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A Stochastic Evolutionary Game Reformulation of the Lux--Marchesi Financial Market Model

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Abstract

The Lux–Marchesi model is a canonical agent-based model of financial markets in which interacting fundamentalists and chartists generate endogenous market instability, including intermittent bubbles, crashes, and clustered volatility. The present study reformulates the Lux–Marchesi mechanism as a finite-population stochastic evolutionary game. Trader types are treated as evolutionary strategies; realized trading performance defines payoff; payoff is mapped into fitness; and strategy revision is represented by payoff-dependent transition probabilities with mutation and finite-population sampling. The resulting model is a coupled stochastic process for trader composition and market states, so price regimes are interpreted as probabilistic outcomes of the joint evolution of strategies and the market environment. Rather than adding a new trader type or claiming immediate empirical calibration, the reformulation supplies a stochastic evolutionary-game layer through which invasion, fixation, stationary distributions, metastability, and stochastic stability can be studied. The analysis develops the three-strategy reformulation, derives a reduced two-strategy birth-death version, states analytical propositions with proofs for key model implications, and reports multi-regime simulations with panel plots of price series and population dynamics. The simulations show that calm markets, mixed adaptive markets, bubbles, crashes, noisy switching, shock responses, and speculative polarization can arise from the same evolutionary transition structure under different parameter regimes.

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1 Introduction

The Lux–Marchesi model occupies an important position in agent-based finance and econophysics because it explains financial instability as an endogenous outcome of trader interaction. In its standard form, a market is populated by fundamentalists, optimistic chartists, and pessimistic chartists. Fundamentalists trade against deviations from fundamental value, whereas chartists trade with market sentiment and trend-following pressure. Price changes are generated by aggregate excess demand, and traders move among behavioral types according to social interaction, recent profitability, and stochastic disturbance. Through these feedbacks, fat-tailed returns, clustered

volatility, bubbles, and crashes can arise without imposing an external news process that already contains the same statistical structure [8, 9].

A large literature has developed related ideas under heterogeneous beliefs, adaptive learning, agent-based computational finance, and evolutionary views of market behavior [1–4]. These approaches share the view that market outcomes depend on the changing composition of trading rules. At the same time, the standard Lux–Marchesi presentation is usually read as a simulation model: the main objects are sample paths of prices, returns, volatility, and trader counts. Such a representation is powerful for generating stylized facts, but it leaves open a complementary theoretical question: how can the same switching mechanism be studied as a finite-population stochastic process with explicit transition probabilities?

A stochastic evolutionary-game formulation provides a natural answer. Trader categories can be interpreted as strategies in a finite population; realized trading performance can be interpreted as payoff; payoff can be converted into fitness; and behavioral switching can be represented as a payoff-dependent revision rule with mutation and finite-population sampling. The joint state is then

$$Y_t = (Z_t, E_t),$$

where Z_t records the population counts of trading strategies and E_t records the market environment, including price, fundamental value, returns, volatility, and mispricing. This representation connects the Lux–Marchesi mechanism to the standard objects of stochastic evolutionary game theory: transition probabilities, fixation probabilities, absorption times, stationary distributions, metastable states, and stochastic stability [5, 10, 12].

The aim is not to replace the economic intuition of the original model. The same trader classes, profitability comparison, herding force, stochastic switching, and endogenous price feedback remain central. The change is in the analytical language. Instead of treating a bubble only as a simulated upward price episode, the reformulation treats it as a region of the joint strategy-price state space in which optimistic chartists have high population share and the price is persistently above fundamental value. Similarly, a crash corresponds to a region dominated by pessimistic chartists, while a calm market corresponds to high fundamentalist share and small mispricing. Under mutation, the relevant long-run object is no longer only an absorbing state or a single simulated trajectory, but a stationary distribution over market regimes.

This perspective also clarifies the scope of the contribution. The model is not intended as the first chartist-fundamentalist model, the first adaptive financial-market model, or a fully calibrated empirical account of observed returns. Its role is to build a bridge between the Lux–Marchesi mechanism and finite-population stochastic evolutionary game theory. The resulting framework makes it possible to ask whether chartists can invade a fundamentalist market, how fixation probabilities behave in a reduced birth-death version, which regimes receive high stationary probability, and how mutation and selection intensity affect the persistence of speculative states.

The analysis proceeds in four steps. First, the standard Lux–Marchesi mechanism is reviewed in a compact form. Second, the trader population is reformulated as a three-strategy stochastic evolutionary game with payoff-based revision. Third, a

reduced two-strategy birth-death version is developed to obtain analytical expressions for transition ratios and fixation probabilities. Fourth, several analytical implications are stated with proofs, and multi-regime simulations are used to check whether the same transition structure can generate calm, mixed, bubble, crash, noisy, shock-driven, cyclical, and polarized market regimes. The simulations are interpreted as proof-of-mechanism evidence rather than empirical validation; calibration and formal stylized-fact testing remain tasks for subsequent work.

The rest of the manuscript is organized as follows. Section 2 reviews the standard Lux–Marchesi model. Section 3 translates the model from agent-based finance into stochastic evolutionary-game language. Section 4 specifies payoffs, fitness, and revision probabilities. Section 5 presents the coupled stochastic evolutionary financial game. Section 6 develops the reduced two-strategy birth-death version. Sections 7 and 8 discuss stationary distributions, stochastic stability, metastability, bubbles, and volatility clustering. Section 9 states analytical implications and proof status. Section 10 reports the multi-regime simulation results. Section 11 concludes.

2 The Standard Lux–Marchesi Model

The standard Lux–Marchesi model is a stochastic multi-agent model of a financial market populated by heterogeneous traders. Its purpose is not to impose empirical regularities directly, but to show how fat-tailed returns, volatility clustering, bubbles, and crashes can arise endogenously from interaction among traders [8, 9]. The model contains three core ingredients: heterogeneous trading rules, stochastic switching among trader groups, and price adjustment through excess demand.

2.1 Trader groups and population accounting

Let the total number of traders be fixed at N. At time t, the population is divided into fundamentalists, optimistic chartists, and pessimistic chartists:

$$N_F(t) + N_+(t) + N_-(t) = N. \tag{1}$$

Fundamentalists believe that the market price should return toward a fundamental value p_t^f. They buy when the asset is undervalued and sell when it is overvalued. Chartists do not primarily trade on fundamental value. Instead, optimistic chartists expect price increases, while pessimistic chartists expect price decreases. The difference

$$M_t = N_+(t) - N_-(t) \tag{2}$$

can be interpreted as a simple measure of chartist sentiment or market mood. A positive value of M_t represents optimistic pressure, while a negative value represents pessimistic pressure.

2.2 Excess demand and price adjustment

The market price is adjusted by a market maker in response to aggregate excess demand. A compact representation is

$$p_{t+1} - p_t = \frac{1}{\lambda} ED_t + \sigma_p \eta_{t+1}, \tag{3}$$

where ED_t is total excess demand, $\lambda > 0$ measures market depth, and η_{t+1} is an exogenous order-flow or price-adjustment shock. A useful schematic decomposition of excess demand is

$$ED_t = D_F(t) + D_C(t), \tag{4}$$

with

$$D_F(t) = \phi_F N_F(t)(p_t^f - p_t), \tag{5}$$

for fundamentalists and

$$D_C(t) = \phi_C \big(N_+(t) - N_-(t)\big) \tag{6}$$

for chartists. The parameters ϕ_F and ϕ_C measure the aggressiveness of fundamentalist and chartist trading. This representation captures the central economic mechanism: fundamentalists pull the price toward fundamental value, while chartists amplify directional market sentiment.

2.3 Switching among trader types

A distinctive feature of the Lux–Marchesi model is that trader types are not fixed. Agents may switch between fundamentalism and chartism, and chartists may switch between optimism and pessimism. The switching mechanism depends on both social interaction and relative attractiveness. In compact form, the transition intensity from type i to type j may be written as

$$q_{ij}(t) = \nu_{ij} \exp\{A_j(t) - A_i(t)\}, \tag{7}$$

where $\nu_{ij} > 0$ is a baseline switching rate and $A_j(t)$ is the attractiveness of type j. Attractiveness typically combines recent profitability, herding pressure, and random behavioral disturbance. Thus, switching is neither purely rational optimization nor pure noise. It is an adaptive behavioral process in which profitable and socially reinforced strategies become more attractive.

Two types of switching are especially important. First, switching between optimistic and pessimistic chartism changes market sentiment and can create trend-following pressure. Second, switching between chartism and fundamentalism changes the balance between destabilizing speculative demand and stabilizing fundamental demand. When chartists become numerous, price movements can become self-reinforcing. When fundamentalists become numerous, mispricing tends to be corrected.

2.4 Endogenous instability

The key implication of the standard model is that instability is endogenous. Large fluctuations do not require large external shocks. A small disturbance can be amplified when it changes profitability or sentiment in a way that attracts more traders into the currently successful rule. If optimistic chartists earn high profits during an upward movement, more traders may adopt optimistic chartism, increasing excess demand and pushing the price further upward. A similar mechanism can operate in reverse during crashes. This feedback generates intermittent regimes: long calm periods may be interrupted by turbulent episodes with high volatility and large price deviations.

For the present paper, the most important point is that the standard model already contains the raw material for an evolutionary interpretation: a finite population, a finite set of trader types, profitability-based adaptation, stochastic switching, and endogenous price feedback. The reformulation developed below does not discard these elements. It rewrites them in the language of stochastic evolutionary game theory by treating trader types as strategies, attractiveness as payoff or fitness, and switching intensities as transition probabilities of a finite-population Markov process.

3 From Agent-Based Finance to Stochastic Evolutionary Games

The purpose of the reformulation is to translate the behavioral switching structure of the Lux–Marchesi model into the formal language of stochastic evolutionary game theory. The economic content remains the same: traders compare rules, some rules become more attractive than others, and aggregate demand moves prices. The mathematical object, however, changes. Instead of treating trader switching only as a simulation mechanism, we treat it as a finite-population stochastic revision process with an explicit transition law.

3.1 Trader types as evolutionary strategies

Let the set of available strategies be

$$S = \{F, C^+, C^-\}, \tag{8}$$

where F denotes fundamentalism, C^+ denotes optimistic chartism, and C^- denotes pessimistic chartism. A trader who uses F evaluates price relative to fundamental value. A trader who uses C^+ expects upward price pressure, while a trader who uses C^- expects downward price pressure. In the original agent-based interpretation these are trader classes. In the evolutionary interpretation they are strategies whose population shares evolve over time.

For a finite population of size N, the population state is

$$Z_t = (N_F(t), N_+(t), N_-(t)), \qquad N_F(t) + N_+(t) + N_-(t) = N. \tag{9}$$

The corresponding frequency vector is

$$X_t = (x_F(t), x_+(t), x_-(t)), \qquad x_i(t) = \frac{N_i(t)}{N}. \tag{10}$$

Thus X_t lies on the three-strategy simplex. The finite-population formulation is important because strategy frequencies change in discrete steps of size $1/N$. Random drift, mutation, imitation errors, and payoff noise can therefore affect the realized path, especially when N is not extremely large or when selection is weak.

3.2 Market environment as a stochastic game state

The Lux–Marchesi mechanism is not a game with a fixed payoff matrix. The payoff of a trading rule depends on the market environment created by previous trading decisions. We therefore define an environmental state

$$E_t = (p_t, p_t^f, r_t, \sigma_t, m_t), \tag{11}$$

where p_t is the market price, p_t^f is the fundamental value, $r_t = p_t - p_{t-1}$ is the price change or return, σ_t is a volatility estimate, and m_t is a sentiment or mispricing indicator. The complete state of the reformulated model is

$$Y_t = (Z_t, E_t). \tag{12}$$

This notation emphasizes that the evolutionary process is coupled to the market. The composition Z_t affects excess demand and hence the next price movement, while the realized price movement affects the subsequent profitability of each strategy. The model is therefore a stochastic evolutionary game with endogenous environmental feedback rather than a population game with constant payoffs.

3.3 Transition law and Markov representation

The central object in the reformulation is the transition probability

$$P(y, y') = \Pr(Y_{t+1} = y' \mid Y_t = y), \tag{13}$$

where $y = (z, e)$ and $y' = (z', e')$. A useful decomposition is

$$P(y, y') = P_Z(z' \mid z, e) P_E(e' \mid z', z, e), \tag{14}$$

where P_Z describes stochastic strategy revision and P_E describes price and market-state updating. The first component is evolutionary: traders switch strategies according to payoff, social reinforcement, mutation, and noise. The second component is financial: prices respond to the excess demand generated by the new or current composition of strategies.

This decomposition is the main formal difference from the standard Lux–Marchesi presentation. In the standard model, switching intensities are specified as part of an

agent-based simulation. In the reformulated model, the same switching forces define a Markov transition kernel. Once the kernel is specified, the model can be studied through the probability distribution of future states rather than through a single simulated trajectory.

3.4 Interpretation of regimes

The state-space formulation also changes the interpretation of market regimes. A fundamentalist calm regime corresponds to states with large x_F and small mispricing. An optimistic bubble regime corresponds to states with large x_+ and positive price deviation from fundamentals. A pessimistic crash regime corresponds to states with large x_- and negative price pressure. A turbulent adaptive regime corresponds to states in which strategy shares change rapidly and volatility is high.

With no mutation, some homogeneous or near-homogeneous strategy states may behave like absorbing or nearly absorbing states. With mutation or experimentation, the process typically becomes irreducible and the long-run object becomes a stationary distribution rather than a single terminal state. This permits the analysis of questions that are not natural in the standard simulation-only framing: which regimes receive high stationary probability, how long the system remains in a bubble-like region before escaping, and whether a fundamentalist or chartist regime is stochastically stable when the mutation rate becomes small.

Consequently, the reformulation should not be understood as replacing the Lux–Marchesi model. It is a change of analytical language. The original model supplies the economic mechanism of heterogeneous trading and endogenous price adjustment. Stochastic evolutionary game theory supplies the probabilistic concepts needed to analyze invasion, persistence, metastability, and long-run regime selection.

4 Payoffs, Fitness, and Revision Probabilities

This section specifies the evolutionary mechanism of the reformulated model. The objective is to keep the Lux–Marchesi intuition of profitability-based switching while expressing it in the language of stochastic evolutionary game theory. The construction has four components: strategy-specific trading demand, realized payoff, payoff-to-fitness conversion, and a finite-population revision rule.

4.1 Trading demands

Let the strategy set be

$$S = \{F, C^+, C^-\}, \tag{15}$$

where F denotes fundamentalists, C^+ denotes optimistic chartists, and C^- denotes pessimistic chartists. A compact specification of individual demand is

$$D_F(t) = \phi_F\big(p_t^f - p_t\big), \tag{16}$$

$$D_{C^+}(t) = \phi_C\, s_t, \tag{17}$$

$$D_{C^-}(t) = -\phi_C\, s_t, \tag{18}$$

where p_t is the market price, p_t^f is the fundamental value, $\phi_F > 0$ is the strength of fundamentalist correction, $\phi_C > 0$ is the chartist position scale, and $s_t \geq 0$ is a trend or sentiment intensity. A simple choice is $s_t = 1$, while a more Lux–Marchesi-like implementation may set s_t increasing in recent absolute returns or in the strength of prevailing sentiment. Fundamentalists buy when price is below fundamental value and sell when price is above it. Optimistic chartists submit positive directional demand, whereas pessimistic chartists submit negative directional demand.

If $Z_t = (N_F(t), N_+(t), N_-(t))$ is the population state, aggregate excess demand is

$$ED_t = N_F(t)D_F(t) + N_+(t)D_{C^+}(t) + N_-(t)D_{C^-}(t) + \xi_t, \tag{19}$$

where ξ_t is an order-flow shock. A market-maker price adjustment rule is then

$$p_{t+1} = p_t + \kappa ED_t + \sigma_p \eta_{t+1}, \qquad \kappa > 0, \tag{20}$$

with η_{t+1} representing exogenous microstructure or news noise. This price equation is intentionally parsimonious. It preserves the central feedback channel: the population composition determines excess demand, and excess demand changes the future price path.

4.2 Realized payoffs

The payoff of a strategy is defined from realized trading performance. For each $i \in S$,

$$\pi_i(t+1) = D_i(t)\Delta p_{t+1} - c_i - \frac{\gamma_i}{2}\widehat{\sigma}_t^2, \qquad \Delta p_{t+1} = p_{t+1} - p_t. \tag{21}$$

The first term is trading profit from holding demand $D_i(t)$ during the price change Δp_{t+1}. The parameter $c_i \geq 0$ is a strategy-specific cost. The term involving $\gamma_i \geq 0$ penalizes exposure to local volatility, where $\widehat{\sigma}_t^2$ may be computed as a rolling or exponentially weighted variance of returns. This specification allows chartists to be rewarded during persistent directional movements but penalized in unstable high-volatility episodes. It also allows fundamentalists to perform poorly during bubbles, even if their rule is stabilizing in the long run.

Because traders usually respond to recent performance rather than a single-period payoff, the model uses a smoothed payoff index:

$$\bar{\pi}_i(t+1) = (1-\alpha)\bar{\pi}_i(t) + \alpha\pi_i(t+1), \qquad 0 < \alpha \leq 1. \tag{22}$$

A high value of α means that agents react quickly to the most recent trading outcome. A low value of α produces slower adaptation and longer persistence of existing regimes.

4.3 Fitness mapping

Evolutionary selection is introduced by converting smoothed payoff into fitness:

$$f_i(t) = \exp\{\beta\bar{\pi}_i(t)\}, \tag{23}$$

where $\beta \geq 0$ is selection intensity. When $\beta = 0$, all strategies have equal fitness and strategy revision is driven only by random experimentation and finite-population drift. When β is small but positive, payoff differences affect switching weakly. When β is large, traders strongly favor strategies with higher recent payoff, so herding and rapid regime movement are more likely. The exponential map is useful because fitness remains positive even when realized profit is negative.

4.4 Revision probabilities with mutation

At each revision event, one trader is selected uniformly from the population. If the selected trader currently uses strategy i, the probability of adopting strategy $j \neq i$ is

$$\rho_{ij}(t) = (1-\mu)\frac{f_j(t)}{\sum_{k\in S} f_k(t)} + \mu\frac{1}{|S|-1}, \qquad i \neq j, \tag{24}$$

where $\mu \in [0,1]$ is the mutation or experimentation rate. The first component represents payoff-biased imitation or adaptive choice. The second component allows random experimentation, model uncertainty, or idiosyncratic belief changes.

The induced transition probability on the finite population state space is

$$P_t(z, z - e_i + e_j) = \frac{z_i}{N}\rho_{ij}(t), \qquad i \neq j, \tag{25}$$

where e_i is the unit vector associated with strategy i. The probability of remaining in the same population state is

$$P_t(z,z) = 1 - \sum_{i\in S}\sum_{j\neq i}\frac{z_i}{N}\rho_{ij}(t). \tag{26}$$

Thus the switching rule becomes an explicit finite-population Markov transition law. Without mutation, homogeneous states can become absorbing. With mutation, the chain can move away from homogeneous states, making the stationary distribution and stochastic stability meaningful long-run objects.

4.5 Interpretation of parameters

The parameters have direct economic interpretations. The coefficient ϕ_F measures the strength of stabilizing fundamentalist demand, while ϕ_C measures the strength of directional chartist demand. The parameter κ controls market depth: high κ means that a given excess demand produces a larger price response. The selection intensity β controls how aggressively traders chase profitable strategies. The mutation rate μ prevents permanent lock-in and permits rare transitions among market regimes. Finally, the memory parameter α governs whether regime changes are abrupt or persistent. In the simulation section, different combinations of these parameters generate calm fundamentalist markets, optimistic bubbles, pessimistic crashes, and noisy adaptive markets from the same evolutionary mechanism.

5 The Coupled Stochastic Evolutionary Financial Game

The previous section defines the local mechanism by which traders evaluate strategies and revise their behavior. The complete reformulated model is obtained by coupling that evolutionary rule to the price-adjustment mechanism of the market. The state of the system is written as

$$Y_t = (Z_t, E_t, H_t), \tag{27}$$

where $Z_t = (N_F(t), N_+(t), N_-(t))$ is the finite-population strategy state, E_t is the market environment, and H_t collects payoff-memory variables such as $\bar{\pi}_F(t)$, $\bar{\pi}_+(t)$, and $\bar{\pi}_-(t)$. Including H_t is useful because traders do not necessarily respond only to one-period profit; they may respond to smoothed recent performance. With this augmented state, the process can be treated as Markovian.

A compact market environment is

$$E_t = (q_t, q_t^f, r_t, \sigma_t, m_t), \tag{28}$$

where $q_t = \log p_t$ is log price, $q_t^f = \log p_t^f$ is log fundamental value, $r_t = q_t - q_{t-1}$ is return, σ_t is a volatility or noise scale, and $m_t = x_+(t) - x_-(t)$ is chartist sentiment. The population state affects the environment through excess demand. A convenient reduced specification is

$$ED_t = N_F(t)D_F(t) + N_+(t)D_+(t) + N_-(t)D_-(t), \tag{29}$$

with

$$D_F(t) = a_F(q_t^f - q_t), \tag{30}$$
$$D_+(t) = a_C + b_C r_t, \tag{31}$$
$$D_-(t) = -a_C + b_C r_t. \tag{32}$$

Fundamentalists buy when price is below fundamental value and sell when price is above it. Optimistic chartists create positive speculative pressure, pessimistic chartists create negative speculative pressure, and the trend-following component $b_C r_t$ allows recent returns to reinforce current chartist demand.

The price update can then be written as

$$q_{t+1} = q_t + \alpha \frac{ED_t}{N} + \sigma_t \varepsilon_{t+1}, \qquad \varepsilon_{t+1} \sim \mathcal{N}(0,1), \tag{33}$$

where $\alpha > 0$ measures price impact. The fundamental value may be fixed for theoretical analysis or allowed to follow a slow stochastic process,

$$q_{t+1}^f = q_t^f + \sigma_f \xi_{t+1}, \qquad \xi_{t+1} \sim \mathcal{N}(0,1). \tag{34}$$

The fixed-fundamental case isolates endogenous market instability, while the stochastic-fundamental case separates endogenous speculation from exogenous fundamental news.

After the price is updated, strategy payoffs are realized and smoothed:

$$\bar{\pi}_i(t+1) = (1-\gamma)\bar{\pi}_i(t) + \gamma\pi_i(t+1), \qquad i \in \{F, +, -\}. \tag{35}$$

Fitness is then computed from $\bar{\pi}_i(t+1)$, and one or more traders revise strategy according to the transition probabilities defined in the previous section. Under a single-revision protocol,

$$Z_{t+1} = Z_t - e_i + e_j \tag{36}$$

with probability $P(i \to j \mid Y_t)$, where e_i is the unit vector for strategy i. Thus the joint model is

$$Z_{t+1} \sim P_Z(\cdot \mid Z_t, E_t, H_t), \tag{37}$$

$$E_{t+1} = G(E_t, Z_t) + \varepsilon_{t+1}, \tag{38}$$

$$H_{t+1} = M(H_t, E_{t+1}, Z_t), \tag{39}$$

where P_Z is the evolutionary transition kernel, G is the market-environment update, and M is the payoff-memory update.

This construction clarifies the feedback loop. Strategy composition determines aggregate demand; aggregate demand moves prices; price changes determine realized payoffs; realized payoffs change fitness; and fitness changes the future composition of strategies. In this sense, the market environment is not an external background variable. It is an endogenous state generated by the current population of strategies, and it simultaneously determines the future evolutionary success of those strategies. This is why the model can be interpreted as a stochastic evolutionary game with environmental feedback, rather than only as a numerical agent-based simulation.

The regime interpretation follows directly from the joint state space. A calm fundamentalist regime can be represented by states with high x_F and small mispricing $|q_t - q_t^f|$. A bubble regime corresponds to high x_+ and positive mispricing $q_t - q_t^f > 0$. A crash regime corresponds to high x_- and negative mispricing. A turbulent regime corresponds to high chartist share, frequent switching between x_+ and x_-, and large $|r_t|$ or σ_t. If mutation and experimentation are present, these regimes are not absorbing in the strict sense. They are recurrent or metastable regions that the process may enter, leave, and revisit.

For a finite discretization of the price and volatility variables, the process has a transition matrix P over the joint state space. With positive mutation and regular noise, the chain is typically irreducible and admits a stationary distribution Π satisfying

$$\Pi = \Pi P. \tag{40}$$

The long-run probability of a market regime $\mathcal{R}$ is then

$$\Pi(\mathcal{R}) = \sum_{y \in \mathcal{R}} \Pi(y). \tag{41}$$

This is the central analytical gain of the reformulation: bubbles, crashes, and calm markets can be studied as probability-weighted evolutionary regimes rather than only as individual simulated time-series episodes.

Table 1 Comparison between the standard Lux–Marchesi model and the stochastic evolutionary-game reformulation.

Dimension	Standard Lux–Marchesi model	Evolutionary-game reformulation
Main identity	Agent-based financial market	Finite-population stochastic evolutionary game
Trader types	Behavioral classes	Strategies in a population state
Switching	Behavioral transition rates	Payoff-dependent revision probabilities
Randomness	Simulation and switching noise	Markov transition structure with mutation and shocks
Market role	Price adjusts to excess demand	Price is an endogenous environmental state
Main output	Price paths and return statistics	Regime probabilities and strategy-state dynamics
Long-run object	Simulated market episodes	Stationary distribution or invariant measure
Instability	Endogenous market turbulence	Metastable evolutionary regime
Equilibrium idea	Emergent simulation pattern	Fixation, recurrence, or stochastic stability

6 A Reduced Two-Strategy Birth-Death Version

The three-strategy formulation is useful for simulation because it separates fundamentalists, optimistic chartists, and pessimistic chartists. For analytical work, however, it is often helpful to reduce the model to two strategies. The optimistic and pessimistic chartist states are combined into a single speculative strategy C, while fundamentalism is denoted by F. The reduced strategy set is therefore

$$S = \{F, C\}. \tag{42}$$

This reduction does not mean that optimistic and pessimistic sentiment is unimportant. Rather, it treats chartism as the broader tendency to trade on recent market direction, social reinforcement, or nonfundamental signals. The reduced model is therefore best understood as a tractable analytical approximation to the full three-strategy stochastic evolutionary financial game.

6.1 Population state and market interpretation

Let i denote the number of chartists in a population of size N. The number of fundamentalists is $N - i$, and the chartist share is

$$x_i = \frac{i}{N}, \qquad i \in \{0, 1, \ldots, N\}. \tag{43}$$

The boundary state $i = 0$ represents a purely fundamentalist market, while $i = N$ represents a purely chartist market. Intermediate states represent mixed markets in which fundamental correction and speculative extrapolation coexist. This one-dimensional state variable is useful because it permits explicit birth-death transition probabilities, fixation probabilities, and absorption-time calculations.

The reduced model should be interpreted as a regime-level approximation. When x_i is small, price pressure is mainly stabilizing because fundamentalists dominate. When x_i is large, price pressure is mainly speculative because chartists dominate. The analytical question is therefore not only whether prices fluctuate, but whether the population process tends to move toward fundamentalist dominance, chartist dominance, or persistent coexistence.

6.2 Reduced payoff and fitness

Let $\bar{\pi}_C(i,t)$ and $\bar{\pi}_F(i,t)$ denote smoothed payoffs of chartism and fundamentalism when the current chartist population is i. A compact payoff difference is

$$\Delta\bar{\pi}(i,t) = \bar{\pi}_C(i,t) - \bar{\pi}_F(i,t). \tag{44}$$

Positive values of $\Delta\bar{\pi}(i,t)$ mean that chartism has recently performed better than fundamentalism; negative values mean that fundamentalism has recently performed better. Payoffs are converted into fitness by the exponential mapping

$$f_C(i,t) = \exp\{\beta\bar{\pi}_C(i,t)\}, \qquad f_F(i,t) = \exp\{\beta\bar{\pi}_F(i,t)\}, \tag{45}$$

where $\beta \geq 0$ is the selection intensity. When β is close to zero, payoff differences have little influence and random drift is important. When β is large, traders respond strongly to recent relative profitability.

6.3 Birth-death transition probabilities

The number of chartists can increase by one, decrease by one, or remain unchanged. A Moran-type revision rule gives the probability that a fundamentalist adopts chartism as

$$T_i^+ = \frac{N-i}{N}\left[(1-\mu)\frac{if_C(i,t)}{if_C(i,t) + (N-i)f_F(i,t)} + \frac{\mu}{2}\right], \tag{46}$$

where $\mu \in [0,1]$ is the mutation or experimentation rate. The first factor, $(N-i)/N$, is the probability that the revising trader is a fundamentalist. The expression inside brackets is the probability that chartism is selected either through payoff-biased imitation or through experimentation.

Similarly, the probability that a chartist adopts fundamentalism is

$$T_i^- = \frac{i}{N}\left[(1-\mu)\frac{(N-i)f_F(i,t)}{if_C(i,t)+(N-i)f_F(i,t)} + \frac{\mu}{2}\right]. \tag{47}$$

The probability of no change is

$$T_i^0 = 1 - T_i^+ - T_i^-. \tag{48}$$

Thus the reduced population process is a birth-death Markov chain on $\{0, 1, \ldots, N\}$. The financial interpretation is direct: T_i^+ measures the local probability that speculative behavior spreads, while T_i^- measures the local probability that fundamentalist behavior recovers.

6.4 Selection gradient and invasion logic

A useful local object is the transition ratio

$$R_i = \frac{T_i^+}{T_i^-}. \tag{49}$$

When $R_i > 1$, the population is locally more likely to move toward a larger chartist share; when $R_i < 1$, it is locally more likely to move toward a smaller chartist share. In the absence of mutation, and away from the boundaries, the payoff-sensitive part of this ratio is governed by the relative fitness term

$$\frac{f_C(i,t)}{f_F(i,t)} = \exp\{\beta\Delta\bar{\pi}(i,t)\}. \tag{50}$$

Hence a temporary run of chartist profitability can increase the probability of further chartist adoption. This is the reduced evolutionary version of speculative reinforcement. Conversely, when mispricing becomes large and fundamentalist profits improve, the transition ratio can move below one, increasing the probability of a return toward fundamentalist dominance.

6.5 Fixation probability

If mutation is absent, $i = 0$ and $i = N$ are absorbing states. The fixation probability of a single chartist mutant in a population of $N - 1$ fundamentalists is

$$\rho_C = \left[1 + \sum_{k=1}^{N-1}\prod_{j=1}^{k}\frac{T_j^-}{T_j^+}\right]^{-1}. \tag{51}$$

Under neutral drift, where chartists and fundamentalists have equal fitness, this probability is

$$\rho_C = \frac{1}{N}. \tag{52}$$

Chartism is favored by selection when $\rho_C > 1/N$. This converts the qualitative question "Can speculative trading invade a fundamentalist market?" into a finite-population probability statement. Likewise, the fixation probability of a fundamentalist mutant in a chartist market can be used to ask whether stabilizing fundamental behavior can invade a speculative regime.

6.6 Role of mutation

When $\mu > 0$, the boundary states are no longer permanently absorbing. A purely fundamentalist market may generate chartist experimentation, and a purely chartist market may generate fundamentalist experimentation. The chain then becomes irreducible under standard regularity conditions and admits a stationary distribution. In economic terms, mutation prevents the model from predicting permanent dominance by one trading rule. Instead, the market moves among regimes, with long-run probabilities determined by payoff differences, selection intensity, experimentation, and price feedback.

The reduced birth-death model is therefore useful for two reasons. First, it provides a transparent analytical approximation to the richer three-strategy model. Second, it shows how the reformed Lux–Marchesi mechanism can be studied with standard stochastic evolutionary quantities: transition ratios, fixation probabilities, absorption times, and stationary distributions. These quantities are the analytical bridge between agent-based financial simulation and stochastic evolutionary game theory.

7 Stationary Distribution and Stochastic Stability

The preceding sections specify the local transition mechanism of the reformulated model. This section explains how that mechanism can be used to study long-run market regimes. When mutation, experimentation, or payoff noise is positive, the process does not normally remain forever in a homogeneous trader state. Instead, it moves repeatedly among strategy compositions and market environments. This makes the stationary distribution the natural long-run object.

Let the full state be

$$Y_t = (Z_t, E_t, H_t), \tag{53}$$

where Z_t is the vector of trader counts, E_t is the market environment, and H_t contains payoff-memory variables. After discretizing the continuous market variables, or after working with an empirical state partition, the model induces a transition matrix P on a finite or countable state space. A stationary distribution Π satisfies

$$\Pi = \Pi P, \qquad \sum_{y \in \mathcal{Y}} \Pi(y) = 1. \tag{54}$$

The quantity $\Pi(y)$ is the long-run probability of observing state y. Thus the analysis is no longer limited to a single simulated price path. It can ask how much probability mass is assigned to fundamentalist, speculative, crash-like, or high-volatility states.

For financial interpretation, it is useful to group microstates into regimes. For example, define a calm fundamentalist regime by

$$\mathcal{F} = \{y : x_F(y) \geq \bar{x}_F,\ |q(y) - q^f(y)| \leq \bar{m},\ \sigma(y) \leq \bar{\sigma}\}, \tag{55}$$

an optimistic bubble regime by

$$\mathcal{B}^+ = \{y : x_+(y) \geq \bar{x}_+,\ q(y) - q^f(y) > \bar{m},\ \sigma(y) > \bar{\sigma}\}, \tag{56}$$

and a pessimistic crash regime by

$$\mathcal{B}^- = \{y : x_-(y) \geq \bar{x}_-,\ q^f(y) - q(y) > \bar{m},\ \sigma(y) > \bar{\sigma}\}. \tag{57}$$

The stationary probability of any regime $\mathcal{R}$ is then

$$\Pi(\mathcal{R}) = \sum_{y \in \mathcal{R}} \Pi(y). \tag{58}$$

This expression is important because it turns qualitative statements about bubbles or crashes into probability statements. A bubble is not merely a visually increasing price segment; it is a region of the joint strategy-price state space that may receive positive long-run probability.

In simulations, the stationary distribution can be approximated by the empirical occupation measure

$$\widehat{\Pi}_T(\mathcal{R}) = \frac{1}{T - T_0} \sum_{t=T_0+1}^{T} \mathbf{1}\{Y_t \in \mathcal{R}\}, \tag{59}$$

where T_0 is a burn-in period. This statistic directly reports how often the market spends time in each regime. Comparing $\widehat{\Pi}_T(\mathcal{F})$, $\widehat{\Pi}_T(\mathcal{B}^+)$, and $\widehat{\Pi}_T(\mathcal{B}^-)$ across parameter values shows how selection intensity, mutation, market depth, and risk adjustment change the long-run structure of the market.

Stochastic stability adds a zero-noise selection criterion. Let Π_μ denote the stationary distribution when the mutation or experimentation rate is μ. A regime $\mathcal{R}$ is stochastically stable if

$$\lim_{\mu \to 0} \Pi_\mu(\mathcal{R}) > 0. \tag{60}$$

This concept is useful when several regimes are locally persistent. The deterministic or low-noise dynamics may allow both fundamentalist and chartist persistence, but rare transitions can still select one regime in the very long run [5, 7, 13]. In the present model, stochastic stability asks whether the market ultimately favors a fundamentalist-calm region or whether speculative chartist regimes remain selected even when experimentation becomes rare.

8 Metastability, Bubbles, and Volatility Clustering

The stationary distribution describes long-run probabilities, but financial instability is also about persistence. A market regime is economically important when the process

remains near it long enough to affect observed returns. This motivates the concept of metastability. A metastable regime is not absorbing, but the process can stay there for a long time before a rare transition moves it elsewhere.

In the reformulated Lux–Marchesi model, metastability arises from feedback between population composition and price movement. Suppose optimistic chartists are numerous. Their positive demand pushes the price upward. The resulting price increase raises the recent payoff of optimistic chartism, which increases the probability that other traders adopt the same strategy. This feedback can keep the system near a bubble-like region even when the price is already above fundamentals. The same logic applies to pessimistic chartists during a crash-like episode. Fundamentalists provide a stabilizing force, but their effect may be delayed if recent trend-following payoffs remain high.

A compact way to measure persistence is the mean residence time. For a regime $\mathcal{R}$, define the first exit time

$$\tau_{\mathcal{R}} = \inf\{s > 0 : Y_{t+s} \notin \mathcal{R} \mid Y_t \in \mathcal{R}\}. \tag{61}$$

The conditional expectation $\mathbb{E}[\tau_{\mathcal{R}}]$ measures the expected duration of that regime. In financial terms, $\mathbb{E}[\tau_{\mathcal{B}^+}]$ approximates bubble persistence, while $\mathbb{E}[\tau_{\mathcal{B}^-}]$ approximates crash-regime persistence. These quantities complement stationary probabilities: a regime may occur rarely but last for a long time once entered, or occur frequently but disappear quickly.

The same idea can be summarized by an empirical regime-transition matrix. If $R_t \in \{\mathcal{F}, \mathcal{B}^+, \mathcal{B}^-, \mathcal{T}\}$ denotes the classified regime at time t, where $\mathcal{T}$ represents a turbulent mixed regime, define

$$\widehat{Q}_{ab} = \Pr(R_{t+1} = b \mid R_t = a). \tag{62}$$

Large diagonal entries of $\widehat{Q}$ indicate persistent regimes. Large off-diagonal entries from $\mathcal{F}$ to $\mathcal{B}^+$ or $\mathcal{B}^-$ indicate frequent transitions from calm markets into speculative instability.

Volatility clustering has a natural interpretation in this framework. Conditional volatility is regime dependent. When the system is in a chartist-dominated region, excess demand is more directional and payoff feedback is stronger, so returns tend to be larger and more persistent in absolute value. When the system is in a fundamentalist-dominated region, price deviations from fundamentals are corrected more strongly, so volatility is lower. Since the population state changes gradually, high-volatility periods occur in clusters rather than as isolated shocks. In compact form,

$$\mathrm{Var}(r_{t+1} \mid Y_t \in \mathcal{B}^+ \cup \mathcal{B}^-) > \mathrm{Var}(r_{t+1} \mid Y_t \in \mathcal{F}). \tag{63}$$

Thus volatility clustering is produced by persistent movement among evolutionary regimes, not only by exogenous volatility shocks.

This interpretation remains consistent with the original Lux–Marchesi insight that turbulent market statistics can be generated endogenously by interacting traders [8, 9].

The reformulation adds a theoretical layer: bubbles, crashes, and volatility clusters are recurrent or metastable subsets of a finite-population stochastic evolutionary process.

9 Analytical Implications and Proof Status of the Research Questions

The reformulation generates questions that are difficult to express in the standard agent-based language alone. Some of these questions can be answered analytically from the stochastic evolutionary structure of the model. Others, especially those concerning empirical stylized facts, require simulation or calibration. This section therefore separates exact implications from simulation-based diagnostics.

Proposition 1 (Payoff-monotonic adoption) *Suppose $\beta > 0$ and $0 \leq \mu < 1$. If strategy i has higher attractiveness than strategy j, so that $A_i(t) > A_j(t)$, then the logit mutation-selection rule assigns a higher adoption probability to i than to j.*

Proof The adoption probability is

$$q_i(t) = (1-\mu)\frac{\exp\{\beta A_i(t)\}}{\sum_{k\in S}\exp\{\beta A_k(t)\}} + \frac{\mu}{3}. \tag{64}$$

For any two strategies i and j, subtracting the two probabilities gives

$$q_i(t) - q_j(t) = (1-\mu)\frac{\exp\{\beta A_i(t)\} - \exp\{\beta A_j(t)\}}{\sum_{k\in S}\exp\{\beta A_k(t)\}}. \tag{65}$$

The denominator is positive. Since $\beta > 0$ and the exponential function is strictly increasing, $A_i(t) > A_j(t)$ implies $\exp\{\beta A_i(t)\} > \exp\{\beta A_j(t)\}$. Hence $q_i(t) > q_j(t)$. □

This proposition proves that the evolutionary rule is payoff-consistent: a trading rule with higher recent attractiveness is selected more frequently. It is therefore meaningful to interpret successful chartist or fundamentalist rules as spreading through the trader population.

Proposition 2 (Mutation prevents permanent exclusion of strategies) *If $\mu > 0$, every strategy receives strictly positive adoption probability at every date. In particular,*

$$q_i(t) \geq \frac{\mu}{3} > 0 \qquad \text{for all } i \in S. \tag{66}$$

Consequently, no strategy is permanently excluded from the revision process.

Proof The logit mutation-selection rule is the sum of a payoff-dependent component and a mutation component. Since the payoff-dependent component is nonnegative,

$$q_i(t) = (1-\mu)\frac{\exp\{\beta A_i(t)\}}{\sum_{k\in S}\exp\{\beta A_k(t)\}} + \frac{\mu}{3} \geq \frac{\mu}{3}. \tag{67}$$

Thus every strategy has positive probability of being chosen whenever a revision event occurs. This removes permanent exclusion at the strategy-revision level. In a finite-population implementation with positive revision probability $\nu > 0$, repeated revision events can reintroduce any strategy with positive probability. □

This result explains why mutation changes the long-run object of analysis. Without mutation, homogeneous states may be absorbing. With mutation, the model is better studied through recurrent movement and stationary probabilities.

Proposition 3 (Existence of a stationary distribution under finite discretization) *Suppose the strategy population is finite and the market environment is either finite or discretized into a finite grid. If mutation is positive and the induced transition matrix is irreducible and aperiodic, then the joint process has a unique stationary distribution* Π *satisfying*

$$\Pi = \Pi P. \tag{68}$$

Proof Under the stated discretization, the joint state space for (Z_t, E_t) is finite. The transition law therefore defines a finite Markov chain with transition matrix P. A finite irreducible and aperiodic Markov chain has a unique stationary distribution, and the distribution of the chain converges to it from any initial state. Hence there exists a unique Π such that $\Pi = \Pi P$. □

This proposition provides the analytical basis for long-run regime probabilities. For example, if $\mathcal{F}$ is the calm fundamentalist region, $\mathcal{B}^+$ the optimistic bubble region, $\mathcal{B}^-$ the crash region, and $\mathcal{T}$ the turbulent region, their stationary probabilities are

$$\Pi(\mathcal{R}) = \sum_{y \in \mathcal{R}} \Pi(y), \qquad \mathcal{R} \in \{\mathcal{F}, \mathcal{B}^+, \mathcal{B}^-, \mathcal{T}\}. \tag{69}$$

Proposition 4 (Fixation probability in the reduced two-strategy model) *Consider the reduced two-strategy model* $S = \{F, C\}$ *with no mutation. Let* $i \in \{0, 1, \ldots, N\}$ *be the number of chartists. If* T_i^+ *and* T_i^- *are the one-step probabilities that the number of chartists increases or decreases, then the probability that chartists fixate starting from* i *chartists is*

$$\rho_C(i) = \frac{1 + \sum_{k=1}^{i-1} \prod_{j=1}^{k} \frac{T_j^-}{T_j^+}}{1 + \sum_{k=1}^{N-1} \prod_{j=1}^{k} \frac{T_j^-}{T_j^+}}. \tag{70}$$

For one chartist mutant, this becomes

$$\rho_C(1) = \left[1 + \sum_{k=1}^{N-1} \prod_{j=1}^{k} \frac{T_j^-}{T_j^+} \right]^{-1}. \tag{71}$$

Proof Let ρ_i denote the probability of eventual chartist fixation starting from i chartists. The boundary conditions are $\rho_0 = 0$ and $\rho_N = 1$. For $1 \le i \le N-1$, first-step analysis gives

$$\rho_i = T_i^+ \rho_{i+1} + T_i^- \rho_{i-1} + (1 - T_i^+ - T_i^-)\rho_i. \tag{72}$$

Rearranging,

$$T_i^+(\rho_{i+1} - \rho_i) = T_i^-(\rho_i - \rho_{i-1}). \tag{73}$$

Define $\Delta_i = \rho_i - \rho_{i-1}$. Then

$$\Delta_{i+1} = \frac{T_i^-}{T_i^+} \Delta_i. \tag{74}$$

Iterating gives

$$\Delta_{k+1} = \left(\prod_{j=1}^{k} \frac{T_j^-}{T_j^+} \right) \Delta_1. \tag{75}$$

Using $\rho_N - \rho_0 = 1$ and summing the increments yields

$$1 = \Delta_1 \left[1 + \sum_{k=1}^{N-1} \prod_{j=1}^{k} \frac{T_j^-}{T_j^+} \right]. \tag{76}$$

Solving for Δ_1 and summing from 1 to i gives the stated formula. □

This proposition answers the invasion question in analytical form. Chartism is favored by selection when

$$\rho_C(1) > \frac{1}{N}, \tag{77}$$

that is, when one chartist mutant fixates more often than under neutral drift. The same formula, with the roles of F and C reversed, gives the fixation probability of fundamentalism in a chartist market.

Proposition 5 (A sufficient condition for chartist advantage) *In the reduced two-strategy model, suppose that $T_i^+ > T_i^-$ for all $i = 1, \ldots, N-1$. Then a single chartist mutant has fixation probability above the neutral benchmark:*

$$\rho_C(1) > \frac{1}{N}. \tag{78}$$

Proof If $T_i^+ > T_i^-$ for all interior states, then

$$0 < \frac{T_i^-}{T_i^+} < 1. \tag{79}$$

Therefore each product in the denominator of $\rho_C(1)$ is smaller than one:

$$\prod_{j=1}^{k} \frac{T_j^-}{T_j^+} < 1. \tag{80}$$

Hence

$$1 + \sum_{k=1}^{N-1} \prod_{j=1}^{k} \frac{T_j^-}{T_j^+} < 1 + (N-1) = N. \tag{81}$$

Taking reciprocals gives $\rho_C(1) > 1/N$. □

This result gives a clean sufficient condition for speculative invasion. It does not require chartists to be globally optimal in every market environment; it only states that if the transition pressure favors chartists at every interior population state, chartist fixation exceeds neutral drift.

Proposition 6 (Residence-time bound for metastable regimes) *Let $\mathcal{R}$ be a regime region, such as an optimistic bubble or pessimistic crash. Suppose that, for every state $y \in \mathcal{R}$, the one-period probability of leaving $\mathcal{R}$ is at most ε, where $0 < \varepsilon < 1$. Then the expected residence time in $\mathcal{R}$ is at least $1/\varepsilon$.*

Proof Let $\tau_{\mathcal{R}}$ be the first exit time from $\mathcal{R}$. If the probability of exit in any period is at most ε, then the probability of remaining in $\mathcal{R}$ for at least m periods is bounded below by $(1-\varepsilon)^m$. Therefore,

$$\mathbb{E}[\tau_{\mathcal{R}}] = \sum_{m\geq 0} \Pr(\tau_{\mathcal{R}} > m) \geq \sum_{m\geq 0} (1-\varepsilon)^m = \frac{1}{\varepsilon}. \tag{82}$$

□

This proposition gives a precise meaning to the statement that bubbles and crashes are metastable. A bubble is persistent when the probability of leaving the optimistic-chartist region is small. A crash is persistent when the probability of leaving the pessimistic-chartist region is small.

Proposition 7 (Proof status of stochastic stability) *For a finite perturbed chain P_μ with stationary distribution Π_μ, a regime $\mathcal{R}$ is stochastically stable if*

$$\liminf_{\mu\to 0} \Pi_\mu(\mathcal{R}) > 0. \tag{83}$$

The reformulated model therefore admits a formal stochastic-stability question once the finite-state approximation and mutation structure are specified.

Proof For each positive mutation rate $\mu > 0$, the finite perturbed chain has a stationary distribution under the conditions of Proposition 3. Stochastic stability is defined by the limiting stationary probability as the perturbation vanishes. If a regime retains positive limiting probability as $\mu \to 0$, then it is selected by rare mutations and is stochastically stable. Determining which regime has this property requires comparing the low-mutation transition paths or resistance costs among regimes. □

The remaining empirical question is different in nature. The model can generate volatility clustering and heavy-tailed returns in simulation, but these properties are not guaranteed by the transition rule alone. They require diagnostics such as tail estimation, autocorrelation of returns, autocorrelation of absolute or squared returns, and comparison with observed data.

Thus, Section 9 answers the theoretical part of the research program. The model proves payoff-monotonic adoption, mutation-driven recurrence, stationary-distribution existence under finite-state conditions, fixation probabilities in the reduced model, and a residence-time interpretation of metastable regimes. The simulation section then addresses the qualitative regime-generation question, while full empirical validation remains a separate calibration exercise.

10 Simulation Design and Multi-Regime Results

The simulation exercise is intended as a proof-of-mechanism test of the reformulated model. The question is whether the same stochastic evolutionary transition structure can generate a wide range of qualitatively distinct market regimes when the parameters governing selection, mutation, social reinforcement, price impact, and initial population shares are changed. If the answer is positive, then the reformulation

Table 2 Research questions, analytical objects, and proof status.

Question	Object	Status in the present model
Can chartism invade a fundamentalist market?	$\rho_C(1)$	Proved in the reduced birth-death model; chartism is favored if $\rho_C(1) > 1/N$.
Can fundamentalism invade a speculative market?	$\rho_F(1)$	Same fixation formula with the roles of F and C reversed.
Does payoff affect adoption?	Logit revision probability	Proved by payoff-monotonicity of the exponential revision rule.
Does mutation prevent permanent exclusion?	Mutation-selection rule	Proved because $q_i(t) \geq \mu/3 > 0$ for every strategy.
Which regime dominates in the long run?	Stationary probability $\Pi(\mathcal{R})$	Proved to be well-defined for a finite irreducible and aperiodic approximation; dominance is computed from Π.
Which regime survives as mutation vanishes?	Stochastic stability	Formally defined by $\liminf_{\mu\to 0} \Pi_\mu(\mathcal{R}) > 0$; identifying the selected regime requires low-mutation comparison.
How long do bubbles and crashes last?	Residence time $\mathbb{E}[\tau_{\mathcal{R}}]$	Bounded analytically by exit probabilities; estimated directly in simulation.
Does the model reproduce stylized facts?	Return diagnostics	Not proved from structure alone; tested by simulation and empirical calibration.

has operational content and is not merely a relabeling of the standard Lux–Marchesi mechanism.

The simulations use the three-strategy population

$$S = \{F, C^+, C^-\},$$

where F denotes fundamentalists, C^+ optimistic chartists, and C^- pessimistic chartists. At each date t, the finite-population state is

$$Z_t = (N_F(t), N_+(t), N_-(t)), \qquad X_t = (x_F(t), x_+(t), x_-(t)).$$

The baseline market environment is generated by a constant fundamental value together with endogenous price fluctuations, except in the shock regime, where the fundamental benchmark is shifted once to test adjustment and recovery. Per-capita demands are specified as

$$d_F(t) = k_F\big(\log p_t^f - \log p_t\big), \qquad d_+(t) = q_C + \eta\,\theta_t, \qquad d_-(t) = -q_C + \eta\,\theta_t,$$

where k_F is the fundamentalist correction strength, q_C is baseline chartist pressure, η is the sensitivity of chartists to recent price trends, and θ_t is a smoothed trend signal.

Aggregate excess demand is

$$D_t = x_F(t)d_F(t) + x_+(t)d_+(t) + x_-(t)d_-(t),$$

and the market-maker price equation is

$$r_{t+1} = \alpha \tanh(D_t) + \sigma_p \varepsilon_{t+1}, \qquad \log p_{t+1} = \log p_t + r_{t+1}.$$

The hyperbolic tangent prevents numerically explosive one-period jumps while preserving the sign of excess demand. Strategy performance is measured by

$$\pi_i(t+1) = d_i(t) r_{t+1} - c_i - \lambda r_{t+1}^2,$$

which is then smoothed according to

$$\bar{\pi}_i(t+1) = \omega \bar{\pi}_i(t) + (1-\omega)\pi_i(t+1).$$

Attractiveness combines payoff memory and social reinforcement:

$$A_i(t+1) = \bar{\pi}_i(t+1) + s x_i(t).$$

The adoption probabilities follow the logit mutation-selection rule

$$q_i(t+1) = (1-\mu)\frac{\exp\{\beta A_i(t+1)\}}{\sum_{j\in S}\exp\{\beta A_j(t+1)\}} + \frac{\mu}{3},$$

and a fraction ν of the population revises in each period, so the expected next-period composition is

$$\tilde{x}_i(t+1) = (1-\nu)x_i(t) + \nu q_i(t+1).$$

The finite population is then updated by multinomial sampling,

$$Z_{t+1} \sim \text{Multinomial}\big(N, \tilde{X}_{t+1}\big),$$

which preserves endogenous stochasticity even when the parameter vector is fixed.

Eight regimes are reported. The first is a *fundamentalist calm* regime with strong stabilizing correction and a high initial fundamentalist share. The second is a *balanced mixed market* in which the three trader groups coexist. The third is an *optimistic bubble* regime with strong social reinforcement and optimistic chartist dominance. The fourth is a *pessimistic crash* regime, which is the bearish counterpart of the bubble case. The fifth is a *high-noise adaptive switching* regime with high mutation and frequent revision. The sixth is a *trend-reversal cycle* regime in which price momentum and optimistic chartist success reinforce each other over long intervals. The seventh is a *shock and recovery* regime with a discrete downward shift in the fundamental benchmark. The eighth is a *speculative polarization* regime in which the market becomes heavily chartist-dominated and mispricing becomes extreme.

Figure 1 shows that the reformulated model can generate multiple qualitatively different price paths from the same conceptual structure. The calm regime remains tightly centered around the fundamental benchmark. The mixed regime fluctuates around the benchmark without a persistent directional drift. The bubble regime exhibits sustained upward divergence driven by optimistic chartist dominance, while the crash regime exhibits sustained downward divergence under pessimistic chartist dominance. The noisy regime remains bounded but highly irregular. The trend-reversal cycle generates a long cumulative appreciation driven by self-reinforcing trend-following behavior. The shock regime shows a discrete downward shift followed by partial recovery. The speculative polarization regime produces the strongest and most persistent upward mispricing because chartists almost completely dominate the market.

Figure 2 confirms that the price regimes are supported by economically meaningful changes in trader composition. In the calm regime, the fundamentalist share remains close to one, and both chartist groups stay near zero. In the bubble regime, optimistic chartists rapidly dominate the population, whereas in the crash regime pessimistic chartists become dominant. In the mixed and noisy regimes no single group fixes permanently; instead, the three groups coexist and the market stays in a fluctuating but bounded state. The trend-reversal cycle shows persistent optimistic chartist dominance after an initial transition. The shock regime exhibits a more balanced population with temporary increases in chartist influence around the adverse movement in fundamentals. The speculative polarization regime converges to near-complete optimistic chartist dominance and therefore yields persistent explosive mispricing.

Table 3 Return and price statistics for the eight-regime simulation report. Prices are indexed to $p_0 = 100$.

Regime	Mean r_t	Std. r_t	Min. price	Max. price	Final price
Fundamentalist calm	−0.000007	0.001816	97.686	101.757	98.893
Balanced mixed market	−0.000008	0.004446	89.254	108.094	98.826
Optimistic bubble	0.002019	0.001974	99.917	2067.569	2067.569
Pessimistic crash	−0.002011	0.002009	4.897	100.000	4.897
High-noise adaptive switching	−0.000007	0.006536	95.112	107.391	98.951
Trend-reversal cycle	0.002372	0.003582	100.000	3508.138	3508.138
Shock and recovery	−0.000145	0.003555	74.637	105.727	80.434
Speculative polarization	0.005978	0.004459	99.219	784078.978	784078.978

Tables 3 and 4 show that the eight regimes are clearly separated in both market behavior and population structure. The calm and mixed regimes remain close to the fundamental benchmark on average, but the mixed regime displays noticeably larger dispersion. The noisy regime has the largest bounded return volatility among the non-explosive cases, which is consistent with high mutation and frequent revision. The bubble, crash, cycle, and polarized regimes all feature very high average chartist shares together with large absolute mispricing. The shock regime is intermediate: it remains partially bounded, but the exogenous shift in the fundamental benchmark moves the price into a different region and temporarily changes the population mix.

The two panel figures together are especially informative. The price panels by themselves show that the reformulated model can generate calm, mixed, explosive, cyclical,

Table 4 Population-composition and mispricing statistics for the eight-regime simulation report.

Regime	Mean x_F	Mean $(x_+ + x_-)$	Max. $\lvert \log(p_t/p_t^f) \rvert$
Fundamentalist calm	0.985	0.015	0.023
Balanced mixed market	0.346	0.654	0.114
Optimistic bubble	0.002	0.999	3.029
Pessimistic crash	0.002	0.998	3.017
High-noise adaptive switching	0.335	0.665	0.071
Trend-reversal cycle	0.036	0.964	3.558
Shock and recovery	0.356	0.645	0.160
Speculative polarization	0.004	0.996	8.967

noisy, and shock-driven paths. The population panels show *why* these paths arise. A bubble corresponds to the dominance of optimistic chartists; a crash corresponds to the dominance of pessimistic chartists; a calm market corresponds to fundamentalist dominance; and the noisy and mixed markets correspond to long-run coexistence with no complete fixation. In this sense, the simulation results are directly aligned with the theoretical reinterpretation of the Lux–Marchesi model as a finite-population stochastic evolutionary game.

The results should nevertheless be interpreted carefully. They are proof-of-mechanism simulations rather than empirical calibration. In particular, the most extreme bubble and polarized regimes are intentionally parameterized to make regime separation obvious, so they should not yet be read as realistic quantitative predictions. For a later empirical version, the next steps would be to impose stronger stabilizing forces, calibrate the trend and price-impact parameters more tightly, estimate stationary regime probabilities on a coarse discretized state space, and compare the model quantitatively with observed stylized facts such as fat tails and volatility clustering.

11 Conclusion

This paper reformulates the Lux–Marchesi financial market model as a finite-population stochastic evolutionary game. The reformulation keeps the central economic mechanism of the original model–heterogeneous fundamentalist and chartist trading, stochastic switching, herding, and endogenous price feedback–but expresses it through the formal objects of stochastic evolutionary game theory. Trader types become strategies, realized trading performance becomes payoff, payoff is mapped into fitness, and strategy revision is governed by mutation-selection transition probabilities. The resulting state variable is the coupled process (Z_t, E_t), where trader composition and the market environment evolve together.

The paper's main contribution is not the introduction of a new behavioral type or a new empirical stylized fact. Its contribution is to provide a theoretical bridge between agent-based financial markets and stochastic evolutionary game theory. This bridge changes the analytical focus from simulated price paths alone to probabilistic questions about invasion, fixation, stationary regime probabilities, metastability, and stochastic stability. In this framework, a bubble is naturally interpreted as a region of the state space dominated by optimistic chartists, a crash as a region dominated by

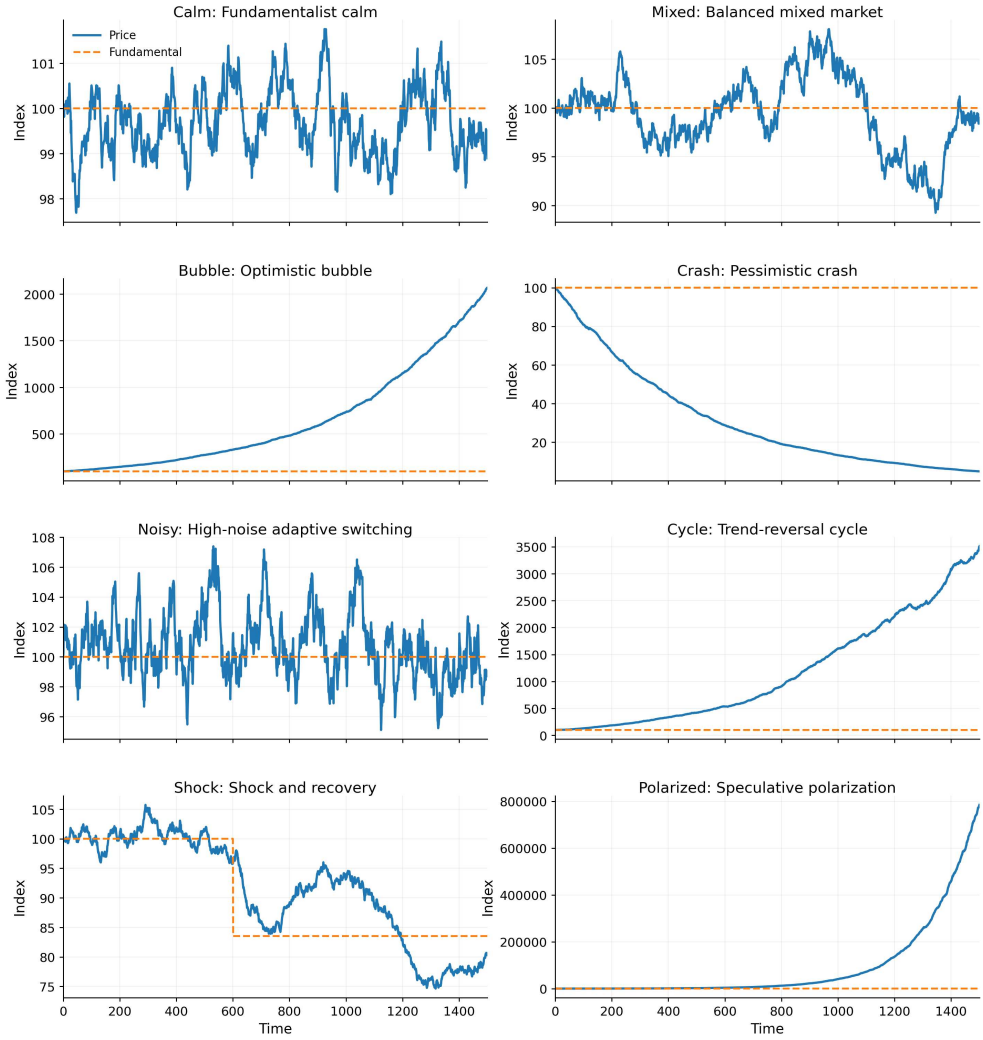


Fig. 1 Panel plot of simulated price series across eight regimes. The solid line is the market price and the dashed line is the fundamental value. The figure is intentionally compact so that the regime comparison can be read directly across subplots.

pessimistic chartists, and a calm market as a region in which fundamentalists remain prevalent and mispricing is small.

Several analytical implications follow directly from the reformulated model. The logit revision rule implies payoff-monotonic adoption: strategies with higher attractiveness receive higher adoption probability. Positive mutation prevents permanent exclusion of any strategy and, under finite-state discretization, supports the existence of a stationary distribution. In the reduced two-strategy birth-death version, chartist fixation probabilities are available in closed form, and the transition-ratio condition identifies when chartist expansion is favored locally. These results show that the

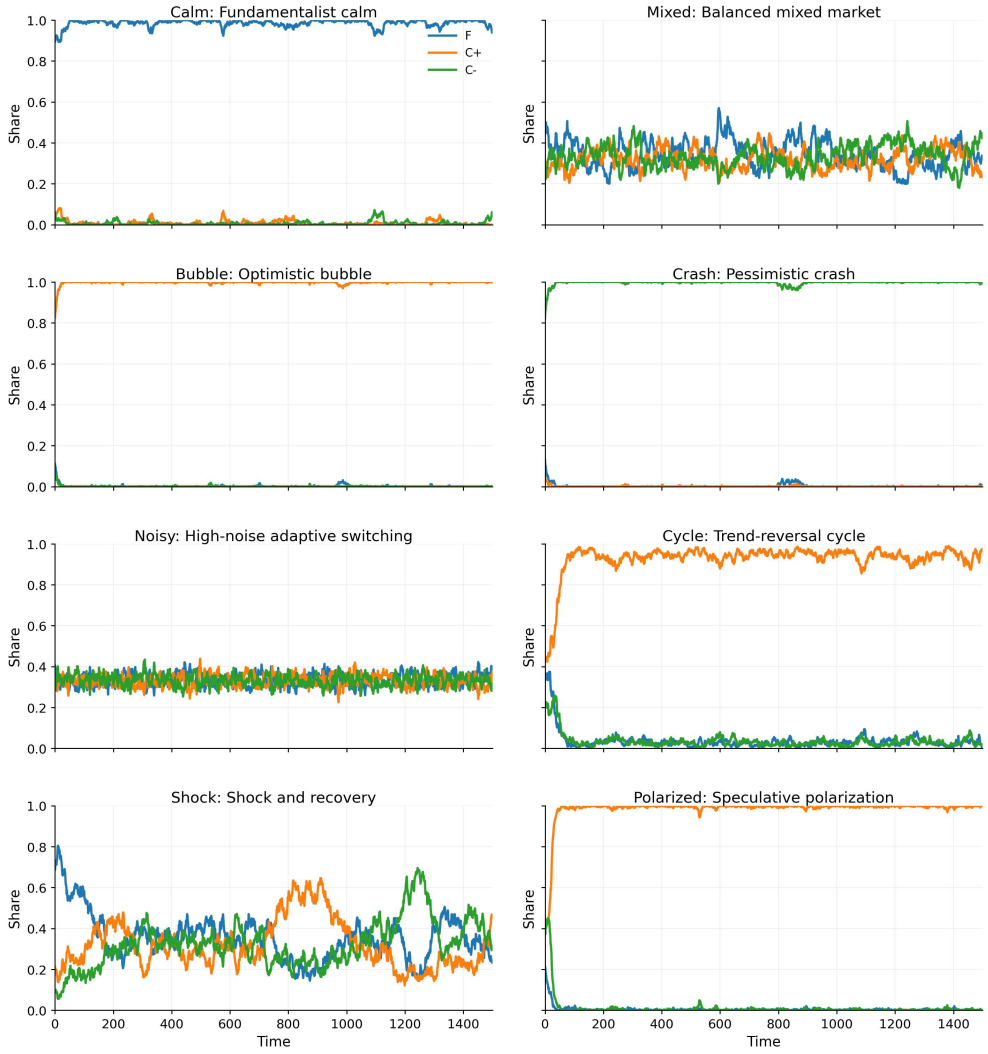


Fig. 2 Panel plot of population dynamics across eight regimes. Each subplot reports the shares of fundamentalists, optimistic chartists, and pessimistic chartists. The population panels provide the evolutionary explanation for the price paths in Figure 1.

reformulation provides more than a simulation vocabulary; it generates mathematical objects that can be analyzed with standard tools from stochastic evolutionary game theory.

The multi-regime simulations provide a complementary proof of mechanism. The same evolutionary transition structure generates fundamentalist calm, balanced mixed markets, optimistic bubbles, pessimistic crashes, high-noise adaptive switching, trend-reversal cycles, shock responses, and speculative polarization. The price panel plots show the diversity of market regimes, while the population panel plots show the evolutionary source of those regimes. In particular, bubbles are associated with optimistic

chartist dominance, crashes with pessimistic chartist dominance, calm markets with fundamentalist dominance, and noisy or mixed markets with persistent coexistence of trader types.

The results should be read with appropriate caution. The simulations are not yet an empirical calibration, and the most extreme regimes are deliberately parameterized to make the mechanisms visible. Future work should estimate coarse transition matrices among regimes, compute stationary distributions and residence times, study the low-mutation limit for stochastic stability, and calibrate the model against observed financial data. It should also compare the reformulated model with the original Lux–Marchesi simulation under matched restrictions and test whether the reformulation reproduces empirical stylized facts such as fat-tailed returns and volatility clustering. Even at the present stage, however, the paper shows that the Lux–Marchesi mechanism can be placed in a stochastic evolutionary-game framework in a coherent and operational way.

Declarations

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Conflict of Interest The authors declare that they have no conflict of interest.

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