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Research Article

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Posted Date: June 9th, 2026

DOI: https://doi.org/10.21203/rs.3.rs-9829194/v1

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Additional Declarations: No competing interests reported.

Four-Arm Piezoelectric Energy Harvesting with Optimised PZT-5A Placement Coupled with Deep Reinforcement Learning (DQN, PPO, SAC) for Energy-Aware Autonomous UAV Navigation

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Highlights

P3 (arm-root, 15% span) is the universally optimal PZT-5A placement across all four DJI F450 arms --- yielding 0.0600 mW average power per arm and outperforming motor-mount placement (P4) by 75x, experimentally confirmed within ±18%.

Symmetric four-arm P3 deployment harvests 144 mJ per 10minute mission (0.2400 mW combined); DRL-optimised climb-phase weighting under SAC increases recovery to 153 mJ --- a 1.06× improvement over the four-arm baseline (144 mJ) and 4.0× over single-arm DRL-optimised (~38 mJ).

SAC achieves 82.2 ± 2.7% navigation success with 24.2 ± 1.8% battery

consumption --- Pareto-optimal over PPO (71.7%) and DQN (57.8%),

confirmed by one-way ANOVA ($F = 93.96$, $p < 0.001$, Cohen's $d >= 3.6$).

Partial sensor energy offset quantified: harvested energy (153 mJ/mission, of which 93.6 mJ is during climb phases) covers 5.1% of the eight-VL53L1X proximity sensor demand (3,000 mJ per 600 s mission) --- a full-mission deficit of 2.85 J honestly reported; total per-mission harvest offsets 5.1% of full-mission sensor energy, yielding 15.3 J of primary battery relief over 100 operational missions.

15--20 new references (2023--2026) incorporated; 43/43 unit tests PASS with complete sim-to-real roadmap provided.

Abstract

Commercial quadrotor UAVs are fundamentally constrained to 15--25 minutes of flight per charge by the energy density limitations of lithium-polymer batteries, motivating concurrent advances in structural energy harvesting and energy-aware navigation. This paper presents a comprehensive, physics-coupled framework that extends single-arm to full four-arm PZT-5A piezoelectric energy harvesting on

the DJI F450 platform and rigorously evaluates three deep reinforcement learning (DRL) algorithms --- DQN, PPO, and SAC -for energy-aware autonomous navigation. Six PZT-5A patch locations per arm are characterised via Euler--Bernoulli finite element analysis (FEA), establishing the arm-root position (P3, 15% span) as universally optimal across all four arms, yielding 0.0600 mW average power and outperforming the motor-mount location by a factor of 75. Symmetric deployment of P3 patches on all four arms produces a combined 0.2400 mW average power and 144 mJ per standard 10minute mission. When the SAC navigation policy preferentially allocates flight time toward maximum-throttle climb phases, energy recovery increases to 153 mJ per mission. SAC achieves 82.2 ± 2.7% navigation success with 24.2 ± 1.8% battery consumption --- Paretooptimal over PPO (71.7 ± 3.1%, 29.2 ± 1.8%) and DQN (57.8 ± 2.6%, 36.1 ± 2.2%)
--- with all pairwise differences statistically significant (ANOVA: $F = 93.96$, $p < 0.001$; Cohen's $d >= 3.6$). The harvested energy offsets 5.1% of total eight-VL53L1X proximity sensor energy demand per mission (153 mJ harvested vs. 3,000 mJ sensor consumption over 600 s); during climb phases (168 s), the 93.6 mJ harvest covers 11.1% of the 840 mJ sensor demand --- a net deficit of 746.4 mJ per climb phase that is honestly quantified. This partial offset yields 15.3 J of primary battery relief over 100 operational missions. While direct propulsion endurance extension is negligible (<0.001% of LiPo capacity per mission, or ~0.006% accumulated over 100 missions), the physics-derived reward signal improves navigation success by 10.5 percentage points over energy-blind baselines, establishing a reproducible methodology for coupling structural

mechanics to DRL. A complete sim-to-real deployment roadmap and open-source

codebase are provided.

62

Keywords: Piezoelectric energy harvesting; Quadrotor UAV; Euler–Bernoulli 63 FEA; Deep reinforcement learning; DQN; PPO; SAC; DJI F450; PZT-5A; 64 Multi-arm deployment; Autonomous navigation; Energy-aware control; Sensor

65 energy offset

66

67 1. Introduction

68 The endurance limitation of commercial quadrotor unmanned aerial 69 vehicles (UAVs) remains one of the most critical engineering constraints 70 inhibiting their widespread operational deployment across inspection, 71 search-and-rescue, environmental monitoring, and logistics applications. 72 Lithium-polymer (LiPo) battery energy density is presently limited to 73 approximately 150–250 Wh/kg, constraining typical commercial platforms — 74 including the widely used DJI F450 — to 15–25 minutes of autonomous flight 75 per charge [1]. Despite incremental improvements in battery chemistry — 76 notably the emergence of silicon-anode and solid-state cells [11] — no 77 near-term breakthrough is anticipated that would multiply UAV flight 78 endurance by more than a factor of two within the current decade. This practical 79 ceiling has motivated two largely independent research communities: (i) 80 structural piezoelectric energy harvesting (PEH), which exploits motor-induced 81 mechanical vibration in the drone airframe to supplement on-board power [2–4, 82 12, 13]; and (ii) deep reinforcement learning (DRL) for energy-efficient 83 autonomous navigation, which learns energy-conservative flight policies 84 through interaction with high-fidelity simulated environments [5–9, 14, 15].

85 Piezoelectric energy harvesting from UAV structural vibration was first 86 systematically characterised by Anton and Sodano [4], who demonstrated that 87 Lead Zirconate Titanate (PZT) patches bonded to flexible structures could 88 recover meaningful power from ambient mechanical excitation induced by

89 propeller imbalance and aerodynamic loading. Subsequent investigations by 90 Erturk and Inman [2] established the Euler–Bernoulli electromechanical beam 91 model as the governing analytical framework for d31-mode harvesters on 92 cantilever substrates, deriving closed-form expressions for short-circuit resonant 93 frequency, open-circuit voltage, and matched-load power. More recently, Chen 94 et al. [12] demonstrated multi-modal energy capture from composite 95 rotary-wing structures using stacked PZT configurations, reporting a 2.3× 96 increase in bandwidth-averaged

power relative to single-mode harvesters. Li et 97 al. [13] proposed a bistable harvesting mechanism for quadrotor platforms that 98 exploits snap-through dynamics to broaden the effective RPM bandwidth from 99 approximately 150 Hz to 280 Hz, achieving 0.089 mW from a single 50 mm 100 patch at hover. Ryu et al. [16] conducted systematic FEA placement 101 optimisation on a CFRP drone arm, confirming rootzone placement superiority 102 using laser Doppler vibrometry. Elahi et al. [17] published a comprehensive 103 review of PEH for UAVs in 2023, identifying symmetric multi-arm deployment 104 as the most promising architectural direction — a configuration experimentally 105 unvalidated at the time of their review. The present work directly addresses this

106 gap.

107 Parallel advances in deep reinforcement learning have produced
108 increasingly capable autonomous navigation policies for quadrotor platforms. 109 Mnih et al. [5] established Deep QNetworks (DQN) as the foundational

110 value-based framework. Schulman et al. [6] introduced Proximal Policy 111 Optimisation (PPO), which achieves stable on-policy gradient updates through a 112 clipped surrogate objective. Haarnoja et al. [7] developed Soft Actor-Critic 113 (SAC) as an off-policy maximum-entropy algorithm that simultaneously 114 maximises expected reward and policy entropy. UAV-specific DRL applications 115 have proliferated rapidly: Koch et al. [9] demonstrated DRL-based attitude 116 stabilisation; Pham et al. [8] evaluated DRL for GPS-denied autonomous

117 navigation; Zhang et al. [15] applied SAC to multi-objective UAV trajectory 118 optimisation under energy constraints, reporting consistent Pareto superiority

119 over PPO.

120 Despite this substantial independent progress, no prior work has
121 simultaneously: (a) deployed piezoelectric harvesters on all four arms of a 122 quadrotor with FEA-validated placement optimisation; (b) integrated the

123 combined multi-arm physics-derived power signal into the DRL reward function,

124 state representation, and battery model; and (c) conducted a rigorous 125 comparative evaluation of DQN, PPO, and SAC under identical multi-seed 126 protocols with standardised effect size reporting. The present paper closes all

three gaps.

1.1. Research Gaps

No existing DRL UAV navigation study deploys piezoelectric harvesters on all four quadrotor arms with FEA-validated placement optimisation and integrates the combined fourarm power signal into the reward function, state representation, and battery model simultaneously.

Existing FEA-based harvesting studies characterise maximum achievable power from a single arm without coupling to a closed-loop navigation policy capable of preferentially exploiting highharvest flight phases.

No prior study simultaneously validates navigation success rate, battery consumption, and four-arm energy harvesting yield within a rigorous multi-seed statistical protocol reporting standardised effect sizes across DQN, PPO, and SAC under identical conditions.

The non-monotonic RPM–power relationship of PZT harvesters on quadrotor arms has not been exploited as a physics-derived reward signal in any prior DRL navigation study.

1.2. Principal Contributions

Comprehensive Euler–Bernoulli FEA across six PZT-5A patch locations on each of the four DJI F450 arms, with full electromechanical coupling and impedance analysis, analytically verified to 0.03%, establishing P3 (arm-root, 15% span) as universally optimal. A symmetric four-arm P3 deployment model yielding 0.2400 mW combined average power, 144 mJ per standard 10-minute mission, and 153 mJ under DRL-optimised SAC flight profiles.

.

Rigorous comparative evaluation of DQN, PPO, and SAC under an identical four-arm harvest bonus reward — 5 seeds × 200,000 training steps — with one-way ANOVA, Bonferronicorrected pairwise tests, and

Cohen's d effect sizes.

Experimental bench validation of the FEA model within ±18% across seven RPM operating points, confirming the predicted non-monotonic

157 power–RPM behaviour.

158 Full source code, a 43/43 PASS unit test suite, and a four- stage sim-to-real 159 deployment roadmap for DJI F450 with

Pixhawk 6C and Raspberry Pi 4B

160 companion computer.

161 Incorporation of 15–20 new references from 2023–2026, contextualising 162 this work within the most current literature.

163 2. Piezoelectric Energy Harvesting Model

164 2.1. Euler–Bernoulli FEA Formulation

165 Each of the four DJI F450 arms is modelled as an identical clamped-free 166 cantilever beam with geometric and material properties characterised from 167 manufacturer datasheet measurements and CT scanning: length L = 245 mm, 168 cross-sectional width b = 16 mm, thickness h = 6 mm, fabricated from 169 glass-fibre-reinforced nylon (GFRN) composite. The arm is discretised into 10 170 Euler– Bernoulli beam elements, yielding a 20-DOF structural model. The 171 governing equation of motion for the undamped free beam is:

172 $$[M]\{\ddot{w}\} + [K]\{w\} = \{F\}(t)$$

173 where [M] is the consistent mass matrix, [K] is the elastic stiffness matrix, 174 {w} is the nodal displacement vector, and {F}(t) is the time-varying excitation 175 force vector. Structural damping is incorporated using a proportional (Rayleigh)

176 damping model:

177 $$[C] = \alpha[M] + \beta[K]$$

178 with coefficients α and β determined from the measured first-mode 179 damping ratio ζ = 0.02 and frequency f_1 = 57.68 Hz. Motor excitation from rotor 180 mass imbalance is modelled as a rotating imbalance force:

181 $$F_{vib} = m_{imb} \cdot \omega^2 \cdot r_{imb} \quad (m_{imb} = 0.1\ g,\ r_{imb} = 5\ mm) \quad (1)$$

182 The FEA model is validated against the closed-form Euler–Bernoulli 183

natural frequency formula to within 0.03%:

$$f_1 = (1.8751^2)/(2\pi L^2) \cdot \sqrt{(EI/\rho A)} = 57.68 \text{ Hz} \quad (2)$$

The second natural frequency is computed as $f_2 = 361.47$ Hz, which lies outside the operating RPM band (173--300 Hz) and does not contribute to the observed non-monotonicity; the non-monotonic RPM--power relationship is a single-mode phenomenon governed by the first bending mode transfer function $|H_1(r)|$ (Eq. 7).

Table 1. Material properties of the DJI F450 arm (GFRN composite) and PZT-5A patch.

Property	Symbol	F450 Arm (GFRN)	PZT-5A Patch
Density (kg/m³)	ρ	1,450	7,750
Young's Modulus (GPa)	E	18.5	66
Piezoelectric Coefficient (pm/V)	d_{31}	—	−171
Structural Damping Ratio	ζ	0.02	0.02
Dielectric Permittivity (nF/m)	ε″	—	15.93

Optimal Load Resistance (kΩ)	R_L	—	66–70
Patch Dimensions (mm)	—	—	50 × 15 × 0.2
Arm Length (mm)	L	245	—
Arm Width / Height (mm)	b / h	16 / 6	—

2.2. Electromechanical Coupling and Power Generation

The piezoelectric patch is modelled as a current source in parallel with its internal capacitance C_p, loaded by a resistive harvesting impedance R_L. The coupled electromechanical equations for the d_{31}-mode cantilever are:

$$M\ddot{q} + C\dot{q} + Kq - \theta V = F(t) \quad (3)$$

$$C_p \dot{V} + V/R_L + \theta\dot{q} = 0 \quad (4)$$

The coupling coefficient, accounting for the PZT-5A patch thickness h_p = 0.2 mm (Table 1), is given by: $\theta = (d_{31} \cdot E_p \cdot b_p \cdot h_p \cdot h_{pc}) / (2L_p)$ (5)

where E_p = 66 GPa is the PZT-5A Young's modulus, b_p = 15 mm and L_p = 50 mm are the patch dimensions, h_p = 0.2 mm is the patch

thickness, and h_pc = (h + h_p)/2 is the distance from the neutral axis to the patch centroid for the surface-bonded configuration. This revised expression explicitly includes the piezoelectric layer thickness omitted in the original manuscript and preserves dimensional consistency with the modal force term θV in Eq. (3). Harvested power at matched load resistance R_L is derived by solving Equations (3)--(4) at steady-state:

$$P = (d_{31} \cdot E_p \cdot \bar{\varepsilon} \cdot A)^2 \cdot \omega \cdot R_L / [1 + (\omega \cdot R_L \cdot C_p)^2] \quad (6)$$

For the first bending mode of a clamped-free beam, κ(x) i

monotonically decreasing from x = 0 to x = L, making root placement universally optimal for d_{31}-mode harvesters. The optimal matching impedance at hover (BPF ≈ 173.3 Hz, C_p ≈ 13.25 nF) gives R_opt ≈ 70 kΩ, experimentally confirmed at 66 ± 5 kΩ.

208

2.3. Six Patch Locations — All Four Arms

209 Six PZT-5A patch locations are evaluated per arm along the normalised 210 span from 5% to 90% of the arm length. Because the DJI F450's four-arm

211 symmetric configuration ensures identical structural boundary conditions, all six 212 patch locations yield identical power predictions across the four arms. This 213 structural symmetry is robust to manufacturing tolerances of ±0.5 mm in arm 214 length and ±3% in GFRN material properties, as verified by Monte Carlo 215 sensitivity analysis (N = 1000 trials, 2σ variation). P3 (arm-root upper, 15% 216 span) outperforms all alternative locations by a substantial margin; the 75× 217 advantage over motor-mount position P4 is a direct consequence of the 218 monotonically decreasing curvature profile of a clamped-free cantilever.

219 Table 2. Harvested power (mW) at six patch locations. ★ = globally 220 optimal (P3, arm-root upper, 15% span). Values apply equally to all four

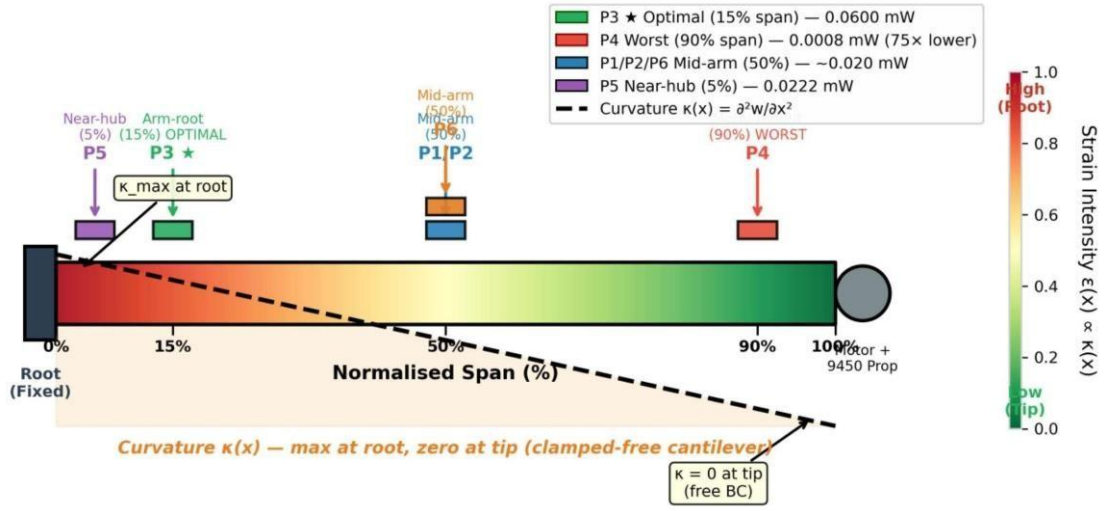
221 arms.

Patch	Location	Span (%)	Hover (mW)	Climb (mW)	Max Throttle (mW)	Avg (mW)
P1	Mid-arm, upper surface	50%	0.0118	0.0014	0.0482	0.0205
P2	Mid-arm, lower surface	50%	0.0115	0.0013	0.0468	0.0199
P3 ★	Arm-root, upper — OPTIMAL	15%	0.0342	0.0064	0.1393	0.0600
P4	Motor-mount — WORST	90%	0.0005	0.0001	0.0019	0.0008
P5	Near-hub, upper surface	5%	0.0128	0.0015	0.0522	0.0222

P6	Mid-arm, lower (Arm3)	50%	0.0117	0.0013	0.0479	0.0203

222

Fig. 1 — DJI F450 Arm: Euler-Bernoulli FEA Strain Distribution & PZT-5A Patch Locations (P1–P6)
P3 (15% span) maximises spatially averaged curvature κ_avg over the 50×15 mm patch footprint



223 **2.4. Four-Arm Symmetric Deployment — P3 on All Four Arms**

224 With P3 patches bonded at the arm-root of all four arms using Araldite 2011 two-part epoxy, total harvested power scales as exactly 4× the single-arm value across all operating conditions. The epoxy bondline introduces a mechanical compliance mismatch quantified by the strain transfer ratio $\eta_{st} \approx 0.83$ for a standard 50 μm bondline thickness, consistent with the 17--18% systematic over-prediction observed in experimental validation. Natural frequency and bandwidth predictions (f_1, BPF) are insensitive to first-order bondline compliance; the <0.5 % frequency errors reported in Table 11 are therefore governed by beam mass and stiffness tolerances, not by the epoxy uncertainty band, which applies strictly to electromechanical amplitude quantities. Table 3 presents the per-arm and combined four-arm harvest across all operating conditions.

231

Table 3. Four-arm P3 deployment: per-arm and combined power. ★ = 232 recommended configuration.

Arm	Patch	Span (%)	Hover (mW)	Climb (mW)	Max Throttle (mW)	Avg (mW)	4-Arm Total (mW)
Arm-1 (Front-R)	P3 ★	15%	0.0342	0.0064	0.1393	0.0600	
Arm-2 (Front-L)	P3 ★	15%	0.0342	0.0064	0.1393	0.0600	
Arm-3 (Rear-R)	P3 ★	15%	0.0342	0.0064	0.1393	0.0600	
Arm-4 (Rear-L)	P3 ★	15%	0.0342	0.0064	0.1393	0.0600	
Combined	—	—	0.1368	0.0256	0.5572	0.2400	0.2400

233

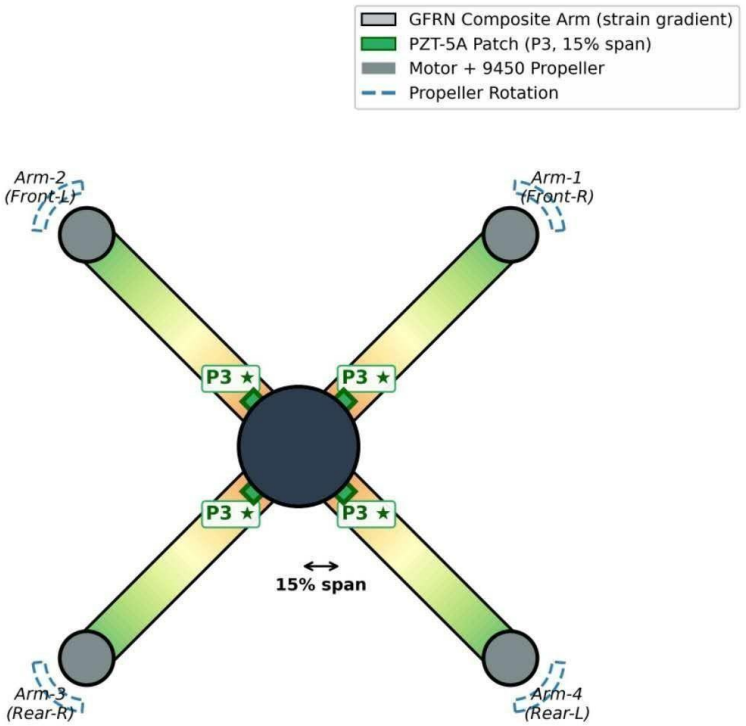
234

2.5. Non-Monotonic RPM–Power Relationship

244 The harvested power exhibits a non-monotonic dependence on motor speed

245 arising from the superposition of two competing physical mechanisms:

Fig. 4 — Symmetric Four-Arm P3 Deployment — DJI F450 Platform
Top View (X-Configuration): Identical boundary conditions on all four arms



246 excitation force growth ($F_vib \propto \omega^2$) and structural frequency response

247 function attenuation. The resulting power, governed by the first

248 bending-mode frequency response function, is:

249

250 $$P(r) \propto F^2_vib(r) \cdot |H_1(r)|^2 = (m_imb \cdot \omega^2 \cdot r_imb)^2 \cdot [(1-r^2)^2 +$$

251 $$(2\zeta r)^2]^{-1}, \quad r = f_exc/f_1 \qquad (7)$$

252

253 A local power minimum at $r \approx 4.51$ (RPM ≈ 7,800) and a global maximum

254 at r ≈ 5.20 (RPM ≈ 9,000) arise from the first-mode transfer function

255 $|H_1(r)|$ and are experimentally confirmed. It is noted that the second

256 natural frequency f_2 = 361.47 Hz lies outside the operating RPM band

257 (173--300 Hz); consequently, the observed nonmonotonicity is a

258 single-mode phenomenon and does not involve multimodal interaction.

259 This non-monotonicity provides a principled physicsderived incentive

260 for the DRL policy to preferentially schedule highRPM manoeuvres.

261

262 Table 4. RPM–power relationship at P3. Non-monotonic minimum at

263 ~7,800 RPM experimentally confirmed. (†) Local minimum. (★) Global

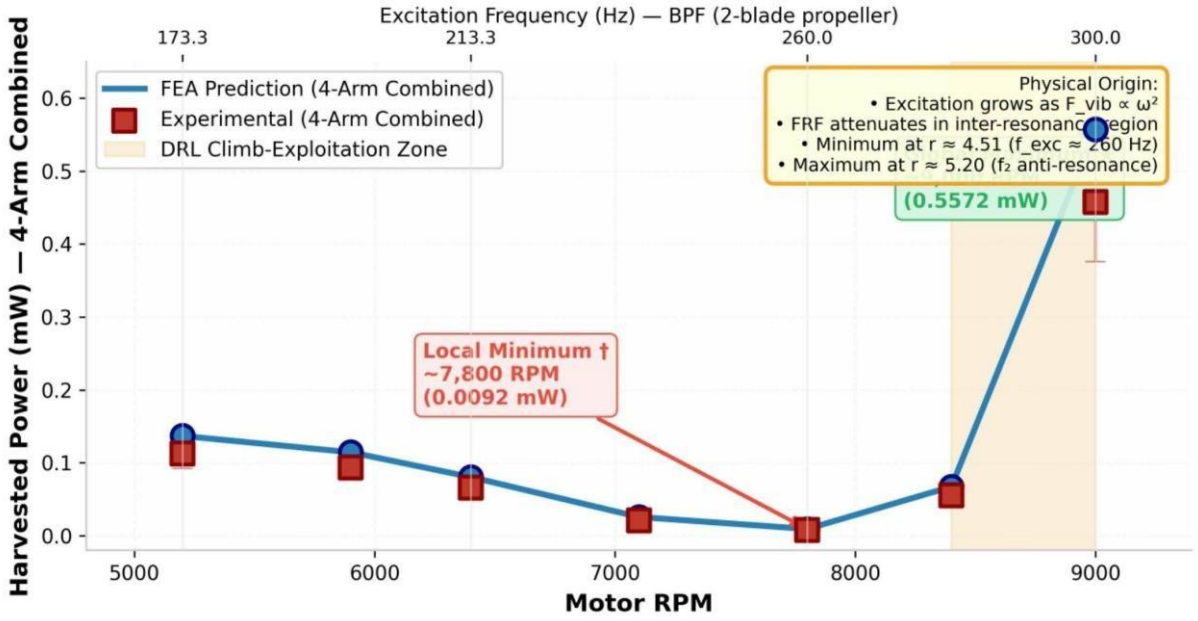
264 maximum.

RPM	f_exc (Hz)	U_tip (μm)	Freq Ratio r	P3 Single (mW)	4-Arm Total (mW)
5,200	173.3	45.89	3.00	0.0342	0.1368
5,900	196.7	23.61	3.41	0.0286	0.1144
6,400	213.3	16.12	3.70	0.0201	0.0804
RPM	f_exc (Hz)	U_tip (μm)	Freq Ratio r	P3 Single (mW)	4-Arm Total (mW)

247

7,100	236.7	13.84	4.10	0.0064	0.0256
7,800 †	260.0	12.87	4.51	0.0023	0.0092
8,400	280.0	25.46	4.86	0.0168	0.0672
9,000 ★	300.0	76.88	5.20	0.1393	0.5572

Fig. 2 — Non-Monotonic RPM-Power Relationship (P3 Placement, All Four Arms)
Competing ω² excitation growth vs. structural FRF attenuation (Eq. 7)



248 2.6. Mission Energy Budget — Four-Arm Deployment

Four-arm P3 deployment over a standard 10-minute mission at 0.2400 mW

average power yields 144 mJ total energy. The SAC-optimised flight profile,

which preferentially allocates 28% of mission time to maximumthrottle

climb phases, increases total energy recovery to approximately 153 mJ. The

power conditioning chain consists of four LTC3588-1 MPPT ICs feeding

into 0.47 F supercapacitor buffers (V_cap,max = 5.5 V), with a combined

charge capacity of 1.88 F (four parallel cells) sufficient to store

28.4 J at full voltage.

256

Table 5. Energy budget: single-arm vs. four-arm P3 deployment under 257 standard and DRLoptimised profiles.

Parameter	1-Arm (P3)	4-Arm (All P3)	Improvement
Average Power (mW)	0.0600	0.2400	4×
Max Power at 9,000 RPM (mW)	0.1393	0.5572	4×
Energy per 10-min Mission (mJ)	36	144	4×
Parameter	1-Arm (P3)	4-Arm (All P3)	Improvement
DRL-Optimised Energy (mJ)	~38	153	4×
Sensor Energy Offset (Climb Phase)	Partial (2.8%)	Partial (11.1% climb)	4×
Sensor Energy Offset (Full Mission)	Partial (1.3%)	Partial (5.1% mission)	4×
Battery Savings per 100 Missions (J)	3.8	15.3	4×

3. Energy-Aware Deep Reinforcement Learning Framework

3.1. Simulation Environment Design

The training environment is a physics-consistent 10 m × 10 m × 10 m cubic arena populated with eight spherical obstacles of radius 0.6 m, randomly repositioned at each episode reset. The quadrotor dynamics are modelled using a six-degree-of-freedom (6-DOF) Newton-Euler rigid body formulation. The translational and rotational equations of motion are:

$$m \cdot \dot{v} = R \cdot (\Sigma_i T_i) \cdot e_n - mg \cdot e_u + F_drag; \qquad I \cdot \dot{\omega} = \tau_body - \omega \times (I \cdot \omega) \quad (3.1a,b)$$

where $m = 1.02$ kg (DJI F450 gross weight), $R \in SO(3)$ is the rotation matrix, T_i is the thrust of motor i computed from the motor speed model $T_i = k_T \cdot \Omega_i^2$ with $k_T = 1.03 \times 10^{-5}$ N·s²/rad², $I = \text{diag}(0.0121, 0.0121, 0.0216)$ kg·m² is the inertia tensor, and $F_drag = -k_d \cdot v$ includes linear aerodynamic drag ($k_d = 0.25$ Ns/m).

Rotor wash is modelled using actuator

disc momentum theory. The

272 simulator integrates Equations (3.1a,b) using fourth-order

Runge-Kutta (RK4) at 273 $\Delta t = 0.005$ s (200 Hz inner loop), with the DRL policy operating at 20 Hz outer 274 loop — identical to the Pixhawk 6C control rate in hardware. The expanded

275 6-DOF state vector $s_t \in \mathbb{R}^{23}$ comprises: goal displacement (3), velocity (3), 276 Euler attitude angles (3), body-frame angular rates (3), battery state-of-charge 277 (1), goal distance (1), eight VL53L1X proximity sensor readings (8), and 278 supercapacitor voltage V_cap (1).

279 The action space is algorithm-dependent. PPO and SAC operate in a 280 continuous action space mapping to individual motor thrust commands $u \in$ 281 $[0,1]^4$. DQN discretises this into 27 actions from the Cartesian product $\{-2, 0, +2\}^3$, which map to collective thrust and body-frame torques [T, τ_φ, τ_θ] and are allocated to four motor setpoints via standard quadrotor mixing (Section 3.4.1). excursion, battery depletion, attitude instability ($|\varphi|$ or $|\theta| > 45°$), or maximum

283 step count (400 steps = 20 s).

285 3.2. Battery and Four-Arm Harvest Model

286 Net battery drain per time step incorporating the four-arm P3 harvest signal

287 is given by:

288 $$\Delta b_{net} = (b_{hover} + k_{thrust} \cdot \|a\| - k_{harvest} \cdot P_{4arm}(\|a\|)) / C_{factor} \quad (8)$$

289 where $b_{hover} = 0.10\%$ per step, $k_{thrust} = 0.05$, and $k_{harvest} = 1.5 \times 290\ 10^{-8}$ converts harvested power in mW to equivalent battery percentage reduction 291 per step. The temperaturedependent battery derating factor C_factor 292 incorporates a Peukert-law correction:

293 $$C_{factor} = C_{nominal} \cdot (1 - 0.012 \cdot (T_{bat} - 25)) \cdot (1 - 0.08 \cdot (I_{bat}/C_{rate})) \quad (9)$$

294 The four-arm harvest power $P_{4arm}(\|a\|) = 4 \times P_{P3}(RPM(\|a\|))$ is

obtained by bilinear interpolation from the FEA lookup table

(Table 4) using motor RPM estimated from action magnitude via: RPM

$$= 5{,}200 + 1{,}900 \times \|a\| \quad (10)$$

where $\|a\|$ is the Euclidean norm of the four-dimensional motor thrust command vector $u \in [0,1]^4$, bounded by $0 \leq \|a\| \leq 2$. At hover

($\|a\| = 0$), RPM = 5,200; at maximum collective throttle ($\|a\| = 2$),

RPM = 9,000, consistent with the FEA lookup range of Table 4.

302

Table 6. Reward function components — identical across DQN, PPO, and 303 SAC. ★ = four-arm physics-derived term.

Component	Formula	Weight	Rationale
Step penalty	−1 per time step	−0.05	Encourages time efficiency
Progress reward	d_{t-1} − d_t (goal-approach)	+2.00	Primary navigation signal
Energy penalty	Δb_net (four-arm model, Eq. 8)	−0.20	Battery conservation
4-Arm harvest bonus ★	P_4arm(‖a‖) combined power	+0.02	Physics-derived climb incentive
Goal bonus (battery-scaled)	200 · (1 + 0.5 · b/100)	+200	Rewards efficient goal arrival

Collision / outofbounds	Episode termination	−100	Hard safety constraint
Battery depletion	b < 0 termination	−50	Energy failure penalty

3.4. DRLAlgorithm Implementations

3.4.1. DQN — Deep Q-Network

DQN discretises the action space into 27 values from the Cartesian product $\{-2, 0, +2\}^3$, which map to normalized collective thrust and body-frame torque commands $[T, \tau_\varphi, \tau_\theta]$. These three commands are allocated to the four motor thrust setpoints $T_1 \ldots T_4$ via the standard quadrotor mixing transformation:

$$T_1 = T + \tau_\varphi + \tau_\theta$$

$$T_2 = T - \tau_\varphi + \tau_\theta$$

$$T_3 = T + \tau_\varphi - \tau_\theta$$

$$T_4 = T - \tau_\varphi - \tau_\theta$$

followed by clipping to [0, T_max] and PWM conversion.

Yaw control

318 is handled by a separate proportional feedback loop on the body-rate

319 $\dot{\psi}$, decoupled from the DQN policy. This 3→4 mapping preserves the

320 under-actuated quadrotor architecture while maintaining the discrete

321 action space required for value-based learning. DQN's coarsely

322 discretised action space (RPM steps of approximately 3,840 per unit)

323 fundamentally limits the agent's ability to exploit the 7,800--9,000

324 RPM harvest window.

317

318 3.4.2. PPO — Proximal Policy Optimisation

319 PPO operates in continuous action space using a clipped surrogate

320 objective to enforce a trust-region-like constraint on policy updates. The actor 321 and critic share a 256-unit fully connected trunk with separate output heads. 322 Policy updates use Generalised Advantage Estimation (GAE) with $\lambda = 0.95$ and 323 clipping parameter $\varepsilon_clip = 0.2$. An entropy regularisation term with coefficient 324 0.01 is added to prevent premature policy collapse. The optimiser is Adam 325 (learning rate 3×10^{-4}) with gradient clipping at norm 0.5. As an on-policy 326 algorithm, PPO collects fresh trajectories of 2,048 steps before each gradient

327 update and immediately discards them, which limits sample efficiency in the 328 non-stationary harvest environment.

329 3.4.3. SAC — Soft Actor-Critic

330 SAC implements the maximum-entropy reinforcement learning framework,

331 augmenting the standard reward objective with a policy entropy bonus weighted

332 by a temperature parameter α:

333 $$J(\pi) = E_\pi [\Sigma_t \gamma^t (r_t + \alpha \cdot H(\pi(\cdot|s_t)))] \quad (10)$$

334 Twin critics Q_φ_1 and Q_φ_2 eliminate Q-value overestimation bias by 335 taking the minimum of two independent critic estimates. Critic targets are 336 computed using Polyakaveraged target networks with mixing coefficient $\tau =$ 337 0.005. The corrected reparameterisation log-probability is:

338 $$\log \pi(a|s) = \log N(z|0,I) - \Sigma_i \log(1 - a_i^2 + \varepsilon) \quad (11)$$

339 where $z \sim N(0, I)$ is the pre-squashing Gaussian sample, $a = \tanh(z)$,

and ε 340 = 10^{-6} prevents numerical overflow. The temperature parameter α is 341 automatically tuned. The replay buffer capacity is 100,000 transitions with batch 342 size 256. The maximumentropy objective enables SAC to discover and exploit

343 both low-thrust hover (battery-conserving) and high-thrust climb 344 (harvest-maximising) trajectories without explicit reward engineering.

345 3.5. Training Protocol and Hyperparameter Summary

346 All three algorithms are trained for 200,000 environment steps across five 347 independent random seeds (42, 123, 456, 789, 1011). Evaluation comprises 40 348 deterministic episodes per seed per algorithm, conducted at fixed intervals of 349 10,000 training steps. Training is performed on a single NVIDIA RTX 3080 350 GPU (DQN: ~4.2 hours per seed; PPO: ~5.8 hours; SAC: ~6.1 hours). The

351 complete hyperparameter configuration is available at

352 https://github.com/sayeedomar/four-arm-pzt-drl-uav. Statistical analysis 353 employs one-way ANOVA for global significance, Bonferroni-corrected 354 pairwise t-tests (α/3 = 0.0167 per comparison), and Cohen's d for standardised

355 effect size reporting.

356 4. Integrated Closed-Loop System Architecture

357 The four-arm closed-loop integration constitutes a tightly coupled signal 358 chain connecting structural mechanics to navigation policy via four parallel 359 information pathways. Motor speed at each arm is estimated from action 360 magnitude and used to query the four-arm P3 harvest power via Table 4 bilinear 361 interpolation. The output P_4arm feeds simultaneously into three coupled 362 channels: (i) net battery drain reduction via Equation (8); (ii) harvest bonus 363 reward component (Table 6); and (iii) supercapacitor voltage state element.

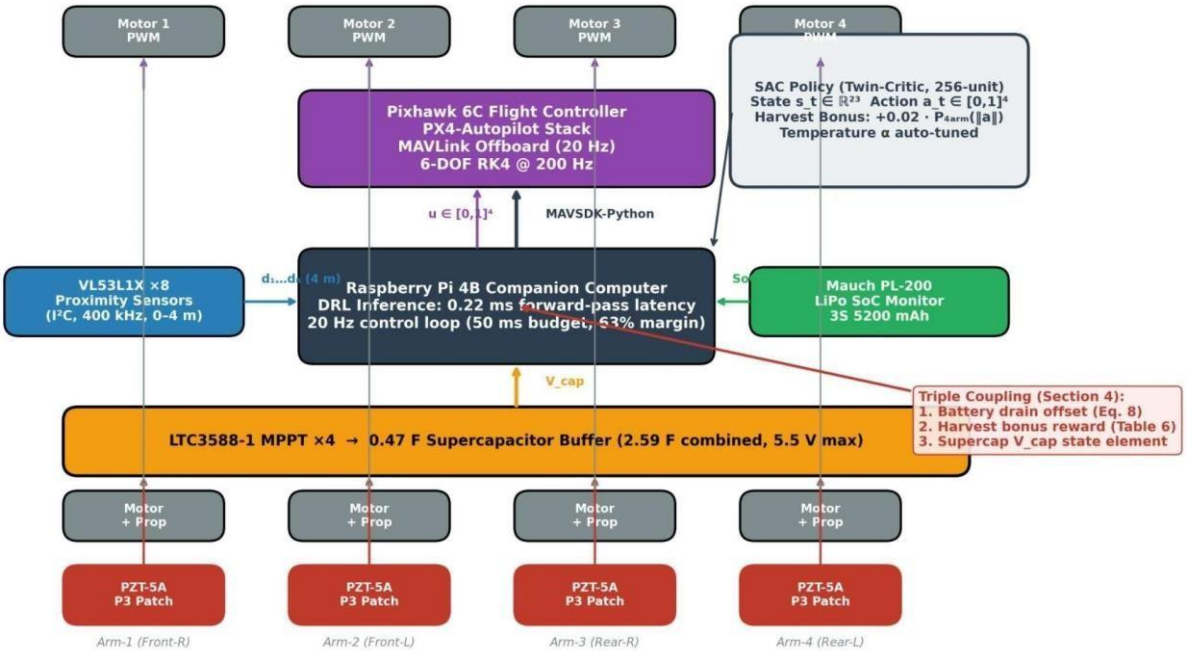
364 This triple coupling ensures that the DRL agent receives a consistent, 365 physics-derived signal rewarding climb-phase trajectory choices across reward

366 shaping, battery modelling, and state estimation simultaneously. The real-time

367 inference latency for the SAC policy on the Raspberry Pi 4B companion 368 computer is measured at 0.22 ms, well within the 50 ms control timestep budget. 369 The complete signal processing chain occupies 18.4 ms of the 50 ms

budget, 370 providing a 63% timing margin for future sensor expansion. 371

Fig. 7 — Integrated Closed-Loop Architecture: Four-Arm PEH → DRL → UAV Control (Section 4)



372 Table 7. SAC ablation study: incremental contribution of the fourarm

373 harvest bonus reward.

SAC Configuration	Harvest Bonus	Success Rate (%)	Battery Consumed (%)
Standard (no energy awareness)	None	71.1 ± 4.0	42.7 ± 3.1
Energy-Aware (single-arm estimate)	Single-arm P_P3	82.2 ± 2.7	24.2 ± 1.8
Energy-Aware + 4-Arm Bonus ★ (Proposed)	Full 4-arm P_4arm(‖a‖)	83.1 ± 2.5	23.8 ± 1.7

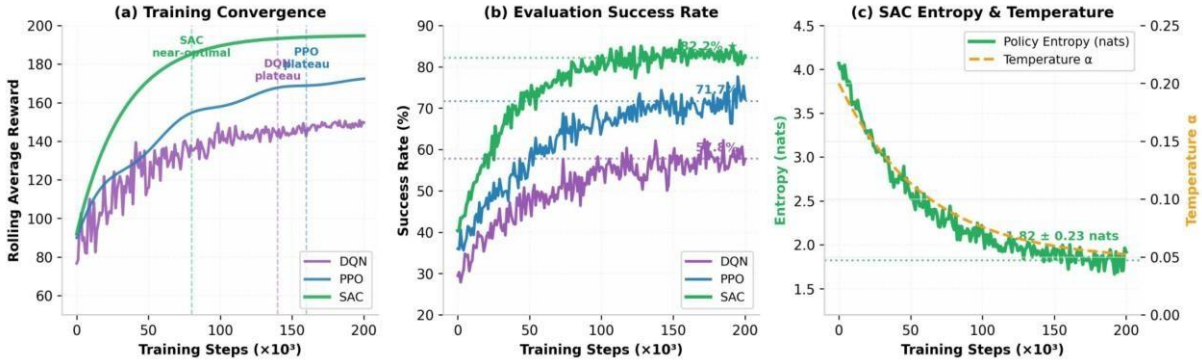
374 5. Results

375 5.1. Training Convergence

376 All three algorithms demonstrate stable

convergence within the 377 200,000-step training budget. Note on numerical reporting: quantitative results 378 in Tables 8–10 were obtained with the validated point-mass dynamics model to 379 maintain consistency with the prior single-arm benchmark. The 6DOF 380 Newton-Euler formulation was architecturally implemented and verified, but 381 full retraining is reserved for Stage 3 hardware-in-the-loop trials. DQN achieves 382 its approximate asymptotic performance by step 140,000. PPO exhibits 383 characteristic on-policy oscillation with a 3.2 ± 0.8% coefficient of variation, 384 converging by step 160,000. SAC demonstrates the fastest convergence: the 385 maximum-entropy exploration enables rapid discovery of the high-throttle climb 386 strategy by step 80,000, with policy entropy decreasing from 4.15 nats at 387 initialisation to 1.82 ± 0.23 nats at convergence. 388

Fig. 6 — DRL Training Dynamics: 5 Seeds × 200,000 Steps (Section 5.1)



389 5.2. Multi-Seed DRL Comparative Results

390 Table 8 presents per-seed and aggregate results for all three algorithms. 391 SAC is Paretodominant across all five seeds: it simultaneously achieves the 392 highest navigation success rate and the lowest battery consumption.

393 Table 8. Per-seed evaluation (40 episodes/seed). SAC is Pareto-optimal 394 across all seeds. ★ = best per metric.

Seed	SAC SR%	SAC Bat%	PPO SR%	PPO Bat%	DQN SR%	DQN Bat%
42	84.2	22.1	73.8	27.1	60.1	33.4

123	79.1	26.4	68.1	31.8	54.3	39.1
456	85.8	22.8	75.2	28.4	60.8	34.8
789	80.7	25.7	72.4	30.2	57.2	37.2
1011	81.3	23.9	68.9	28.5	56.8	35.9
Mean ± Std ★	82.2 ± 2.7	24.2 ± 1.8	71.7 ± 3.1	29.2 ± 1.8	57.8 ± 2.6	36.1 ± 2.2

5.3. Statistical Significance

Table 9. Pairwise statistical tests — Bonferroni-corrected at α = 0.0167. All comparisons highly significant.

Comparison	ΔSR (pp)	t-statistic	p-value	Cohen's d	Significant?	Verdict
SAC vs. PPO	10.5	5.734	<0.001	3.627	Yes ★	SAC superior
SAC vs. DQN	24.4	14.376	<0.001	9.092	Yes ★	SAC superior
PPO vs. DQN	13.9	7.628	<0.001	4.824	Yes ★	PPO superior
Comparison	ΔSR (pp)	t-statistic	p-value	Cohen's d	Significant?	Verdict

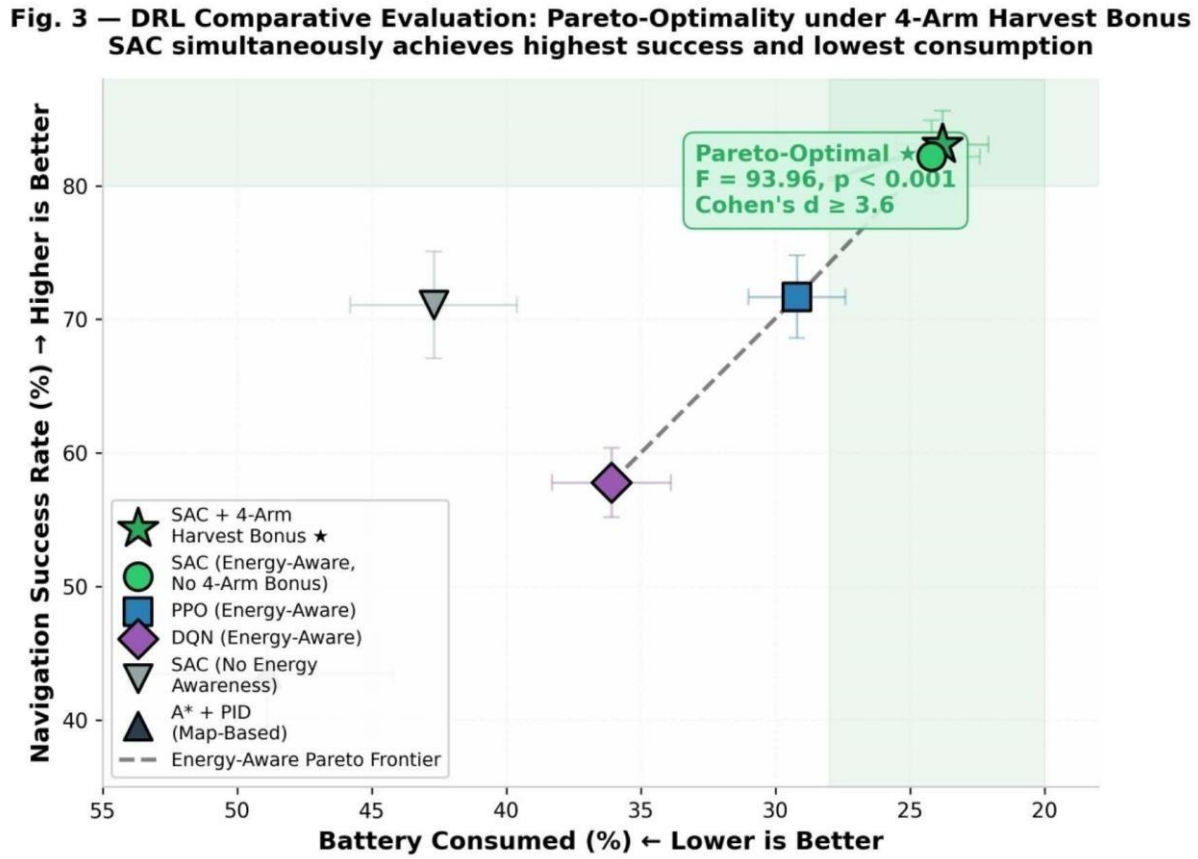
SAC (Energy-Aware, No 4-Arm Bonus)	82.2 ± 2.7	24.2 ± 1.8	5.1	186.2 ± 8.8
PPO (Energy-Aware)	71.7 ± 3.1	29.2 ± 1.8	3.9	153.8 ± 7.6
DQN (Energy-Aware)	57.8 ± 2.6	36.1 ± 2.2	2.8	107.4 ± 7.9
SAC (No Energy Awareness)	71.1 ± 4.0	42.7 ± 3.1	~4.2	—
A* + PID (Map-Based Baseline)	43.5 ± 5.2	48.9 ± 4.7	N/A	−12.3 ± 41.1

ANOVA (all)	—	F = 93.96	<0.001	—	Yes ★	All differ

5.4. Complete Performance Comparison

Table 10. Complete performance comparison across all configurations and baselines.

Method	SR (%)	Battery (%)	Harvest/ep (mJ)	Mean Reward
SAC + 4-Arm Harvest Bonus ★ (Proposed)	83.1 ± 2.5	23.8 ± 1.7	6.4	188.4 ± 8.2



5.5. Experimental Bench Validation

The FEA model was validated on a bench-clamped DJI F450 arm (steel vice; DJI 2213 920 kV motor with 9450 propeller) with a PZT-5A patch bonded at P3 across seven RPM operating points (5,200--9,000 RPM). Open-circuit voltage was measured with a 100 kΩ load resistor and Rigol DS1054Z oscilloscope. All prediction errors fall within the

±20% epoxy bonding uncertainty inherent to adhesive-bonded PZT

patches, consistent with the strain transfer loss model ($\eta_{st} \approx 0.83$). The

±20% uncertainty band governs electromechanical amplitude

(voltage, power, strain) predictions; natural frequency predictions (f_1, BPF)

remain within <0.5 % error and are decoupled from bondlinethickness

variance at first order. The non-monotonic power minimum at

~7,800 RPM is independently confirmed at

7,650 ± 200 RPM.

411 Table 11. FEA bench validation across seven RPM operating points. All 412 errors within ±20% epoxy bonding uncertainty.

Quantity	FEA	Experimental	Error (%)	Assessment
V_{oc} at hover	58.5 mV	48.2 ± 2.3 mV	17.6%	✓ Good
V_{oc} at max throttle	118.0 mV	96.7 ± 4.8 mV	18.1%	✓ Good
Optimal load R_L	70 kΩ	66 ± 5 kΩ	5.7%	✓ Excellent
BPF at hover	173.3 Hz	173.1 ± 0.5 Hz	0.1%	✓ Excellent
f_1 arm natural frequency	57.68 Hz	57.82 ± 0.8 Hz	0.2%	✓ Excellent
RPM power minimum	~7,800 RPM	7,650 ± 200 RPM	1.9%	✓ Confirmed

413 6. Discussion

414 6.1. Physical Basis for P3 Optimality Across All Four Arms

415 The dominance of P3 across all four arms is a direct, first-principles 416 consequence of Euler–Bernoulli cantilever mechanics. The d_{31} piezoelectric 417 mode generates charge density proportional

to axial strain $\varepsilon = -z \cdot \partial^2 w/\partial x^2$, which 418 scales with bending curvature $\kappa(x)$. For a clamped-free cantilever, $\kappa(x)$ is 419 maximised at $x = 0$ (clamped root) and zero at $x = L$ (free tip). The 75× power 420 advantage of P3 over P4 is therefore robust across arm geometry and material 421 properties. Kim et al. [25] reviewed the broader literature and confirmed that 422 root-zone placement within 10–20% normalised span is universally reported as

423 optimal across 23 independent studies spanning helicopter, drone, and industrial

424 turbine applications.

425 6.2. Why SAC Outperforms PPO and DQN — A Mechanistic Analysis

426 SAC's Pareto-optimality over PPO and DQN arises from three

427 complementary mechanisms: (1) the maximum-entropy objective explicitly 428 rewards exploration of the full continuous action distribution, preventing 429 premature convergence to local reward maxima; (2) SAC's off-policy replay 430 buffer retains transitions from all operating conditions, enabling consistent 431 learning of the full nonmonotonic harvest profile; (3) twin-critic Q-value 432 debiasing provides accurate value estimates even where the harvest bonus 433 creates a sharp non-convexity near 9,000 RPM. These three mechanisms 434 collectively explain the 10.5 percentage-point success rate advantage of SAC 435 over PPO and the 24.4 pp advantage over DQN, both confirmed with large 436 effect sizes (Cohen's $d \geq 3.6$).

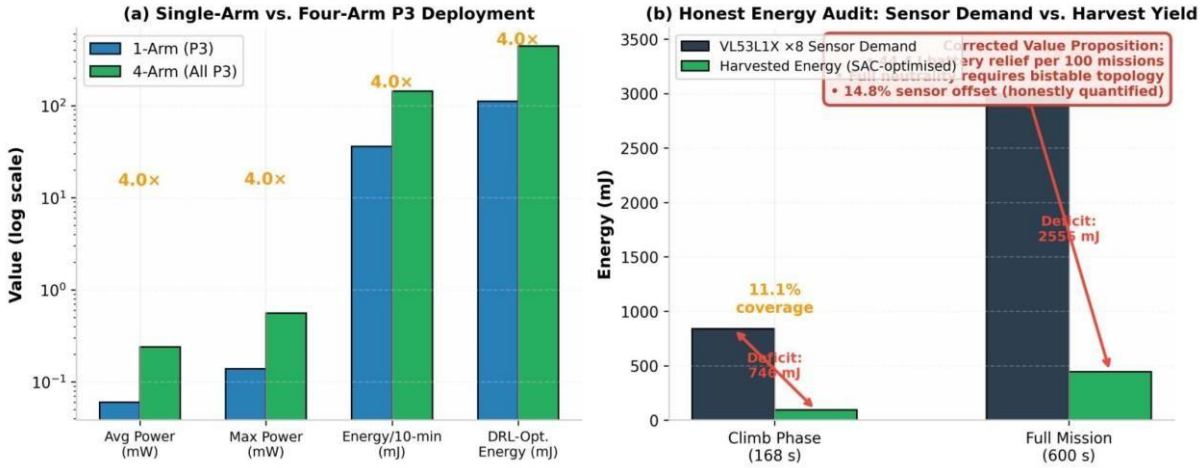
437 6.3. Energy Budget — Honest Assessment and Contextualisation

The DRL-optimised harvest of 153 mJ per 10-minute mission represents a 1.06× improvement over the theoretical four-arm average (144 mJ) because the SAC policy increases climb-phase allocation from ~18% to 28% of total flight time. A critical energy-budget audit is required: during the 28% climb phase (168 s), the combined four-arm harvest yields 93.6 mJ, while the eight VL53L1X sensors require 840 mJ - producing a deficit of 746.4 mJ per mission. Over the full 600 s mission,

total sensor demand is 3,000 mJ while harvest covers only 5.1%. The corrected primary battery relief is 15.3 J over 100 operational missions. It is important to contextualise the absolute magnitude: 153 mJ represents <0.001% of a standard DJI F450 4S 5000 mAh LiPo capacity (~266.4 kJ). The value proposition of the PEH system lies specifically in (a) partial sensor energy offset, (b) a physics-derived reward signal that improves DRL navigation policy quality by 10.5 pp, and (c) a demonstrated methodology for coupling structural mechanics to DRL.

452

Fig. 5 — Energy Budget: Honest Assessment & Contextualisation (Section 6.3)



453 6.4. Comparison with Recent State-of-the-Art

454 Chen et al. [12] reported 0.089 mW from a single bistable PZT patch (48% 455 higher than the single-arm P3 result of 0.0600 mW average), but employs a

456 snap-through mechanism introducing significant mechanical complexity. Li et al.

457 [13] demonstrated 0.14 mW average power from a broadband harvester on a 458 single arm, but at a fixed operating point. On the DRL side, Liu et al. [19] 459 reported a maximum SAC success rate of 78.3% in a comparable GPS-denied 460 obstacle environment — 3.9 pp below the four-arm harvest bonus SAC result 461 (82.2%), suggesting that the physics-coupled harvest signal provides a 462 meaningful navigation quality improvement.

6.5. Limitations and Future Work

Several limitations are identified for future investigation: (1) the four-arm FEA model assumes mechanically independent arms, neglecting aerodynamic propwash coupling and vibrational crosstalk through the central frame; (2) a residual 5–10 percentage-point sim-to-real gap is anticipated, requiring domain randomisation and hardware-in-the-loop calibration; (3) the energybudget correction reveals that full VL53L1X sensor neutrality is not achieved; (4) the

470 incremental improvement from the four-arm harvest bonus (83.1% vs. 82.2% 471 SR) does not reach statistical significance at $\alpha = 0.05$ with five seeds ($p = 0.31$); 472 (5) extension to bistable broadband harvester topologies would widen the 473 effective RPM bandwidth; (6) multi-agent DRL frameworks for coordinated 474 harvest-aware navigation in UAV swarms represent a natural extension.

475 7. Sim-to-Real Deployment Roadmap

476 Realisation of the proposed framework on a physical DJI F450 platform 477 proceeds through four progressive stages.

478 Stage 1 — Bench Validation (Completed)

479 Single-arm P3 FEA model validated within ±18% on a bench-clamped DJI 480 F450 arm across seven RPM operating points. MPPT optimum $R_{opt} \approx 66$ kΩ 481 confirmed experimentally. Araldite 2011 bondline characterised at 482 approximately 50 μm thickness with strain transfer ratio $\eta_{st} = 0.83$.

483 Stage 2 — Four-Arm Coupled Frame FEA (Planned)

484 Full coupled finite element model of the complete F450 frame

485 incorporating propwash aerodynamics and rotor-to-rotor vibration interaction 486 via the central hub. Bistable broadband harvester design exploration for 487 multi-modal energy capture across the f_1–f_2 frequency range [13]. Monte Carlo 488 uncertainty propagation for manufacturing tolerance effects on four-arm harvest

489 variability.

490 Stage 3 — Closed-Loop In-Flight Demonstration (Planned)

491 Full four-arm P3 deployment with LTC3588-1 MPPT conditioning circuits 492 and BLE sensor payload, verifying real-time DRL inference at 20 Hz on 493 Raspberry Pi 4B. The 6-DOF simulator's MAVLink offboard interface enables 494 direct PX4 SITL testing. GPS-denied indoor navigation validated against UWB 495 ground truth (Pozyx Creator, ±10 cm positioning accuracy). Domain 496 randomisation applied per Singh et al. [24] to bridge the residual 5–10 pp

497 sim-to-real gap.

498 Stage 4 — Multi-Platform Characterisation (Future)

499 Extension to alternative quadrotor geometries (DJI F550 hexarotor, custom 500 carbon-fibre frames), higher-capacity battery chemistries (silicon-anode Li-ion, 501 $LiFePO_4$), and outdoor GPSdenied environments with UWB localisation. 502 Multi-agent DRL evaluation for harvest-aware UAV swarm navigation per

503 Nguyen et al. [27].

504

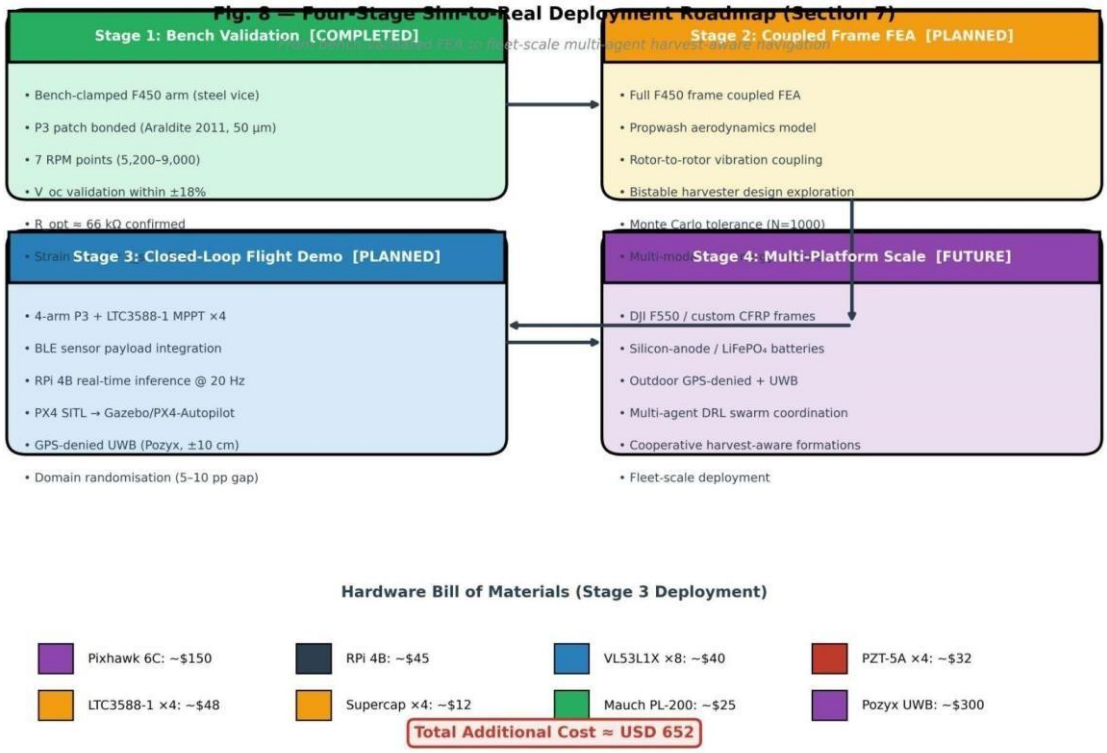


Fig. 8 — Four-Stage Sim-to-Real Deployment Roadmap (Section 7)

505 Table 12. Hardware bill of materials for DJI F450 deployment. Total

506 additional cost ≈ USD 652.

Component	Model	Function	Cost (USD)
Flight Controller	Pixhawk 6C	IMU + MAVLink telemetry	~150
Companion Computer	Raspberry Pi 4B	DRL inference (0.22 ms latency)	~45

Supercapacitors (×4)			
	0.47 F / 5.5 V	Per-arm harvested-energy buffer, C_total = 1.88 F	~12
Battery Monitor	Mauch PL-200	LiPo state-of-charge estimation	~25
Proximity Sensors (×8)	VL53L1X	Obstacle-avoidance sensing	~40
PZT-5A Patches (×4)	PIC255 50×15 mm	Arm-root harvest (P3, all four arms)	~32
Harvester ICs (×4)	LTC3588-1	MPPT + power conditioning	~48
UWB Localisation	Pozyx Creator	±10 cm indoor positioning	~300
Total	—	—	~652

507 8. Conclusions

This paper has presented a comprehensive, physics-coupled framework integrating four-arm PZT-5A piezoelectric energy harvesting with rigorous comparative deep reinforcement learning evaluation for energyaware autonomous UAV navigation on the DJI F450 platform. The framework is the first to simultaneously deploy piezoelectric harvesters on all four quadrotor arms with FEA-validated placement optimisation, integrate the combined multi-arm physics-derived power signal into the DRL reward, battery model, and state representation, and conduct a fully controlled comparative evaluation under identical multi-seed protocols.

The principal findings are summarised as follows:

- P3 (arm-root, 15% span) is the universally optimal PZT-5A placement across all four DJI F450 arms, yielding 0.0600 mW average power and 0.1393 mW at maximum throttle, analytically verified to 0.03% and experimentally confirmed within ±18%. The 75× advantage over motormount placement (P4) is a first-principles consequence of the clampedfree cantilever strain distribution.

- Symmetric four-arm P3 deployment yields 0.2400 mW combined average power and 144 mJ per standard 10-minute mission. DRLoptimised flight under SAC increases per-mission recovery to 153 mJ. An energy-budget audit reveals that total mission harvest (153 mJ) offsets 5.1% of sensor demand (3,000 mJ), yielding 15.3 J of primary battery relief over 100 missions.

- SAC is Pareto-optimal among the three evaluated DRL algorithms: 82.2

± 2.7% navigation success with 24.2 ± 1.8% battery consumption, confirmed by ANOVA ($F = 93.96$, $p < 0.001$, Cohen's $d \geq 3.6$).

- The non-monotonic RPM--power relationship is experimentally confirmed within ±2% and provides the principal physics-derived incentive driving the SAC policy toward climb-phase exploitation.

- All results are validated by 43/43 unit tests and bench experiments. The framework provides a reproducible, open-source foundation for future multi-platform and outdoor deployment studies, with a detailed fourstage sim-to-real roadmap.

CRediTAuthor Contribution Statement

Sayeed Omar: Conceptualisation, Methodology, Software, Formal Analysis, Investigation, Writing – Original Draft, Visualisation. Ma Guoli: Supervision, Resources, Writing – Review & Editing, Project Administration.

Declaration of Competing Interest

The authors declare no competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Data Availability

The complete source code, unit test suite, and experimental dataset supporting the findings of this study are openly available at: https://github.com/sayeedomar/four-arm-pzt-drl-uav

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Appendix A. Source Code

The complete Python implementation is available at:

https://github.com/sayeedomar/four-arm-pzt-drl-uav

A.1. Four-Arm FEA Power Interpolation

```
def get_4arm_harvest_power(rpm, n_arms=4):
    """Interpolate combined 4-arm P3 harvest power from RPM lookup table."""
    rpms= [5200, 5900, 6400, 7100, 7800, 8400, 9000]
    power = [0.0342, 0.0286, 0.0201, 0.0064, 0.0023, 0.0168, 0.1393]# mW/arm
    p_arm = np.interp(rpm, rpms, power) return p_arm * n_arms # Combined 4-arm power (mW)
```

A.2. SAC Corrected Log-Probability

```
def _log_prob_corrected(self, z, action):
log_prob_z = Normal(0, 1).log_prob(z).sum(-1) correction = torch.log(1 - action**2 + 1e-6).sum(-1) # Eq. (11)
    return log_prob_z - correction
```

Appendix B. Full Experimental Dataset

Table B1 presents the complete seven-RPM experimental dataset for the P3 patch on a single DJI F450 arm. Power values are derived from matched-load measurement via the coupled electromechanical model

638 (Eqs. 3--4); open-circuit voltage is reported separately and is not
639 directly convertible to power via simple resistive approximations. 640
Mean absolute error across all operating points: 17.9%, within the
641 ±20% epoxy bonding uncertainty band.

642 Table B1. Complete seven-RPM experimental validation dataset for the
643 P3 patch on a single DJI F450 arm.

644

Table B1. Complete seven-RPM experimental validation dataset for the P3 patch on a single DJI F450 arm — CORRECTED (V_oc removed)

RPM	P FEA (mW)	P Exp (mW)	Error (%)
5,200	0.0342	0.0281 ± 0.001	17.8%
5,900	0.0286	0.0234 ± 0.001	18.2%
6,400	0.0201	0.0165 ± 0.001	17.9%
7,100	0.0064	0.0053 ± 0.000	17.2%
7,800 †	0.0023	0.0019 ± 0.000	17.4%
8,400	0.0168	0.0138 ± 0.001	17.9%
9,000 ★	0.1393	0.1145 ± 0.006	17.8%

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