Supplementary Material

A de Sitter region at every black-hole core: discrete causal-set evidence and a canonical regular continuum metric

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1 Canonical numerical inputs

The model fixes the de Sitter core scale to the inverse of the spin-2 fakeon-pole mass and uses no other free parameter beyond the ADM mass M and the nonlocality cutoff Λ. The headline numerical constants of the canonical sector are collected in Table 1.

Table 1: Canonical inputs of the regular black-hole construction. All quantities are independent rationals or zeros of the canonical entire form factor; none is fit to data.

Symbol	Definition	Value
z_1	first positive real zero of $\Pi_{TT}(z)$	$2.41483889\ldots$
$m_{2,\,\text{pole}}$	spin-2 fakeon-pole mass	$\Lambda\sqrt{z_1} \approx 1.5540\,\Lambda$
ℓ_{can}	canonical de Sitter core scale	$1/m_{2,\,\text{pole}}$
$(M\Lambda)_{\text{min}}$	minimal horizon mass threshold	$0.83641\ldots$
$\mathcal{K}(0)$	Kretschmann scalar at the centre	$96\,M^2/\ell_{\text{can}}^6$
r_{SEC}	SEC-violation radius (inner core)	$2^{-1/3}\,\ell_{\text{can}} \approx 0.7937\,\ell_{\text{can}}$
β_{PPN}	post-Newtonian β	1 (exact)
γ_{PPN}	post-Newtonian γ	1 (exact)
$\delta a_{1/r^4}$	lapse-correction leading order	$\mathcal{O}(1/r^4)$

2 Sprinkling protocol for the empirical closure

The fundamental causal-set demonstration uses a finite-N Poisson sprinkling of the Schwarzschild interior region truncated at radial distance ε_r from the central singularity. The protocol parameters that underlie the headline scaling $\deg_{\max} \sim N^{1.04\pm0.02}$ and the bounded curvature proxy saturating at ≈ 8.4 are listed in Table 2.

3 Figure-regeneration script

The script `make_figures.py` bundled with this supplement regenerates each of the six paper figures from the canonical data and the closed-form continuum expressions. Invocation from the directory containing the script:

```
pythonmake_figures.py
```

The script writes `fig1_overview.pdf` through `fig6_eps_deep.pdf` into the current working directory at publication-quality resolution. Each figure is independent and can be regenerated separately by uncommenting the corresponding block at the bottom of the script. Required packages: `numpy`, `matplotlib`, and `scipy` (for the Brent root used to locate z_1).

Table 2: Sprinkling protocol parameters used in the discrete-level closure. All cells contribute to the fit reported in the main text.

Parameter	Range / value	Notes
Background	Schwarzschild	$M = 1$ in geometric units
Inner cutoff ε_r	$10^{-2}\,M$	deep-IR truncation
Outer cutoff	$2M$	inside event horizon
N-grid	$\{10^4,\, 3{\cdot}10^4,\, 10^5,\, 3{\cdot}10^5\}$	log-spaced
Seeds per N	5	independent Poisson realisations
Causal predicate	metric light-cone	evaluated exactly
Curvature proxy	$\deg_{\max} / \langle \deg \rangle$	local-degree ratio
Reference family	Hayward effective metric	saturates at ≈ 3.3

4 Provenance of the canonical zero z_1

The first positive real zero $z_1 = 2.41483889\ldots$ of the canonical transverse–traceless form factor $\Pi_{TT}(z)$ is the load-bearing numerical input of the construction. Its location was determined by a bracketed Brent root finder applied to a high-precision (`mpmath` dps ≥ 50) implementation of the entire form factor; the value was independently cross-checked against the same zero recorded in the project canonical-validator module, with agreement to 1.3×10^{-12}. The same constant controls the spin-2 fakeon-pole mass $m_{2,\,\mathrm{pole}} = \Lambda\sqrt{z_1}$ and, through it, the de Sitter core scale $\ell_{\mathrm{can}} = 1/m_{2,\,\mathrm{pole}}$, the minimal horizon mass threshold $(M\Lambda)_{\min} = 0.836$, and the Ori e-folding rate $\kappa_- \to m_{2,\,\mathrm{pole}}$ at large $M\Lambda$.

5 Limitations carried into the supplement

The reproducibility material covers the figure pipeline and the canonical numerical constants in full. It does *not* provide a first-principles discrete computation of inner-horizon dynamics, a nonlinear-perturbative proof of stability, or extensions to rotating or charged backgrounds; each of these is explicitly listed as an open question in the main manuscript and remains so in the supplement.