Online Resource 2

Title: Coordinate transformation for the dynamic model of the human-exoskeleton system.

Description: Additional material provides the detailed derivation of the transformations between relative and absolute coordinate frames used in the dynamic model. It includes the mathematical relationships required to convert joint angular positions and the corresponding motor torques between both representations. This information supports reproducibility, clarifies the transformation procedure, and facilitates the implementation of the proposed modeling framework.

Article: Dynamic modeling of human–exoskeleton interaction: A simulation framework for gait rehabilitation.

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Authors: Sergey González-Mejía and José Miguel Ramírez-Scarpetta

Corresponding author: Universidad del Valle, sergey.gonzalez@correounivalle.edu.co

Coordinate frame transformation for the dynamic model of the human-exoskeleton system

The dynamic model of the human-exoskeleton system can be described by the relative or absolute angular positions of the corresponding joints. The difference lies in the size of the mathematical expressions that describe the movement and the relationship in terms of motor torques. For the biped in a double support phase, Fig. 1, the angular deviations with respect to two coordinate frames, one frame is with respect to the vertical axis indicating an absolute angular deviation, and the other frame is based on the angular deviation with respect to the adjacent segment, in this case, the top or bottom segment.

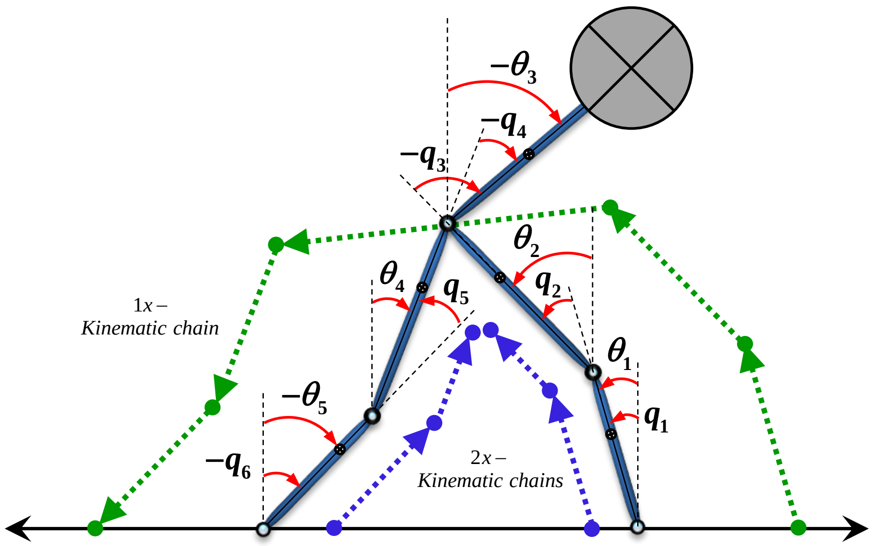

Fig. 1 Biped in double support phase with two coordinate frames

Where $i = 1\ to\ 6$ correspond to the right ankle, right knee, right hip, left hip, left knee, and left ankle, respectively; $q_i = [q_1, q_2, q_3, q_4, q_5, q_6]^T$ represents the vector of relative angular positions; $\tau_{q_i} = [\tau_{q_1}, \tau_{q_2}, \tau_{q_3}, \tau_{q_4}, \tau_{q_5}, \tau_{q_6}]^T$ the motor torques of the six joints of the biped, and $\tau_{m_i} = [\tau_{m_1}, \tau_{m_2}, \tau_{m_3}, \tau_{m_4}, \tau_{m_5}, \tau_{m_6}]^T$ the motor torques associated with relative angular positions. Similarly, $j = 1\ to\ 5$ denote the right ankle, right knee, hips, left knee, and left ankle, respectively; $\theta_j = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5]^T$ the vector of absolute angular positions; and $\tau_{\theta_j} = [\tau_{\theta_1}, \tau_{\theta_2}, \tau_{\theta_3}, \tau_{\theta_4}, \tau_{\theta_5}]^T$ the motor torques corresponding to absolute angular positions.

The dynamic model with respect to relative angular positions is described by Equation (1). When the variables θ_i are expreseed in terms of the q_i, two kinematic chains are generated – two blue lines in Fig. 1, Equation (2):

$$M(q)\ddot{q} + C(q,\dot{q})\dot{q} + G(q) = \tau_q \tag{1}$$

$$\begin{array}{ll} \textit{Kinematic chain No.}1 & \textit{Kinematic chain No.}2 \\ \begin{cases} \theta_1 = q_1 \\ \theta_2 = q_1 + q_2 \\ \theta_3 = q_3 + q_2 + q_1 \end{cases} & \begin{cases} \theta_5 = q_6 \\ \theta_4 = q_6 + q_5 \\ \theta_3 = q_6 + q_4 + q_5 \end{cases} \end{array} \tag{2}$$

For absolute angular positions, the dynamic model is described by Equation (3). If the variables q_i are expreseed in terms of the θ_i, one kinematic chains is generated – one green line in Fig. 1, Equation (4).

$$M(\theta)\ddot{\theta}+C(\theta,\dot{\theta})\dot{\theta}+G(\theta)=\tau_{\theta} \tag{3}$$

$$\begin{cases} q_1=\theta_1 \\ q_2=\theta_2-\theta_1 \\ q_3=\theta_3-\theta_2 \\ q_4=\theta_3-\theta_4 \\ q_5=\theta_4-\theta_5 \\ q_6=\theta_5 \end{cases} \tag{4}$$

Therefore, this procedure allows transformation between coordinate frames corresponding to the angular positions at the joints, q and θ, as required in the dynamic model. Based on the method proposed in [1], this research extends the approach to enable the transformation of the joint torques in terms of τ_q and τ_θ. From the expression $\tau_{\theta_i}=\sum_{j=1}^{6}\tau_{q_j}\left(\partial_{q_j}/\partial_{\theta_i}\right)$; $i=1,2,\dots5$ and Equation (4), the transformation of the coordinate frame from relative, τ_q, to absolute torques, τ_θ, is obtained. Therefore, a system of 5–Equations with 6–Unknowns is derived, Equation (5).

$$\begin{cases} \tau_{\theta_1}=\tau_{q_1}-\tau_{q_2} \\ \tau_{\theta_2}=\tau_{q_2}-\tau_{q_3} \\ \tau_{\theta_3}=\tau_{q_3}+\tau_{q_4} \\ \tau_{\theta_4}=-\tau_{q_4}+\tau_{q_5} \\ \tau_{\theta_5}=-\tau_{q_5}+\tau_{q_6} \end{cases} \tag{5}$$

From the Equation (5), it is possible to transform the coordinate frame of the absolute to relative torques if it is assumed that $\tau_{q_3}=\tau_{\theta_3}$ and $\tau_{q_4}=\tau_{\theta_3}$, Equation (6).

$$\begin{cases} \tau_{q_1}=\tau_{\theta_1}+\tau_{\theta_2}+\tau_{\theta_3} \\ \tau_{q_2}=\tau_{\theta_2}+\tau_{\theta_3} \\ \tau_{q3}=\tau_{\theta_3} \\ \tau_{q4}=\tau_{\theta_3} \\ \tau_{q_5}=\tau_{\theta_4}+\tau_{\theta_3} \\ \tau_{q_6}=\tau_{\theta_5}+\tau_{\theta_4}+\tau_{\theta_3} \end{cases} \tag{6}$$

For the transformation of the coordinate frame from absolute to relative torques, two mathematical relations are used, $\tau_{q_i}=\sum_{j=1}^{3}\tau_{\theta_j}\left(\partial_{\theta_j}/\partial_{q_i}\right)$; $i=1,2,3$ and $\tau_{q_i}=\sum_{j=5}^{3}\tau_{\theta_j}\left(\partial_{\theta_j}/\partial_{q_i}\right)$; $i=4,5,6$, also, including Equation (2). Therefore, a system of 6-Equations with 5-Unknowns is derived, Equation (6), without assuming $\tau_{q_3}=\tau_{\theta_3}$ and $\tau_{q_4}=\tau_{\theta_3}$. From Equation (6), also it is possible to transform the coordinate frame from relative to absolute torques if it is assumed that $\tau_{\theta_3} = \tau_{q_3} + \tau_{q_4}$, then getting Equation (5).

In summary, the transformations between coordinate frames for both angular positions and torques will allow a better analysis according to the established purposes. For this research, the coordinate frame that is described by Equation (7) is used, which is expressed in terms of motor torques.

$$M\left(q_m\right)\ddot{q}_m + C\left(q_m, \dot{q}_m\right)\dot{q}_m + G(q_m) = \tau_{m_i} \tag{7}$$

References

1. Tzafestas, S., Raibert, M., Tzafestas, C.: Robust sliding-mode control applied to a 5-link biped robot. J. Intell. Robot. Syst. 15, 67–133 (1996). https://doi.org/10.1007/BF00435728