties and an experiment. Finally, section six provides restrictions, while section seven presents the conclusions and future scopes.

2. Related scientific work

MFRT idea and simulation connects several established research areas such as quantum resource theories, coherence theory, reference-frame theory, quantum channels and superchannels, temporal-mode photonics, and quantum signal processing. The purpose is not to replace the aforementioned fields, but to explain the complementary role of MFRT among them. The central differentiation is that most existing theories study quantum resources inside a chosen frame. MFRT studies the frame itself as a programmable resource. In other words, MFRT does not only ask "what is the state" but also poses the question "what is the used modal-frame (measurement) language of the device, and the cost to change that language to another".

2.1 Quantum resource theories

Quantum resource theories provide a broad approach to formalizing usefulness under restrictions. The basic idea is that not every operation is allowed for free. Once the allowed operations are fixed, anything that cannot be created by those operations becomes a resource. Chitambar and Gour's review describes this as a general framework covering many different quantum effects, from entanglement to coherence and thermodynamic resources [4]. In simple terms, a resource theory asks three kinds of questions relevant to (1) what objects are being studied, (2) which operations are considered free, and (3) which properties or capabilities become useful or costly under those restrictions. A standard way to write a resource theory would be given by equation 1. This implies that a resource theory is defined by what we study, what we consider easy or free, and what requires additional physical ability. A simple example is related to thermodynamics, where heat at a useful temperature can be a resource. In entanglement theory, entangled states are resources. In MFRT, the ability to implement or convert a modal measurement frame is the resource. In simple terms, the useful capability is not only the quantum state itself, but also the device's ability to choose, reshape, or translate the modal "language" in which the signal is measured or processed.

$$R=(S, S_{free}, O_{free})$$
$$\text{Resource theory} = (\text{objects, free objects, free operations}) \quad (1)$$

where S is the set of objects under study, S_{free} is the subset of objects considered free, and O_{free} is the set of free operations. Consequently, a free operation must transform free objects into free objects. In other words, if something has no resource before the operation, it should not gain resource for free after the operation.

2.2 Coherence as a resource

Quantum coherence is one of the closest existing ideas to MFRT. Baumgratz, Cramer, and Plenio introduced a rigorous framework for quantifying coherence relative to a fixed basis, and Streltsov, Adesso, and Plenio later reviewed coherence as a quantum resource across many physical settings [1, 5]. In coherence theory, the basis is chosen first. A state is called incoherent if it is diagonal in that basis (equation 2). The meaning, in simple words, is that the state is only a classical mixture of the basis states. There are probabilities p_i, but there is no quantum superposition between different basis states.

$$\rho_{inc} = \sum_i p_i \mid i\rangle\langle i \mid \tag{2}$$

where ρ_{inc} is an incoherent quantum state, p_i refers to the probability of being in basis state i, $|i\rangle$: the i-th basis state, and $|i\rangle\langle i|$ is the projector onto that basis state.

For example, if a photon exists in an “early” or “late” time bin, then an incoherent mixture would mean the incoherent quantum state described by equation 3. This describes a photon that is early with some probability and late with some probability, but not in a coherent superposition of early and late. A coherent time-bin state would instead be given by equation 4. This condition cannot be characterized as “early" or late.” It is a quantum superposition of both possibilities. So, MFRT depends on this logic, but changes the question. Coherence theory usually assumes that the basis is already fixed. MFRT asks what happens when the basis itself can be programmed, compared, or converted. Here, “programmed” means that the device can be physically adjusted to select which modes it reads or processes. For example, this happens by changing a pulse shape, an interferometer setting, or a mode converter.

$$\rho_{inc} = \sum_i p_i \mid i\rangle\langle i \mid \tag{3}$$

$$|\psi\rangle = \frac{|\text{early}\rangle + |\text{late}\rangle}{\sqrt{2}} \tag{4}$$

2.3 Asymmetry and reference-frame theories

Another important connection of MFRT is with asymmetry and reference-frame theory. Bartlett, Rudolph, and Spekkens examined how the absence of a shared reference frame affects quantum information processing. In such cases, the source and the receiver may not share the same phase reference, spatial orientation, or modal frame [6]. Marvian and Spekkens developed the theory of “modes of asymmetry”, showing how states, measurements, and channels can be decomposed into components associated with symmetry properties. Their work is especially important because it already treats “modes” as structured mathematical objects under restricted processing [7]. A simple way to express a frame mismatch is through an overlap given by equation 5. This value reports how much mode $|\xi_i\rangle$ from one frame overlaps with mode $|\eta_j\rangle$ from another frame.

$$M_{ji} = \langle \eta_j \mid \xi_i \rangle \tag{5}$$

where $|\xi_i\rangle$ refers to the i-th mode in the first frame, $|\eta j\rangle$ refers to the j-th mode in the second frame, and M_{ji} is the overlap between the two modes.

According to Born's rule [8], the measurable probability is given by equation 6. It means that a source signal in mode i of modal frame ξ gives the probability that a detector using modal frame η reports outcome j.

$$P_{j|i} = |\langle \eta_j \mid \xi_i \rangle|^2 \tag{6}$$

In the case of the two modes being identical, then $P_{j|i}=1$ (for the matching mode), and the detector recognizes the signal perfectly. If the two modes are orthogonal, then $P_{j|i}=0$. This means that a detector associated with mode η_j, has zero probability of detecting a source signal in mode ξ_i. MFRT is related to reference-frame theory, but its emphasis is different. Reference-frame theory often studies what happens when a reference frame is missing or misaligned. MFRT focuses on programmable modal frames in devices such as temporal, spectral, spatial, polarization, or path modes. The question is not only whether the "two parties share a frame" but also "the cost of converting one device frame into another".

2.4 Quantum channels and superchannels

Quantum channels describe physical processes that transform quantum states. Superchannels describe transformations of channels themselves. Chiribella, D'Ariano, and Perinotti introduced the broader formalism of quantum supermaps and quantum combs, while later work developed resource theories of quantum channels and channel comparison by superchannels [9]. The aforementioned are important for MFRT because a modal frame is not treated only as a list of vectors. It is represented as a measurement channel (equation 7). This equation means that frame ξ acts as a measurement device. It takes an input quantum state ϱ, checks how much of it lies in each modal direction, and outputs a classical label i. For example, if the frame is F_ξ=(|early⟩, |late⟩) and the photon is definitely in the early time bin, then the channel outputs "early" with probability 1 and "late" with probability 0.

$$M_\xi(\rho) = \sum_i \mathrm{Tr}(\Pi_i^\xi \rho) \mid i\rangle\langle i \mid \tag{7}$$

where ρ is the input quantum state, M_ξ the measurement channel associated with frame ξ, $\Pi_i^\xi = |\xi i\rangle\langle \xi i|$ the projector onto mode i, $\mathrm{Tr}(\Pi_i^\xi \rho)$ the probability of obtaining outcome i, and $i\rangle\langle i$ the classical record of outcome i, which refers to the stored detector result, such as n_1, n_2, early, or late. For a rotated receiver frame, the corresponding probable labels of n_1 and n_2 are linear combinations of the early and late modes.

As far as a superchannel is concerned, it has the general form given by equation 7. In other words, a superchannel modifies a device by placing a pre-processing operation before it and a post-processing operation after it. It is like changing the input preparation and output processing around a measurement device. For example, if N is a detector, then E_{pre} may reshape the incoming signal before detection, while E_{post} may relabel or process the detector outcomes. MFRT uses this channel perspective to define frame-conversion distance, i.e., how close we can get to a target measurement frame using only free transformations.

2.5 Temporal-mode photonics and quantum pulse gates

Temporal-mode photonics provides one of the strongest experimental motivations for applying MFRT. Brecht, Reddy, Silberhorn, and Raymer described photon temporal modes as a complete framework for quantum information science. Their work shows that temporal modes can form high-dimensional bases for photonic quantum information [2]. Here, "high-dimensional bases" means that the photon is not limited to two modes, such as $|0\rangle$ and $|1\rangle$. Instead, it can be encoded in many distinguishable modes, for example, several different time bins or frequency modes. These modes form a larger basis and can carry more quantum information per photon.

Quantum pulse gates could also be relevant to MFT because they can select and manipulate specific temporal modes. Ansari and co-workers demonstrated temporal-mode measurement tomography of a quantum pulse gate and reconstructed measurement operators in a seven-dimensional temporal-mode space, where a quantum pulse gate is a mode-selective optical device. In simple terms, it acts like a temporal-mode filter. It can pick out one chosen time-shaped mode of a photon from many possible temporal modes. They also showed high-dimensional temporal-mode state tomography after calibration [3], where there is an experimental reconstruction of a photon's quantum state (the photon is described using many distinguishable temporal modes). The basic idea of temporal modes can be described by equation 8. This means that a photon can be described as a combination of different temporal modes. A simple example is the two time bins of a photon. So, $|\psi\rangle = (|\text{early}\rangle + |\text{late}\rangle)/\sqrt{2}$. This means that the photon is in a coherent superposition of arriving early and arriving late.

$$|\psi\rangle = \sum_k c_k \, |t_k\rangle \tag{8}$$

where $|t_k\rangle$ is the k-th temporal mode, c_k is the complex weight of that temporal mode, and $|\psi\rangle$ refers to the full photon state.

MFRT complies with this platform because temporal modes are not only theoretical labels. They can be shaped, selected, and measured by real optical devices such as quantum pulse gates. High-selectivity quantum pulse gating has also been demonstrated using all-optical Ramsey interferometry [10].

2.6 Modal frames and quantum signal processing

The term quantum signal processing already has an established meaning in quantum algorithms, especially in relation to polynomial transformations and quantum singular value transformation (QSVT). QSVT and quantum signal processing show that quantum systems can implement structured transformations of encoded information. MFRT is different because it does not transform singular values of an operator (positive numbers that describe how strongly the operator stretches or suppresses independent directions of a state space), but studies how modal frames themselves affect information access, compatibility, and conversion cost. A transformation is a powerful framework in quantum algorithms. Simply, it allows a quantum computer to apply a controlled mathematical filter to the singular values of a matrix encoded inside a quantum circuit. It is therefore a form of quantum signal processing in the algorithmic sense. The "signal" is not a temporal or optical waveform, but a numerical structure encoded in a quantum operation. MFRT uses the term quantum signal processing in a different, physical-modal sense. Here, the signal is carried by temporal, spectral, spatial, polarization, or path modes, and the processing concerns how these modes are encoded, converted, reconstructed, or measured.

Recent work, on Quantum Signal Processing (QSP) and Quantum Singular Value Transformation (QSVT), confirms that quantum signal processing is a powerful and active term in quantum information which is the information carried by a quantum system (it may be encoded in a qubit state, in the temporal mode of a photon, in its frequency, polarization, path, phase, or even in correlations between several quantum systems). So, MFRT should clearly specify that it studies modal signal processing rather than only algorithmic polynomial transformations [11]. The modal-processing idea can be expressed through equation 9. In a simpler manner, a quantum signal is first encoded into modal components and later decoded. If the decoder correctly reverses the encoder, the output can recover the intended signal. For example, in a time-frequency photonic system, a device may spread information across several orthogonal temporal or spectral modes. A second device may then recombine these modes before final detection. This resembles multicarrier signal processing, but in the quantum case, coherence must be preserved. This analogy is close to orthogonal frequency division multiplexing (OFDM) in classical communications, where information is distributed across orthogonal subcarriers [12]. OFDM ideas have even been considered in quantum-key-distribution contexts, showing that orthogonal multicarrier schemes can be potentially applied in quantum communications [13].

MFRT does not claim that a quantum signal can be destructively measured first and then perfectly rebuilt later. Instead, it emphasizes coherent processing before the final measurement. This is the key difference.

$$|\psi_{out}\rangle = U_{dec}\, U_{enc}\, |\psi_{in}\rangle \tag{9}$$

where, $|\psi_{in}\rangle$ is the input quantum signal, U_{enc} is the coherent modal encoder, U_{dec} is the coherent modal decoder, and $|\psi_{out}\rangle$ refers to the reconstructed output signal.

2.7 Semantic and task-oriented communication as a future scope

A final related area is semantic communications. In semantic communications, the objective is not merely to transmit every bit or symbol accurately, but to preserve the information so that it is meaningful for a task. Surveys and tutorials describe semantic communications as a shift from raw symbol transmission toward meaningful or task-relevant information transfer [14].

So the probable connection with MFRT definitely has a future direction. A modal frame may not always preserve every detail of a quantum signal. In some tasks, it may be capable of preserving the information needed for sensing, estimation, classification, or decision-making. This task-oriented idea is expressed through equation 10. The simple meaning behind it is that instead of asking which frame preserves everything, the question would be relevant to which frame preserves the fact that matters for the task. For example, in a quantum sensing task, we may not need to reconstruct the full quantum state. We may only need to estimate a phase, detect a weak signal, or decide whether a target is present. The best modal frame may be the one that preserves that useful feature most efficiently. For this reason, semantic quantum modal processing is a promising future scope, but it is not the central claim of the present paper.

$$F_{best} = \operatorname{argmax}_{F} I_{task(F)} \tag{10}$$

where F_{best} is the best modal frame, argmax is the option that gives the highest value, and $I_{task(F)}$ is the task-relevant information preserved by F.

2.8 Position of MFRT

The related work can be summarized in Table 1. The theoretical gap addressed by MFRT is therefore filled, while possible future implementations could be identified as areas for further research. Existing theories usually study resources on a fixed basis or channels as general objects. MFRT focuses on programmable modal measurement frames and is relevant to how much information they preserve, how they can be converted, and how they can support coherent modal signal processing.

Table 1: Related work versus the proposed MFRT framework

Existing field	Main question	Relation to MFRT
Quantum resource theories	What is useful under restricted operations?	MFRT applies this logic to modal frames.
Coherence theory	How much superposition exists in a fixed basis?	MFRT asks what happens when the basis itself is programmable.
Reference-frame theory	What happens when parties lack a shared frame?	MFRT studies device-level modal frame mismatch and conversion.
Channel resource theory	How can channels be compared and transformed?	MFRT treats measurement frames as measurement channels.
Temporal-mode photonics	How can photons be encoded in temporal modes?	MFRT provides a theoretical language for applying modal frame ideas to real photonic systems.
Quantum signal processing	How can quantum systems implement controlled signal transformations?	MFRT adds modal-frame information and conversion cost.
Semantic communication	How can task-relevant meaning be preserved?	MFRT may support future task-oriented modal-frame optimization.

3. Modal Frame Resource Theory for Quantum Signal Processing

3.1 General concept

Modal Frame Resource Theory has its origins in a simple idea. In a programmable quantum system, the way a device reads or processes a signal is not fixed once and for all. It can be shaped. A quantum signal may be carried by time modes, frequency modes, spatial modes, polarization modes, or path modes. A device has the option to choose which of these modal "languages" to use. In MFRT, this choice is not treated as a passive mathematical setting. It is treated as a controllable physical structure. So, this structure is called a modal frame.

In simple terms, a quantum state is the signal. A modal frame is the language used by the device to read, process, or reconstruct that signal. If the signal and the device use compatible modal frames, the information can be read clearly. If the frames are mismatched, part of the information may become inaccessible. If we want to reshape one frame into another, we need physical control. MFRT is proposed to quantify these three ideas: compatibility, preserved information, and conversion cost. A simple diagram is illustrated in Figure 1.

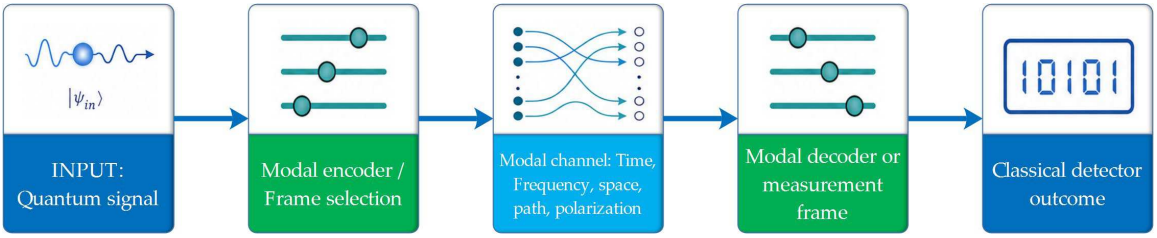


Figure 1. MFRT architecture: A quantum signal is encoded, transmitted, processed, and finally measured through programmable modal frames.

3.2 Modal frame

A modal frame is the set of modes used by a device to describe or process a quantum signal. This can be addressed mathematically by equation 11. For example, if a photon is described by two time bins, the frame may be F_ξ = {|early⟩,|late⟩}. The device can distinguish whether the photon belongs to the "early" or "late" time mode. If the device uses more time bins, the frame becomes larger, with F_ξ equal to { $|t_1\rangle$, $|t_2\rangle$,…, $|t_d\rangle$ }, resulting in a larger modal alphabet.

$$F_\xi = \{ |\xi_1\rangle, |\xi_2\rangle ,\ldots, |\xi_d\rangle \} \tag{11}$$

where F_ξ is the modal frame, $|\xi_i\rangle$ is the i-th mode of the frame, d is the number of available modes, and consequently, F_ξ contains d possible modes.

3.3 Ideal modal frames in MFRT-Pure

In the ideal version of the theory, called MFRT-Pure, we assume that the modal frame is complete and that its modes are perfectly distinguishable. This is expressed by equation 12. This means that different modes do not overlap. If i=j, the overlap is 1. If i≠j, the overlap is 0. In turn, equation 13 means that the frame is complete. It contains all the modes needed to describe the system. For example, if the only possible time-bin modes are "early" and "late," then an ideal detector can distinguish them perfectly (i.e., ⟨early|late⟩=0). Therefore, this means that the two modes are orthogonal and they can be treated as two distinct modal states.

$$\langle \xi_i \mid \xi_j \rangle = \delta_{ij} \tag{12}$$

$$\sum_{i=1}^{d} \mid \xi i\rangle\langle \xi i \mid = 1 \tag{13}$$

3.4 Quantum signal in a modal frame

A quantum signal can be distributed across the modes of a frame. This is expressed by equation 14, where the quantum signal $|\psi\rangle$ is expressed as a combination of the modes $|\xi_i\rangle$. E.g., as previously mentioned $|\psi\rangle = (|early\rangle+|late\rangle)/\sqrt{2}$. This means that the photon is not simply early or late before measurement. It is in a coherent superposition of the two time-bin modes (Figure 2). This point is important. In MFRT, we are not only interested in which modal outcome is detected. We are also interested in how the modal structure carries phase, coherence, and useful information before the final measurement.

$$\mid \psi\rangle = \sum_{i=1}^{d} c_i \mid \xi_i\rangle \tag{14}$$

where $|\psi\rangle$ is the quantum signal, c_i the complex weight of mode i, $|\xi_i\rangle$ is one mode of the frame.

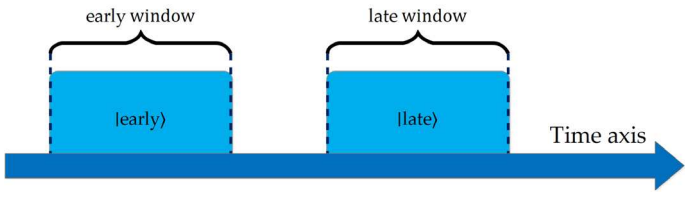


Figure 2. A photon may be described using two distinguishable temporal modes, "early" and "late." Before detection, it may also be prepared in a quantum combination of both modes, rather than being assigned to only one of them.

3.5 Modal measurement channel

A modal frame becomes operational when a device uses it to perform a measurement. The measurement channel associated with frame F_ξ is given by the already explained equation 7 ($M_\xi(\rho) = \sum_i \mathrm{Tr}(\Pi_i^\xi \rho) \mid i\rangle\langle i \mid$). The frame itself defines how the device measures the signal. Each mode of the frame corresponds to one possible detector outcome. The channel takes the input state ϱ, checks its overlap with each mode of F_ξ, and outputs the corresponding classical measurement probabilities. So, this equation declares that the device measures the input state ϱ using the modal frame F_ξ . Each mode of the frame corresponds to one possible detector outcome. The measurement is "perfect" only in the ideal case where the source signal is in one of the same modes that the detector is designed to measure.

3.6 Frame mismatch

The next step is to compare two modal frames (Figure 3). Suppose the signal is prepared in frame F_ξ, but the receiver measures it in another frame F_η . The key quantity is the overlap between their modes. This is expressed by equation 6 ($P_{j|i} = |\langle \eta_j \mid \xi_i \rangle|^2$). This gives the probability that a signal prepared in mode $|\xi i\rangle$ is detected as mode $|\eta_j\rangle$. So, if the modes are identical, the probability is 1. If they are orthogonal, the probability is 0. If they partially overlap, the probability lies between 0 and 1. In simple words, the overlap reveals how well one modal frame recognizes another.

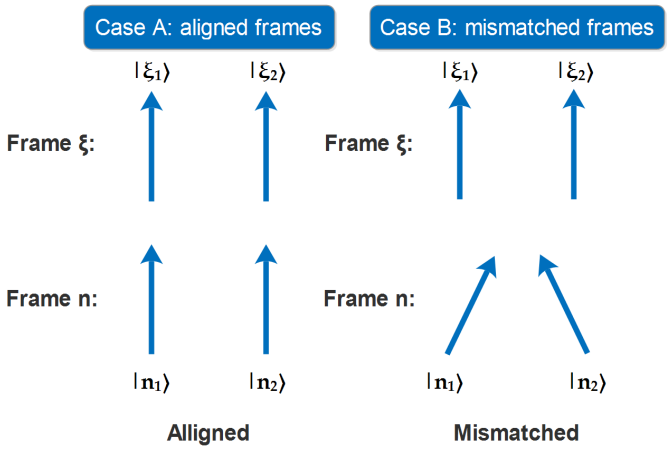


Figure 3. Two modal frames may be aligned or mismatched. When they are aligned, the detector can identify the source mode clearly. When they are mismatched, the measurement outcome becomes uncertain.

3.7 Modal frame information

MFRT can pose a practical question relevant to the quantity of the encoded information remaining accessible when the source and measurement frames are different. So, we define modal frame information based on equation 15. This quantity reports how much information about the prepared modal label can still be recovered when measuring in another frame. Modal frame information tells us how much of the sender's modal encoding can still be recovered by the receiver. Here, "preparation" means that the source creates a quantum signal using a chosen set of modes. For example, in time-bin encoding, a photon source may prepare a photon in an "early" mode, a "late" mode, or a coherent combination of the two. "Measurement in another frame" means that the receiver does not read the signal using the same modal alphabet as the source. If the two alphabets do not match, part of the original information may become uncertain or lost. If the two frames (modal alphabets) are identical, the detector can identify the prepared mode without uncertainty, and the accessible information reaches its maximum value, $I_F = \log_2 d$. Here, d is the number of

distinguishable modes in the frame (equation 15). For example, d=2 gives I_F=1 bit, while d=4 gives I_F=2 bits. But when the two frames are completely mutually unbiased, the measurement result is fully random with respect to the preparation label and I_F=0. For a two-mode system, the maximum value is I_F=1 bit. This means that one binary modal choice can be recovered perfectly. In simple words, modal frame information measures how well two modal frames can translate the same quantum signal.

$$I_F(\xi:\eta) = \log_2 d - \frac{1}{d}\sum_i H(P_{\cdot|i}) \tag{15}$$

where I_F (ξ : η) is the modal frame information, d is the number of modes, and $P_{\cdot|i}$ is the full probability distribution of detector outcomes when mode i was prepared. H is Shannon entropy, which measures uncertainty.

In the case that the detector is assured for each input mode, then H=0 → I_F=1. The notation $P_{\cdot|I}$ denotes the full list of detector probabilities when the source prepares mode i. For example, in a two-mode time-bin system, if the source prepares the "early" mode, then $P_{\cdot|early}$ = ($P_{early\,|\,early}$, $P_{late\,|\,early}$). If the source and detector use the same frame, this distribution is (1,0), meaning that the detector identifies the mode without uncertainty. If the detector uses a fully mismatched frame, the distribution may become (1/2, 1/2), meaning that both outcomes are equally likely. Thus, for a two-mode system, the maximum modal frame information is I_F=1 bit, while a fully uncertain measurement gives I_F=0. Notably, H is the Shannon entropy of the detector-outcome distribution. For a probability distribution P = (p_1,…,p_d), it is defined as $H(P) = -\sum_j p_j log_2 p_j$. In simple terms, it measures how uncertain the detector outcome is. If one outcome is certain, H=0. If all outcomes are equally likely, the uncertainty is maximal [15].

3.8 Free operations

We must define the operations that are free to provide a resource theory. In MFRT, free operations are changes that do not create a genuinely new modal structure. They may rename modes or add phases, but they cannot mix modes into new superpositions (quantum combination of modes). This restriction is essential. By creating a new superposition, the modal frame changes. Since MFRT aims to measure the cost of changing modal frames, such a change cannot be considered free. A free operation can be expressed using equation 16 [16]. For example, in a time-bin measurement, "early" and "late" are two possible detector outcomes. Renaming these outcomes is free in the sense that no physical change is made to the signal or to the measurement device. However, creating (|early⟩+|late⟩)/√2 is not free. This creates a coherent modal superposition and requires physical control, such as an interferometer, pulse shaper, or quantum pulse gate.

$$U_{free} = \sum_i e^{i\phi_i}\ |P_{\mathbf{free}}(i)\rangle\langle i| \tag{16}$$

where U_{free} is a free unitary operation, $e^{i\phi_i}$ adds a phase to mode i, |i⟩ denotes the i-th mode of the chosen reference frame, and $P_{free}(i)$ is the new label assigned to that mode.

3.9 Frame-conversion distance

If two frames are different, it could be interesting to answer how difficult it is to convert one into the other. We define the frame-conversion distance by equation 17. This measures the deviation from the target measurement frame M_η, starting from M_ξ, using only free operations. So, if Δ_{MF}=0, conversion can be conducted freely. Instead, if Δ_{MF}>0, then extra physical control is needed. For

example, converting an early/late detector into a detector that measures coherent superpositions of early and late, this cannot be done by relabeling outcomes. So, renaming detector outcomes is not enough. It changes only the names of the results, not the physical measurement. It requires a real optical transformation. This is exactly the type of resource that MFRT aims to quantify.

$$\Delta_{\mathrm{MF}}(\xi \to \eta) = \inf_{\Theta \in O_{\mathrm{free}}} \| \Theta(M\xi) - M\eta \|_{\diamond} \tag{17}$$

where Δ_{MF} is the frame-conversion distance, M_{ξ} is the starting measurement channel, M_{η} is the target measurement channel, Θ is an allowed free transformation, O_{free} is the set of free operations, and $\|\cdot\|_{\diamond}$ is a standard distance between quantum channels. If $\Delta_{MF}(\xi \to \eta) = 0$, then the target frame can be reached freely from the starting frame. If $\Delta_{MF}(\xi \to \eta) > 0$, then free operations are not enough, and a modal transformation is required.

The previous definition (expressed by equation 17) is introduced here for MFRT, but it is based on standard tools from quantum information theory. These are channel distances, diamond norm distinguishability, free operations in resource theories, and superchannels. Specifically, the symbol Θ means a free transformation acting on the measurement channel. In MFRT, a free transformation is not allowed to create a genuinely new modal frame. It may only perform operations such as relabeling outcomes or adding phases. In general, free operations may rename detector outcomes, reorder existing modes, or add mode-wise phase factors without mixing different modes. Such operations preserve the modal structure. So, this follows the general resource-theoretic idea that free operations must not create the resource being quantified. Quantum resource theories use this logic widely, i.e., once the free operations are fixed, anything that cannot be generated by them is considered a resource [4]. So, Θ is more specifically a superchannel. A quantum channel maps quantum states to quantum states. A superchannel maps quantum channels to quantum channels. In simple words, a superchannel modifies a device, not only a state. For example, it may place a signal-processing step before a measurement device and a classical processing step after it. This is why superchannels are important mathematical tools for studying transformations between measurement devices. Gour's work on the comparison of quantum channels by superchannels develops this type of channel-to-channel transformation framework [17]. Regarding the diamond norm ($\|\cdot\|_{\diamond}$), this is a standard way to measure the distance between quantum channels. In simple words, it tells us how distinguishable two quantum devices are in the best possible experiment, even if the input signal is allowed to be entangled with an auxiliary system. If the diamond distance is zero, the two channels behave identically. If it is large, the two channels can be distinguished more easily. Watrous presents the diamond norm in the context of channel distances and channel discrimination [18].

3.10 Coherent modal signal processing

MFRT also supports a broader signal-processing view. A modal frame is not only useful at the final measurement stage. It can also be part of a coherent encoder–decoder architecture. In this architecture, a quantum signal is first distributed across selected modal components, such as time, frequency, spatial, or path modes, and then recombined before final detection. The term "coherent" means that the phase relations between the modes are preserved during the process, so the signal is transformed rather than destructively measured. Taking into account equation 9 ($|\psi_{out}\rangle = U_{dec}\ U_{dec}\ |\psi_{in}\rangle$), we consider the input quantum signal that is first encoded into modal components and later decoded before final measurement. In the ideal MFRT-Pure case, the decoder is the inverse of the encoder, $U_{dec} = U_{enc}^{\dagger}$, so that $U_{dec}U_{enc}=1$. This means that the modal encoding can be perfectly reversed before the final measurement. In realistic MFRT-Noisy implementations, the decoder may only approximate this inverse relation $U_{dec} \simeq U_{enc}^{\dagger}$, because of loss, noise, phase

errors, or imperfect mode selectivity. Consequently, if $U_{dec} \simeq U_{enc}^{\dagger}$, then the decoder approximately reverses the encoder. In simple terms, the signal is first distributed across modal components and then recombined before the final measurement. This avoids an intermediate destructive measurement that would erase the phase relations between the modes. This idea is similar theoretically to multicarrier signal processing, such as OFDM, where information is distributed across orthogonal carriers and recombined by a receiver. The quantum case is more delicate because phase and coherence must be preserved.

3.11 MFRT-Pure and MFRT-Noisy

To keep the theory clear, we separate the two types. MFRT-Pure studies the ideal case. It assumes that (p1) the modes are perfectly distinguishable, (p2) the frame is complete, (p3) each detector outcome corresponds to one mode, and (p4) there is no detector noise or loss. MFRT-Noisy studies realistic implementations. It includes (n1) detector noise, (n2) optical loss, (n3) imperfect mode selectivity, (n4) overlapping modes, and (n5) classical post-processing. This distinction is important because information can be lost for different reasons. In MFRT-Pure, loss comes from frame mismatch. In MFRT-Noisy, loss can also come from imperfections in the experimental device. Consequently, this separation allows us to wonder whether the information loss is caused by the choice of modal frame or by the physical implementation.

3.12 *Summary of the proposed theory scheme*

The central principle is that *a modal frame is not only a way to observe a quantum signal. It is a controllable structure that determines how information is read, preserved, processed, reconstructed, or lost*. This gives MFRT its beginning, middle, and end. The beginning is the modal frame, the middle is information and conversion cost, and then the end is the programmable quantum signal processing. The proposed MFRT scheme has a clear structure shown below:

1. We define a modal frame as the set of modes used by a device.
2. We represent that frame operationally as a measurement channel.
3. We quantify how well two frames match through their transition probabilities.
4. We define modal frame information as the amount of accessible information preserved across frames.
5. We define frame-conversion distance as the cost of reshaping one frame into another.
6. Finally, we extend the idea toward coherent modal signal processing, where modal frames are used not only for measurement but also for encoding, transmission, and reconstruction.

4. Innovation of the proposed MFRT scheme

The innovation of MFRT lies in the level at which the resource is defined. Most quantum resource theories start after a measurement basis has already been chosen. They ask how much coherence, entanglement, or asymmetry a state contains within that chosen structure. MFRT moves one step earlier, as it treats the modal frame itself as the object to be quantified:

- In simple terms, MFRT does not only ask: *What information is carried by the quantum state?*
- It also asks: *What modal language is used to read, process, or reconstruct that information?*

This fact is the main novelty of the framework. The measurement basis is no longer treated only as a fixed background setting. It becomes a controllable structure with its own information value and conversion cost. Specifically, the proposed scheme introduces three connected contributions:

1. It defines modal frame information, which quantifies how much of the encoded signal remains accessible when the source and receiver use different modal frames. It turns frame mismatch into a measurable information-theoretic quantity.
2. It defines frame-conversion distance, which quantifies how difficult it is to reshape one measurement frame into another using only free operations. This turns the physical ability to change a measurement language into a resource-theoretic cost.
3. It extends the role of modal frames beyond final detection. In the coherent signal-processing view, modal frames can be used for encoding, transmission, decoding, and reconstruction before the final measurement. This connects MFRT with quantum signal processing and shows how the modal framework may be applied to photonic and matter-wave systems.

Therefore, the proposed innovation is not a replacement for standard quantum theory. It is a new organizational layer for programmable quantum systems: *MFRT considers the modal frame as a controllable interface between the quantum signal and the device.*

4.1 Physical interpretation and potential impact

The physical impact of MFRT is that it transfers the attention from fixed detectors to programmable modal analyzers. So, a quantum device is not limited to reading a signal in one predefined basis. Instead, the device may be equipped with pulse shapers, interferometers, quantum pulse gates, spatial mode sorters, or programmable photonic circuits that allow it to choose and reshape the modal frame.

This does not mean that an unknown quantum signal can be destructively measured in many incompatible ways and then perfectly reconstructed. Such a claim would conflict with the no-cloning principle and with measurement disturbance. The correct interpretation is different because the signal may first be processed coherently across several modal degrees of freedom, while phase relations are preserved, and only then measured. In this sense, MFRT points toward enhanced quantum analyzers. These kinds of devices may not merely detect a photon, but choose the modal language in which the photon is read, processed, or reconstructed. This could support future systems for temporal-mode photonics, quantum communication, quantum sensing, quantum imaging, and semantic quantum signal processing.

So, *the goal is not to measure every property at once, but to engineer/construct the modal frame that preserves the properties that matter*.

5. Simulation and Results

For the purpose to present the practical usage in accordance with the conceptual formulation of modal frame resource theory (MFRT), a simple but indicative numerical simulation (based on Octave-compatible code) of two modes is shown. The purpose is to extend the formulation to simulation using a clear method. Notably, the goal is not to reproduce a full complexity system relevant to quantum optical experiments but to present and prove the heart of MFRT. This applies to the case when the source and the receiver use different modal frames; the potentiality of accessing modal information changes in a predictable and quantifiable way.

The simulation includes a source operating in a two-mode time-bin frame, with basis states such as |early⟩ and |late⟩. The receiver operates in another modal frame {|η1⟩, |η2⟩} which is a rotated version of the source frame by an angle θ. The receiver modes are expressed by equations 18 and 19. Also, the receiver does not always measure directly in the original frame (early / late). Instead, it may operate in a partially or fully mismatched modal frame. The transition probabilities

are then given by equations 20 through 23. So, if we send |early⟩, the receiver provides an outcome of η_1 with probability of $\cos^2\theta$ and outcome η_2 with probability of $\sin^2\theta$. Therefore, their sum equals unity, which is given by equation 24. Instead, in the case of transmitting |late⟩, the probabilities are reversed and given by equations 22 and 23, while their sum is equal to unity (equation 25). Hence, the overall probability table is given by equation 26.

$$|\eta_1\rangle = \cos\theta\,|early\rangle + \sin\theta\,|late\rangle \tag{18}$$

$$|\eta_2\rangle = -\sin\theta\,|early\rangle + \cos\theta\,|late\rangle \tag{19}$$

$$P_{1|1} = P(\eta_1 \mid early) = |\langle \eta_1 \mid early \rangle|^2 = \cos^2\theta \tag{20}$$

$$P_{2|1} = P(\eta_2 \mid early) = |\langle \eta_2 \mid early \rangle|^2 = sin^2\theta \tag{21}$$

$$P_{1|2} = P(\eta_1 \mid late) = |\langle \eta_1 \mid late \rangle|^2 = \sin^2\theta \tag{22}$$

$$P_{2|2} = P(\eta_2 \mid late) = |\langle \eta_2 \mid late \rangle|^2 = \cos^2\theta \tag{23}$$

$$P_{1|1} + P_{2|1} = P(\eta_1 \mid early) + P(\eta_2 \mid early) = \cos^2\theta + \sin^2\theta = 1 \tag{24}$$

$$P_{1|2} + P_{2|2} = P(\eta_1 \mid late) + P(\eta_2 \mid late) = \sin^2\theta + \cos^2\theta = 1 \tag{25}$$

$$P = \begin{bmatrix} P(\eta_1 \mid early) & P(\eta_1 \mid late) \\ P(\eta_2 \mid early) & P(\eta_2 \mid late) \end{bmatrix} = \begin{bmatrix} \cos^2\theta & \sin^2\theta \\ \sin^2\theta & \cos^2\theta \end{bmatrix} \tag{26}$$

These probabilities form the simplest possible modal transition between two frames. Based on them, a low computed MFRT score is defined from equations 27 and 28. In this context, a relationship can be produced by measuring how different the two outcome probabilities are from equation 27. The produced score is used as a simple but extremely useful indicator of modal compatibility. High values indicate strong correlation between source labels and receiver outcomes, whereas low values indicate strong ambiguity.

$$D(\theta) = |P(\eta_1 \mid early) - P(\eta_2 \mid early)| = |cos^2\theta - sin^2\theta| = |\cos(2\theta)| \tag{27}$$

$$S_{MFRT}(\theta) = 100\,|\cos(2\theta)| \tag{28}$$

The simulation workflow of the proposed MFRT model (two-mode) is illustrated in Figure 4. The simulation starts with the definition of the source time-bin modal frame and the rotated receiver frame. Then, for each mismatch of angle θ, the corresponding receiver modes are

produced, and the transition probabilities are calculated. Based on these probabilities, the MFRT score is computed as a simple compatibility indicator. The results are stored and visualized.

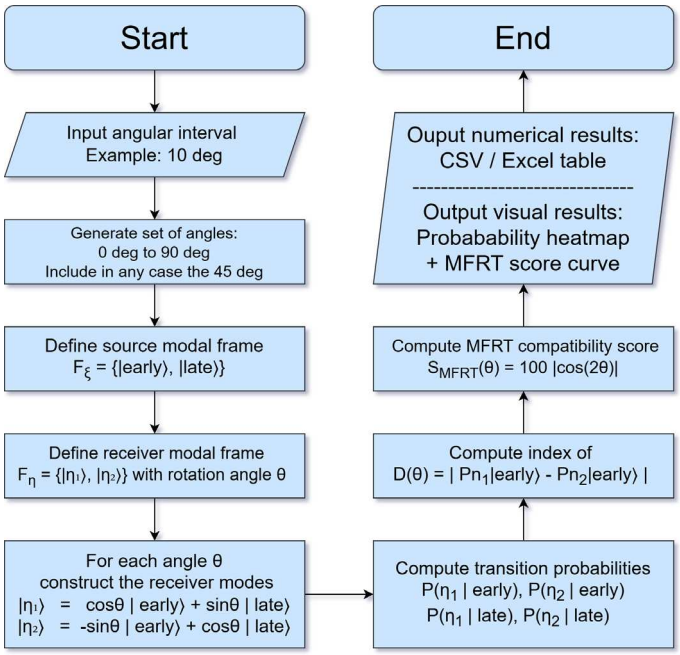


Figure 4. Flow chart of an MFRT procedure. The algorithm defines the source and receiver, computes the transition, evaluates the MFRT compatibility score, and generates numerical and graphical outputs.

5.1 Results and Discussion

An illustrated result is shown in Figure 5. There, the figure presents the probabilities as a transition in color map format, while Figure 6 shows the MFRT score as a function of the frame mismatch in terms of angle θ. The results reveal a clear and symmetric structure. When θ=0°, the modal frame of the receiver is perfectly aligned with the source frame. In this case, we have either $|\eta_1\rangle$=|early⟩ or $|\eta_2\rangle$=|late⟩, while the corresponding transition probabilities become fully deterministic, such as:

- $P(\eta_1 \mid early)$=1, $P(\eta_2 \mid early) = 0$
- $P(\eta_1 \mid late) = 0$, $P(\eta_2 \mid late) = 1$

The aforementioned provide the fact that the receiver identifies the source modal label without ambiguity. Consequently, the MFRT score reaches its maximum value of 100.

Examining the fact that θ increases, the receiver modal frame gradually deviates from the source frame. Also, the detector outcomes become less informative about the original source mode. This corresponds to a gradual loss of modal distinguishability. The heatmap (Figure 5) clearly visualizes this evolution. This is evident as the bright and dark regions progressively mix. Nevertheless, the most critical point occurs at θ=45°. At this angle, the receiver modes become equal-weight superpositions of the source modes as shown in equation 29. Accordingly, the probabilities are given by equation 30. In this case, the receiver outcomes show that a decision cannot be made whether the source emitted |early⟩ or |late⟩. This corresponds to maximum ambiguity, and the MFRT score falls to zero. This is the point of the strongest modal mismatch in this simulation.

$$|\eta_1\rangle = \frac{|early\rangle+|late\rangle}{\sqrt{2}} \text{ and } |\eta_2\rangle = \frac{-|early\rangle+|late\rangle}{\sqrt{2}} \quad (29)$$

$$\text{For } \theta = 45° : \begin{cases} P(\eta_1 \mid early) = P(\eta_2 \mid early) = 0{,}5 \\ P(\eta_1 \mid late) = P(\eta_2 \mid late) = 0{,}5 \end{cases} \tag{30}$$

Before and after 45°, the score decreases and rises, respectively, in a symmetric manner. This happens because the receiver modal frame starts approaching an effective label-swapped version of the source frame. At $\theta \in \{0°, 90°\}$, the system is fully deterministic, although with exchanged modal interpretation. Therefore, the score is equal to 100. This symmetric behavior confirms that the score exhibits modal compatibility at the same angular deviations from 45° rather than merely angular distance in a geometric sense.

Overall, the simulation demonstrates that even in a two-mode condition, the relation between source and receiver frames can be quantified in a simple and physically interpretable way. The heatmap (Figure 5) provides a direct visual description of modal transition structure, while the scoring curve provides a compact summary of frame compatibility.

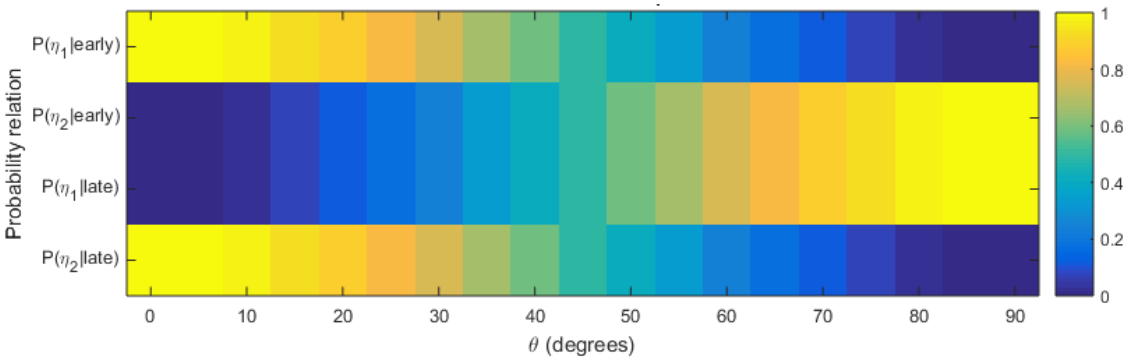


Figure 5. Probabilities as a transition in color map format

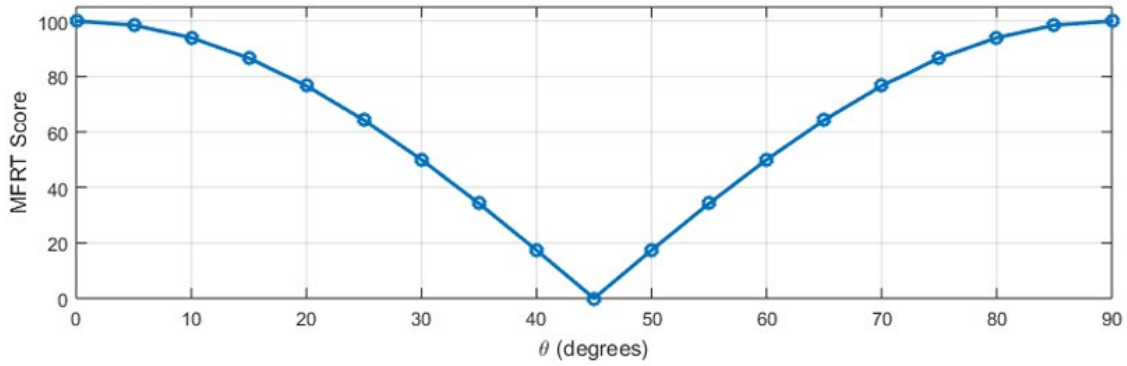


Figure 6. MFRT score as a function of the frame mismatch

5.2 Innovation of the simulated scenario

The innovation of this simulation does not depend on numerical complexity, but instead on a representative example with a conceptual role within MFRT. First, the simulation introduces a modal frame-centered viewpoint. Standard quantum simulations often highlight the evolution of a quantum state under a fixed measurement basis. Here, the state condition remains simple, but the emphasis is given toward the relation between two modal frames. In this sense, the simulation is not only state-oriented, but also frame-oriented. Second, the simulation provides a direct operational interpretation of modal mismatch. Instead of giving abstractly various clues about basis dependence, the simulation shows clearly how a source modal frame and a receiver frame

become correlated, partially mismatched, or fully ambiguous. This makes the MFRT concept measurable. Third, the introduced score from equation 28 (i.e., $SMFRT(\theta)=100|\cos(2\theta)|$) acts as a simple prototype of a modal compatibility indicator. It is not presented as a universal or final metric for all quantum systems, but as a useful first measure that translates modal mismatch into a normalized score. This is important as it gives MFRT a usable numerical form without using complicated methods. Fourth, the simulation supports the broader MFRT interpretation that a modal frame is not just a background basis, but a physically relevant structure. Its compatibility and conversion can be quantified. In other words, the simulation implements the main MFRT claim that different modal frames do not merely provide alternative descriptions of the same signal. They change how much of the encoded information remains accessible.

Hence, this simulation serves as a first proof of concept for MFRT. It shows, under the simplest possible quantum conditions, the way that modal compatibility may be quantified and visualized. Its main strength is interpretability. The source–receiver system mismatch is directly connected to observable transition probabilities and to a useful compatibility score. So, the simulation provides a useful bridge between the theoretical ideas of MFRT and future implementations.

6. Restrictions

MFRT does not replace standard quantum mechanics and does not claim in any case that measurement disturbance can be ignored. It also does not indicate, even imply, that an unknown quantum state can be destructively measured in several incompatible modal frames and then perfectly reconstructed. The coherent reconstruction idea applies only when modal processing is performed before the final measurement and quantum coherence is preserved. MFRT also does not assume that modal frames can be chosen arbitrarily. Specifically, a modal frame must be correlated to modes that can be physically defined, controlled, or approximated by an available quantum device. Therefore, this framework is most meaningful and in cases such as programmable platforms where modal axes can be shaped, selected, or converted within realistic experimental conditions.

The present formulation focuses mainly on ideal modal frames, where each outcome corresponds to a perfectly distinguishable mode. Noisy implementations, imperfect detectors, losses, and various drawbacks require further development within MFRT-Noisy. The simple simulation of two modes and the proposed scoring formulation of $S_{MFRT}(\theta)=100|\cos(2\theta)|$ should be considered as a proof-of-concept demonstration, but not as a universal MFRT metric. It corresponds to modal-frame compatibility based on a useful example, but other systems may require entropy-based, channel-based, or experimentally calibrated measures. As the proposed frame-conversion distance is introduced, its mathematical properties, operational meaning, and experimental estimation should be further studied. Particularly, future work should examine how this distance behaves under realistic conditions (e.g., noise, partial mode overlap).

7. Conclusions and future scopes

This work proposed the Modal Frame Resource Theory (MFRT) as a simple framework for treating programmable quantum measurement bases as resources. The main idea is that a modal frame is not only a mathematical basis, but a controllable structure used by a device to read, process, or reconstruct a quantum signal. MFRT defines two main quantities. The modal frame information measures how much encoded information remains accessible across different modal frames, while frame-conversion distance measures the difficulty of converting one modal frame into another

under specifically allowed free transformations. The framework also supports a coherent signal-processing view, where modal frames may assist encoding, transmission, decoding, and reconstruction before final measurement. Thus, MFRT attracts attention from fixed detectors to programmable modal analyzers. Hence, it provides a resource-based theoretical language for quantum signal processing platforms.

The conducted simulation provides a first proof of concept for the proposed theory. It shows that the compatibility between a source modal frame and a receiver modal frame can be translated into transition probabilities and a useful MFRT score for comparison and other purposes. Although this score is not proposed as a universal metric, it demonstrates the way that modal-frame mismatch can be visualized and connected to information accessibility.

Future work may extend MFRT in various directions. First, the coherent modal-processing view can be developed toward a quantum analogue of multicarrier signal processing, similar to OFDM. There, the information is distributed across orthogonal modal channels and recombined before final detection. Second, experimental implementations may be explored using temporal-mode photonics, quantum pulse gates, interferometers, and other apparatus. Third, MFRT may be connected with semantic and task-oriented quantum communication, where the goal is not to preserve every detail of a quantum signal, but to preserve the modal information that is useful for sensing, communication, or decision-making. Also, a fourth future implementation is modal frame-assisted quantum cryptography. If a transmitter and a receiver share a private programmable modal frame, they may encode and decode quantum signals in a modal alphabet that is difficult for an eavesdropper to identify without the correct frame. This idea is related to high-dimensional QKD, reference-frame-based cryptography, and quantum data locking [19, 20, 21, 22], but MFRT may provide a specific resource-theoretic language for quantifying the value and conversion cost of the shared modal frame. Importantly, the security should not rely on hiding the protocol itself. The protocol may be public, but the private resource would be the selected modal frame or the parameters defining it. So, the development of private modal-frame quantum communication is future work. Fifth, a first future direction is the extension of the simulation framework beyond the two-mode example. Future studies may examine, e.g., higher-dimensional modal frames, noisy channels, imperfect mode selectivity, and partial frame overlap. This would help connect the theoretical MFRT definitions with practical benchmarking tools for programmable quantum systems and signal-processing platforms.

References

1. T. Baumgratz, M. Cramer, and M. B. Plenio, "Quantifying coherence," *Phys. Rev. Lett.*, vol. 113, no. 14, p. 140401, 2014. DOI: 10.1103/PhysRevLett.113.140401
2. B. Brecht, D. V. Reddy, C. Silberhorn, and M. G. Raymer, "Photon Temporal Modes: A Complete Framework for Quantum Information Science," *Phys. Rev. X.*, vol. 5, no. 4, 2015. DOI:10.1103/PhysRevX.5.041017
3. V. Ansari, G. Harder, M. Allgaier, B. Brecht, and C. Silberhorn, "Temporal-mode measurement tomography of a quantum pulse gate," *Phys. Rev. A*, vol. 96, no. 6, 2017. DOI: 10.1103/PhysRevA.96.063817
4. E. Chitambar and G. Gour, "Quantum resource theories," *Rev. Mod. Phys.*, vol. 91, no. 2, 2019. DOI: 10.1103/RevModPhys.91.025001
5. A. Streltsov, G. Adesso, and M. B. Plenio, "*Colloquium*: Quantum coherence as a resource," *Rev. Mod. Phys.*, vol. 89, no. 4, 2017. DOI: 10.1103/RevModPhys.89.041003

6. S. D. Bartlett, T. Rudolph, and R. W. Spekkens, "Reference frames, superselection rules, and quantum information," *Rev. Mod. Phys.*, vol. 79, no. 2, pp. 555–609, 2007. DOI: 10.1103/RevModPhys.79.555
7. I. Marvian and R. W. Spekkens, "Modes of asymmetry: The application of harmonic analysis to symmetric quantum dynamics and quantum reference frames," *Phys. Rev. A*, vol. 90, no. 6, 2014. DOI: 10.1103/PhysRevA.90.062110
8. N. P. Landsman, "Born rule and its interpretation," in *Compendium of Quantum Physics*, Berlin, Heidelberg: Springer Berlin Heidelberg, 2009, pp. 64–70. DOI: 10.1007/978-3-540-70626-7_20
9. G. Chiribella, G. M. D'Ariano, and P. Perinotti, "Quantum circuit architecture," *Phys. Rev. Lett.*, vol. 101, no. 6, p. 060401, 2008. DOI: 10.1103/PhysRevLett.101.060401
10. D. V. Reddy and M. G. Raymer, "High-selectivity quantum pulse gating of photonic temporal modes using all-optical Ramsey interferometry," *Optica*, vol. 5, no. 4, p. 423, 2018. DOI: 10.1364/OPTICA.5.000423
11. X. Lu, Y. Liu, and H. Lin, "Quantum signal processing and quantum singular value transformation on U (N)," *Quantum*, vol. 10, no. 2048, p. 2048, 2026. DOI: 10.22331/q-2026-03-27-2048
12. S. K. Chronopoulos, "Flexible and reconfigurable OFDM implementation in DSP platform for various purposes and applications," *Sensors (Basel)*, vol. 24, no. 9, p. 2732, 2024. DOI: 10.3390/s24092732
13. D. Motlagh and N. Wiebe, "Generalized Quantum Signal Processing," *arXiv [quant-ph]*, 2023. DOI: 10.48550/arXiv.2308.01501
14. Z. Qin, X. Tao, J. Lu, W. Tong, and G. Y. Li, "Semantic Communications: Principles and Challenges," *arXiv [cs.IT]*, 2021. DOI: 10.48550/arXiv.2201.01389
15. C. E. Shannon, "A mathematical theory of communication," *ACM SIGMOBILE Mob. Comput. Commun. Rev.*, vol. 5, no. 1, pp. 3–55, 2001. DOI: 10.1145/584091.584093
16. P. Li and Y. Li, "Generalized physically incoherent operations," *Phys. Rev. A*, vol. 109, no. 6, 2024. DOI: 10.1103/PhysRevA.109.062424
17. G. Gour, "Comparison of quantum channels by superchannels," *IEEE Trans. Inf. Theory*, vol. 65, no. 9, pp. 5880–5904, 2019. DOI: 10.1109/TIT.2019.2907989
18. J. Watrous, *The theory of quantum information*. Cambridge, England: Cambridge University Press, 2018. pp. 164–196. DOI: 10.1017/9781316848142
19. C. H. Bennett and G. Brassard, "Quantum cryptography: Public key distribution and coin tossing," *Theor. Comput. Sci.*, vol. 560, pp. 7–11, 2014. DOI: 10.1016/j.tcs.2014.05.025
20. H. Yu *et al.*, "Quantum key distribution implemented with d-level time-bin entangled photons," *Nat. Commun.*, vol. 16, no. 1, p. 171, 2025. DOI: 10.1038/s41467-024-55345-0
21. W.-Y. Liang *et al.*, "Proof-of-principle experiment of reference-frame-independent quantum key distribution with phase coding," *Sci. Rep.*, vol. 4, no. 1, p. 3617, 2014. DOI: 10.1038/srep03617
22. G. Chiribella, L. Maccone, and P. Perinotti, "Secret quantum communication of a reference frame," *Phys. Rev. Lett.*, vol. 98, no. 12, p. 120501, 2007. DOI: 10.1103/PhysRevLett.98.120501