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Observation of Parametric Resonance Phenomenon in a Cold Cesium Atoms

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We report the first observation of motional excitation induced by parametric resonance of cold ^{133}Cs atoms in a dual-species magneto-optical trap (MOT) simultaneously trapping ^{133}Cs and ^{85}Rb atoms. By modulating the cooling laser's intensity at approximately twice the resonant frequency of the trap, the atoms inside the trap separate into two atomic clouds, oscillating with a π phase difference due to the trap potential's nonlinearity, resulting in a finite oscillation length. This behavior reflects general nonlinear dynamics and is described as a Hopf bifurcation or limit cycle motion. The measured oscillation amplitude was compared with theoretical calculations derived from simple Doppler cooling theory, changing modulation amplitude, magnetic field gradient, and saturation parameter. In addition, we investigate the noise-induced switching dynamics by measuring the temporal evolution of atomic populations localized in each of the two attractors under resonant symmetry lifting induced by an effective bias field. Notably, this dual-species MOT system is expected to provide a new platform for studying nonlinear and nonequilibrium dynamics between cold rubidium and cesium atomic ensembles.

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I. INTRODUCTION

The study of collective phase transitions has long been a central theme in statistical physics, with the Ising model serving as one of the most fundamental paradigms for understanding symmetry breaking and critical phenomena. In particular, the exact solution of the two-dimensional Ising model by Onsager [1] established a cornerstone for equilibrium critical phenomena, revealing universal behavior governed by short-range interactions and strong fluctuations. The exact solvability and rich structure of the model have been further elaborated in subsequent works [2]. In addition, the duality relation introduced by Kramers and Wannier provides an exact determination of the critical temperature [3].

Beyond single systems, considerable efforts have been devoted to coupled Ising systems, where interactions between multiple subsystems give rise to richer collective behavior. Early studies of bilayer and multilayer Ising models demonstrated that interlayer coupling modifies the critical temperature while preserving universality [4]. Subsequent works showed that competing intra- and interlayer interactions can induce tricritical points and complex phase diagrams [5–7]. More recent studies, including decorated and frustrated bilayer Ising models, have further revealed that coupling between layers can generate new phase boundaries and emergent ordering phenomena [8, 9]. These results establish that coupling between otherwise well-understood critical systems provides a powerful route to uncover emergent collective behavior.

In parallel, cold-atom systems have emerged as a versatile platform for realizing controlled nonequilibrium phase transitions. In particular, periodically modulated magneto-optical traps have enabled the realization of ideal mean-field symmetry-breaking transitions in ensembles of cold atoms [10, 11]. In these systems, atoms behave as nonlinear oscillators under parametric driving, exhibiting bistability and spontaneous symmetry breaking between two dynamical states. The theoretical framework for such phenomena has been developed in the context of nonequilibrium statistical mechanics and large fluctuations [12, 13].

Previous studies have successfully demonstrated ideal mean-field transitions in single-species cold-atom systems, such as rubidium-based ensembles, the extension to interacting multi-system configurations remains largely unexplored. From the perspective of statistical physics, this problem is closely analogous to that of coupled Ising models, where the interplay between intra-system ordering and inter-system coupling leads to emergent phenomena including modified critical temperatures, multicriticality, and hybridized collective modes [17]. Realizing such a system experimentally requires the independent implementation of multiple mean-field critical systems and the ability to introduce controlled interactions between them.

As a crucial step toward this goal, it is necessary to establish a second, independent platform exhibiting an ideal mean-field transition. In this context, cesium atoms provide a promising candidate due to their favorable atomic structure and suitability for cold-atom manipulation [18]. In particular, realizing a parametric resonance

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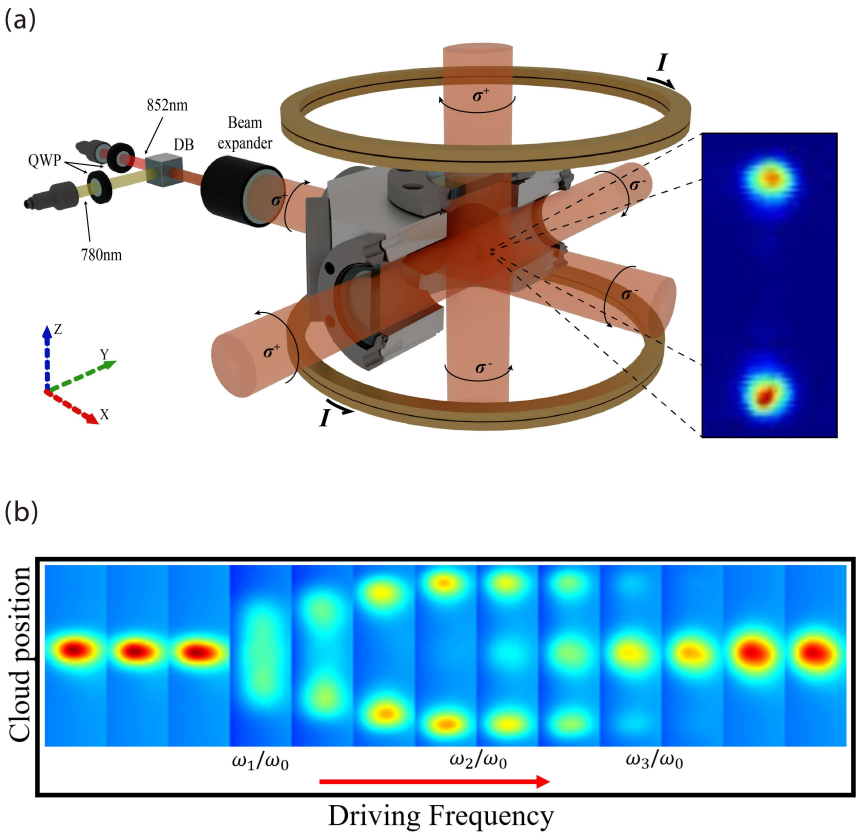


FIG. 1: (a) Schematic of ^{133}Cs–^{85}Rb dual-species magneto-optical trap apparatus. QWP, quarter-wave plate; DB, Dichroic beam splitter. For clarity, most optical components are omitted. (b) Typical motional amplitude of atomic cloud versus a driving frequency: The captured atomic images are taken at specific timings when the atoms reach their maximum amplitude, using an EMCCD. ω_1 and ω_2 are called a super-critical Hopf bifurcation and a sub-critical Hopf bifurcation, respectively.

system based on cold cesium atoms enables the formation of a nonlinear dynamical system analogous to previously demonstrated rubidium-based systems.

In this work, we report the realization of a parametrically driven nonlinear dynamical system using cold cesium atoms and investigate its fundamental properties. By implementing periodic modulation in a magneto-optical trap, we induce parametric resonance and characterize the resulting dynamical states, including bistability, noise-induced switching. Through detailed analysis, we demonstrate that the cesium-based system exhibits the essential features required for an ideal mean-field transition system.

In this paper, we briefly describe a dual-species MOT system as a platform that enables the study of nonlinear and nonequilibrium dynamics between cold rubidium and cesium atomic ensembles in Sec. II. In Section III, we present a study of the nonlinear dynamical properties of periodically modulated cold atoms composed of magneto-optically trapped ^{133}Cs atoms. The measured oscillatory motion is compared with theoretical predictions based on a one-dimensional Doppler cooling model, under variations of the modulation amplitude, magnetic field gradient, and saturation parameter, showing good agreement with the theoretical framework. Furthermore, we investigate the noise-induced switching dynamics by measuring the temporal evolution of atomic populations localized in each of the two attractors under resonant symmetry lifting induced by an effective bias field. The final section briefly summarizes the results.

II. EXPERIMENT SETUP

A ^{133}Cs–^{85}Rb dual-species MOT apparatus comprises an ultra-high vacuum stainless chamber, magnetic fields, and 780nm and 852nm cooling and repumping laser beams. For both the two atomic species, the intensities of the cooling lasers propagating in opposite directions along the z-axis are independently controlled by acousto-optic modulators (AOMs). We realize a parametric resonance of ^{133}Cs cold atoms by modulating intensities of the cooling lasers counter-propagating along the z-axis (anti-Helmholtz coil axis) as shown in Fig. 1(a) [11]. The cooling laser intensities along the z-axis and xy-axis are 0.271 mW/cm^2 and 1.353 mW/cm^2, respectively, and laser detunings δ_z, δ_x, and δ_y are -2.872 Γ, -3.963 Γ, and -3.887 Γ, respectively. To minimize the sub-Doppler cool-

ing effect [19, 20], the detunings along each axis were set differently. The magnetic field gradient b is 0.1137 T/m, and the repumping laser intensity is 0.406 mW/cm^2. The typically measured trap frequency and damping coefficient of the trapped atoms are 40.04 Hz and 50.08 s^{-1}.

The laser intensity is periodically modulated by the acousto-optic modulator and the modulation amplitude h is also controlled by it. As the modulation frequency ω of the laser intensity is around twice trap frequency ω_0, the external motion of ^{133}Cs cold atoms is excited and separated into two atomic clouds which are periodically oscillating with exactly opposite phases, as shown in Fig. 1(b). The movement of the atoms in Fig. 1(b) was captured with the EMCCD (Photometrics, Cascade 128+) at the time when the amplitude of the periodic atomic motion reached its maximum, while varying the modulation frequency. In the range of $\omega < \omega_1$, there is only one stable attractor at the origin, but as the modulation frequency increases, the limit cycle motion occurs at the frequency $\omega = \omega_1$, which is called a supercritical Hopf bifurcation. On the contrary, in the range of $\omega_2 < \omega < \omega_3$, as we decrease the modulation frequency, the union of the limit cycle motions and a stable attractor at the origin becomes only the limit cycle motions without the stable attractor at the origin at the frequency $\omega = \omega_2$, which is called a sub-critical Hopf bifurcation [21–23].

In general, in the single-particle system, the hysteresis phenomena is clearly observed as the modulation frequency increases across $\omega = \omega_2$ and decreases across $\omega = \omega_3$. Namely, if the thermal noise due to the random recoil of spontaneously emitted photon can be neglected, atoms in the limit cycle motions mostly remains in the same state as the frequency increases, and atoms in a stable attractors also stays in the same state as the frequency decreases. However, because the noise cannot be neglected in practice, a noise-induced transition effect is observed in which a single particle undergoes transitions between limit cycle states and a stable attractors over time. This allows us to calculate the probability distribution for each state based on these transitions. But, in our many-body system, instead of analyzing the time-dependent transitions of a single particle, we can directly measure the probability distribution in each state by counting the number of atoms distributed in each state in a single measurement, as it is an ensemble system. Thus, Figure 1(b) illustrates the atoms distributed in each state due to thermal fluctuations and demonstrates an example of the hysteresis phenomenon in a many-body system.

III. DISCUSSION AND RESULTS

A. Analysis of limit cycle motions

We experimentally investigate the nonlinear dynamical behavior of parametrically resonant cold cesium atoms. Since a detailed quantitative comparison between experimental results and theoretical analysis has already been carried out for parametrically resonant cold rubidium atoms [24, 26], such quantitative analysis is not included here. We briefly review the theoretical description of cold atomic paramatric resonance in a magento-optical trap.

The nonlinear equation of motion of ^{133}Cs atoms along the z-axis is described by the simple Doppler theory of a magneto-optical trap, which is expanded to the third order [26] and given by

$$\ddot{z} + \gamma\dot{z} + \omega_0^2(1 + h\cos\omega t)z + B_0(z + \eta\dot{z})^3 = 0, \qquad (1)$$

where $\eta = \hbar k/\mu_B b$, and ω_0, γ, h, and B_0 are the trap frequency, damping coefficient, modulation amplitude, and nonlinear coefficient, respectively. ω_0, γ, and B_0 are given by

$$\begin{aligned} \omega_0 &= \sqrt{\frac{8k\mu_B b s_0(-\delta_z/\Gamma)}{m_a(1+4(\delta_z/\Gamma)^2)^2}}, \\ \gamma &= \frac{\hbar k}{\mu_B b}\omega_0^2, \\ B_0 &= \frac{8\mu_B^2 b^2(4(\delta_z/\Gamma)^2-1)}{\hbar^2\Gamma^2(4(\delta_z/\Gamma)^2+1)^2}\omega_0^2. \end{aligned} \qquad (2)$$

where k is the wave vector, μ_B is the Bohr magneton, m is the mass of an atom, Γ is the decay rate of the excited state of ^{133}Cs atom.

Following Eq. (2), under identical conditions of δ_z/Γ, b, and s_0, the ratio of the trap frequencies of ^{133}Cs and ^{85}Rb atoms is found to be $\omega_0/\omega_r \approx 0.77$, which ^{85}Rb trap frequency ω_r. As such, the measured results fall within a range comparable to the previously reported trap frequencies of ^{85}Rb, indicating that the trap parameters of the two atomic species can be precisely tuned within a similar regime by controlling the laser detuning, intensity, and magnetic field gradient. In addition, the damping coefficient is given by $\gamma = (\hbar k/\mu_B b)\omega_0^2$, and under the same conditions, the ratio of the damping coefficients of ^{133}Cs to ^{85}Rb is $\gamma/\gamma_r \approx 0.54$, which ^{85}Rb damping coefficient γ_r. If the experimental conditions are adjusted such that the trap frequencies of the two atomic species are identical, the ratio becomes $\gamma/\gamma_r (= k/k_r) \approx 0.92$, which ^{85}Rb wave vector k_r, depending only on the ratio of the wave vectors of the two atoms. Furthermore, the ratio of the nonlinear coefficients of ^{133}Cs to ^{85}Rb is $B_0/B_r (= k\Gamma_r/k_r\Gamma) \approx 1.039$, which ^{85}Rb nonlinear coefficient B_r and decay rate of the excited state Γ_r. This indicates that the key parameters governing the system's dynamics can be flexibly tuned, either matched or deliberately differentiated, thereby providing a platform to explore interactions between ideal mean-field systems realized in two independent nonequilibrium atomic ensembles composed of different species.

The frequencies ω_1, ω_2, and ω_3 derived from the Eq. (1)

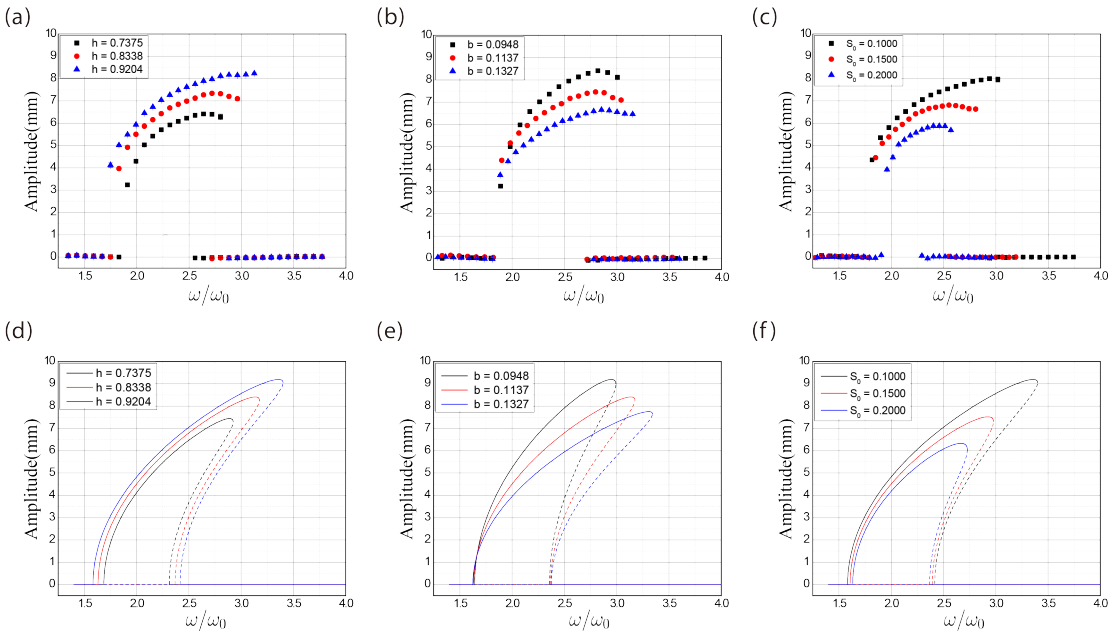


FIG. 2: Atomic motional amplitude versus (a,d) modulation amplitude h, (b,e) magnetic field gradient b, and (c,f) saturation parameter s_0: Panels (a), (b), and (c) show the experimental results, and the corresponding experimental conditions are given by $(s_0, b) = (0.1000, 0.1137\ \mathrm{T/m})$, $(h, s_0) = (0.8338, 0.1000)$, and $(h, b) = (0.9204, 0.1137\ \mathrm{T/m})$, respectively. The theoretical results shown in panels (d), (e), and (f) were obtained under the same corresponding experimental conditions. The stable (unstable) solutions of the Eq. (1) are represented as solid (dotted) lines in (d), (e), and (f).

are theoretically given by

$$\omega_1 = 2\omega_0 - \frac{\omega_0}{2}\sqrt{h^2 - h_T^2}, \tag{3}$$

$$\omega_2 = 2\omega_0 + \frac{\omega_0}{2}\sqrt{h^2 - h_T^2}, \tag{4}$$

$$\omega_3 = \omega_0\left(1 + \frac{h}{2h_T}\sqrt{4 + h_T^2}\right), \tag{5}$$

where $h_T = 2\gamma/\omega_0$, which is a threshold value of the modulation amplitude above which the parametric resonance can occur. The nontrivial steady state solutions of the Eq. (1) for the motional amplitude R is given by

$$R^2 = \frac{16\omega_0^2}{3B_0(4 + h_T^2)^2} \times \left(4(\zeta - 2) - h_T^2 \pm \sqrt{h^2(4 + h_T^2) - 4h_T^2(\zeta - 1)^2}\right), \tag{6}$$

where $\zeta = \omega/\omega_0$ [25]. The upper (lower sign) denotes the stable (unstable) solutions, where are represented as solid (dotted) lines in Figs. 2 (d), (e), and (f).

To analyze experimentally the Hopf bifurcation phenomenon of parametric resonance of ^{133}Cs cold atoms in Fig. 1(b), we measured the changes in the amplitude of limit cycle motions with variations in modulation frequency ω, by adjusting the modulation amplitude h, magnetic field gradient b, and saturation parameter s_0 as shown in Fig. 2. Figure 2 shows the experimental results of the amplitude dependence on modulation frequency, measured by varying (a) modulation amplitude, (b) magnetic field gradient, and (c) saturation parameter. Figure 2(d), (e), and (f) represent the theoretical results corresponding to Fig. 2(a), (b), and (c), respectively. The experimental results are qualitatively consistent with the theoretical results and show that the observed motional amplitude versus ω/ω_0 exhibits behavior similar to that previously reported for ^{85}Rb [25, 26]. The experimental results confirm that the two atomic species exhibit closely analogous nonlinear dynamical behavior.

B. Noise-induced switching dynamics

Figure 3(a) shows the image sequence during the noise-induced switching between the two periodic states S_1 and S_2 in the modulation frequency range of $\omega_1 < \omega < \omega_2$. Switching between periodic states S_n $(n = 1, 2)$ typically necessitates a significant fluctuation and overcoming an effective activation barrier [29]. The switching rate $W_{nm} (n \neq m, n = m = 1, 2)$ for the noise-induced transition from the n state to the m state is sensitive exponentially and dependent of the activation energy R_n of the n state and thermal temperature T such as $W_{nm} = C\exp(-R_n/k_BT)$.

In the absence of an additional external perturbation, the noise-induced switching rates between the two states, W_{12} and W_{21}, become identical in this modulation frequency regime, resulting in an equal number of atoms occupying the steady states S_1 and S_2. However, when an effective DC bias field is applied from the S_1 state

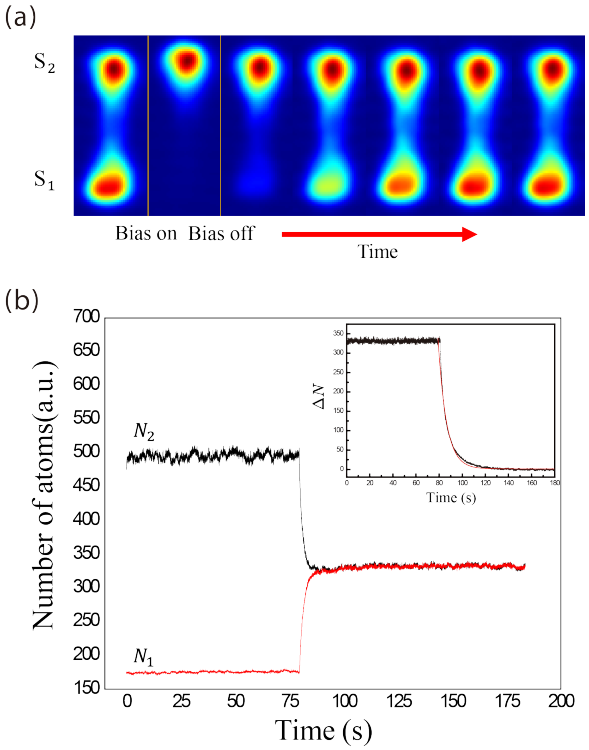


FIG. 3: Initialization for noise-induced switching dynamics: the typical experimental conditions are given by $(h, s_0, b) = (0.8338, 0.1000, 0.1137\ \mathrm{T/m})$; (a) Preparation of atomic population into a given stable state S_2; the image sequence for the transition, (b) Typical time evolution of the switching populations in each state S_1 and S_2 after Bias-off; the transition rate W is obtained from a fitting result (red line) with the population difference $\Delta N(= N_2 - N_1) = N_0 \exp(-2Wt)$ as shown in the inset.

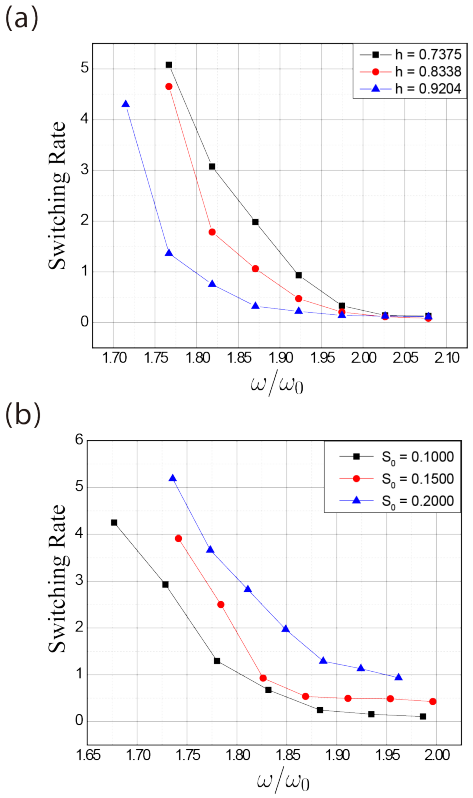


FIG. 4: Dependence of switching rate W on the driving frequency ω under different conditions of (a) modulation amplitude h and (b) saturation parameter s_0: The experimental conditions for panels (a) and (b) are $(s_0, b) = (0.1000, 0.1137\ \mathrm{T/m})$ and $(h, b) = (0.9204, 0.1137\ \mathrm{T/m})$, respectively.

toward the S_2 state, as shown in the Fig. 3(a), W_{12} becomes larger than W_{21}, allowing more atoms to be prepared in the S_2 state. After this state preparation, when the bias field is turned off, W_{12} and W_{21} become identical again, and the system undergoes a relaxation process toward an equal population distribution between the S_1 and S_2 states. Therefore the effective DC bias field is applied in a single direction to ensure that all the atoms occupy only one (S_2) of the two stable states [30]. And then as soon as turning off it, we can directly measure the intercloud switching rates W by analyzing the decay time of the difference in cloud populations, $\Delta N(= N_2 - N_1) = N_0 \exp(-2Wt)$, as shown in Fig. 3(b).

As a consequence, Figure 4 represents the dependence of switching rate W on the driving frequency ω under different conditions of modulation amplitude h and saturation parameter s_0. From the experimental results, it is observed that the switching rate decreases as the modulation amplitude increases (Fig. 4(a)), whereas it increases with increasing saturation parameter (Fig. 4(b)). It can be confirmed that the observed switching rate regime is similar to the range (0.1 to 5 s^{-1}) previously reported for ^{85}Rb atoms, depending on the experimental parameters.

Overall, the experimental findings suggest that the observed limit-cycle motions and noise-induced switching dynamics in the parametrically driven ^{133}Cs system exhibit strong similarities to the nonlinear behaviors previously reported in ^{85}Rb atoms, where signatures of ideal mean-field transitions were demonstrated. These results indicate that the nonequilibrium cesium-atom system also possesses the essential characteristics required for realizing ideal mean-field transitions.

IV. CONCLUSIONS

We report the first observation of parametric resonance in magneto-optically trapped ^{133}Cs atoms, including the emergence of limit-cycle motion and Hopf bifurcation behavior. By systematically varying the modulation frequency, modulation amplitude, and magnetic-field gradient, the characteristics of the limit-cycle oscillations were experimentally investigated through measurements of their oscillation amplitudes. The observed results qualitatively agree well with theoretical calculations based on simple Doppler cooling theory for a two-level atomic system. In addition, the noise-induced

switching dynamics were studied by monitoring the temporal evolution of atomic populations localized in the two attractors under resonant symmetry breaking induced by an effective bias field. The experimental findings further reveal that ^{133}Cs atoms exhibit nonlinear dynamical behavior remarkably similar to that previously observed in magneto-optically trapped ^{85}Rb atoms.

This work presents a variety of promising research opportunities enabled by a dual-MOT system in which rubidium and cesium atoms are simultaneously trapped and independently manipulated within the same experimental platform. In particular, the coexistence of two distinct cold-atom ensembles provides a unique environment for investigating interaction effects arising from collisions and collective coupling between nonlinear oscillators composed of different atomic species. Since each trapped atomic ensemble can exhibit nonlinear dynamical behavior under periodic modulation, the dual-MOT configuration offers an attractive platform for studying coupled nonequilibrium systems, synchronization phenomena, symmetry-breaking processes, and noise-induced collective dynamics in multicomponent cold-atom systems. Future work will focus on experimentally and theoretically investigating these interaction mechanisms in greater detail, with particular emphasis on interspecies coupling effects, collective nonlinear oscillations, noise-induced switching dynamics, and emergent nonequilibrium behaviors in coupled dual-MOT systems.

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