

A Influence of the domain size

We tested the effect of defining a smaller domain as described in Table 1. Bedrock elastic properties remain the same as in Model 1.

Name	Reference	Small
Resolution (m)	50	50
X_{max} (km)	30	20
X_{min} (km)	-20	-20
Y_{max} (km)	20	20
Y_{min} (km)	-20	20
Depth (km)	-10	10

Table 1: Dimension characteristics of the *Reference* and *Small* modeled domains.

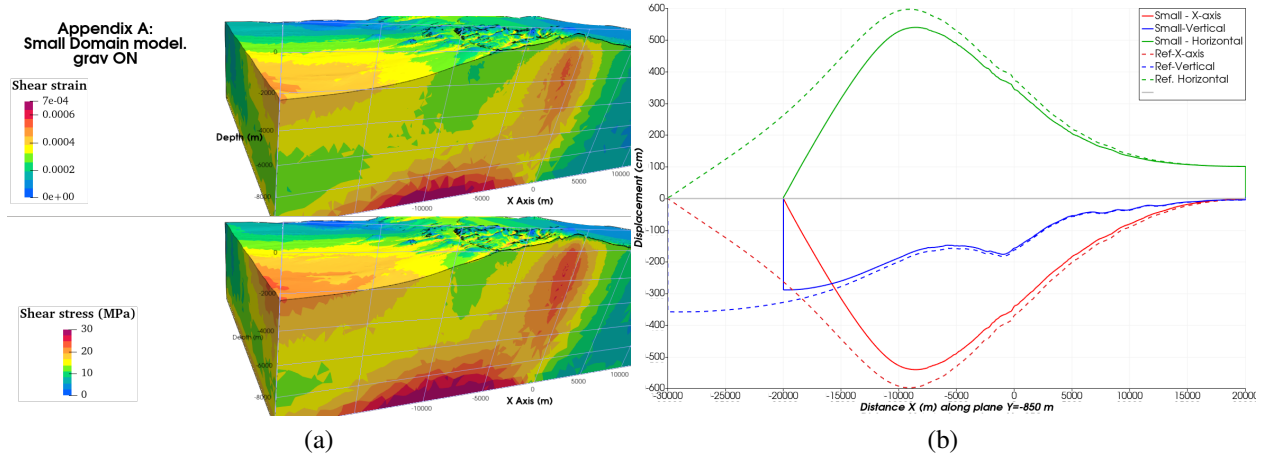


Figure A.1: (a) Shear stain (top) and stress (bottom) at the western boundary of the *Small* model. (b) Horizontal, vertical, and X-axis displacements for the *Reference* (dashed) and *Small* (solid) models.

Shear stress and strain display similar magnitudes as the *Reference* model. Nevertheless, a local maximum develops at the extreme western boundary of the domain, reaching values similar to those beneath the summit. This behavior reflects the boundary effect associated with a reduced boundary (from 30 km to 20 km).

The resulting *Small* model exhibits a $\sim 10\%$ decrease in the total Horizontal (X+Y) displace-

9 ment, as well as in the X-component, and a ~20% decrease in the vertical displacement relative to
10 the Reference model.

11 Because the walls (borders) of the modeled domain are closer to the edifice's submit, the gravity
12 force is balanced over a smaller distance, in the way that these borders “prevent” lateral spreading
13 due to the topographic-bathymetric gradient. Hence the resulting displacements are lower than in
14 the Reference model (*Section 4.1*).

B Influence of the effective strength

B.1 Preliminary models

In this supplementary section we explore different crustal strength configurations to illustrate their influence on surface displacement and plastic strain. Results indicate that the parameters that control most the depth extent of failure and the magnitude of surface displacements are the cohesion c and tension T .

Name	φ_{bd} ($^{\circ}$)	c_{bd} (MPa)	T_{bd} (MPa)	u_{max}^z (m)	PDF (m)
P10A	10	10	10	+2	<100
P10B	10	8	5	-8	1500
P15A	15	10	10	+1.5	<100
P15B	15	8	5	+6	1000
P20A	20	10	10	+1.6	200
P20B	20	8	5	+5	300

Table 2: Plastic strength parameters of *Section B* tests: friction angle (φ_{bd}), cohesion (c_{bd}), and tension (T_{bd}). Results are given as: surface displacement (u_{max}^z ; negative when subsidence and positive when uplift), and the maximum depth of plastic failure away from the surface.

B.2 Complementary results for Models 2 and 3

The best model cases describing the extreme behaviors of Mt. Pelée are models P20A and P10B. They correspond to Model 2 and Model 3 shown in Sections 4.2 and 4.3. Here we present their extended results:

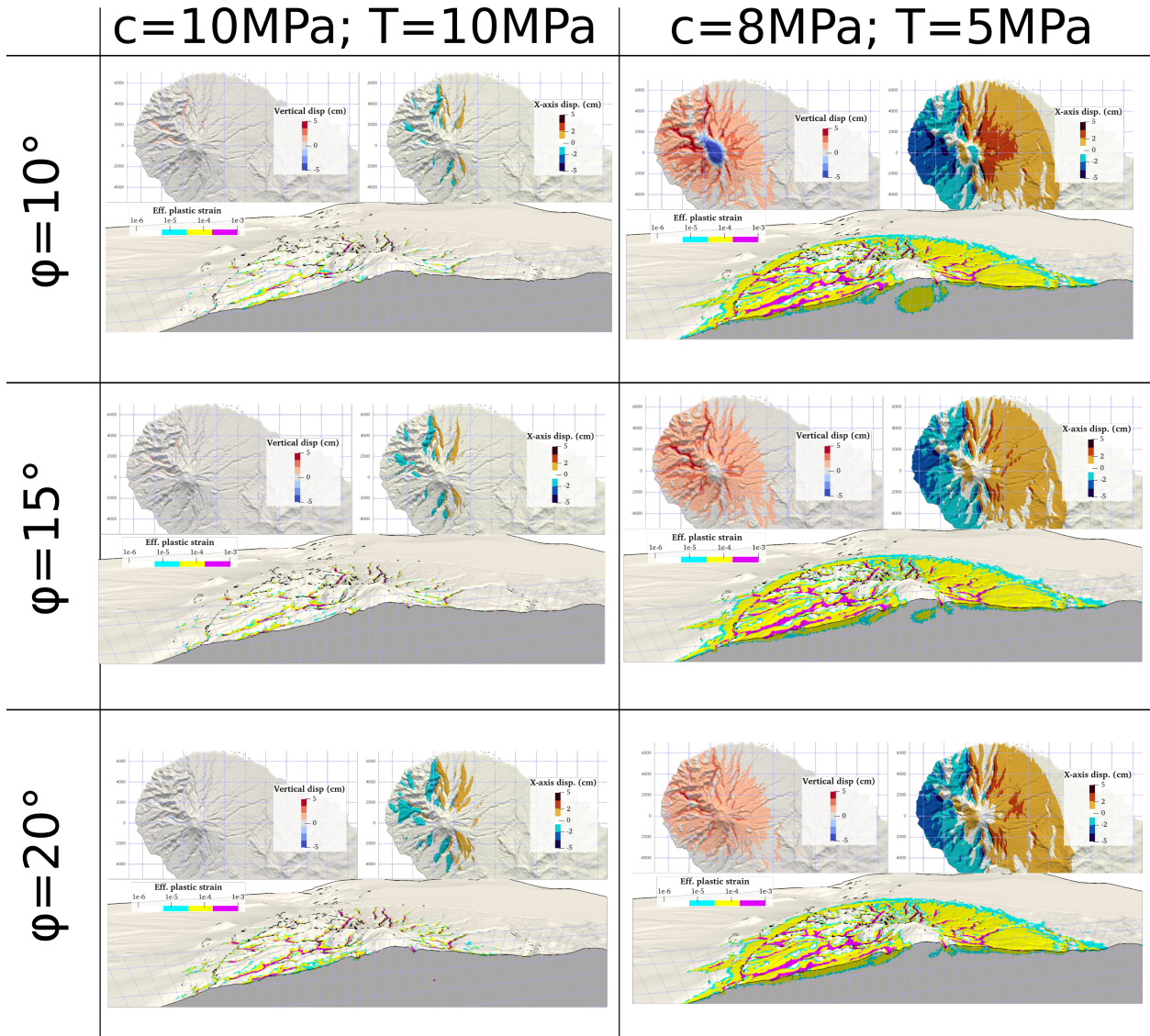
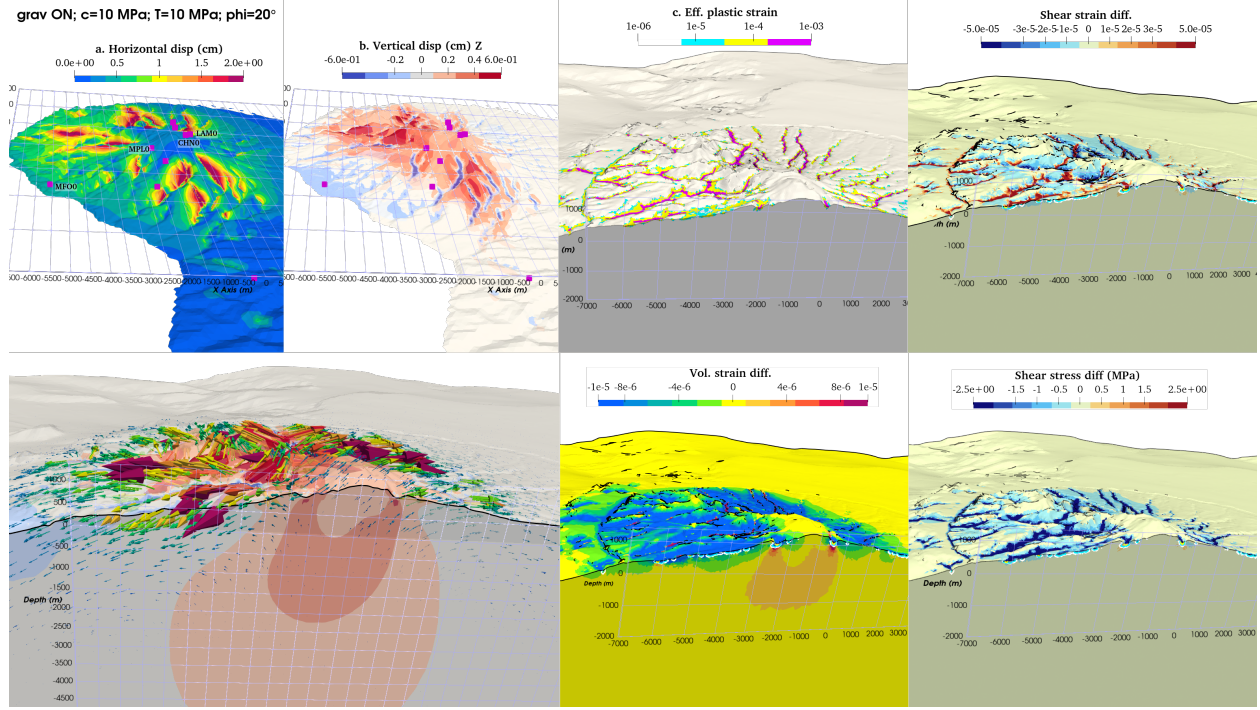


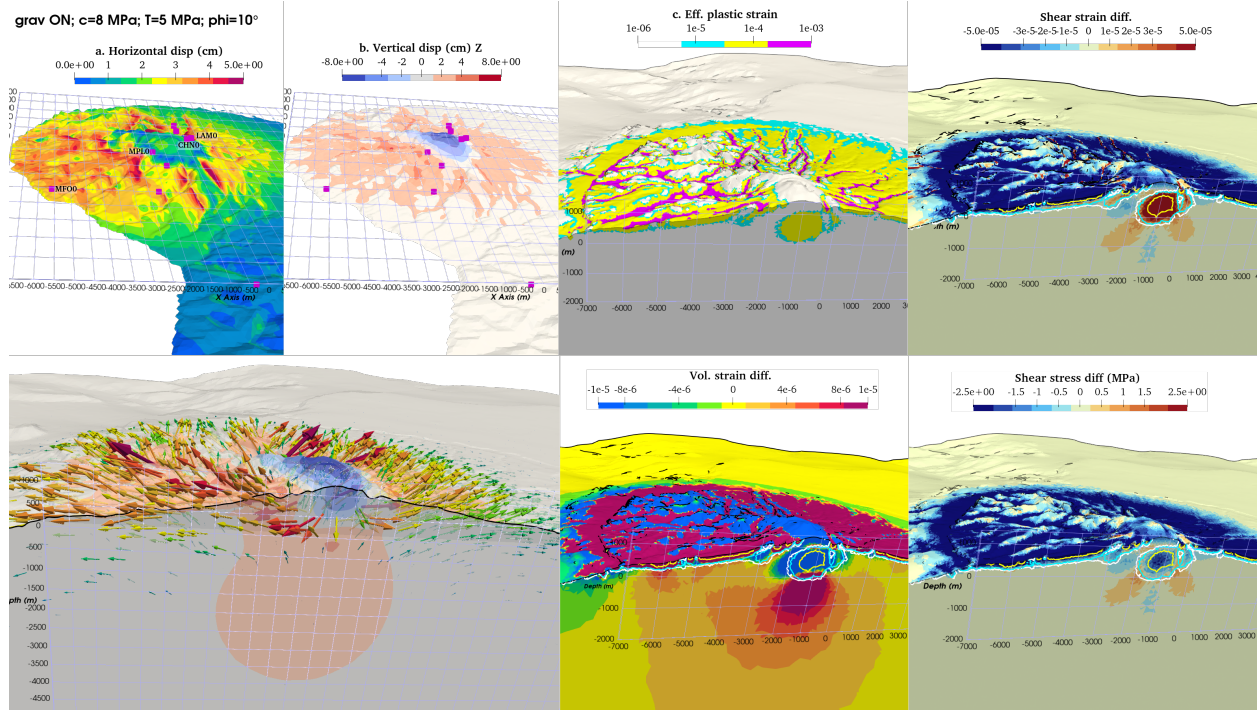
Figure B.1: Resulting figures for all preliminary elasto-plastic models in Table 2. Each figure is built as follows: (Top) Map view of vertical (left) and X-axis (right) displacements; (Bottom) Effective plastic strain.

C Results for an inflating compliant source without gravity

Surface deformation measured above inflating magma chambers is usually compared with an analytical solution of an expanding source in an elastic half-space. Mogi (1958) determined an



(a) Model 2 *strong* effective strength ($\phi=10^\circ$, $c=8$ MPa, $T=5$ MPa)



(b) Model 3 *weak* effective strength ($\phi=10^\circ$, $c=8$ MPa, $T=5$ MPa)

Figure B.2: Extended results for (a) Model 2 and (b) Model 3.

analytical solution relating 3D surface deformation with a point-dilation source :

$$u_{Mogi}^z = \frac{(1 - \nu)a^3 \Delta P}{G} \frac{d}{R^3}, \quad (1)$$

where ν is the Poisson's ratio and G the shear modulus, equal to 0.27 and 15.6 GPa respectively in our case study. a is the radius of the pressurized sphere and R , the distance to the source with $R = \sqrt{x^2 + d^2}$ where x is the horizontal distance to the source and d its depth. Mogi's analytical solution is valid for $a \ll d$, hence [McTigue \(1987\)](#) developed a correction factor (Eq. 2) to gain accuracy for sources close to the surface:

$$u_{McTigue}^z = u_{Mogi}^z \left[1 + \left(\frac{a}{d} \right)^3 \left(\frac{1 + \nu}{2(-7 + 5\nu)} + \frac{15d^2(-2 + \nu)}{4R^2(-7 + 5\nu)} \right) \right]. \quad (2)$$

The distance between the top of our hypothesized ellipsoid and the surface is $z=900$ m. We assume $a=135$ m, therefore $d=1035$ m. The overpressure (ΔP) is set to 10 MPa.

We show the results of an elastic case with the Mount Pelée topography, Fig. [C.1](#); considering the topography instead of a flat surface adds complexity to the comparison of vertical displacements with the analytical solutions (i.e. [Mogi, 1958](#); [McTigue, 1987](#)), producing two peaks of vertical displacement above the shallow inflating source. The vertical displacement can be roughly described by a linear behavior of ca. 1 mm per 10 MPa of inflation.

Then we account for an elasto-plastic crust with the following parameters: $\varphi=5^\circ$, $c=5$ MPa, $T=3$ MPa. We choose to use a crustal strength weaker than the one of our Model 3 (*Section 4.3*), in order to exaggerate the difference between a purely elastic model and an extremely (non-realistic) weak crust. In Fig. [C.1](#), the difference between Mogi's and McTigue's analytical solutions are similar, as well as between our elastic and elasto-plastic tests with 10 MPa overpressure. We observe a difference of 5% in between both behaviors when applying an overpressure of 20 MPa.

These tests show that the influence of elasto-plasticity becomes minor with respect to elasticity when gravity is not accounted for.

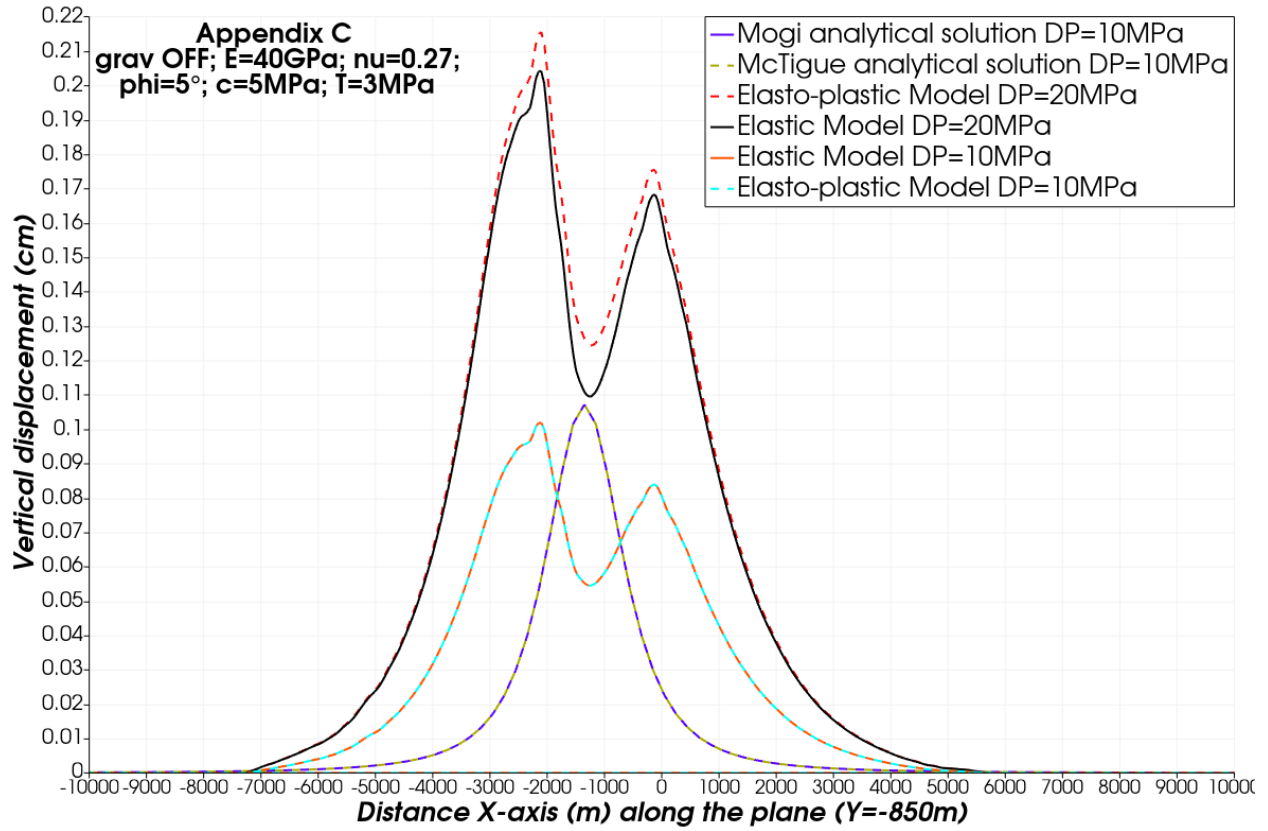


Figure C.1: Vertical displacements between analytical solutions (Mogi and McTigue), elastic and elasto-plastic Models for an overpressure of 10 and 20 MPa.

D Influence of a compliant source embedded in a gravity field

D.1 Young modulus of the source

Here we show the influence of a compliant source in the gravity field for a purely elastic model. The vertical displacement differences (with respect to Model 1) induced by test models in which the compliant source has a Young modulus 100 times and 1000 times weaker than the bedrock, respectively ($E_{s1} = 0.40$ GPa and $E_{s2}=0.04$ GPa, respectively) are shown in Figure D.1.1 and D.1.2.

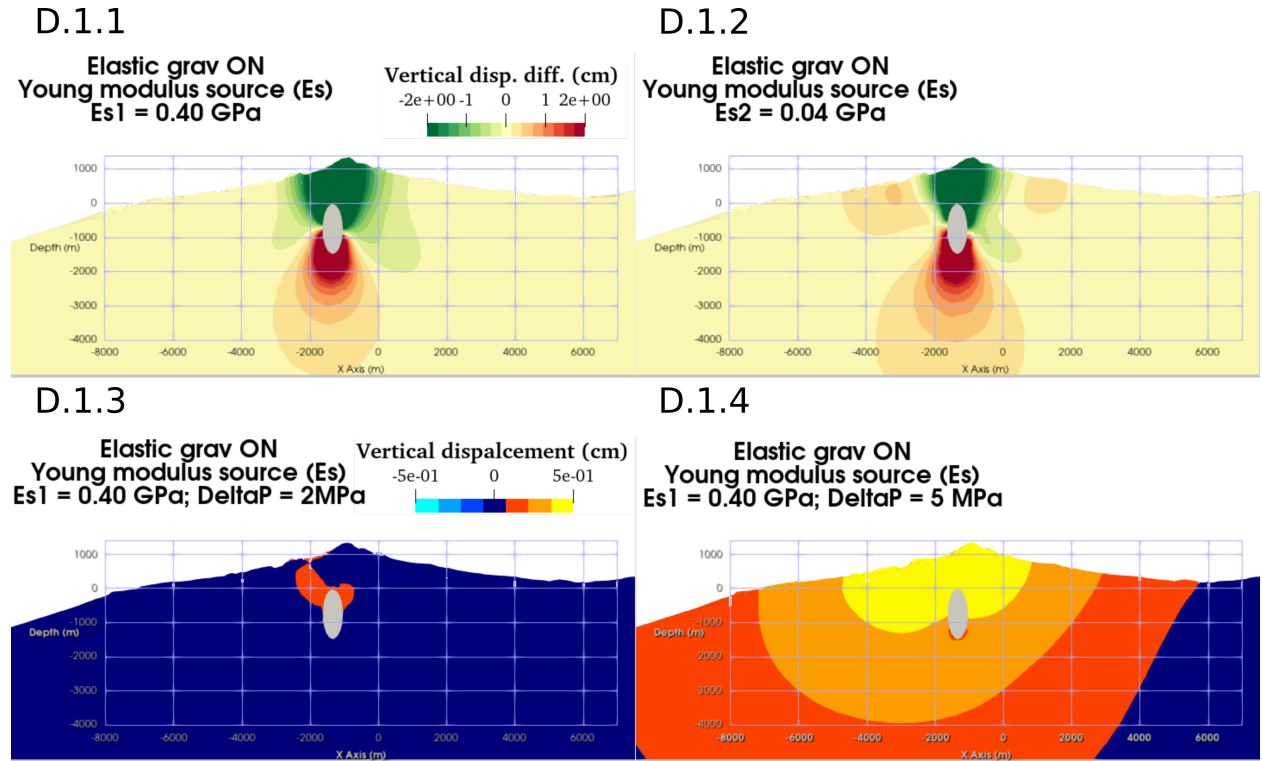


Figure D.1: Elastic model in the gravity field, displaying differences between $E_{s2}=0.04$ GPa and $E_{s1}=0.40$ GPa, with respect to the mechanical state in Model 1. a-b-c) Map view of Horizontal, X-axis and vertical displacements. e-f-g) cross-sections at $Y=-850$ m of shear strain, X-axis and vertical displacement.

Considering a weaker Young modulus for the source domain than in the surrounding bedrock generates subsidence. When comparing the surface vertical displacement produced with E_{s1} and

with E_{s2} , we can see that E_{s2} subsides 1 cm more than E_{s1} , above the source. Also, the presence of a weaker Young modulus generates a surface uplift of ca. 5 mm at 2 km laterally away from the source (Fig. D.1.2).

D.2 Complementary tests for an overpressurized compliant source

In this section, we test the effect of an inflating compliant source ($E_{s1} = 0.40$ GPa) and explore the influence of the overpressure as a tool to counteract the subsidence induced by the compliant source. Finding such equilibrium allows us to establish a *new mechanical equilibrium* for Model 4 (Section 5).

Figures D.1.3 and D.1.4 show the vertical displacement response to an overpressure (ΔP) of 2 MPa and 5 MPa, respectively. When considering $\Delta P=2$ MPa, the vertical response around the source is almost null, with an uplift component of less than 1 mm. In contrast, when applying $\Delta P=5$ MPa, the uplift is significantly larger, reaching 6 mm above the source. Hence hereafter in the main text, the term *mechanical equilibrium* used to consider the existence of a compliant shallow source is the result of a purely elastic model with an applied overpressure equal to $\Delta P=2$ MPa applied from the source's walls.

References

- McTigue, D., 1987: Elastic stress and deformation near a finite spherical magma body: resolution of the point source paradox. *Journal of Geophysical Research: Solid Earth*, **92 (B12)**, 12 931–12 940, doi:10.1029/JB092iB12p12931.
- Mogi, K., 1958: Relations between the Eruptions of Various Volcanoes and the Deformations of the Ground Surfaces around them. *Bulletin of the Earthquake Research Institute*, **36**, 99–134.