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Superluminal Propagation of Light and Anomalous Phase Shift in a Λ -Type Atomic Medium

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Abstract

Light-matter interaction between a light field and an atomic medium could be used to manipulate the medium's dispersive properties, leading to subluminal and superluminal regimes. Utilizing the same idea, this work investigates the propagation of structured superluminal light and anomalous phase shifts in a Λ -type three-level atomic medium, where a structured control field governs the atomic system. The absorption and dispersion properties are modulated by the azimuthal quantum number (ℓ) of the control field. The maxima and minima of the absorption spectrum, along with the corresponding normal and anomalous dispersion peaks, are enhanced with a (2ℓ) dependence on the azimuthal quantum number in the medium. The negative group index and anomalous phase shift vary within the ranges $-1000 \leq n_g \leq -900$ and $-6.5 \times 10^5 \leq \phi \leq -5.6 \times 10^5$ rad, respectively. The corresponding group velocity lies in the range $-3.33 \times 10^5 \leq v_g \leq -3.0 \times 10^5$ m/s within the spatial region $-2\lambda \leq x, y \leq 2\lambda$. Furthermore, the anomalous phase shift is significantly enhanced in regions exhibiting superluminal behavior, following the (2ℓ) azimuthal quantum number dependence. The findings of this work may have potential applications in optical storage devices, imaging, coding, and the design of advanced photonic technologies.

Keywords: Superluminal propagation, Anomalous phase shift, Density matrix formalism, Λ -type atomic system, Nonlinear medium.

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1. Introduction

Superluminal and subluminal light propagation describe situations where the group velocity of light traveling through a medium becomes either greater than (superluminal) or less than (subluminal) the speed of light in a vacuum (c). These phenomena arise from changes in the refractive index and dispersion characteristics of the medium through which light propagates. In recent years, there has been growing interest in exploring both extremely fast (superluminal) and extremely slow (subluminal) light propagation, particularly with respect to the group velocity. This field has attracted considerable attention due to its significant applications in nonlinear optics [1, 2], optical switching [3], quantum communication [4], and enhanced sensitivity [5, 6, 7]. Numerous research efforts have focused on controlling the propagation of subluminal and superluminal light by adjusting various physical parameters [8].

The parameters of the driving field can effectively control the group velocity of light, enabling it to propagate either faster or slower than its vacuum speed [9]. Arif and his colleagues [10] explored this concept by investigating the propagation of light pulses ranging from extremely slow to exceptionally fast, achieved by varying the intensity of a Laguerre field, a structured light field characterized by orbital angular momentum within a four-level atomic system. Similarly, Zaman and his research team [11] analysed the subluminal (slower-than-light) and superluminal (faster-than-light) behaviour of light beams by employing three probe fields in a five-level atomic configuration. In another significant contribution, Tarhan [12] examined both ultrafast and ultraslow light propagation in an optomechanical medium that exhibits electromagnetically induced transparency (EIT). This quantum phenomenon renders otherwise opaque materials transparent to specific probe light frequencies. This investigation utilized both a coupling laser and a probe laser to explore the resulting light-matter interactions. Further studies have implemented Laguerre Gaussian (LG) beams, a type of laser beam with a spiral phase structure, to investigate subluminal and superluminal propagation within four-level atomic configurations, focusing on how light transmission correlates with group velocity in a dispersive medium, where light speed varies with frequency [13, 14]. A theoretical model based on the density-matrix formalism has also been developed to control the propagation speed of light pulses, thereby enabling smooth transitions between subluminal and superluminal regimes in an N-type four-level atomic system [15]. Research in this field has further demonstrated the remarkable tunability of light velocity, with experiments reporting propagation speeds ranging from 90.9 m/s to 799.78 m/s [16]. Such investigations span a wide range of quantum systems, including quantum dots [17], graphene [18], quantum circuits [19], atomic media [20, 21], Fermi gases, and photonic crystals [22], highlighting the interdisciplinary signifi-

cance of controlling light propagation at both subluminal and superluminal scales.

In quantum physics, structured light is light whose electromagnetic field has been deliberately manipulated or designed to form waveforms other than plane waves. Such light has been of greater control in quantum studies and experiments due to its intricate spatial and phase structure, orbital angular momentum, and exotic polarization properties [23]. In classical optics, light is generally characterized by its wavelength, amplitude, phase, and polarization. Structured light, however, extends this concept by enabling the deliberate shaping and control of light into specific forms or patterns. These patterns, such as grids, dots, or lines, go beyond a simple uniform beam, allowing for sophisticated spatial arrangements of light. The real-life applications of such lighting patterns are pervasive. They can be used to provide examples in 3D scanning, where they assist sensors and cameras in scanning and interpreting the shapes of various objects, and in entertainment, where they produce amazing visual effects in laser shows. In short, structured light refers to the process of shaping light into a specific pattern to enhance visual appeal. In the early years of the 21st century, several new devices were developed, including digital micromirrors [24, 25], liquid crystal spatial light modulators [26], and more advanced geometric phase components such as quarter-wave plates (q-plates) [27], along with other spin-orbit systems [28, 29, 30]. These innovations enabled most laboratories to generate structured light directly from the source [31] or tailor it for specific applications. Over the last two to three decades, the concepts of sculpted, molded, shaped, or structured light [32, 33, 34, 35] have become essential to optical research. Specialties such as data processing, micro-scale manipulation, optical sorting, accurate optical delivery, and the creation of optically controlled micromachines have advanced to a very high level. Among the most prominent is the application of light beams with orbital angular momentum, which not only has fundamental and practical significance [36, 37, 38] but has also spurred extensive research in quantum optics. One of the earliest to write about the concept of structured light was Thomas Young [39], more than two centuries ago. In 1803, he demonstrated that plane light waves could interfere with one another when passing through a pair of slits in a cardboard sheet, creating fringes, a one-dimensional representation of light's intensity structure. As explained by Young, the arrangement and the distance between these fringes were substantial evidence of the nature of light as a wave. These principles formed the basis of work on interference patterns, which has since become the basis of interferometry, a technique that not only has given rise to work in fields like structural engineering and medical imaging, but also to the understanding of photon behaviour and the observation of gravitational waves. Robert Wood noted, some hundred years ago, that polarized fringes could be produced when two beams of light with perpendicular polarizations interfered

[40]. Light shaping is an operation with a long history, almost as long as that of lasers [41, 42, 43]. The first works on this topic, employing computer-generated holograms and diffractive optics, were the most significant [44, 45]. Since structured light is highly efficient at capturing three-dimensional surfaces, it is widely used across many fields. Organized light systems are commonly used to scan objects and generate precise 3D models, which are helpful for digital and industrial inspection and reverse engineering. Moreover, facial recognition systems use structured light to capture the finer geometries of an individual's 3D face. A popular one is Apple's Face ID, which uses an infrared projector with structured light to map facial features in 3D. It is also relevant in robotic systems, where structured light is used to improve perception and maneuverability by capturing 3D information. This technology can be used in virtual reality (VR) to provide accurate motion tracking and spatial mapping. It has found application in medicine for detailed surface imaging of the human body, supporting dental scanning and orthopaedic measurements. The idea of structuring can be generalized to light rays in arbitrary directions and even to other types of waves, yielding controlled patterns as many modes interact within a system. There is a broader discussion of the use of structured light, including imaging, medical diagnostics and therapy, optical communications, quantum computing, and light-matter interactions [46].

Despite the significant advancements that have been made in the research of structured light and optical vortices, the current research has concentrated chiefly on the study of intensity and phase distributions but has devoted little attention to how the complex susceptibility (χ) of the medium, specifically the real $\text{Re}(\chi)$ and imaginary ($\text{Im}(\chi)$) parts affect the group index (n_g), and propagation properties of light through dispersive or resonant mediums. The complex relationship between dispersion, absorption, and topological beam characteristics, such as the azimuthal quantum number (ℓ), has not been well investigated. Moreover, the full correlation between the microscopic medium's responses and the macroscopic field distributions has not been fully determined. Such an inability to achieve integration limits the ability to control and tune structured beams required for new photonic and quantum technologies with precision.

The current investigation is motivated by the need to develop a coherent theoretical and visual model that is quantitative in its relation to the medium's complex susceptibility to the active behavior of structured light. This work will attempt to clarify how the material's dispersion and absorption impact the phase evolution, energy transport and vortex dynamics in optical fields by concurrently analyzing the functions of $\text{Re}(\chi)$, $\text{Im}(\chi)$, group index (n_g) and phase shift (ϕ) with higher azimuthal quantum numbers (ℓ). The lessons learned from this strategy are intended to drive technological breakthroughs in the design of engineered,

structured beams with highly controlled phase and propagation characteristics, leading to new developments in slow-light devices, nonlinear photonic modulation, and vortex-enabled optical information processing.

This work is organized as follows: In Sec. 2, we explain the system model and its working principle along with the Hamiltonian and analytical calculations. In Sec. 3, we discuss the numerical results based on the real and imaginary parts of susceptibility and superluminal propagation of light and anomalous phase shift in a Λ -Type atomic medium. In the last section, we present our concluding remarks.

2. Theoretical Model and Dynamics

In this section, we investigate the Λ -type cesium atomic medium for subluminal and superluminal light propagation. The proposed atomic configuration consists of a three-level Λ -type medium driven by a probe and a structured control field to manipulate superluminal propagation and anomalous phase shifts as shown in Fig. 1. In our proposed atomic system, as depicted in Fig. 1, the electric probe field having Rabi frequency δ_p establishes a connection between states $|2\rangle$ and $|3\rangle$, while the structured control field, operating at a Rabi frequency δ_1 , links states $|1\rangle$ and $|3\rangle$. The energy decay rates from state $|3\rangle$ to states $|1\rangle$ and $|2\rangle$ are represented by γ_{31} and γ_{32} , respectively. The probe and the structured control fields are detuned from their respective resonance frequencies expressed as $\zeta_p = \omega_p - \omega_{23}$ and $\zeta_1 = \omega_1 - \omega_{13}$, where ω_p and ω_1 are the angular resonance frequencies of the probe and structured control fields, respectively. The self and interaction Hamiltonian for the proposed system is given by [47]:

$$H_0 = \hbar[\omega_1|1\rangle\langle 1| + \omega_2|2\rangle\langle 2| + \omega_3|3\rangle\langle 3|] \quad (1)$$

$$H_{int} = -\frac{\hbar}{2} \left[\delta_1 e^{-i\zeta_1 t} |1\rangle\langle 3| + \delta_p e^{-i\zeta_p t} |2\rangle\langle 3| \right] + H.c., \quad (2)$$

where δ_1 and δ_p are the Rabi frequencies of the structured control and probe fields, respectively, and $H.c.$ represents the Hermitian conjugate. The optical Bloch equations, which govern the density-matrix dynamics of an atomic system interacting with electromagnetic fields, determine its temporal evolution. The three-level atomic system's optical Bloch equations are expressed as follows [48]

$$\frac{d}{dt}(\rho) = -\frac{i}{\hbar}[H_{int}, \rho] - \frac{1}{2} \sum_{ij} \gamma_{ij} (\eta_i \eta_j^\dagger \rho + \rho \eta_i \eta_j^\dagger - 2\eta_i^\dagger \rho \eta_j). \quad (3)$$

Here $\eta_j^\dagger$ and η_i represent creation and annihilation operators, while ρ denotes the density matrix operator. Simplifying the Master equation by introducing the explicit time independence in the form $\rho_{ij} = \tilde{\rho}_{ij} e^{-i\zeta_{ij} t}$,

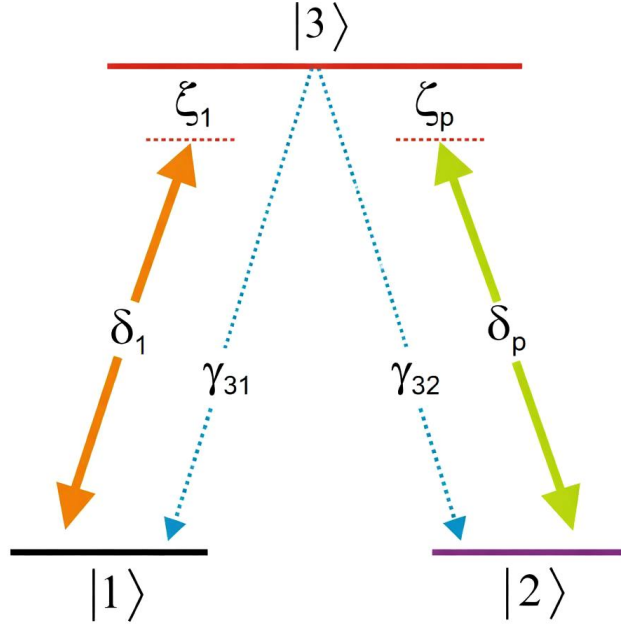


Figure 1: Λ -type three-level atom driven by a weak probe field δ_p of frequency ω_p and a strong structured light δ_1 of frequency ω_1 . γ_{32} and γ_{31} are the decay rates from the excited level $|3\rangle$ to lower levels $|2\rangle$, and $|1\rangle$, whereas ζ_p and ζ_1 are the probe and structured light detunings, respectively.

we obtained the following two coupled rate equations given below [48]:

$$\frac{d}{dt}(\tilde{\rho}_{23}) = \left(i\zeta_p - \frac{\gamma_{23}}{2}\right)\tilde{\rho}_{23} + \frac{i}{2}(\tilde{\rho}_{33} - \tilde{\rho}_{22})\delta_p - \frac{i}{2}\delta_1\tilde{\rho}_{21}, \quad (4)$$

$$\frac{d}{dt}(\tilde{\rho}_{21}) = \left[i(\zeta_p - \zeta_1) - \frac{1}{2}(\gamma_{31} + \gamma_{32})\right]\tilde{\rho}_{21} - \frac{i}{2}\delta_1^*\tilde{\rho}_{23} + \frac{i}{2}\delta_p\tilde{\rho}_{31}. \quad (5)$$

The initial elements of the density matrix are determined by using perturbation theory, under the assumption that the population in the ground state $|2\rangle$ is approximately 1, i.e., $(\tilde{\rho}_{22}^{(0)} \approx 1)$. In contrast, the population of the atoms in other states is assumed to be zero, i.e., $(\tilde{\rho}_{11}^{(0)} = \tilde{\rho}_{33}^{(0)} = 0)$. Subsequently, the resulting system of equations is solved using the matrix inversion method [49]:

$$W = -Q^{-1}B, \quad (6)$$

where W and B are column matrices of order 2×1 , and Q is a square matrix of order 2×2 . The susceptibility is a complex function that connects the absorption and dispersion profiles of the probing field to the imaginary and real parts of the complex susceptibility χ , respectively. The polarization of the atomic medium under consideration is denoted by the expression $p = \epsilon_0\chi E$, which helps to illustrate the susceptibility of the

atomic medium. The polarization is defined as $P = |\xi_{23}|^2 \tilde{\rho}_{23}$ where ξ_{23} is the dipole matrix element, and ρ_{23} refers to the coherence of the probe field, indicating how the field interacts with the atomic medium and is given. The coherence of the associated probe and the dipole matrix element between the ground state $|2\rangle$ and the excited state $|3\rangle$ are

$$\tilde{\Lambda}_{23} = \frac{-i(\gamma_{31} + \gamma_{32} + 2i(\zeta_1 - \zeta_p))\delta_p}{|\delta_1|^2 + (\gamma_{31} + \gamma_{32} + 2i(\zeta_1 - \zeta_p))(\gamma_{32} + 2i\zeta_p)}, \quad (7)$$

$$\xi_{23} = \sqrt{\left(\frac{3\gamma_{32}\hbar\epsilon_0\lambda^3}{8\pi^2}\right)} \quad (8)$$

The optical susceptibility χ defines the response of the atomic medium to the probe field and is obtained by the comparison of two polarizations, which is expressed as follows:

$$\chi = -\frac{2n\xi_{23}^2}{\epsilon_0\hbar\delta_p}, \quad (9)$$

where n is the atomic density per m^{-3} , $\hbar$ is the reduce Plank constant ϵ_0 is the permittivity of free-space and ξ_{23} is dipole matrix element and determined from Einstein coefficients. The coupling strength may vary spatially with the structured light, making the electric susceptibility depend on x , y , and z . This would indicate that susceptibility varies with position rather than being uniform. This implies an anisotropic nature, with susceptibility varying across the medium, resulting in spatially non-uniform polarization responses. The group index n_g , which characterizes superluminal propagation, is given by

$$n_g = n_r + \omega_0 \frac{\partial n}{\partial \zeta_p}, \quad (10)$$

where ω_0 is the carrier frequency of the probe field. Subluminal propagation generates a normal phase shift and superluminal propagation generates an anomalous phase shift ϕ which is given by;

$$\phi = \frac{2\pi}{\lambda} n_g L \quad (11)$$

where L is the propagation length of the structured surface plasmon waves.

2.1. Laguerre-Gaussian Field Configuration

To study superluminal propagation and anomalous phase shifts in structured light carrying orbital angular momentum (OAM), and their dependence on the azimuthal quantum number, we model the control field using a Laguerre-Gaussian (LG) mode. This configuration introduces both radial and angular dependence via the azimuthal angle θ and transverse distance r , modulated by a Gaussian envelope. The LG beam

specifically addresses transitions involving the G_1 state in the atomic model. The spatially varying Rabi frequency is given by [50]:

$$\delta_1(r, \theta) = \delta_0 \left(\frac{r}{R} \right)^\ell e^{i\ell\theta} e^{-r^2/R_1^2}, \quad (12)$$

where δ_0 is the uniform amplitude of the Rabi frequency, ℓ is the azimuthal quantum number corresponding to the OAM of the beam, and R and R_1 denote the beam waist and Gaussian envelope radius, respectively. The azimuthal phase term $e^{i\ell\theta}$ encodes the helical phase structure of the OAM, while the radial factor $(r/R)^\ell$ produces a central vortex with vanishing intensity at $r = 0$. The angle θ is defined continuously over $0 \leq \theta < 2\pi$ using the four-quadrant arctangent:

$$\theta = 2 \tan^{-1} \left(\frac{y}{x + \sqrt{x^2 + y^2}} \right)$$

The spatial modulation of $\delta_1(r, \theta)$ directly imprints the intensity and phase profile of the LG beam onto the complex atomic susceptibility $\chi = \text{Re}(\chi) + i \text{Im}(\chi)$. Radially, the factor $(r/R)^\ell$ enlarges the dark vortex core with increasing ℓ , where $\delta_1 \rightarrow 0$ and quantum coherence is suppressed. In this core, atoms effectively behave as uncoupled two-level systems, showing strong absorption ($\text{Im}(\chi) \gg 0$) and normal dispersion.

Outside the core, in the bright annular region, the Rabi frequency is nonzero, restoring coherence. Azimuthally, the phase term $e^{i\ell\theta}$ causes a total phase accumulation of $2\pi\ell$ around the beam axis. In a phase-sensitive atomic medium, this generates ℓ alternating regions of constructive and destructive quantum interference along a single azimuthal loop. Regions with steep positive dispersion slopes ($\partial n / \partial \zeta_p > 0$) correspond to EIT phenomenon, where absorption is suppressed ($\text{Im}(\chi) \rightarrow 0$) and the group index is large and positive ($n_g \gg 1$), yielding subluminal propagation ($v_g < c$). Neighboring regions with steep negative dispersion slopes ($\partial n / \partial \zeta_p < 0$) exhibit anomalous dispersion, reducing the group index below unity ($n_g < 1$) or even making it negative ($n_g < 0$), enabling superluminal propagation ($v_g > c$) where the pulse envelope advances faster than the speed of light in vacuum.

3. Results and Discussion

In this section, we investigate the superluminal propagation of light and the anomalous phase shift associated with structured light fields in a three-level Λ -type cesium atomic system. Specifically, we demonstrate how the azimuthal quantum number ℓ governs the medium's overall optical response. The absorption spectrum, dispersion, negative group index, and anomalous phase-shift profiles are systematically analyzed. The

atomic decay rate is set to $\gamma = 2\pi \times 6.07$ MHz (corresponding to the cesium D_2 line), and all frequency-dependent parameters in our model are scaled relative to γ . The remaining physical parameters held constant throughout the numerical simulations are: atomic density $n = 10^{18}$ atoms/m³, central carrier frequency $\omega_0 = 1 \times 10^3 \gamma$, speed of light $c = 3 \times 10^8$ m/s, vacuum permittivity $\epsilon_0 = 8.854 \times 10^{-12}$ F/m, and the reduced Planck constant $\hbar = 1.054 \times 10^{-34}$ J · s.

In Fig. 2, the plots are traced for the absorption profile, as a function of normalized spatial coordinates ($x\lambda^{-1}$) and ($y\lambda^{-1}$) within a wavelength range of $(-2\lambda \leq x, y \leq +2\lambda)$, which gives a picture of the spatial variation of light matter interaction in the (x, y) plane. The imaginary part of the susceptibility is related to the absorption damping spectrum. Absorption profile exhibits periodic behavior with variation in $x\lambda^{-1}$ and $y\lambda^{-1}$. The probe beam produces structured light due to the periodic oscillation. At the azimuthal quantum number of the structured control field ($\ell = 1, 2$), the absorption profile exhibits two maxima and two minima. One maximum occurs at the boundary between the first and second quadrants, while the other occurs at the boundary between the third and fourth quadrants. The minima occur at the boundary between the first and third quadrants. At the same time, the other one appears at the border of the first and third quadrants, as depicted in Fig. 2a. The dipole-like patterns of the maxima and minima are a reflection of the azimuthal symmetry of the optical vortex, where the absorption oscillations are in mirror images. At the azimuthal quantum number of structured control field ($\ell = 2, 2\ell = 4$), the absorption profile demonstrates four maxima and four minima as shown in Fig. 2b. When the azimuthal quantum number of the structured control field is tuned to ($\ell = 3, 2\ell = 6$), the absorption profile reveals six maxima and six minima, as revealed in Fig. 2c. Finally, for the azimuthal quantum number of the structured control field ($\ell = 4, 2\ell = 8$), the absorption profile shows eight maxima and eight minima as shown in Fig. 2d.

In this λ -type atomic configuration, the ability to steer light propagation stems from the physical mechanism of coherent population trapping, leading to the formation of a dark state. When both the probe and structured control fields are tuned to exact resonance (zero detuning), the strong control field induces quantum coherence between the two lower ground states, trapping the atomic population in a coherent superposition known as a dark state. This destructive quantum interference suppresses probe absorption, producing Electromagnetically Induced Transparency along with a steep positive dispersion slope ($\frac{\partial n}{\partial \zeta_p} > 0$). As a result, the group index becomes large and positive ($n_g \gg 1$), leading to subluminal (slow) light propagation with $v_g < c$. On the other hand, superluminal propagation can be achieved by engineering the system to enter an anomalous dispersion regime, via, for instance, an absorption or gain doublet, where the dispersion

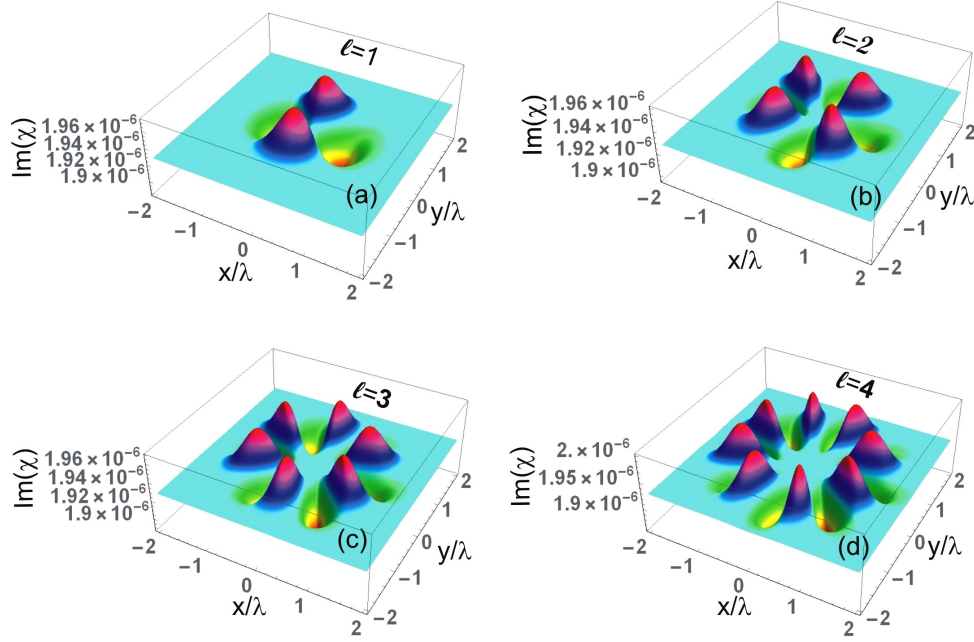


Figure 2: Spatial distribution of the atomic absorption, represented by the imaginary part of the susceptibility $\text{Im}(\chi)$, across the transverse coordinates $x\lambda^{-1}$ and $y\lambda^{-1}$ for azimuthal quantum numbers: (a) $\ell = 1$, (b) $\ell = 2$, (c) $\ell = 3$, and (d) $\ell = 4$. The plots illustrate the modulation of absorption maxima and minima. The chosen parameters are $\gamma_{32} = \gamma_{31} = 2\gamma$, $\zeta_1 = \zeta_2 = 0$, $\delta_0 = 1\gamma$, and a beam waist of $R = 0.8\lambda$.

slope becomes sharply negative ($\frac{\partial n}{\partial \zeta_p} < 0$). This negative slope pushes the group index below unity ($n_g < 1$) or into negative values ($n_g < 0$), causing the pulse envelope to travel superluminally ($v_g > c$) or exhibit a negative group velocity, while the information velocity remains fundamentally bounded by c .

In Fig. 3, the plots are traced for the dispersion profile, as a function of normalized spatial coordinates ($x\lambda^{-1}$) and ($y\lambda^{-1}$) within a wavelength range of $(-2\lambda \leq x, y \leq 2\lambda)$, which gives a picture of the spatial variation of light matter interaction in the $(x - y)$ plane. The fundamental part of susceptibility is related to the dispersion spectrum. The dispersion profile exhibits periodic behavior with variation in $x\lambda^{-1}$ and $y\lambda^{-1}$. The probe beam produces structured light due to the periodic oscillation. At the azimuthal quantum number of structured control field ($\ell = 1, 2\ell = 2$), the dispersion profile exhibits two normal peaks and two anomalous peaks. The two normal peaks occur at the first and third quadrants. In comparison, the anomalous peaks appear at the second and fourth quadrants as depicted in Fig. 3a. By the fine-tuning of the azimuthal quantum number of the structured control field ($\ell = 2, 2\ell = 4$), the dispersion profile reveals four normal peaks and four anomalous peaks as shown in Fig. 3b. Finally, at the azimuthal quantum number of structured control field ($\ell = 3, 2\ell = 6$), the dispersion profile represents six normal peaks and six anomalous peaks as

represented in Fig. 3c. For the azimuthal quantum number of structured control field ($\ell = 4, 2\ell = 4$), the dispersion profile demonstrates eight normal peaks and eight anomalous peaks as shown in Fig. 3d.

The increase in the number of spatial oscillations in both the real and imaginary parts of χ with higher azimuthal quantum number ℓ arises from the combination of azimuthal phase accumulation and the interference effects that depend on the relative phase of the atomic responses. The control beam in a structured light field carries an azimuthal phase factor $e^{i\ell\phi}$, so the optical phase varies by $2\pi\ell$ over one full rotation ($\phi = 0$ to 2π) around the propagation axis. In a phase-sensitive atomic medium, this local phase shift continuously alters the relative phase between the driving fields. Consequently, the interference condition alternates between constructive and destructive exactly ℓ times around the beam circumference. This cyclic modulation converts the otherwise smooth azimuthal intensity profile into a discrete pattern with ℓ petals, leading to a corresponding increase in the number of alternating extrema in absorption and dispersion proportional to ℓ .

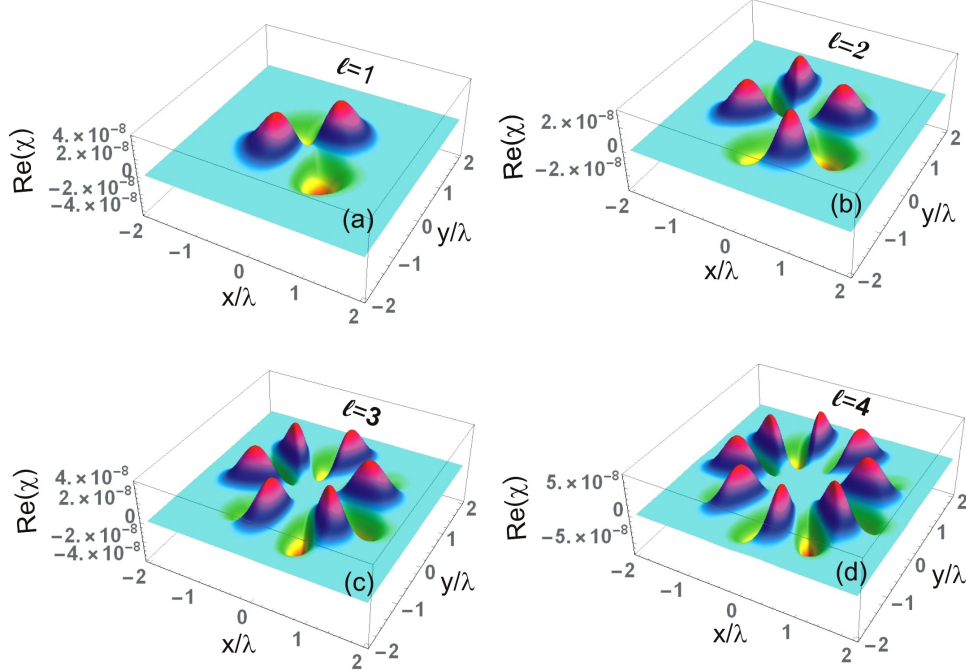


Figure 3: The real part of susceptibility as a function of the spatial coordinates ($x\lambda^{-1}$) and ($y\lambda^{-1}$) to reveal the absorption maxima and minima. The parameters have values given as: $\gamma_{32} = \gamma_{31} = 2\gamma$, $\zeta_1 = 0\gamma$, $\zeta_2 = 0\gamma$, $\delta_0 = 1\gamma$. The beam waist has a value of $R = 0.8\lambda$, and the azimuthal quantum number (a) $\ell=1$ (b) $\ell=2$ (c) $\ell=3$ (d) $\ell=4$.

In Fig. 4, the plots are traced for group index profile (n_g) as a function of spatial coordinates ($x\lambda^{-1}$) and ($y\lambda^{-1}$), within a wavelength range of $(-2\lambda \leq x, y \leq 2\lambda)$. The group index is related to the group velocity. The group index profile exhibits periodic behavior as a function of $x\lambda^{-1}$ and $y\lambda^{-1}$. The probe

beam produces structured light due to the periodic oscillation. when the group index is greater than 1 i.e. $[n_g > 1]$, then this is called subluminal propagation. In this case, the speed at which the pulse propagates will be less than the speed of light, i.e., $[v_g < c]$. On the other hand, when the group index is smaller than 1, i.e., $[n_g < 1]$, then this is called superluminal propagation. In this case, the speed at which the pulse propagates through the atomic medium appears to be faster than the speed of light, i.e., $[v_g > c]$. In Fig. 4a, we investigate a small, fast superluminal propagation behavior with a negative group index; this does not imply a violation of causality [51]. The cause of this effect is due to the peculiar dispersion properties of the medium, which causes this unusual behavior. Superluminal propagation is evidenced by a negative envelope of the wave (v_g), which yields a negative group index. This causes a negative delay time ($t_d = t_m - t_v$), in which (t_m) denotes the duration of the pulse in the medium, and (t_v) represents the duration of the pulse in vacuum. Here, the pulse takes less time to travel through the medium than through a vacuum. The negative envelope of the wave (n_g) therefore makes the pulse travel more rapidly through the medium, with a time in the medium less than in vacuum, though at the same distance (L). At the azimuthal quantum number of structured control field ($\ell = 1, 2\ell = 2$), the negative group index profile exhibits two superluminal maxima and two superluminal minima. One maximum occurs at the boundary between the first and fourth quadrants, while the other occurs at the boundary between the third and second quadrants. The minimum occurs at the boundary between the first and second quadrants. At the same time, the other one appears at the border of the third and fourth quadrants as depicted in Fig. 4a. For the azimuthal quantum number of the structured control field ($\ell = 2, 2\ell = 4$), the negative group index profile shows four maxima and four minima as shown in Fig. 4b. By the fine-tuning of the azimuthal quantum number of the structured control field ($\ell = 3, 2\ell = 6$), the negative group index profile reveals six maxima and four minima as shown in Fig. 4c. Finally, at the azimuthal quantum number of structured control field ($\ell = 4, 2\ell = 8$), the negative group index profile illustrates eight maxima and eight minima as demonstrated in Fig. 4d. Overall the group index profile varies between $(-900 \leq n_g \leq -1000)$, and the group velocity $v_g = \frac{c}{n_g}$ alters between $(-3.33 \times 10^5 \leq v_g \leq -3 \times 10^5)\text{m/sec}$ within the position range of $-2\lambda \leq x, y \leq 2\lambda$.

In Fig. 5, the plots are traced for the anomalous phase shift profile ($\phi = \frac{2\pi}{\lambda}n_gL$) as a function of spatial coordinates ($x\lambda^{-1}$) and ($y\lambda^{-1}$), within a wavelength range of $(-2\lambda \leq x, y \leq 2\lambda)$. The anomalous phase shift is associated with a negative group index. Anomalous phase shift profile exhibits periodic behavior with variation in $x\lambda^{-1}$ and $y\lambda^{-1}$. The probe beam produces structured light due to the periodic oscillation. At the azimuthal quantum number of structured control field ($\ell = 1, 2\ell = 2$), the anomalous phase shift profile

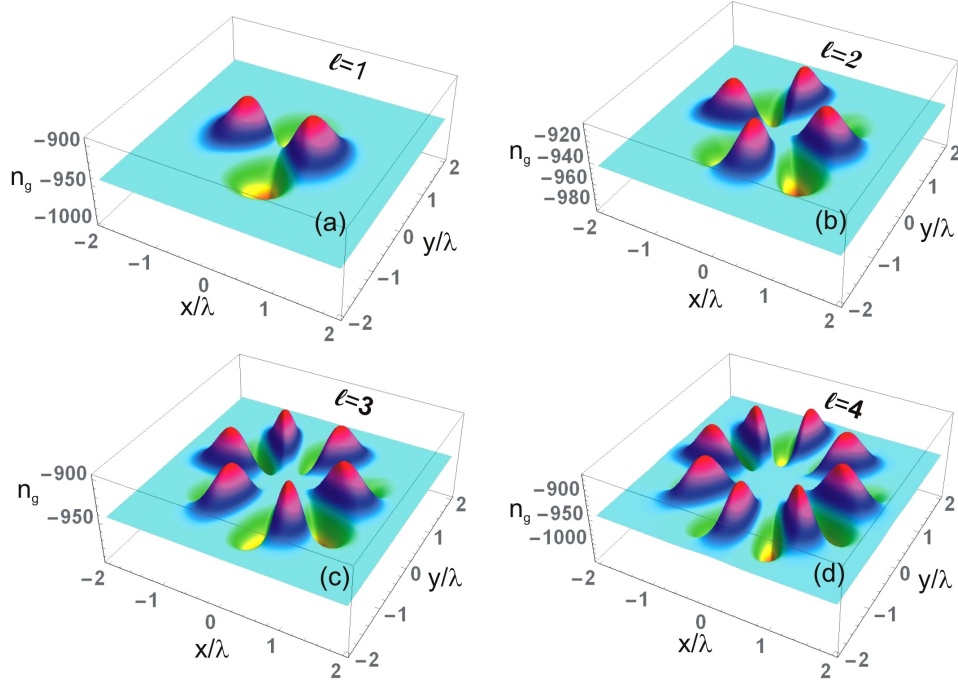


Figure 4: The plots are traced for the group index profile as a function of spatial coordinates ($x\lambda^{-1}$) and ($y\lambda^{-1}$) to investigate absorption maxima and minima. The parameters values are given as: $\gamma_{32}=\gamma_{31}=2\gamma$, $\zeta_1=0\gamma$, $\zeta_2=0\gamma$, $\delta_0=1\gamma$, and beam waist $R=0.8\lambda$ (a) $\ell=1$ (b) $\ell=2$ (c) $\ell=3$ (d) $\ell=4$.

exhibits two maxima and two minima. One maximum occurs at the boundary between the first and fourth quadrants, while the other occurs at the boundary between the third and second quadrants. The minimum occurs at the boundary between the first and second quadrants. At the same time, the other one appears at the border of the third and fourth quadrants as depicted in Fig. 5a. For the azimuthal quantum number of structured control field ($\ell = 2, 2\ell = 4$), the anomalous phase shift profile reveals four maxima and four minima as revealed in Fig. 5b. By the fine tuning of the azimuthal quantum number of the structured control field ($\ell = 3, 2\ell = 6$), the anomalous phase shift profile illustrates six maxima and six minima as illustrated in Fig. 5c. Finally, at the azimuthal quantum number of structured control field ($\ell = 4, 2\ell = 8$), the anomalous phase shift profile shows eight maxima and eight minima as shown in Fig. 5d. Overall, the anomalous phase shift profile varies in the range of $(-560 \times 10^3) \leq \phi \leq -650 \times 10^3$ rad, within the position range of $-2\lambda \leq x, y \leq 5\lambda$.

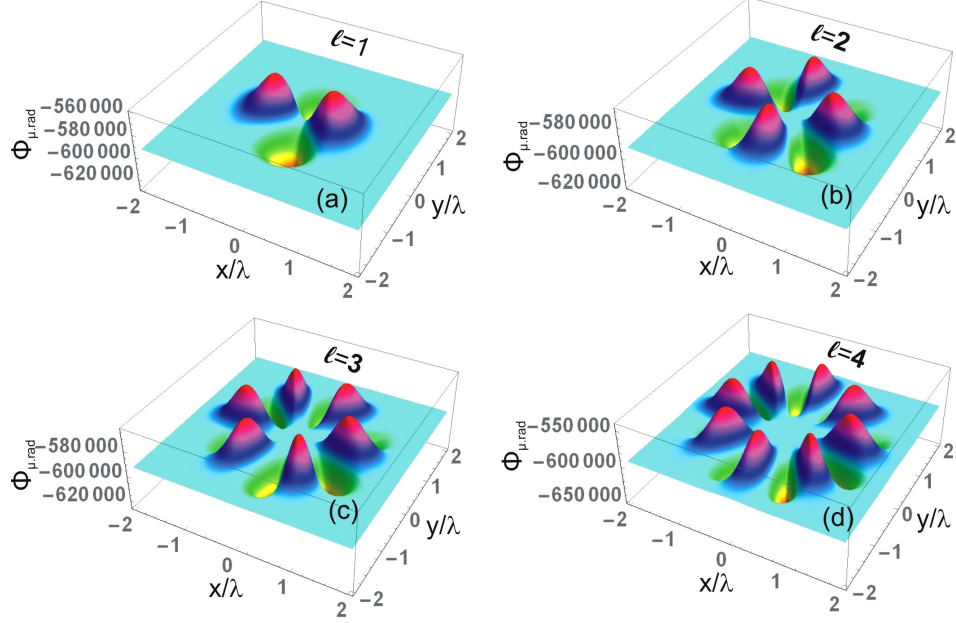


Figure 5: The plots are traced for the anomalous phase shift as a function of spatial coordinates ($x\lambda^{-1}$) and ($y\lambda^{-1}$), to investigate absorption maxima and minima. The chosen parameters are: $\gamma_{32}=\gamma_{31}=2\gamma$, $\zeta_1 = 0$, $\zeta_2 = 0$, $\delta_0=1\gamma$, and beam waist $R=0.8\lambda$ (a) $\ell = 1$ (b) $\ell = 2$ (c) $\ell = 3$ (d) $\ell = 4$.

4. Conclusion

The superluminal propagation of light and anomalous phase shift influenced by a structured light control field have been investigated in a Λ -type cesium atomic system. The density-matrix formalism is used to calculate the coherence term of the probe field and the electric susceptibility. The imaginary part of the electric susceptibility is associated with the absorption spectrum, and the fundamental part is related to the dispersion spectrum. The refractive index and dispersion are associated with the group index. Anomalous dispersion generates a negative group index and superluminal propagation as a consequence of the quantum interference between different fields. The maxima and minima of the absorption profile and the corresponding dispersion profile normal/ anomalous peaks are enhanced according to 2ℓ azimuthal quantum number dependence in the medium. The negative group index profile varies between $-900 \leq n_g \leq -1000$, and the associated group velocity $v_g = \frac{c}{n_g}$ alters between $-3.33 \times 10^5 \leq v_g \leq -3 \times 10^5$ m/sec, similarly the anomalous phase shift alters between $-560,000 \leq \phi \leq -620,000$ rad, both group index and anomalous phase shift lie within a wavelength range of $(-2\lambda \leq x, y \leq 2\lambda)$. The 2ℓ azimuthal quantum number conditions in the medium also enhance the superluminal propagation and anomalous phase shift. These modified results have crucial applications in storage devices, nonlinear optics, quantum information processing, imaging,

and technology design.

Competing interests

The author has no relevant financial or non-financial interests to disclose.

Ethical approval

This declaration is not applicable.

Authors' contributions

Najm Uddin: Conceptualization, Writing- Original draft. **Latif Ur Rahman and O. A. I. Elzibar:** Software, Validation, and Funding acquisition. **Naglaa Mohammed:** Project administration, Supervision. **Muhammad Idrees:** Formal analysis, Data curation, and Supervision. **Osman Abdalla Adam Osman:** Visualization. **Muhib Ullah:** Writing-review and editing, Supervision.

Availability of data and materials

No Data is associated with the manuscript.

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