Research Square

Preprints are preliminary reports that have not undergone peer review.
They should not be considered conclusive, used to inform clinical practice,
or referenced by the media as validated information.

Bianchi Type - VI0 Bulk Viscous string cosmological Model in the presence of attractive Massive Scalar field

P. Pavan Kumar

Department of Mathematics, Sri GCSR College, Rajam, Andhra Pradesh, Department of Mathematics, JNTUGV, Vizianagaram, Andhra Pradesh, India-535003

R. L Naidu

lakshumnaidu.r@gmrit.edu.in

GMRIT Deemed to be University, Rajam, Andhra Pradesh, India-532127

Shaik Kalesha Vali

JNTUGV, Vizianagaram, Andhra Pradesh, India-535003

Research Article

Keywords: Bianchi-Type metric, cosmological models, General relativity, Bulk Viscosity, Massive scalar fild

Posted Date: May 20th, 2026

DOI: https://doi.org/10.21203/rs.3.rs-9755285/v1

License: This work is licensed under a Creative Commons Attribution 4.0 International License.
Read Full License

Additional Declarations: The authors declare no competing interests.

Bianchi Type - VI_0 Bulk Viscous string cosmological Model in the presence of attractive Massive Scalar field

P. Pavan Kumar[1,3], R. L Naidu[2], Shaik Kalesha Vali[3]

[1] Department of Mathematics, Sri GCSR College, Rajam, Andhra Pradesh, India –532127, Mail Id: 21022pmt04@jntugv.edu.in

[2] Department of Mathematics, GMRIT Deemed to be University, Rajam, Andhra Pradesh, India-532127, Mail Id: lakshumnaidu.r@gmrit.edu.in

[3] Department of Mathematics, JNTUGV, Vizianagaram, Andhra Pradesh, India-535003, Mail Id: vali.maths@jntugvcev.edu.in

Corresponding author: R. L. Naidu, Email: lakshumnaidu.r@gmrit.edu.in

Abstract: In this investigation we are intended to study Bianchi Type - VI_0(BT- VI_0) cosmological model of the early universe with bulk viscosity having one-dimensional cosmic strings attached with an attractive massive scalar field in general relativity. To find the exact solution of Einstein field equations we use physically relevant conditions. Also, used the barotropic equation of the state for the pressure and density and bulk viscous pressure is assumed to be proportional to energy density. With the help of graphical plots, we have discussed some physical and kinematical properties of the model.

Keywords: Bianchi-Type metric, cosmological models, General relativity, Bulk Viscosity, Massive scalar fild

PACS Numbers: 04.20.–q; 04.20. Jb; 03.50. De; 05.70. Ce

1. Introduction

Einstein's general theory of relativity is the beautiful logical physical theory which effectively describes gravitational phenomena. It is also useful in discussing cosmology and evolution of our universe. However, it is said this does not satisfactorily explain the recent scenario of accelerated expansion of the universe confirmed by physical experiments (Reiss 1998, Perlmutter 1999). The concept of bulk viscosity plays a vital role in cosmology since it has significant role in getting accelerated expansion of the universe which is known as inflationary phase.

Ellis (1971) has pointed out that during the evolution of the universe bulk viscosity arises. Misner (1998) explained that when neutrinos decouple from the cosmic fluid during the formation of galaxies and particle creation of the universe, viscosity plays a significant role (Hu 1983). Also, shortly after the big bang, spontaneous symmetry breaking produced the topological defects known as cosmic strings, domain walls, and monopoles. Strings are lines with particles attached to them. Strings are very important in cosmology, since they help the galaxy formation at the early stages of the evaluation of the universe. Hence the study of bulk viscosity and cosmic strings models in cosmology in general relativity has attracted several researchers.

Many authors have investigated bulk viscous cosmological models in Einstein's gravitation theory (Barrow 1996; Padmanabhan and Chitre 1987; Pavan et al, 1991; Marteus et al. 1995; Lima et al. 1993; Roy and Tiwari 1983; Mohanty and Pradhan 1992; Singh and Shri Ram 1996, Singh 2005). Stachel (1980), Letlier (1983), Vilenkin et al. (1981), Krori et al. (1990), Mohanta and Mukharjee (2001), and Bhattacharya and Baruah (2001) have all investigated various aspects of string cosmological models in general relativity. Wang (2004, 2005, 2006), Bali and Dave (2002), Bali and Pradhan (2007), and Tripathy et al. (2009, 2010) have all investigated Bianchi type cosmological models in the presence of cosmic strings and bulk viscosity in general relativity.

Scalar fields are significant in cosmology since they help to discuss the early phases of evaluating the universe. The existence of scalar fields is supported by particle physics. For example, the Higgs mechanism explaining the mass of a particle is a massive scalar field (Zeldovich 1986). The quintessence scalar field explains the recently discovered concept of accelerated universe expansion. It also contributes to inflation during early appraisal of the cosmos. There are two sorts of scalar fields: zero mass and attractive massive. Massless scalar fields interact over vast distances, whereas massive scalar fields interact over short distances. Mass-less and massive scalar field models are being extensively explored. This section focuses on the work done on enormous scalar field models. Naidu (2018), Reddy et al. (2019), Naidu et al. (2019), Aditya et al. (2019), Aditya and Reddy (2019), Singh and Rani (2015), and Shanti et al. (2016) are among the authors who have discussed various aspects of Bianchi type cosmological models with different matter sources coupled with attractive massive scalar fields in general relativity and modified gravity theories.

Our research focuses on the BT – VI_0 cosmological model, which includes bulk viscous cosmic strings and an attracting massive scalar field in general relativity. This model has not been studied previously. The BT-VI_0 space time, a generalization of FRW space time, is important to consider while evaluating the universe. The work is organised as follows: Section 2 provides the field equation for Einstein's theory of gravity with a bulk viscous string source linked with an attractive massive scalar field. Section 3 focuses on achieving an exact solution for the field equations and the BT-VI_0 model. Section 4 is devoted to computing the model's physical and kinematic features and discussing them in cosmology using graphical representations. Section 5 contains conclusions.

2. Metric and Field Equations

We take the spatially homogeneous and anisotropic metric in the form

$$ds^2 = dt^2 - A^2dx^2 - B^2e^{-2\alpha x}dy^2 - C^2e^{2\alpha x}dz^2 \qquad (1)$$

where A, B, C are functions of cosmic time t only and α is a non-zero constant.

Einstein's field equations with bulk viscous string source and in the presence of massive scalar field are (we use gravitational units c=1,8πG=1)

$$R_{ij} - \frac{1}{2}g_{ij}R = -(T_{ij}) + \emptyset_{,i}\emptyset_{,j} - \frac{g_{ij}}{2}(\emptyset_{,k}\emptyset^{,k} - M^2\emptyset) \qquad (2)$$

Here T_{ij} is the energy momentum tensor of bulk viscous fluid containing one-dimensional cosmic string is

$$T_{ij} = (\rho + \bar{p}) - \bar{p}g_{ij} - \lambda x_i x_j, \qquad \bar{p} = p - 3\zeta H \qquad (3)$$

$\bar{p}$ is the bulk viscous pressure, $\zeta(t)$ is the bulk viscous coefficient, H is the Hubble parameter of the model, ρ is the energy density of the system, λ is the string tensor density and M is the mass of the scalar field $\emptyset$ which satisfies the Klein-Gordon equation

$$g^{ij}\emptyset_{;ij} + M^2\emptyset = 0 \text{ and } u_i u^i = -x_i x^j = 1 \qquad (4)$$

where semicolon denote covariant differentiation, u_i is the four velocity of the fluid and x_i is the string direction which we take along x-axis.

Now using equations (3) and (4) Einstein field equations (2) for the metric (1) can explicitly, written as

$$\frac{\ddot{B}}{B}+\frac{\ddot{C}}{C}+\frac{\dot{B}\dot{C}}{BC}-\frac{\alpha^2}{A^2}=-\bar{p}+\lambda+\frac{1}{2}\left(\dot{\phi}^2-M^2\phi^2\right) \quad (5)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{C}}{C}+\frac{\dot{A}\dot{C}}{AC}-\frac{\alpha^2}{A^2}=-\bar{p}+\frac{1}{2}\left(\dot{\phi}^2-M^2\phi^2\right) \quad (6)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\alpha^2}{A^2}=-\bar{p}+\frac{1}{2}\left(\dot{\phi}^2-M^2\phi^2\right) \quad (7)$$

$$\frac{\dot{A}\dot{B}}{AB}+\frac{\dot{B}\dot{C}}{BC}+\frac{\dot{A}\dot{C}}{AC}-\frac{\alpha^2}{A^2}=\rho+\frac{1}{2}\left(\dot{\phi}^2+M^2\phi^2\right) \quad (8)$$

$$\alpha\left(\frac{\dot{B}}{B}-\frac{\dot{C}}{C}\right)=0 \quad (9)$$

and $$\ddot{\phi}+\dot{\phi}\left(\frac{\dot{A}}{A}+\frac{\dot{B}}{B}+\frac{\dot{C}}{C}\right)+M^2\phi=0 \quad (10)$$

where an overhead dot denotes differentiation with respect to cosmic time t.

3. Solutions and the model

Eqn. (9) yields immediately

$$B=c_1C$$

where c_1 is a constant of integration which can be set equal to unity without loss of generality so that we have

$$B=C \quad (11)$$

Use of Eqn. (11) in the set of equations (5) – (8) and (10) we get

$$2\frac{\ddot{B}}{B}+\left(\frac{\dot{B}}{B}\right)^2-\frac{\alpha^2}{A^2}=-\bar{p}+\lambda+\frac{1}{2}\left(\dot{\phi}^2-M^2\phi^2\right) \quad (12)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\alpha^2}{A^2}=-\bar{p}+\frac{1}{2}\left(\dot{\phi}^2-M^2\phi^2\right) \quad (13)$$

$$\frac{\ddot{A}}{A}+\frac{\ddot{B}}{B}+\frac{\dot{A}\dot{B}}{AB}-\frac{\alpha^2}{A^2}=-\bar{p}+\frac{1}{2}\left(\dot{\phi}^2-M^2\phi^2\right) \quad (14)$$

$$2\frac{\dot{A}\dot{B}}{AB}+\left(\frac{\dot{B}}{B}\right)^2-\frac{\alpha^2}{A^2}=\rho+\frac{1}{2}\left(\dot{\phi}^2+M^2\phi^2\right) \quad (15)$$

and $$\ddot{\phi}+\dot{\phi}\left(\frac{\dot{A}}{A}+2\frac{\dot{B}}{B}\right)+M^2\phi=0 \quad (16)$$

Equations (12) to (15), are three independent field equations with six unknowns $A, B, \bar{p}, \rho, \lambda$ and $\emptyset$. Hence to obtain a determinate solution we use the following physically relevant conditions.

(i) The shear scalar σ^2 is proportional to the expansion scalar θ which gives a relation between metric potentials (Collins et al. 1980)

$$A=(BC)^m \quad (17)$$

where, m is an arbitrary constant and maintains the anisotropy of the metric.

(ii) Following Johri and Sudharsan (1989) and Johri and Desikan (1994) we take a power law between scalar field $\emptyset$ and the average scale factor $a(t)$ given by

$$\emptyset \propto [a(t)]^n$$

So that $$\emptyset=\emptyset_0 a^n \quad (18)$$

where $\emptyset_0$ and n are constants (Aditya and Reddy 2018)

Now in order to obtain a physically realistic model we consider the power law assumption for the average scale factor so that we have (Berman, M.S. and Gomide, F. M.1988)

$$a=(ABC)^{\frac{1}{3}}=(t+t_0)^{\beta} \quad (19)$$

where t_0 and $\beta>0$ are constants.

(iii) For a barotropic fluid, combined effect of paper pressure and bulk viscous pressure can be expressed as

$$\bar{p}=p-3\zeta H=\varepsilon\rho \text{ where } \varepsilon=\varepsilon_0-\beta \text{ and } p=\varepsilon_0\rho \quad (20)$$

Where ϵ_0 and β are constants.

Use Eq. (11) and (17) in (19) we get

$$A = (t+t_0)^{\frac{3\beta m}{m+1}}, \quad B = (t+t_0)^{\frac{3\beta}{2(m+1)}}, \quad C = (t+t_0)^{\frac{3\beta}{2(m+1)}} \tag{21}$$

Using Eqns. (1) and (21), BT- VI_0 bulk viscous model in the presence of massive scalar field can be written as

$$ds^2 = dt^2 - (t+t_0)^{\frac{6\beta m}{m+1}}dx^2 - (t+t_0)^{\frac{3\beta}{(m+1)}}e^{-2\alpha x}dy^2 - (t+t_0)^{\frac{3\beta}{(m+1)}}e^{2\alpha x}dz^2 \tag{22}$$

4. Physical and Kinematical Parameters:

This section is devoted to the physical discussions of the dynamical parameters of the model (22).

Spatial Volume is

$$V = a^3(t) = ABC = (t+t_0)^{3\beta} \tag{23}$$

The average Hubble parameter is

$$H = \frac{\dot{a}}{a} = \frac{\beta}{t+t_0} \tag{24}$$

The expansion scalar is

$$\theta = 3H = \frac{3\beta}{t+t_0} \tag{25}$$

The Shear scalar is

$$\sigma^2 = \frac{1}{2}\sigma^{ij}\sigma_{ij} = \frac{1}{2}\sum_{i=1}^{3}(H_i - H)^2 = \frac{3\beta^2(2m-1)^2}{4(m+1)^2(t+t_0)^2} \tag{26}$$

The average anisotropy parameter is

$$A_h = \frac{1}{3}\sum_{i=1}^{3}\left(\frac{H_i - H}{H}\right)^2 = \frac{3}{2}\left(\frac{2m-1}{m+1}\right)^2 \tag{27}$$

and the scalar field in the model is given by

$$\emptyset = \emptyset_0(t+t_0)^{n\beta} \tag{28}$$

With the help of Eqns. (21) and (28) Eqn. (15) gives the energy density

$$\rho = \frac{9\beta^2(4m+1)}{4(m+1)^2(t+t_0)^2} - \frac{\alpha^2}{(t+t_0)^{\frac{6\beta m}{m+1}}} - \frac{1}{2}\emptyset_0{}^2 n^2\beta^2(t+t_0)^{2n\beta-2} - \frac{1}{2}\emptyset_0{}^2 M^2(t+t_0)^{2n\beta} \quad (29)$$

With the help of Eqns. (21) and (28), Eqn. (13) gives the Bulk viscous pressure

$$\bar{p} = -\frac{3\beta[4m^2(3\beta-1)+3m(3\beta-2)-2]}{4(m+1)^2(t+t_0)^2} + \alpha^2(t+t_0)^{\frac{-6\beta m}{m+1}} + \frac{1}{2}\emptyset_0{}^2 n^2\beta^2(t+t_0)^{2n\beta-2} - \frac{1}{2}\emptyset_0{}^2 M^2(t+t_0)^{2n\beta} \quad (30)$$

Eqns. (12), (13) and (21) gives the string tension density

$$\lambda = \frac{(9-45m)\beta^2+6\beta(m+1)+9\beta^2}{4(m+1)^2(t+t_0)^2} \quad (31)$$

Using Eqn. (20),

$$p = \epsilon_0\rho = \epsilon_0\left[\frac{9\beta^2(4m+1)}{4(m+1)^2(t+t_0)^2} - \frac{\alpha^2}{(t+t_0)^{\frac{6\beta m}{m+1}}} - \frac{1}{2}\emptyset_0{}^2 n^2\beta^2(t+t_0)^{2n\beta-2} - \frac{1}{2}\emptyset_0{}^2 M^2(t+t_0)^{2n\beta}\right] \quad (32)$$

Using Eqn. (20) $\bar{p} = p - 3\zeta H = \epsilon\rho$ which gives bulk viscous coefficient

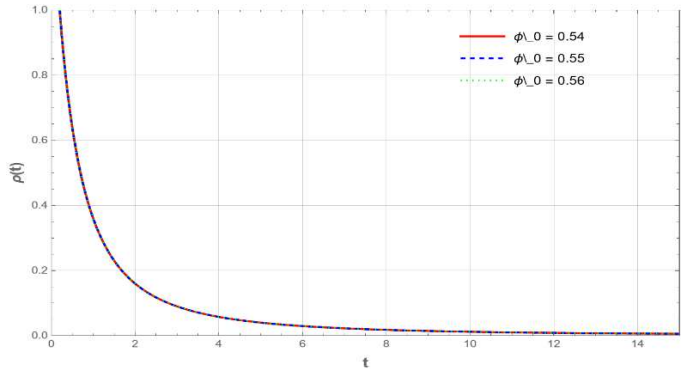
$$\zeta = \frac{p-\bar{p}}{3H} = \frac{3\beta\epsilon_0(4m+1)+4m^2(3\beta-1)+3m(3\beta-2)-2}{4(m+1)^2(t+t_0)} - \frac{(\epsilon_0+1)}{3\beta}\alpha^2(t+t_0)^{1-\frac{6\beta m}{m+1}} - \left(\frac{\epsilon_0-1}{6\beta}\right)M^2\emptyset_0{}^2(t+t_0)^{1+2n\beta} - \left(\frac{\epsilon_0+1}{6}\right)n^2\beta\emptyset_0{}^2(t+t_0)^{2n\beta-1} \quad (33)$$

State finder parameters are

$$r = \frac{\ddot{a}}{aH^3} = 1 - \frac{3}{\beta} + \frac{2}{\beta^2}, \qquad s = \frac{r-1}{3\left(q-\frac{1}{2}\right)} = \frac{2-3\beta}{3\beta(\beta-2)} \quad (31)$$

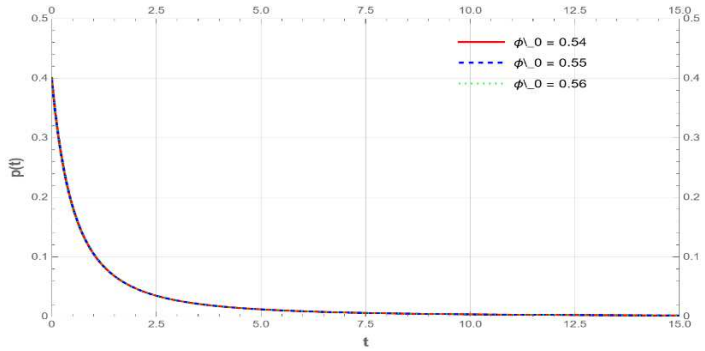
We, now, focus our attention on the behaviour of physical and kinematical parameters of our model using graphical representation of the parameters.

Fig-1:Plot of Energy density ρ versus cosmic time t, for n=-1, β=0.8, $\emptyset_0$ =0.5, t_0 =1, M=0.1, m=2, α=0.1.



From the Fig-1, it may be observed that the energy density ρ(t) remains positive and monotonically decreasing throughout cosmic evolution. At late times, ρ is dominated by the leading t^{-2} term, indicating a physically viable, decaying energy density consistent with an expanding and accelerating universe, while scalar field and anisotropic contributions decay rapidly and become negligible.

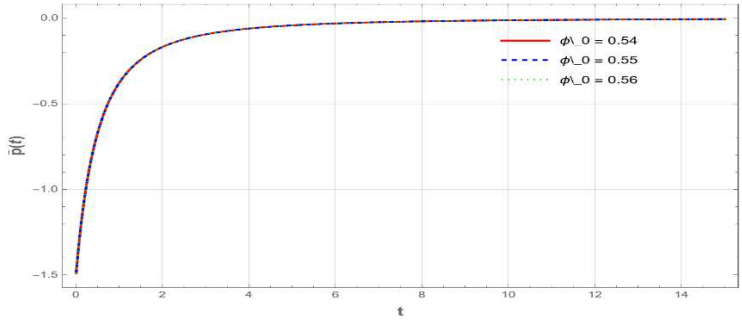
Fig-2:Plot of Pressure p versus cosmic time t, for n=-1, β=0.8, $\emptyset_0$ =0.5, t_0 =1, M=0.1, m=2, α=0.1, $\epsilon_0 = 0.3$



From the Fig-2, it may be observed that, the pressure p(t) remains positive and monotonically decreasing with cosmic time, closely following the decay behaviour of the energy density. At late times, the dominant t^{-2} contribution ensures a smoothly decreasing positive pressure, indicating a physically consistent viscous fluid that supports accelerated expansion while avoiding future singular behaviour.

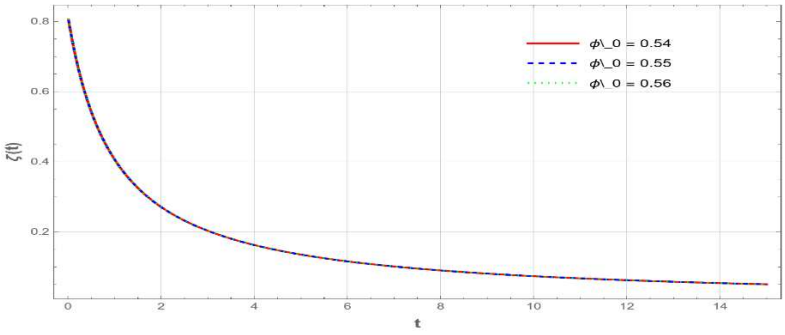
Fig-3: Plot of Bulk viscous pressure ($\bar{p}$) versus cosmic time t, for n=-1, β=0.8, $\emptyset_0$ =0.5, t_0 =1, M=0.1, m=2, α=0.1, $\epsilon_0 = 0.3$

.



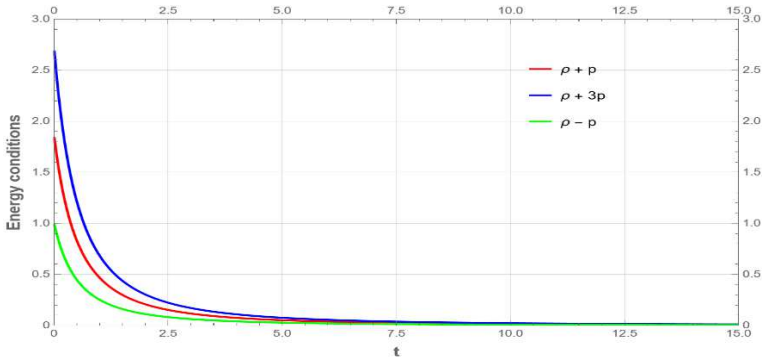
From the Fig-3, it may be observed that the bulk viscous pressure $\bar{p}$ remains negative throughout the cosmic evolution, dominated at late times by the inverse–square decay term $(t + t_0)^{-2}$. This persistent negativity of $\bar{p}$ signifies an effective dissipative pressure driving accelerated expansion, while the scalar-field and anisotropic contributions decay rapidly and become dynamically insignificant at late times..

Fig-4: Plot of Bulk viscous coefficient (ζ)versus cosmic time t, for n=-1, β=0.8, $\emptyset_0$ =0.5, t_0 =1, M=0.1, m=2, α=0.1, $\epsilon_0 = 0.3$



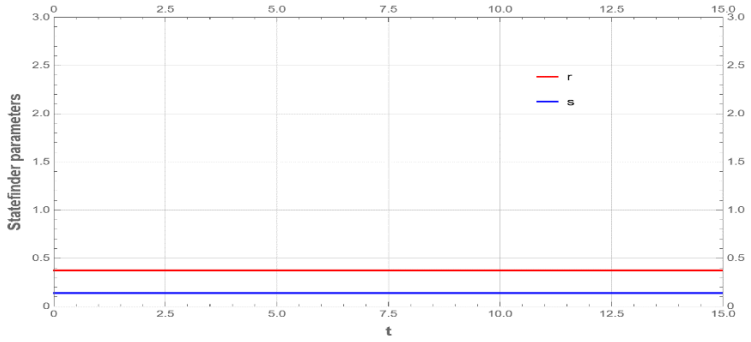
From the Fig-4, it may be observed that the bulk viscosity coefficient ζ(t) remains positive and monotonically decreasing with cosmic time, dominated at late times by the leading inverse–time term $(t + t_0)^{-1}$. This positive definite behaviour of ζ(t) confirms the presence of physically admissible dissipative effects, which gradually weaken with expansion and support a realistic viscous-driven accelerated cosmological evolution.

Fig-5: Plot of Energy conditions ρ+p, ρ-p and ρ+3p versus cosmic time t, for n=-1, β=0.8, $\emptyset_0$ =0.5, t_0 =1, M=0.1, m=2, α=0.1, $\epsilon_0 = 0.3$



From the Fig-5, it may be observed that the combinations ρ+p, ρ-p remain positive throughout the evolution, ensuring the validity of the null and weak energy conditions. Meanwhile, ρ+3p at early times but decreases monotonically and approaches small positive values at late times, indicating a transition toward an accelerated expansion regime driven by effective viscous effects while marginally satisfying the strong energy condition.

Fig-6: Plot of state finder parameters r, s versus cosmic time t, for β=0.6



From the Fig-6, it may be observed that the state finder parameters indicating a clear deviation from the standard ΛCDM fixed point (r=1, s=0). This behaviour signifies a quintessence-like accelerating universe**,** where the cosmic expansion is smoother than the ΛCDM model but still driven by an effective dark-energy–dominated phase.

5. Conclusions:

In this work, we have investigated a spatially homogeneous and anisotropic Bianchi type VI_0 cosmological model filled with a bulk viscous fluid containing one–dimensional cosmic strings in the presence of an attractive massive scalar field within the framework of general relativity. Exact solutions of the Einstein field equations were obtained by adopting a physically viable law for the average scale factor, which leads to a time-dependent deceleration parameter and allows a detailed study of the dynamical evolution of the universe.

The analysis of physical parameters reveals that the energy density and isotropic pressure remain positive and decrease monotonically with cosmic time, while the bulk viscous pressure stays negative, providing an effective mechanism for accelerated expansion. The bulk viscosity coefficient is found to be positive and decaying, confirming the thermodynamic consistency of the model. Furthermore, the scalar field and anisotropic contributions diminish rapidly at late times, ensuring convergence toward an isotropic universe.

The energy conditions are shown to be satisfied during the cosmic evolution, and the behavior of the state finder parameters (r, s) indicates a departure from the standard ΛCDM scenario toward a quintessence-like accelerated phase. Overall, the present model successfully describes a transition from an early anisotropic universe to a late-time accelerating universe, making it a viable cosmological model compatible with current observational expectations.

Acknowledgements:

The authors are thankful to Prof. D. R. K. Reddy, Retd. Professor of Andhra University, for his helpful discussions in the preparation of the manuscript.

Funding: Authors received no financial support from any agency to this research work and publication.

References:

[1] A. G. Riess et al. Astron. J. **116**, 1009 (1998)

[2] S. Perlmutter et al. Astrophys. J. **517**, 565 (1999)

[3] G.F.R. Ellis Relativistic cosmology **47,** 104 (1971)

[4] C.W. Misner and Watt, K. "ADM revisited for binary neutron stars", "Richard Arnowitt fest: A symposium on Supersymmetry and Gravitation", 13-22, April (1998)

[5] B.L. Hu Advances in Astrophys. world scientific, Singapore (1983)

[6] J.D. Barrow class. Quantum Grav. **13**, 2965 (1996)

[7] T. Padmanabhan and S.M. Chitre Phys. Lett. A, **120**, 433 (1987)

[8] D. Pavan et al. Class. Quantum. Grav. **8**, 357 (1991)

[9] R. Marteus class. Quantum Grav. **12**, 7455 (1995)

[10] J.A.S. Lima et al. Phys. Rev. D **53**, 4287 (1993)

[11] S. R. Roy and O.P. Tiwari ind. J. Pure Appl. Math. **14**, 233 (1983)

[12] G. Mohanty and B.D. Pradhan Int. J. Theor. Phys. **31**, 151 (1992)

[13] J.K. Singh and S. Ram Astrophys. Space Sci. **236**, 277 (1996)

[14] J.K. Singh Nuovo Cimento 1208, 1251 (2005)

[15] J. Stachel Phys. Rev. D 21, 1271 (1980)

[16] P.S. Letlier Phys. Rev. D **28**, 2414 (1983)

[17] A. Vilenkin Phys. Rev. D **23**, 852 (1981)

[18] K.D. Krori et al. Gen. Relativ. Gravit **22**, 125 (1990)

[19] P. Mahanta and A. Mukharjee: Indian J. Pure Appl. Math. 32, 199 (2001)

[20] R. Bhattacharya and K. Baruah Ind. J. Pure Appl. Math. **32**, 199 (2001)

[21] X.X. Wang Chin. Phys. Lett. **21**, 1205 (2004)

[22] X.X. Wang Chin. Phys. Lett. **22**, 29 (2005)

[23] X.X. Wang Chin. Phys. Lett. **23**, 1702 (2006)

[24] R. Bali and Sh. Dave Asrophys. Space Sci. **282**, 461 (2002)

[25] R. Bali and A. Pradhan Gen. Relati. Quant. Cosmology chin. Phys. Lett. **24**, 585 (2007)

[26] S.K. Tripathy, *et al. Astrophys Space Sci* **321**, 247–252 (2009

[27] S.K. Tripathy, et al., Astrophys Space Sci, **325**, 93–97 (2010)

[28] Y.A.B. Zeldovich gov. Sci. Rev E Astrophys. Space Phys. **51**, 37 (1986)

[29] R.L. Naidu et al. can. J. Phys. **97**, 330 (2018)

[30] D.R.K. Reddy et al. J. Dyn. Syst. Geomm. Theories. **17**, 1 (2019)

[31] Naidu R.L. et al. Heliyon **5**, e01645 (2019)

[32] Y. Aditya et al. Astrophys. Space Sci.**364,** 190 (2019)

[33] Y. Aditya and D.R.K Reddy Astrophys. Space Sci.**364,** 3 (2019)

[34] J.K. Singh and S. Ram Int. J. Theor. Phys. **54**, 45 (2015)

[35] M.V. Shanthi et al. Can. J. Phys **94**, 578 (2016)

[36] C.B. Collins et al., *Gen. Rel. Grav.*, **12**, 805 (1980)

[37] V.B. Johri, and R. Sudharsan, Aust. J. Phys. 42, 215 (1989)

[38] V.B. Johri and K. Desikan, Pramana, **42** 473 (1994)

[39] M.S. Berman and F.M. Gomide: *Gen. Rel. Grav.*, **20**, 191 (1988)