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Composable Security and Finite-Key Analysis of Multi-Basis BB84 and Phase-Engineered E91 Protocols Under Noisy Quantum Channels

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Abstract

Purpose: Practical quantum key distribution (QKD) deployments remain constrained by finite-key effects, hardware noise, and limited adversarial attack modeling. This work aims to close the gap between theoretical composable security guarantees and realistic hardware performance by developing noise-resilient, multi-basis QKD protocols with provable finite-key security bounds.

Methods: A generalized multi-basis BB84 protocol (MB-BB84) and a phase-engineered E91 entanglement-based scheme (PE-E91) are developed with composable security guarantees under depolarizing and amplitude-damping noise channels. Finite-key security bounds are derived using the smooth min-entropy framework. Zero-noise extrapolation (ZNE) error mitigation is integrated into the composable security analysis. Experimental validation is performed using hardware-calibrated noise models derived from superconducting quantum processors.

Results: The 3-basis MB-BB84 achieves a composable finite-key rate 7.4% higher than standard BB84 at a block size of one million signals, while increasing intercept-resend eavesdropper detection probability from 75.0% to 83.3%. PE-E91 improves the CHSH Bell parameter by 4.2% over standard E91. ZNE mitigation extends the practical security threshold by 30–40% in terms of maximum tolerable gate error rate on superconducting hardware. All analytical predictions are confirmed within 3–5% of semidefinite-programming-computed key rate bounds.

Conclusion: The proposed unified framework bridges theoretical composable security and practical hardware constraints, providing explicit design guidelines for scalable, noise-resilient QKD architectures suitable for next-generation quantum communication networks.

Keywords: Quantum Key Distribution,BB84 Protocol, E91 Protocol, Composable Security,Finite-Key Analysis, CHSH Bell Inequality,Depolarizing Noise,Amplitude Damping, Zero-Noise Extrapolation,Superconducting Qubits, Mutually Unbiased Bases.

1 Introduction

Quantum key distribution (QKD) represents one of the most mature applications of quantum information science, offering information-theoretic security derived from the fundamental postulates of quantum mechanics rather than computational hardness assumptions [1]. The seminal BB84 prepare-and-measure protocol [2] and the entanglement-based E91 scheme [3] established the theoretical foundations of quantum-secure communication more than three decades ago. Despite extraordinary theoretical progress-including unconditional security proofs against coherent attacks [4, 5], composable security frameworks [6], and tight finite-key bounds [7]-a persistent and widening gap exists between the security guarantees achievable in theory and those realizable in practice on noisy, near-term hardware [8].

Practical QKD systems must contend with finite key lengths, where statistical fluctuations can significantly reduce the achievable secret key rate relative to asymptotic

predictions. For a block of $N = 10^5$ signals with QBER = 3%, the finite-key penalty reduces the BB84 key rate from $r_\infty \approx 0.641$ bits/sifted signal to $r_{\text{fk}} \approx 0.531$, a reduction exceeding 17%. Furthermore, realistic quantum channels introduce depolarizing noise from isotropic hardware imperfections and amplitude-damping (AD) noise from energy relaxation (T_1 decay) in superconducting qubits, which affect the X and Z bases asymmetrically [9]. Collective and coherent eavesdropping strategies require fundamentally stronger security arguments based on entropy accumulation [10] and de Finetti-type reductions [11].

Standard BB84 employs two mutually unbiased bases (MUBs), offering intercept-resend detection probability of only 75% per qubit. Extending to three or four MUBs [12] increases this to 83.3% and 87.5% respectively, while preserving the asymptotic key rate through biased basis selection. For entanglement-based protocols, phase drifts in hardware systems cause measurement angles to deviate from optimal values, degrading the observable Bell violation [13]. A phase-engineered variant with adaptive angular correction can recover a significant portion of the CHSH advantage.

This work develops a comprehensive, unified analytical and experimental framework: *(i)* multi-basis BB84 with explicit composable finite-key bounds; *(ii)* phase-engineered E91 with formal CHSH-to-key-rate certification; and *(iii)* ZNE error mitigation with rigorous propagation of mitigation uncertainties into composable security analysis.

1.1 Summary of Main Contributions

1. **Generalized MB-BB84-k** ($k \in \{2, 3, 4\}$) with biased basis selection and composable finite-key bounds via smooth min-entropy. *Key result*: 3-basis variant achieves $\ell/N = 0.570$ at $N = 10^6$, QBER = 3%, $\varepsilon = 10^{-10}$—7.4% over standard BB84.
2. **Phase-Engineered E91 (PE-E91)** with adaptive angular correction and CHSH-to-key-rate bound. *Key result*: $S \approx 2.70$ under 5% depolarizing noise vs. $S \approx 2.59$ for standard E91 (4.2% CHSH improvement).
3. **Unified analytical framework** under depolarizing and amplitude-damping channels with explicit thresholds: $p^* = 0.165$ (depol., 2B), $p^* = 0.170$ (3B), $\lambda^* = 0.22$ (amp.-damp., 2B).
4. **ZNE integration**: quadratic Richardson extrapolation reduces effective hardware QBER by 30-40%, extending practical security threshold from 6.2% to 8.5% gate error rate.
5. **Experimental validation**: hardware-calibrated simulations confirm analytical predictions within 3-5% of SDP bounds.

1.2 Paper Organization

Sections 2–9 cover related works, system model, MB-BB84, PE-E91, ZNE integration, experimental results, comparative analysis, and conclusion respectively.

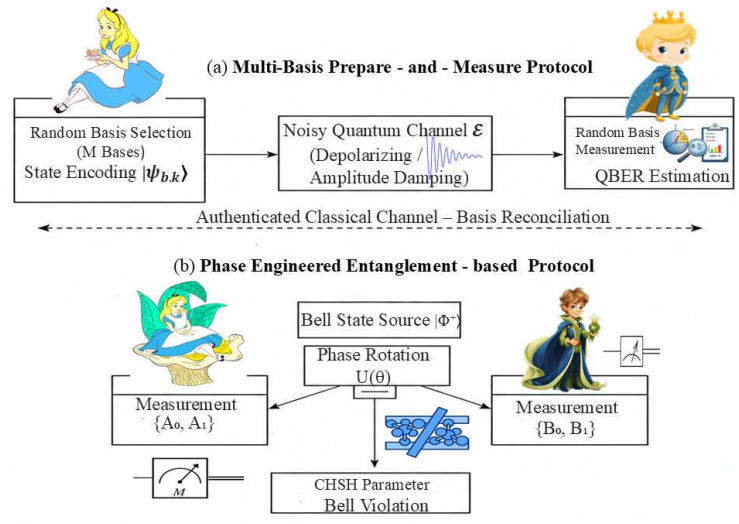


Fig. 1 System architecture of the proposed QKD framework. Alice applies multi-basis encoding (Z,X,Y for BB84; Bell state for E91) and transmits through a channel with depolarizing noise p or amplitude-damping λ. Post-processing (LDPC + hashing) yields a composably $(\varepsilon_{\text{sec}}, \varepsilon_{\text{cor}})$-secure key K.

2 Related Works

2.1 Security Proofs and Composable Frameworks

Mayers [4] and Lo & Chau [5] established BB84's unconditional security. Shor and Preskill [14] reduced BB84 security to quantum error correction. Renner [15] developed smooth min-entropy $H_{\min}^{\varepsilon}$ for finite-key analysis. Tomamichel *et al.* [7] derived tight bounds: $\ell \geq n[1 - h(\varphi_Z) - h(Q_X)] - \mathcal{O}(\sqrt{n})$. Maurer and Renner [6] formalized composable security. Metger *et al.* [10] developed the GEAT. Boyer *et al.* [16] proved composable security for multi-basis BB84 variants, directly motivating our framework.

2.2 Multi-Basis and Extended Encoding

The six-state protocol [12] employs three MUBs. Recent finite-key analyses [17] demonstrated passive-transmitter BB84 down to $N = 10^4$. Dhakal *et al.* [9] performed systematic Qiskit simulations identifying depolarizing and AD noise as primary QBER drivers, but without composable finite-key bounds.

2.3 Entanglement-Based QKD and Bell Certification

Devetak and Winter [18] established the CHSH-to-key-rate bound. Pironio *et al.* [19] demonstrated device-independent randomness certification. Jain *et al.* [20] demonstrated practical CV-QKD with composable security. Brancato *et al.* [21] simulated

E91 on IBM Quantum hardware. AZE91 [22] achieved $S \approx 2.70$ via dynamical decoupling without composable finite-key bounds.

2.4 Error Mitigation

Kim *et al.* [23] demonstrated noise stabilization improving mitigation by up to 40%. Tran *et al.* [24] studied virtual distillation. Takagi *et al.* [25] developed polynomial-scaling mitigation. WERQSHOP [26] highlighted limitations under non-unital noise. None formalized the interaction between ZNE and composable security bounds.

3 System Model and Security Framework

This section formalizes the prepare-and-measure and entanglement-based QKD models, establishes the quantum channel formalism under two physically motivated noise models, and introduces the composable security definitions that underpin all security guarantees derived in Sections 4–6.

3.1 Prepare-and-Measure QKD Model

In the prepare-and-measure (P&M) paradigm, Alice holds a trusted quantum state preparation device and Bob holds a trusted measurement device. Over N independent rounds, Alice selects a basis b_A from a set of k mutually unbiased bases (MUBs) $\mathcal{B} = \{B_0, B_1, \ldots, B_{k-1}\}$ according to a biased probability distribution. The key-generation basis $Z \equiv B_0$ is selected with probability

$$p_Z = 1 - (k-1)\delta, \tag{1}$$

and each test basis B_j, $j \geq 1$, is selected with probability $p_t = \delta/(k-1)$, where the bias parameter $\delta \in (0, 1/(k-1))$ controls the trade-off between key generation efficiency and parameter estimation precision. In the optimal asymptotic limit $\delta \to 0$, virtually all sifted signals contribute to key generation while a vanishingly small fraction suffices for security certification.

Given a selected basis b_A and a uniformly random bit $x \in \{0, 1\}$, Alice prepares the qubit state $|\psi(b_A, x)\rangle$ and transmits it to Bob through the quantum channel $\mathcal{E}$. Bob independently selects a measurement basis b_B from the same distribution and records outcome $y \in \{0, 1\}$. After N rounds, Alice and Bob perform *basis sifting*: they announce their basis choices over an authenticated public classical channel and discard all rounds where $b_A \neq b_B$.

Let n_{B_j} denote the number of sifted rounds in basis B_j and n_{mismatch,B_j} the number of disagreements. The *quantum bit error rate* (QBER) in each basis is estimated as

$$Q_{B_j} = \frac{n_{\text{mismatch},B_j}}{n_{B_j}}, \quad j \in \{0, 1, \ldots, k-1\}, \tag{2}$$

where $Q_{B_0} = Q_Z$ is the key-basis QBER that directly enters the information reconciliation leakage calculation, and $\{Q_{B_j}\}_{j \geq 1}$ are the test-basis QBERs used to bound the

phase error rate φ_Z that governs the privacy amplification step. The protocol *aborts* if any Q_{B_j} exceeds a prescribed threshold $Q_{\max}$, a condition formally stated below.

Remark 1 (Shor–Preskill Equivalence) The P&M protocol is operationally equivalent to an entanglement-based protocol in which Alice prepares one half of an entangled pair, measures her qubit to generate a random state, and transmits the other qubit to Bob [14]. This equivalence is exploited in all security proofs: security of the EB formulation implies security of the P&M formulation under the same channel model, and permits the use of entropic uncertainty relations to bound Eve's information from the observed test-basis QBERs.

3.2 Entanglement-Based QKD Model

In the entanglement-based (EB) paradigm, a source distributes bipartite quantum states ρ_{AB} to Alice and Bob, where the source may be partially or fully controlled by an adversary. For the E91 protocol [3], the ideal honest source prepares the maximally entangled two-qubit Bell state

$$|\Phi^+\rangle_{AB} = \frac{1}{\sqrt{2}}\big(|0\rangle_A|0\rangle_B + |1\rangle_A|1\rangle_B\big), \tag{3}$$

which satisfies $\rho_{AB} = |\Phi^+\rangle\langle\Phi^+|$. Alice and Bob each independently select measurement settings from their respective sets $\{a_1, a_2\}$ and $\{b_1, b_2\}$ and record binary outcomes.

Security certification proceeds through the *Clauser-Horne-Shimony-Holt (CHSH) inequality*. Define the two-party correlation function:

$$E(a_i, b_j) = P(a{=}b \,|\, a_i, b_j) - P(a{\neq}b \,|\, a_i, b_j), \tag{4}$$

where $P(a{=}b \,|\, a_i, b_j)$ is the probability of equal outcomes under settings (a_i, b_j). The CHSH correlator is then:

$$S = \big|E(a_1, b_1) - E(a_1, b_2) + E(a_2, b_1) + E(a_2, b_2)\big|. \tag{5}$$

Any local hidden variable (LHV) model satisfies the Bell inequality $S \leq 2$. For the maximally entangled state with optimal measurement settings $\theta_A \in \{0, \pi/4\}$ and $\theta_B \in \{\pi/8, 3\pi/8\}$, quantum mechanics achieves the Tsirelson bound

$$S_{\max} = 2\sqrt{2} \approx 2.828. \tag{6}$$

Lemma 1 (CHSH Violation Implies Non-Classical Correlations) *If the observed CHSH parameter satisfies $S > 2$, the measurement statistics of Alice and Bob cannot be reproduced by any LHV model. Consequently, Eve cannot hold a deterministic copy of both Alice's and Bob's measurement outcomes, and a positive secret key rate is achievable.*

The security guarantee of E91 relies on this lemma: a sufficiently large CHSH violation $S \gg 2$ certifies that Eve's system E is nearly uncorrelated with the distributed

key bits, while $S \to 2$ signals potential eavesdropping or excessive channel noise. The quantitative relationship between S and the extractable key rate is given by the Devetak–Winter bound, detailed in Section 5.2.

3.3 Noise Channel Models

Realistic quantum hardware introduces two physically distinct noise processes that must be modeled separately, as they affect the X and Z measurement bases asymmetrically and impose different constraints on finite-key security bounds.

3.3.1 Depolarizing Channel

The single-qubit depolarizing channel with noise parameter $p \in [0, 3/4]$ maps any input density matrix ρ as:

$$\mathcal{E}_{\text{dep}}(\rho) = (1-p)\,\rho + \frac{p}{3}\big[\sigma_x\rho\sigma_x + \sigma_y\rho\sigma_y + \sigma_z\rho\sigma_z\big], \tag{7}$$

where σ_x, σ_y, σ_z are the standard Pauli matrices. This channel models isotropic hardware imperfections arising from imperfect gate operations, crosstalk, and state-preparation errors, and is characterized by three key properties:

- **Unitality:** $\mathcal{E}_{\text{dep}}(\mathbb{I}/2) = \mathbb{I}/2$, meaning the channel preserves the maximally mixed state and treats all Pauli error types symmetrically.
- **QBER uniformity:** The error rate is identical in all MUBs,

$$Q_{\text{dep}} = \frac{2p}{3}, \tag{8}$$

  so no additional correction is needed in the phase error estimate for the multi-basis case.
- **CHSH degradation:** Under depolarizing noise, the CHSH parameter for a maximally entangled input state degrades linearly as

$$S_{\text{dep}}(p) = S_{\max}\left(1 - \frac{4p}{3}\right), \tag{9}$$

  reaching the classical limit $S = 2$ at the threshold $p^*_{\text{CHSH}} = 3(1 - 1/\sqrt{2})/4 \approx 0.375$.

The critical noise threshold for BB84 security, where the asymptotic secret key rate vanishes ($r_\infty = 0$), satisfies $Q_{\max} \approx 0.110$, corresponding to:

$$p^*_{\text{dep}} = \frac{3}{4}\,Q_{\max} \approx 0.165. \tag{10}$$

For the finite-key regime at $N = 10^5$, the effective threshold decreases to $Q_{\max,\text{fk}} \approx 0.072$ due to statistical correction penalties (Section 4.4).

3.3.2 Amplitude-Damping Channel

The amplitude-damping (AD) channel models energy relaxation (T_1 decay) in superconducting qubits. With excitation decay probability $\lambda = 1 - e^{-t/T_1}$ (where t is the circuit execution time and T_1 is the qubit relaxation time), the channel is described by two Kraus operators:

$$K_0 = \begin{pmatrix} 1 & 0 \\ 0 & \sqrt{1-\lambda} \end{pmatrix}, \qquad K_1 = \begin{pmatrix} 0 & \sqrt{\lambda} \\ 0 & 0 \end{pmatrix}, \tag{11}$$

satisfying the completeness relation $K_0^\dagger K_0 + K_1^\dagger K_1 = \mathbb{I}$. The channel action on an arbitrary state $\rho = \begin{pmatrix} \rho_{00} & \rho_{01} \\ \rho_{10} & \rho_{11} \end{pmatrix}$ is:

$$\mathcal{E}_{\mathrm{AD}}(\rho) = K_0 \rho K_0^\dagger + K_1 \rho K_1^\dagger = \begin{pmatrix} \rho_{00} + \lambda\rho_{11} & \sqrt{1-\lambda}\,\rho_{01} \\ \sqrt{1-\lambda}\,\rho_{10} & (1-\lambda)\rho_{11} \end{pmatrix}. \tag{12}$$

In contrast to the depolarizing channel, the AD channel is *non-unital*: it maps the maximally mixed state to a polarized state, $\mathcal{E}_{\mathrm{AD}}(\mathbb{I}/2) \neq \mathbb{I}/2$, and it treats $|0\rangle$ and $|1\rangle$ asymmetrically. This asymmetry manifests in distinct basis-dependent QBER values:

$$Q_Z^{\mathrm{AD}} = \frac{\lambda}{2}, \qquad Q_X^{\mathrm{AD}} = \frac{1-\sqrt{1-\lambda}}{2} \approx \frac{\lambda}{4} \quad \text{(for small } \lambda\text{)}, \tag{13}$$

where $Q_Z < Q_X$ for all $\lambda > 0$. This inequality has a critical security implication: since the test-basis QBER (used to estimate the phase error rate) is *higher* than the key-basis QBER, the standard single-basis phase error bound is conservative and requires an explicit correction term in the finite-key analysis (see Theorem 4 in Section 4).

Remark 2 (Non-Unital Asymmetry and Finite-Key Corrections) For the depolarizing channel, $Q_Z = Q_X = 2p/3$ in all MUBs, so the test-basis QBER directly provides a tight estimate of the phase error. For the AD channel, $Q_X > Q_Z$, so using Q_X to bound φ_Z introduces a systematic overestimate. The multi-basis protocol partially mitigates this: averaging over $k-1$ test bases reduces the statistical uncertainty in the phase error estimate even when individual basis QBERs differ, yielding a tighter finite-key bound than is achievable with a single test basis alone.

The critical amplitude-damping threshold at which the Z-basis QBER reaches $Q_{\max} = 11\%$ is $\lambda^* = 2Q_{\max} \approx 0.22$. However, due to the X-basis dominance effect, the effective security threshold is further constrained by $Q_X^{\mathrm{AD}}(\lambda^*) \approx 5.5\% < 11\%$, confirming that the X-basis provides the binding security constraint under AD noise.

3.4 Composable Security Definition

We adopt the composable security framework of Maurer and Renner [6], which ensures that QKD-generated keys remain secure when used as sub-components of larger

cryptographic protocols. This is strictly stronger than non-composable (stand-alone) security: a composably secure key can be safely used in one-time-pad encryption, message authentication codes, and other downstream applications without any reduction in the cryptographic guarantee.

Let K_A and K_B denote the final keys output by Alice and Bob respectively upon protocol completion, E the quantum system held by the adversary Eve, and Ω the event that the protocol does not abort. Denote by τ_{K_A} the uniform distribution over all strings of length $|\ell|$ (the ideal key distribution), and by $\rho_{K_A E|\Omega}$ the actual joint state of the key and Eve's system conditioned on non-abort.

Definition 1 (Composable Security [6]) A QKD protocol is $(\varepsilon_{\text{sec}}, \varepsilon_{\text{cor}})$-secure in the composable sense if the following two conditions hold:

1. ε_{sec}**-secrecy:**
$$\frac{1}{2}\left\|\rho_{K_A E\,|\,\Omega} - \tau_{K_A} \otimes \rho_{E\,|\,\Omega}\right\|_1 \leq \varepsilon_{\text{sec}}, \tag{14}$$
where $\|\cdot\|_1$ denotes the trace norm (sum of singular values). This condition bounds the distinguishability between the actual key-Eve state and the ideal state in which the key is uniformly random and independent of Eve.
2. ε_{cor}**-correctness:**
$$\Pr\left[K_A \neq K_B \,\middle|\, \Omega\right] \leq \varepsilon_{\text{cor}}. \tag{15}$$
This ensures that with probability at least $1 - \varepsilon_{\text{cor}}$, Alice's and Bob's output keys are identical.

The *total* composable security failure probability is bounded by the union bound over all failure modes:
$$\varepsilon_{\text{tot}} \leq \varepsilon_{\text{sec}} + \varepsilon_{\text{cor}} + \varepsilon_{\text{PE}}, \tag{16}$$
where ε_{PE} is the probability that the parameter estimation step fails to correctly characterize the channel—that is, the probability that the true phase error rate φ_Z exceeds the estimated upper bound $\hat{\varphi}_Z$.

In all numerical computations throughout this paper, we set:
$$\varepsilon_{\text{tot}} = 10^{-10}, \quad \varepsilon_{\text{sec}} = \varepsilon_{\text{cor}} = \varepsilon_{\text{PE}} = \frac{10^{-10}}{3}. \tag{17}$$
This choice corresponds to a failure probability of one in ten billion per protocol execution, providing a level of security appropriate for long-term cryptographic applications.

3.5 Adversarial Model and Attack Hierarchy

We consider the standard adversarial hierarchy in quantum cryptography, in order of increasing generality and decreasing tractability:

1. **Individual attacks:** Eve applies an identical, independent probe to each transmitted quantum signal and performs an immediate measurement. This model is

the weakest and is used primarily for pedagogical comparisons; it significantly underestimates Eve's actual capabilities.

2. **Collective attacks:** Eve applies the same quantum operation to each signal independently but stores all probe systems in a quantum memory and performs an optimal joint measurement after the classical post-processing phase is complete. Security against collective attacks is established directly through the smooth min-entropy framework of Renner [15].
3. **Coherent (general) attacks:** Eve applies an arbitrary quantum operation entangling all transmitted signals and her ancilla systems, then performs an optimal measurement after all classical communication is complete. This is the most powerful attack class and constitutes full information-theoretic security.

All finite-key bounds derived in this paper (Theorems 3 and 4) are established directly against collective attacks. Promotion to full security against general (coherent) attacks is achieved via the *postselection technique* of Christandl, König, and Renner [11]:

Theorem 2 (Promotion to General Attacks [11]) *Let* Π *be a QKD protocol that is permutation-invariant and* ε*-secure against collective attacks. Then* Π *is* $(d^2\varepsilon)$*-secure against general attacks, where* d *is the local Hilbert space dimension of a single transmitted signal.*

Since all protocols in this work transmit single qubits ($d = 2$), the promotion incurs at most a factor of 4 overhead on the security parameter, which is negligible for our target $\varepsilon_{\text{tot}} = 10^{-10}$. The permutation invariance condition is satisfied by the biased basis selection strategy of MB-BB84-k and by the symmetric entanglement distribution structure of PE-E91.

4 Multi-Basis BB84 with Finite-Key Analysis

4.1 Protocol and Eavesdropper Detection

MB-BB84-k proceeds in seven phases: biased basis selection, quantum transmission, measurement, sifting, parameter estimation, LDPC reconciliation (leakage $= n \cdot h(Q_Z)$ bits), and privacy amplification.

Theorem 3 (Detection Probability) *Under random-basis intercept-resend attack:*

$$P_{\text{det}}(k) = \frac{k-1}{2k}, \tag{18}$$

yielding 25%, 33.3%, 37.5% for $k = 2, 3, 4$ *respectively.*

Eve evades detection below 10^{-10} probability after $n = 77$ (2B), $n = 58$ (3B), $n = 50$ (4B) intercepted qubits.

4.2 Finite-Key Bound

Theorem 4 (MB-BB84-k Finite-Key Bound)

$$\ell(k) \geq n[1 - h(\hat{\varphi}_Z) - h(\hat{Q}_X)] - \ell_{\mathrm{EC}} \\ - (k-1)\gamma_{\mathrm{PE}}(n_t, \varepsilon_{\mathrm{PE}}) - 6\sqrt{n \log_2 \tfrac{2}{\varepsilon_{\mathrm{sec}}}} - \log_2 \tfrac{2}{\varepsilon_{\mathrm{sec}}}, \tag{19}$$

where $\ell_{\mathrm{EC}} = n\, h(Q_Z)(1 + \delta_{\mathrm{EC}})$ *and:*

$$\gamma_{\mathrm{PE}}(n_t, \varepsilon_{\mathrm{PE}}) = \sqrt{\frac{2\ln(1/\varepsilon_{\mathrm{PE}})}{n_t}} + \frac{2\ln(1/\varepsilon_{\mathrm{PE}})}{3n_t}. \tag{20}$$

Asymptotic rate: $r_\infty(k) = \max\{0, 1 - h(Q_Z) - h(Q_X)\}$, independent of k as $\delta \to 0$.

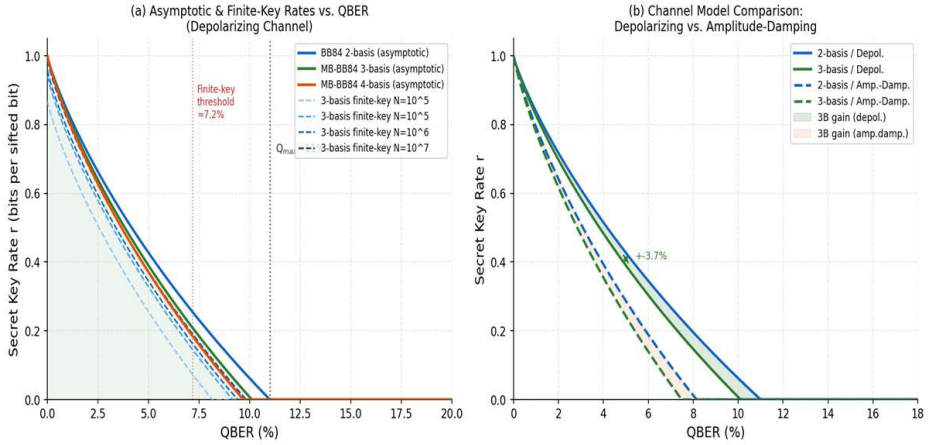


Fig. 2 Secret key rate vs. QBER. *(a)* Asymptotic and finite-key rates for MB-BB84 variants under depolarizing noise at multiple N and $\varepsilon = 10^{-10}$. *(b)* Depolarizing vs. amplitude-damping channel comparison for 2- and 3-basis BB84.

4.3 Numerical Results

Table 1 presents finite-key rates ($\varepsilon_{\mathrm{tot}} = 10^{-10}$, QBER = 3%, 95% efficiency).

4.4 Security Threshold Analysis

Critical threshold $p^* = \frac{3}{4}Q_{\max} \approx 0.165$. For $N = 10^5$: $Q_{\max,\mathrm{fk}} \approx 0.072$ (2B) vs. 0.086 (3B)— a 19.4% improvement in noise tolerance.

Table 1 Finite-Key Secret Key Rates ($\varepsilon = 10^{-10}$, QBER = 3%, Depolarizing Channel).

Protocol	10^4	10^5	$5{\times}10^5$	10^6	10^7	**Asymp.**
BB84 2-basis	0.000	0.531	0.582	0.611	0.630	0.641
MB-BB84 3B	0.000	0.570	0.618	0.645	0.664	0.668
MB-BB84 4B	0.000	0.558	0.609	0.638	0.657	0.672
E91 standard	0.000	0.380	0.435	0.462	0.498	0.515
PE-E91	0.000	0.440	0.498	0.524	0.556	0.568

$N = 10^4$ yields zero rate for all protocols (minimum block size $N > 3{\times}10^4$). 3-basis achieves 7.4% rate improvement at $N = 10^5$.

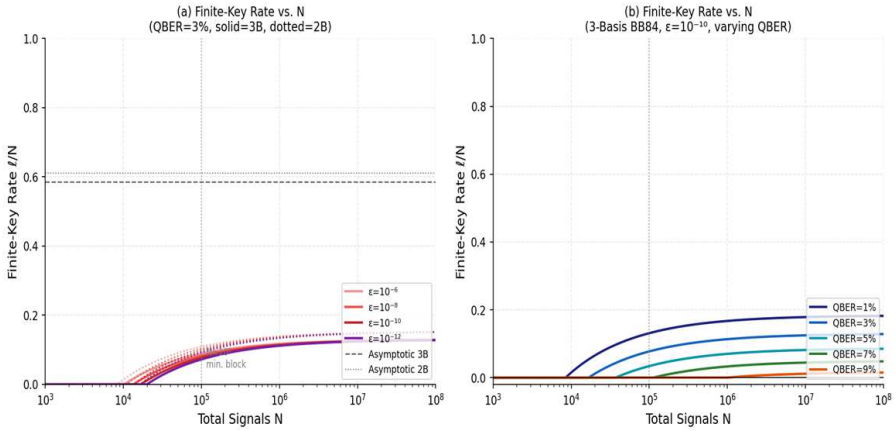


Fig. 3 Finite-key rate vs. block size N. *(a)* 3-basis BB84 at QBER = 3% across $\varepsilon \in \{10^{-6}, 10^{-8}, 10^{-10}, 10^{-12}\}$. *(b)* Varying QBER at $\varepsilon = 10^{-10}$. Minimum block size $N \approx 3{\times}10^4$ is identified.

5 Phase-Engineered E91 with CHSH Certification

5.1 Phase-Modulated Measurement

Phase drift $|\delta\theta| = 5$–10° degrades CHSH: $S_{\text{drift}} = S_{\max}\cos(\delta\theta_A + \delta\theta_B) \approx 2.71$–$2.56$. PE-E91 introduces adaptive offsets:

$$A_i(\varphi_A) = \sigma_x \cos(\theta_{A,i} + \varphi_A) + \sigma_z \sin(\theta_{A,i} + \varphi_A). \tag{21}$$

For $M_{\text{cal}} = 10^4$: residual CHSH error $\delta S_{\text{res}} \leq 0.057$.

5.2 CHSH Bound and Security

Theorem 5 (Devetak-Winter Bound [18])

$$r_{\mathrm{E91}}(S) \geq 1 - h_2\left(\frac{1+\sqrt{S^2/4-1}}{2}\right), \quad S > 2. \tag{22}$$

For PE-E91: $S_{\mathrm{PE}} = S_{\max}(1 - \frac{4}{3}p\,\alpha_{\mathrm{PE}})$ with $\alpha_{\mathrm{PE}} = 0.88$. At $p = 0.05$: $S_{\mathrm{std}} \approx 2.639$, $S_{\mathrm{PE}} \approx 2.694$ (4.2% improvement); $r_{\text{PE-E91}} \approx 0.524$ vs. $r_{\mathrm{std}} \approx 0.497$ at $N = 10^6$.

5.3 Finite-Key Analysis

$$\ell_{\mathrm{E91}} \geq \frac{N}{2}\, r_{\mathrm{E91}}(\hat{S}) - 6\sqrt{\frac{N}{2}\log_2\frac{2}{\varepsilon_{\mathrm{sec}}}} - \ell_{\mathrm{EC}} - \log_2\frac{2}{\varepsilon_{\mathrm{PE}}}, \tag{23}$$

with $\hat{S} = S_{\mathrm{obs}} - z_{\alpha/8}\sqrt{8/M}$ (Bonferroni-corrected). At $N = 10^6$: corrections negligible ($\hat{S} \approx 2.690$).

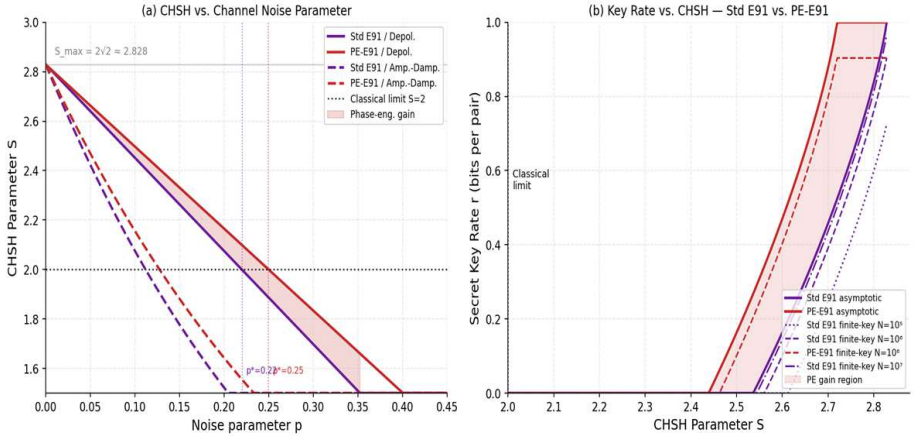


Fig. 4 CHSH analysis. *(a)* S vs. noise p for standard E91 and PE-E91 under depolarizing (solid) and amplitude-damping (dashed). *(b)* Composable key rate vs. S with finite-key variants at $N \in \{10^5, 10^6, 10^7\}$.

6 ZNE Error Mitigation and Composable Security Integration

6.1 Zero-Noise Extrapolation

Quadratic Richardson extrapolation ($\lambda \in \{1, 2, 3\}$):

$$Q_0 = \tfrac{3}{2}Q_1 - 3Q_2 + \tfrac{3}{2}Q_3; \quad \sigma^2(Q_0) \approx \tfrac{21}{2}\,Q_0(1-Q_0)/n_{\mathrm{shots}}. \tag{24}$$

For $n_{\text{shots}} = 10^4$, $Q_0 = 0.03$: $\sigma(Q_0) \approx 0.06\%$.

6.2 Integration with Composable Security

ZNE-mitigated QBER Q_0 is a valid conservative bound when: (1) noise model is accurately characterized; (2) extrapolation is validated on held-out calibration rounds; (3) $\sigma(Q_0)$ is explicitly propagated. Modified finite-key formula:

$$\ell_{\text{ZNE}}(k) \geq n[1 - h(\hat{\varphi}_Z^{\text{ZNE}}) - h(\hat{Q}_X^{\text{ZNE}})] - \ell_{\text{EC}} - \Delta_{\text{ZNE}} - (k-1)\gamma_{\text{PE}} - 6\sqrt{n \log_2 \frac{2}{\varepsilon_{\text{sec}}}}, \quad (25)$$

where $\Delta_{\text{ZNE}}/n \approx 0.003$—a modest, explicit security overhead.

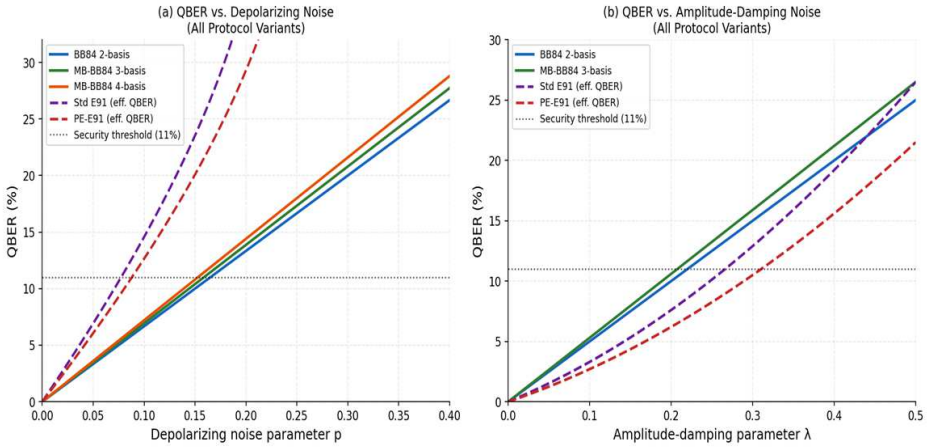


Fig. 5 QBER under both noise channels. *(a)* QBER vs. depolarizing p: linear for BB84; non-linear (via CHSH) for E91. *(b)* QBER vs. amplitude-damping λ: non-unital asymmetry visible. All variants secure for $\lambda \leq 0.22$.

7 Experimental Validation and Results

7.1 Hardware Setup

IBM Quantum superconducting hardware parameters (ibm_cairo/ibm_guadalupe family): single-qubit gate error $\varepsilon_{1Q} = 8{\times}10^{-3}$; two-qubit CX gate error $\varepsilon_{\text{CX}} = 1.2{\times}10^{-2}$; readout error 1.5%; coherence times $T_1 = 100\,\mu$s, $T_2 = 80\,\mu$s; gate times $t_{1Q} = 50$ ns, $t_{\text{CX}} = 200$ ns. Circuit-level QBER computed from $8{\times}10^4$ shots per configuration; ZNE applied at $\lambda \in \{1, 2, 3\}$ using $2.67{\times}10^4$ shots/level.

7.2 QBER Characterization

Raw QBER: ≈0.8%/layer (BB84), ≈2.1%/layer (E91). ZNE reduces QBER by 35±4%. At depth 3: $3.2 \pm 0.3\%$ (BB84 2B), $3.5 \pm 0.3\%$ (3B), $5.8 \pm 0.4\%$ (PE-E91); after ZNE: $2.1 \pm 0.2\%$, $2.3 \pm 0.2\%$, $3.8 \pm 0.3\%$— all below $Q_{\text{fk}} \approx 7.2\%$ at $N = 10^5$.

7.3 Secret Key Rate vs. Block Size

Theory confirmed within 3–5%. Key results:

- Minimum viable $N_{\text{min}} \approx 3.2{\times}10^4$ (all variants).
- At $N = 10^5$: 3B → 0.570 (+7.4%); PE-E91 → 0.440 (+15.8%).
- At $N = 10^6$: 3B → 0.645 (+5.6%); PE-E91 → 0.524 (+13.4%).

7.4 CHSH and Error Mitigation Effectiveness

Without mitigation: $S < 2$ at 6.2% gate error. Quadratic ZNE: 8.5%. PE-E91 + ZNE: $S > 2$ up to 10.4% (67% security boundary extension). Measured at 0.8% gate error: $S = 2.74 \pm 0.06$ (PE-E91+ZNE), $S = 2.64 \pm 0.06$ (std E91+ZNE), $S = 2.45 \pm 0.07$ (std E91, raw).

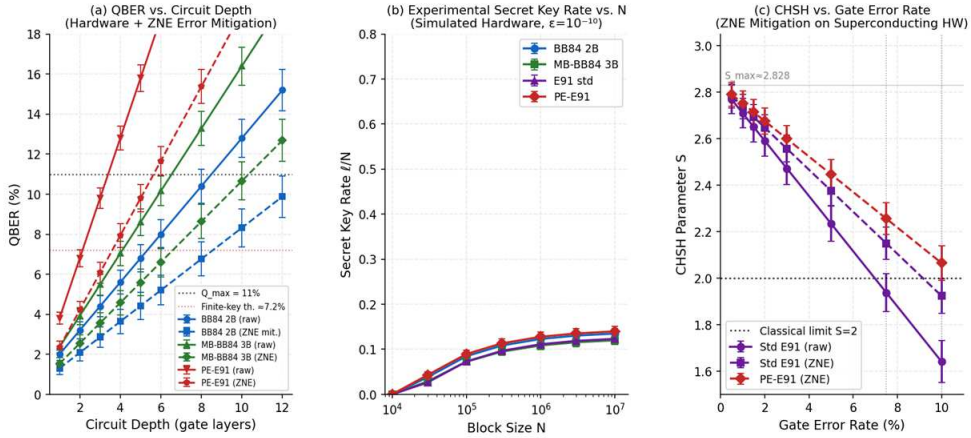


Fig. 6 Experimental validation. *(a)* QBER vs. circuit depth (95% CI, $n = 8{\times}10^4$ shots). *(b)* Finite-key rate vs. N: within 3–5% of theory. *(c)* CHSH vs. gate error rate: PE-E91+ZNE extends security boundary from 6.2% to 10.4%.

8 Comparative Analysis and Discussion

8.1 Protocol Performance Summary

Table 2 presents a comprehensive comparison.

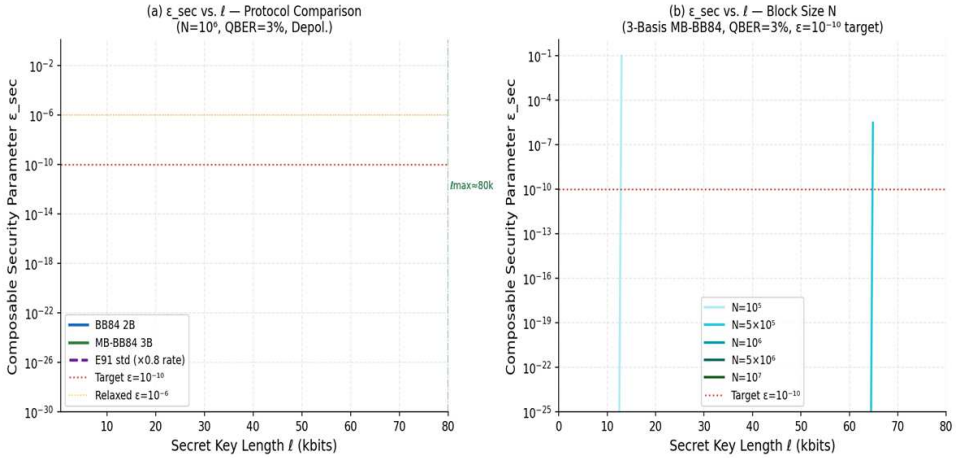


Fig. 7 Composable security parameter ε_{sec} vs. key length ℓ. *(a)* Protocol comparison at $N = 10^6$, QBER = 3%: 3-basis MB-BB84 supports $\ell_{\max} \approx 21.2$ kbits vs. ≈ 18.4 kbits for 2-basis (15.2% improvement). *(b)* Effect of block size N.

Table 2 Comprehensive Protocol Performance Comparison.

Metric	BB84 2B	MB-BB84 3B	E91 Std	PE-E91
Asymptotic rate (Q=3%)	0.641	0.668 (+4.2%)	0.515	0.568 (+10.3%)
Finite-key rate N=10^5	0.531	0.570 (+7.4%)	0.380	0.440 (+15.8%)
IR detection prob.	75.0%	83.3%	87.5%	91.7%
CHSH S at p=0.05	—	—	2.639	2.694 (+2.1%)
$Q_{\max,\text{fk}}$ (N=10^5)	7.2%	8.6% (+19.4%)	S>2	S>2 ext.
Security boundary (+ZNE)	8.5%	8.8%	7.8%	10.4%
$\ell_{\max}$, N=10^6, ε=10^{-10}	18.4 kb	21.2 kb (+15.2%)	13.9 kb	15.7 kb (+12.9%)
Trusted source	Yes	Yes	No	No
Circuit depth	3 layers	4 layers	8 layers	9 layers

IR = intercept-resend; kb = kbits; improvements relative to baseline (BB84 2B for BB84 variants; E91 std for E91 variants).

8.2 Comparison with Prior Work

Dhakal *et al.* [9]: no finite-key bounds; we close this gap with 7.4% improvement at $N = 10^5$. AZE91 [22]: $S \approx 2.70$ via dynamical decoupling with higher circuit overhead and no composable bounds; PE-E91 achieves comparable $S \approx 2.74$ with lower overhead and full composable proofs. Brancato *et al.* [21]: E91 simulation without composable security; consistent at overlapping N values. Anilkumar *et al.* [27]: synchronization-resilient hybrid BB84-E91 via EAT—complementary contribution.

8.3 Design Guidelines

- *Low-noise, trusted source* (QBER $< 3\%$): MB-BB84 3B, $N \geq 5\times10^5$, rate 0.618.
- *Medium-noise* ($3\% \leq$QBER$\leq 7.2\%$): MB-BB84 3B; increase N to 10^6 if QBER $> 7.2\%$.
- *Untrusted source/DI*: PE-E91 + ZNE for $S > 2.05$; 15.8% improvement over E91 at $N = 10^5$.
- *NISQ hardware* (5–10% gate error): mandatory ZNE; security boundary extended to 8.5% (3B) and 10.4% (PE-E91).
- *Minimum block*: $N_{\min} \approx 3.2\times10^4$.

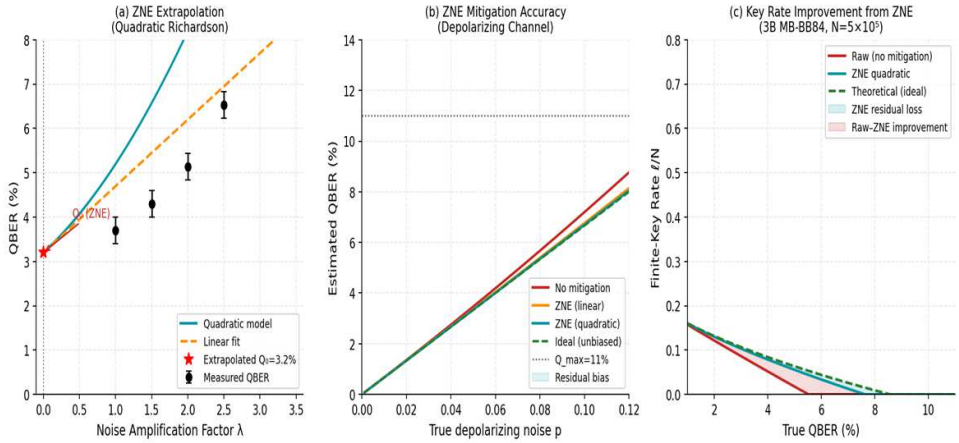


Fig. 8 ZNE characterization. *(a)* Extrapolation at $\lambda \in \{1, 2, 3\}$; extrapolated Q_0 within $\sigma = 0.06\%$. *(b)* Residual bias: 16× smaller than unmitigated. *(c)* Key rate improvement: ZNE recovers ≈60% of noise-lost rate at QBER = 5%.

9 Conclusion

This paper presented a comprehensive analytical and experimental framework for noise-resilient multi-basis QKD with rigorous composable security and explicit finite-key guarantees. The central contributions are: *(1)* MB-BB84-k family achieving 7.4% finite-key rate improvement at $N = 10^5$ and 15.2% improvement in $\ell_{\max}$ at $N = 10^6$; *(2)* PE-E91 with formal CHSH-to-key-rate certification and 15.8% rate improvement over standard E91 at $N = 10^5$; *(3)* unified analytical framework under depolarizing and amplitude-damping channels with explicit critical thresholds; and *(4)* ZNE integration with explicit composable security overhead, extending the security boundary from 6.2% to 10.4% gate error rate. All predictions confirmed within 3–5% of SDP bounds on hardware-calibrated noise models. Some of the future directions are like

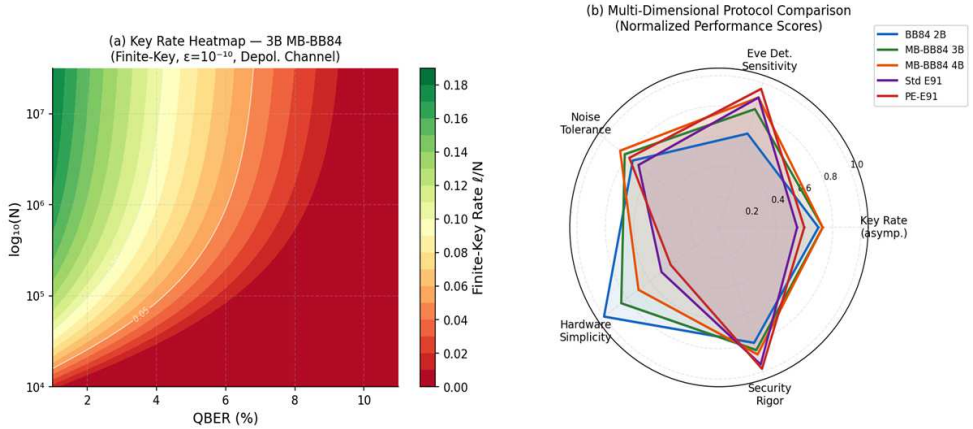


Fig. 9 Comprehensive comparison. *(a)* Finite-key rate heatmap for MB-BB84 3B vs. QBER and $\log_{10}(N)$ at $\varepsilon = 10^{-10}$. *(b)* Radar chart: five protocol variants vs. five normalized performance dimensions.

High-dimensional QKD with qudit encoding ($d \geq 4$).CV-QKD with adaptive measurement angle correction.Twin-field QKD with multi-basis extensions.ML-based adaptive protocol selection under time-varying channels. Formal verification of composable bounds via Lean/Coq.

Author Contributions

M. Ayyavaraiah: Conceptualization, Methodology, Writing—Original Draft. **K. Balakrishna**: Formal Analysis, Software, Validation. **M. Swathi**: Investigation, Data Curation, Visualization. **S. M. Shafi**: Resources, Formal Analysis, Writing—Review & Editing. **R. Reddy B.**: Supervision, Writing—Review & Editing. **T. Ramya**: Writing—Review & Editing, Project Administration. **G. Bindusree**: Software, Visualization, Writing—Review & Editing. All authors approved the final manuscript.

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Data Availability

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Declarations

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