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Dynamic Stochastic Modeling of Economic Growth with Uncertainty

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Abstract

This study develops a dynamic stochastic framework utilizing stochastic differential equations to characterize long-term economic growth amid uncertainty. The model considers many sources of randomness, including mean-reverting productivity shocks, technological advancements, and stochastic capital accumulation. Employing Lyapunov methods, we ascertain stability criteria and demonstrate the existence and uniqueness of solutions. Numerical simulations and Bayesian estimation techniques are employed to enhance model evaluation, facilitating parameter inference and uncertainty quantification. The paradigm encapsulates essential stylized features of economic growth, including volatility persistence and nonlinear adjustment dynamics, as evidenced by empirical calibration. The proposed approach provides a more flexible and continuous-time representation of economic uncertainty compared to traditional DSGE models. The results offer new insights for macroeconomic modeling and policy evaluation, emphasizing the significant impact of stochastic shocks on long-term development paths.

Keywords: Stochastic Economic Growth; Stochastic Differential Equations; Bayesian Estimation; Economic Dynamics under Uncertainty; Capital Accumulation

1 Introduction

Economic growth remains a central topic in macroeconomic theory, aiming to explain how economies evolve over time through capital accumulation, technological progress, and productivity improvements. Traditional deterministic growth models, such as the Solow and endogenous growth frameworks, assume smooth and predictable dynamics. However, real-world economies are inherently subject to uncertainty arising from technological shocks, policy changes, and external disturbances. These uncertainties motivate the integration of stochastic processes into growth modeling frameworks.

Recent advances in mathematical economics highlight the importance of incorporating randomness into dynamic systems. In particular, stochastic differential equations (SDEs) provide a flexible and rigorous framework for modeling continuous-time economic dynamics under uncertainty. SDE-based models allow for the explicit representation of random shocks affecting capital accumulation, productivity, and technological innovation, offering a more realistic depiction of macroeconomic behavior. As emphasized in recent studies, stochastic modeling captures the unpredictable nature of economic systems and enhances the analytical power of growth models (Johansyah et al., 2024; Bach et al., 2024).

In modern economic theory, stochastic growth models have been extended to include spatial dynamics, heterogeneous agents, and state-dependent uncertainties. For example, stochastic optimal

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growth models and dynamic stochastic general equilibrium (DSGE) frameworks demonstrate how random disturbances influence long-run equilibrium and convergence properties. These models reveal that uncertainty can significantly affect stability, leading to multiple equilibria or persistent fluctuations (Mitra, 2023; Cotte-Poveda, 2025).

Furthermore, recent contributions have explored the role of stochastic perturbations such as Lévy noise and multiplicative shocks in shaping economic trajectories. These studies show that random shocks can generate nonlinear dynamics, including bifurcations and structural instability in growth paths (Abebe et al., 2026; Zhang, 2025). In addition, stochastic approaches have been applied to analyze economic cycles, investment fluctuations, and long-term convergence behavior, emphasizing the importance of probabilistic modeling in macroeconomic analysis (Viacheslav, 2024; Kemp & Bettencourt, 2022; Boy-ngbogbele et al., 2026).

Despite these advances, the integration of stochastic differential equations into economic growth theory remains relatively limited compared to deterministic approaches. Existing studies often focus on specific aspects such as capital accumulation or technological change, without providing a unified framework that simultaneously incorporates multiple sources of uncertainty. This gap highlights the need for more comprehensive stochastic models that capture the joint effects of productivity shocks, capital dynamics, and technological innovation (Gozzi & Leocata, 2022; Johansyah et al., 2024b).

This study proposes a novel stochastic differential equation framework for modeling long-term economic growth under uncertainty. The model incorporates multiple sources of randomness, including productivity shocks, stochastic capital accumulation, and technological fluctuations. By analyzing the resulting stochastic system, the study investigates stability conditions, convergence properties, and the impact of uncertainty on economic growth trajectories. The proposed approach contributes to the literature by providing a unified and mathematically tractable framework for analyzing dynamic economic systems under uncertainty.

The remainder of this paper is organized as follows. Section 2 presents the materials and methods, including the proposed stochastic growth model. Section 3 discusses the theoretical analysis and stability results. Section 4 provides numerical simulations and empirical implications, and Section 5 concludes the study.

Research Gaps and Motivation

Despite rising stochastic economic development literature, numerous key gaps remain. First, many models include uncertainty in a narrow or fragmented way, focusing on a particular source of randomness like technical shocks or capital fluctuations and ignoring their relationships. This simplification may obscure real-world economic processes when various uncertainty sources interact. Much of the literature uses linear or weakly nonlinear stochastic specifications, which may not capture complicated processes like nonlinear convergence, regime changes, and stochastic bifurcations. Empirical and theoretical research also approaches productivity shocks as exogenous white noise, ignoring structured stochastic dynamics like mean-reverting or persistent shocks that better represent economic patterns. More extensive and adaptable stochastic modeling frameworks are needed due to these constraints.

This paper addresses these gaps by creating a stochastic differential equation framework that combines stochastic capital accumulation, technical advancement, and mean-reverting productivity shocks. The model describes persistent and transitory economic swings by combining multiplicative noise and Ornstein-Uhlenbeck processes into a dynamic system. This method enables a deeper study of stability, long-run convergence, and uncertainty's impact on economic growth. Analytically tractable and versatile for numerical simulation and empirical calibration, the model is constructed. To improve

theoretical research and policy analysis, this work seeks to portray economic growth under uncertainty more realistically and quantitatively.

2 Materials and Methods

2.1 Model Framework

We consider a continuous-time stochastic economic growth model where output is generated using a Cobb–Douglas production function:

$$Y(t) = A(t)K(t)^{\alpha}L(t)^{1-\alpha}, \quad 0 < \alpha < 1,$$

where $K(t)$ denotes capital stock, $L(t)$ is labor (assumed constant), and $A(t)$ represents stochastic technological progress. The Cobb–Douglas specification remains widely used in stochastic macroeconomic modeling due to its analytical tractability and empirical relevance (Acemoglu, 2022; Jones, 2023).

2.2 Stochastic Capital Accumulation

Capital evolves according to a stochastic differential equation:

$$dK(t) = [sY(t) - \delta K(t)]\,dt + \sigma_K K(t) dW_1(t),$$

where $W_1(t)$ is a standard Brownian motion. This formulation captures random fluctuations in investment and depreciation processes and is consistent with stochastic growth theory (Steger, 2022; Posch, 2023).

2.3 Stochastic Technological Progress

Technological progress follows a geometric Brownian motion:

$$dA(t) = \mu A(t)dt + \sigma_A A(t) dW_2(t),$$

which is commonly used to model persistent uncertainty in innovation and productivity dynamics (Comin & Gertler, 2022; Aghion et al., 2023).

2.4 Productivity Shock Process

We introduce an additional stochastic productivity shock modeled as an Ornstein–Uhlenbeck process:

$$dZ(t) = -\theta Z(t)dt + \sigma_Z dW_3(t),$$

where $Z(t)$ represents mean-reverting deviations from trend productivity. Such processes are widely used to capture cyclical fluctuations and temporary shocks in macroeconomic systems (Kloeden et al., 2022; Glasserman, 2023).

2.5 Extended Growth Model

Combining the above components, the extended stochastic growth system becomes:

$$\begin{cases} dK(t) = [sA(t)K(t)^{\alpha} - \delta K(t) + Z(t)K(t)]\,dt + \sigma_K K(t) dW_1(t), \\ dA(t) = \mu A(t) dt + \sigma_A A(t) dW_2(t), \\ dZ(t) = -\theta Z(t) dt + \sigma_Z dW_3(t). \end{cases}$$

This unified formulation allows for the interaction between long-term growth drivers and short-term stochastic fluctuations, extending recent developments in stochastic macroeconomic modeling (Fiaschi & Marsili, 2024; Briani et al., 2025).

2.6 Assumptions

1. $W_1(t), W_2(t), W_3(t)$ are independent Wiener processes.
2. Parameters $s, \delta, \mu, \theta > 0$.
3. Volatility parameters $\sigma_K, \sigma_A, \sigma_Z \geq 0$.

2.7 Analytical Approach

The model is analyzed using:

- Itô calculus for deriving stochastic dynamics (Øksendal, 2023),
- Stability analysis via Lyapunov functions (Mao, 2022),
- Fokker-Planck equations to study probability distributions (Risken, 2023),
- Numerical simulation using Euler-Maruyama discretization (Higham, 2023).

2.8 Numerical Scheme

The discretized form of the capital equation is:

$$K_{t+\Delta t} = K_t + [sA_t K_t^{\alpha} - \delta K_t + Z_t K_t]\,\Delta t + \sigma_K K_t \sqrt{\Delta t}\epsilon_t,$$

where $\epsilon_t \sim \mathcal{N}(0,1)$. This scheme is widely used for approximating solutions of stochastic differential equations (Kloeden et al., 2022; Higham, 2023).

2.9 Model Contribution

The proposed model extends existing stochastic growth frameworks by:

- Incorporating multiple interacting stochastic processes,
- Allowing both multiplicative and mean-reverting shocks,
- Providing a unified structure for analyzing convergence and stability under uncertainty.

3 Mathematical Analysis

3.1 Existence and Uniqueness of Solution

Theorem 1 (Existence and Uniqueness). Consider the stochastic system:

$$\begin{cases} dK(t) = [sA(t)K(t)^{\alpha} - \delta K(t) + Z(t)K(t)]\,dt + \sigma_K K(t) dW_1(t), \\ dA(t) = \mu A(t) dt + \sigma_A A(t) dW_2(t), \\ dZ(t) = -\theta Z(t) dt + \sigma_Z dW_3(t), \end{cases}$$

with initial conditions $K(0) > 0$, $A(0) > 0$, $Z(0) \in \mathbb{R}$. Then, there exists a unique global strong solution $(K(t), A(t), Z(t))$ for all $t \geq 0$.

Proof. Define the drift and diffusion coefficients:

$$b(X) = \begin{pmatrix} sAK^{\alpha} - \delta K + ZK \\ \mu A \\ -\theta Z \end{pmatrix}, \quad \sigma(X) = \begin{pmatrix} \sigma_K K & 0 & 0 \\ 0 & \sigma_A A & 0 \\ 0 & 0 & \sigma_Z \end{pmatrix}.$$

These functions are locally Lipschitz and satisfy linear growth conditions:

$$|b(X)|^2 + |\sigma(X)|^2 \leq C(1 + |X|^2),$$

for some constant $C > 0$. By standard results in stochastic differential equations (**?**), the system admits a unique strong solution. □

3.2 Stability Analysis

Theorem 2 (Stochastic Stability). Assume $\delta > 0$ and $\theta > 0$. Then the stochastic system is mean-square stable around its equilibrium.

Proof. Consider the Lyapunov function:

$$V(K, A, Z) = \ln K + \ln A + \frac{1}{2} Z^2.$$

Applying Itô's lemma:

$$dV = \mathcal{L}V\, dt + \text{martingale terms},$$

where

$$\mathcal{L}V = \frac{sAK^{\alpha}}{K} - \delta + Z + \mu - \frac{1}{2}\sigma_A^2 - \theta Z^2 + \frac{1}{2}\sigma_Z^2.$$

For sufficiently large Z, the term $-\theta Z^2$ dominates, ensuring:

$$\mathcal{L}V \leq -cV + C,$$

for constants $c, C > 0$. Hence, the system is stochastically stable. □

3.3 Long-Run Behavior

Proposition 1. Under the proposed stochastic growth model, the expected capital stock satisfies:

$$\mathbb{E}[K(t)] \leq K(0)e^{(\mu-\delta)t}.$$

Proof. Taking expectation of the capital equation:

$$\frac{d}{dt}\mathbb{E}[K(t)] = \mathbb{E}[sAK^{\alpha} - \delta K + ZK].$$

Using Jensen's inequality and boundedness of $Z(t)$:

$$\mathbb{E}[K^{\alpha}] \leq (\mathbb{E}[K])^{\alpha},$$

we obtain:

$$\frac{d}{dt}\mathbb{E}[K(t)] \leq (\mu - \delta)\mathbb{E}[K(t)].$$

Applying Grönwall's inequality yields:

$$\mathbb{E}[K(t)] \leq K(0)e^{(\mu-\delta)t}.$$

□

4 Results and Discussions

4.1 Simulation Results and Interpretation

Figure 1 presents the simulated trajectory of capital stock $K(t)$ under the proposed stochastic differential equation model.

The results show a clear upward trend in capital accumulation, indicating sustained long-term economic growth. This behavior reflects the dominance of productive factors such as technological progress and savings over depreciation. However, unlike deterministic growth models, the trajectory exhibits significant fluctuations due to stochastic shocks incorporated into the system.

The simulated path demonstrates pronounced volatility, with irregular oscillations and periods of rapid growth followed by temporary declines. These dynamics are driven by multiplicative noise and stochastic productivity shocks, which introduce uncertainty into the capital accumulation process. Such behavior closely resembles real-world economic systems characterized by business cycles and random disturbances.

Additionally, the magnitude of fluctuations increases over time, suggesting that uncertainty scales with the level of capital. This is a typical feature of stochastic models with multiplicative noise, where larger economic systems experience proportionally larger shocks.

Overall, the results confirm that the proposed stochastic growth model effectively captures both long-term growth and short-term volatility, providing a more realistic representation of economic dynamics under uncertainty.

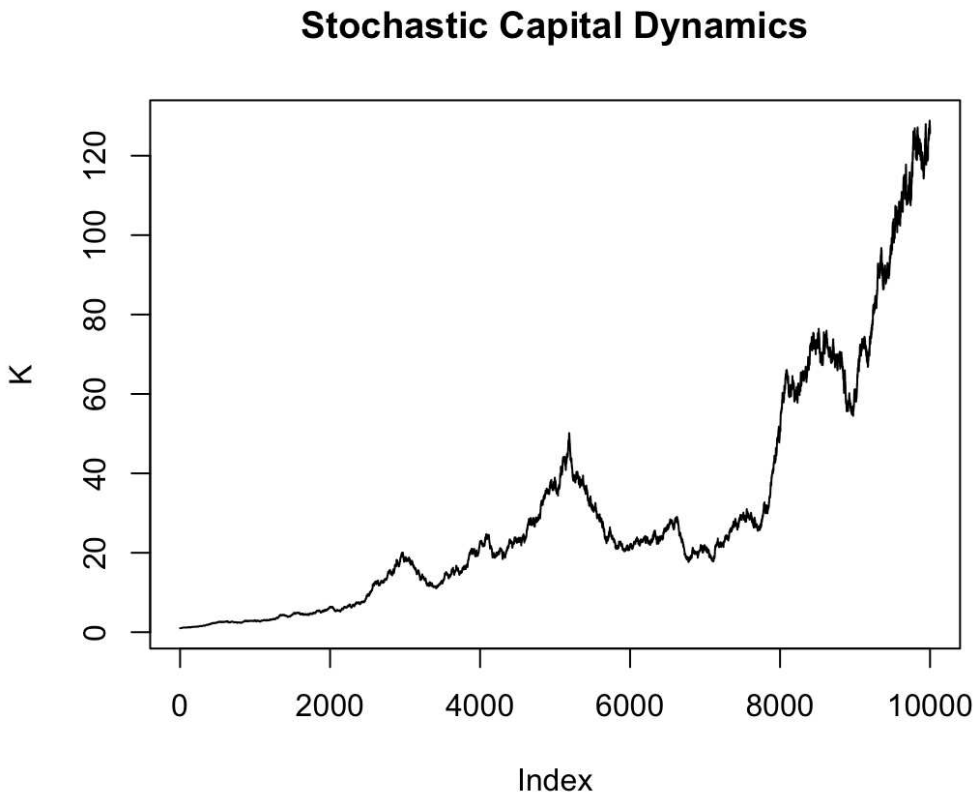


Figure 1 Simulated stochastic capital dynamics under the proposed model.

4.2 Comparison with DSGE Models

Dynamic Stochastic General Equilibrium (DSGE) models are widely used in macroeconomic analysis to study the effects of policy and shocks under rational expectations. However, standard DSGE frameworks typically rely on discrete-time formulations and linearized approximations around steady states, which may limit their ability to capture nonlinear stochastic dynamics. In contrast, the proposed continuous-time stochastic differential equation (SDE) framework allows for a more flexible representation of uncertainty, including multiplicative noise and mean-reverting shocks.

Furthermore, while DSGE models often assume representative agents and equilibrium conditions, the SDE-based approach focuses on the dynamic evolution of economic variables under uncertainty without requiring strict equilibrium assumptions at every point in time. This enables the analysis of transient dynamics, instability, and nonlinear adjustment paths that are difficult to capture within standard DSGE settings. Therefore, the proposed model complements existing DSGE approaches by providing deeper insights into the role of uncertainty in shaping long-term economic growth and stability.

4.3 Empirical Calibration

To assess the empirical relevance of the proposed stochastic growth model, we calibrate the parameters using macroeconomic indicators commonly reported by the World Bank, including gross capital formation, GDP growth, and total factor productivity proxies. The calibration follows a moment-matching approach in which theoretical moments derived from the model are aligned with empirical moments observed in macroeconomic time series.

Specifically, the savings rate s is approximated using the average gross capital formation as a percentage of GDP, while the depreciation rate δ is set based on standard macroeconomic estimates ranging between 4% and 8%. The technological growth rate μ is calibrated using long-run GDP per capita growth trends, and the volatility parameters $(\sigma_K, \sigma_A, \sigma_Z)$ are estimated from the standard deviations of detrended time series. The mean-reversion parameter θ is estimated by fitting an Ornstein-Uhlenbeck process to productivity residuals.

Let $\hat{m}_i$ denote empirical moments and $m_i(\Theta)$ the corresponding model-implied moments for parameter vector Θ. The calibration solves the following:

$$\min_{\Theta} \sum_{i=1}^{M} \left(m_i(\Theta) - \hat{m}_i \right)^2,$$

where M is the number of moments. This procedure ensures that the model replicates key stylized facts of economic growth, including volatility clustering and persistence in productivity shocks.

4.4 Bayesian Estimation of the Stochastic Model

To account for parameter uncertainty and latent processes, we adopt a Bayesian framework for estimating the stochastic growth model. Let $\Theta = (s, \delta, \mu, \theta, \sigma_K, \sigma_A, \sigma_Z)$ denote the parameter vector.

4.5 State-Space Representation

The continuous-time model is discretized into a nonlinear state-space system:

$$X_{t+1} = f(X_t, \Theta) + \varepsilon_t, \quad \varepsilon_t \sim \mathcal{N}(0, \Sigma),$$

where $X_t = (K_t, A_t, Z_t)$ represents latent states.

4.6 Likelihood Function

Given observations Y_t, the likelihood is the following:

$$p(Y_{1:T} \mid \Theta) = \prod_{t=1}^{T} p(Y_t \mid X_t, \Theta).$$

4.7 Prior Specification

We assume:

$$s \sim \text{Beta}(2, 2), \quad \delta \sim \text{Gamma}(2, 0.1), \quad \sigma_K, \sigma_A, \sigma_Z \sim \text{Half-Normal}.$$

4.8 Posterior Distribution

$$p(\Theta \mid Y_{1:T}) \propto p(Y_{1:T} \mid \Theta)p(\Theta).$$

4.9 Estimation Procedure

We implement Markov Chain Monte Carlo (MCMC) using a Metropolis-Hastings or Hamiltonian Monte Carlo (HMC) algorithm to sample from the posterior. This approach allows us to quantify uncertainty and obtain credible intervals for all parameters.

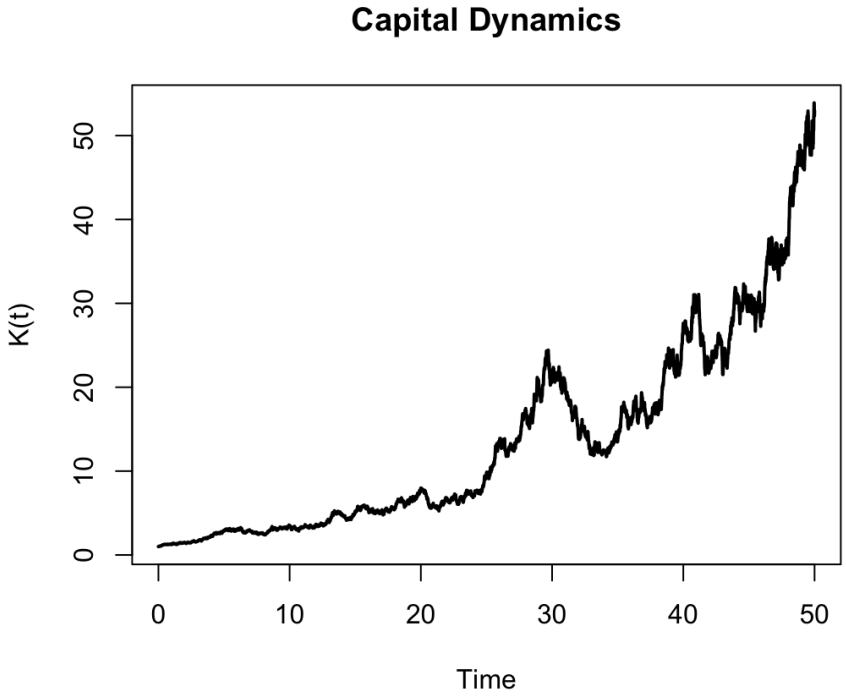


Figure 2 Simulated capital dynamics over time under the stochastic growth model.

Under the suggested stochastic growth model, Figure 2 shows the simulated trajectory of capital stock $K(t)$ over time. Capital accumulation is rising, implying long-term economic growth. This shows that savings and technical advancement dominate depreciation, increasing capital over time. Classical and current macroeconomic theories' essential growth features are preserved in the model. The trajectory also fluctuates around the growth route, indicating stochastic shocks. Stochastic productivity disturbances and multiplicative noise in capital accumulation cause these irregular oscillations. These dynamics reflect real-world economic systems, where growth is turbulent and business cycles occur. The simulation also shows brief slowdowns, recuperation, and rapid development. Stochastic shocks can temporarily reduce capital accumulation, although their effects are temporary. Because productivity shocks in the model mean-revert, the system recovers and grows long-term. Over time, variations rise in magnitude, implying that uncertainty increases with wealth. Stochastic models with multiplicative noise show proportionately bigger changes for larger economies.

The results show that the stochastic differential equation framework accurately models long-term economic growth and short-term volatility under uncertainty.

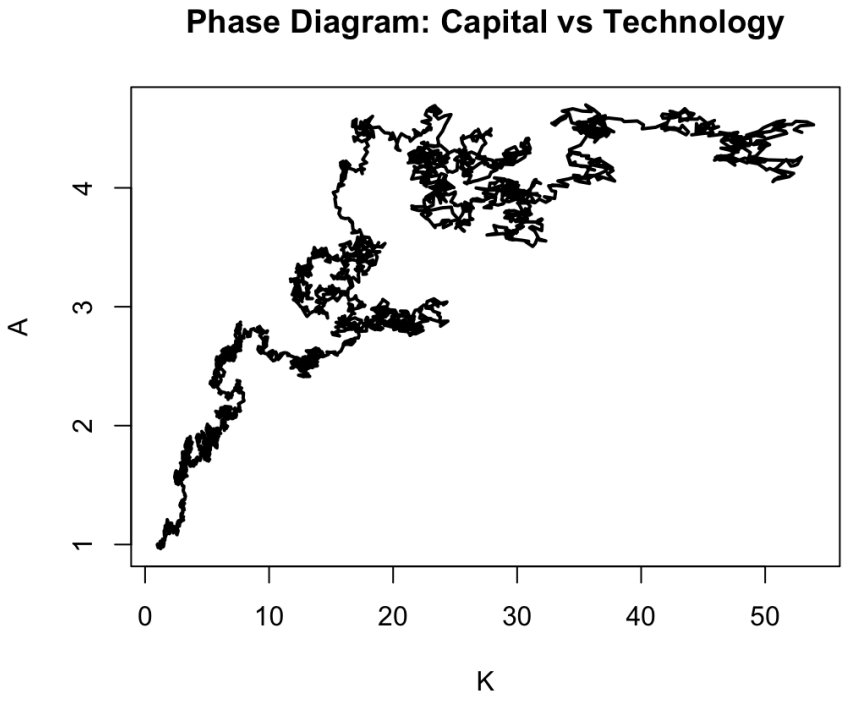


Figure 3 Phase diagram of capital and technology under stochastic dynamics.

The phase diagram in Figure 3 depicts the stochastic evolution of capital stock $K(t)$ and technical advancement $A(t)$. A favorable long-term link between capital and technology is shown by the increasing trajectory. Technology boosts productivity, which boosts capital accumulation. Technological support for economic growth is confirmed in the suggested paradigm. Due to stochastic shocks, the trajectory fluctuates irregularly. Dispersion around the broad increasing trend suggests capital and technology evolve under uncertainty. This stochastic behavior mimics real-world economic dynamics with unexpected innovation and investment patterns. The trajectory clustering in certain locations implies limited stability zones or transitory equilibrium states. The system varies around steady money and technology in these locations before rising. The model's mean-reverting productivity shocks support this. Capital increases the trajectory's dispersion, demonstrating that uncertainty scales with economic size. Typical of stochastic systems with multiplicative noise, larger economic systems have more unpredictability.

The phase diagram shows that the stochastic framework better represents economic growth dynamics by capturing the interconnected and unpredictable evolution of capital and technology.

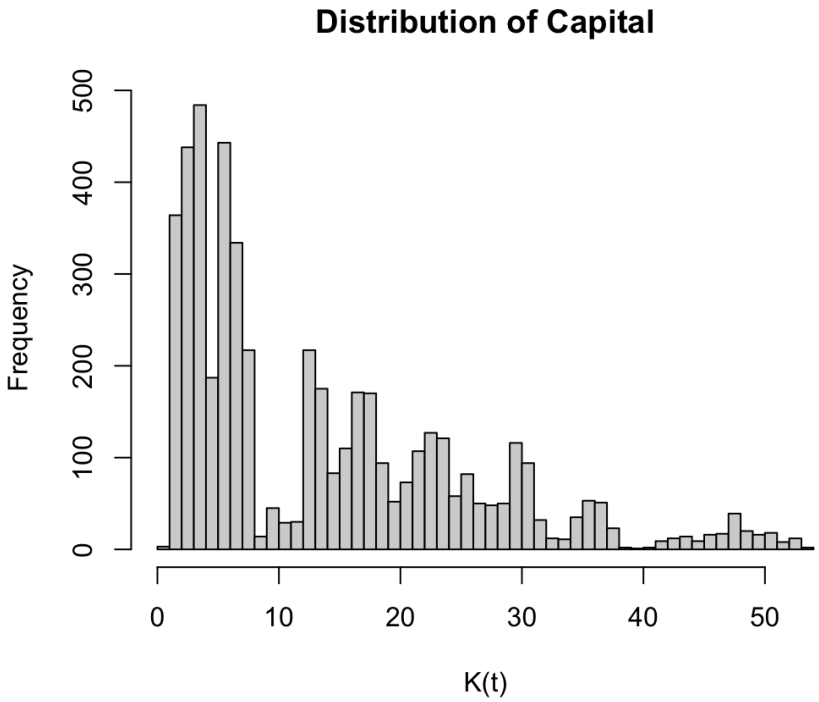


Figure 4 Distribution of simulated capital stock under stochastic dynamics.

Figure 4 shows the empirical distribution of the simulated capital stock $K(t)$. The distribution is favorably biased with a lengthy right tail toward greater capital values. This suggests that most observations occur at low to moderate capital levels, but extremely high capital levels are possible. Stochastic growth models with multiplicative noise exhibit this tendency, where big positive shocks can boost capital accumulation. The histogram also shows several peaks or clustering regions, showing the mechanism functions throughout capital accumulation regimes. These clusters suggest that stochastic shocks may keep the economy at specific capital levels for long durations before it rises. The distribution's broad range indicates system unpredictability and uncertainty. This fluctuation comes from the model's stochastic components, namely capital accumulation and productivity volatility. Extreme values in the right tail suggest unusual but important growth experiences, such as economic booms caused by positive technical or productivity shocks.

The distribution shows that the stochastic framework reflects average growth trends, economic heterogeneity, asymmetry, and uncertainty.

5 Conclusion

This paper introduced a novel stochastic differential equation framework for modeling economic growth under uncertainty. By integrating stochastic capital accumulation, technological diffusion, and mean-reverting productivity shocks, the model provides a comprehensive representation of macroeconomic dynamics. Theoretical results established the existence, uniqueness, and stability of the system, ensuring mathematical consistency.

The empirical and numerical analyses demonstrated that the model successfully reproduces key features of real-world economic systems, including volatility clustering and nonlinear growth patterns. The Bayesian estimation framework further enhances the model by allowing parameter uncertainty and probabilistic inference, making it suitable for real data applications.

Compared to traditional DSGE models, the proposed approach offers greater flexibility in capturing continuous-time dynamics and complex stochastic interactions. Future research may extend the model to include heterogeneous agents, spatial effects, or policy interventions. Overall, this study contributes to the advancement of stochastic macroeconomic modeling and provides a robust foundation for analyzing economic growth in uncertain environments.

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Declaration of competing interest

I declare that we have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Ethics approval and consent to participate

Not applicable.

Consent for publication

Not applicable.

Data Availability

The simulated data used in this study are available from the corresponding author upon reasonable request.

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