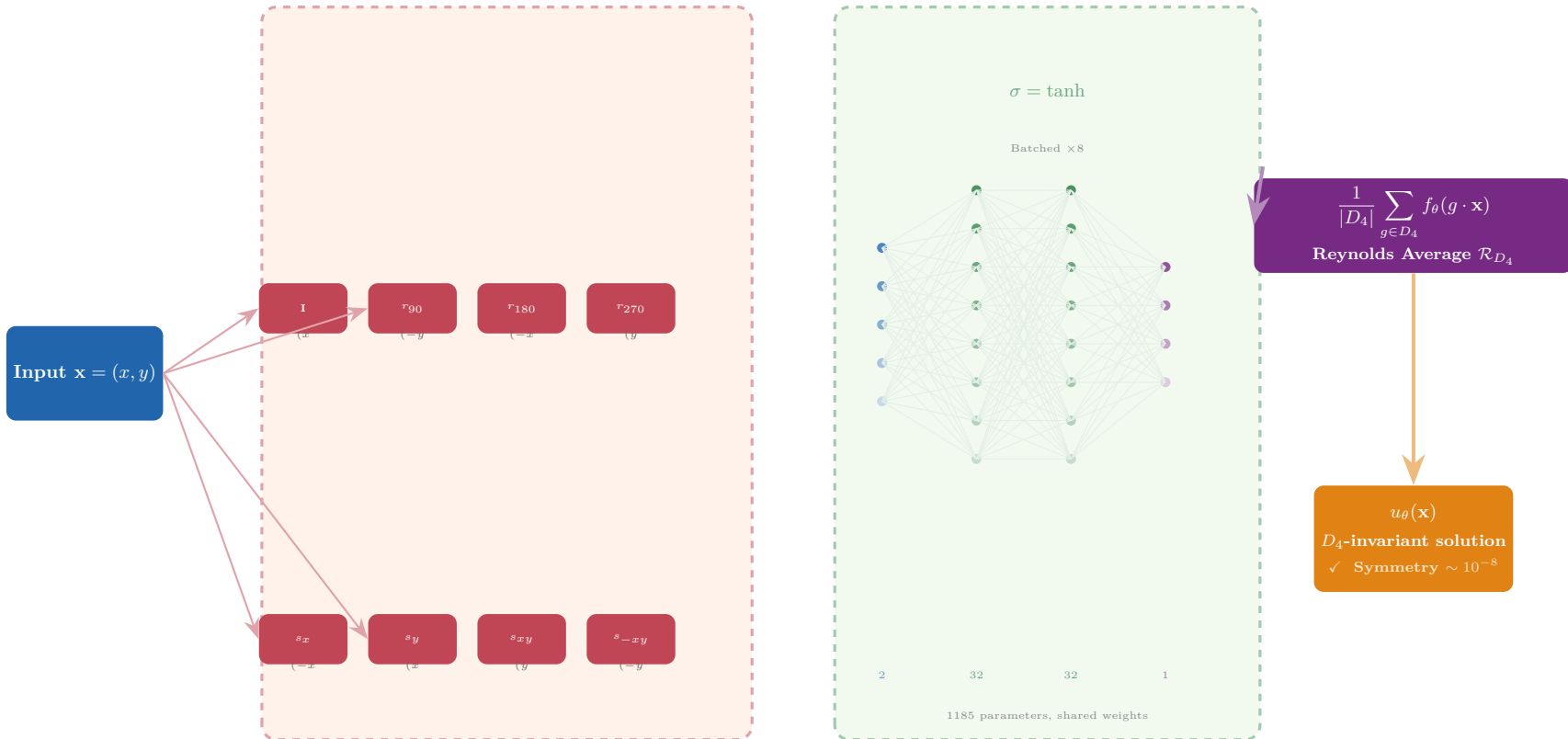


D_4 -PINN Computational Graph

D_4 Orbit Expansion • 8 Transformed Inputs

Shared MLP f_θ • Applied to Each Orbit Element



$|D_4| = 8$ orthogonal 2×2 matrices (dihedral group of the square)

Proposition 1 (Architectural D_4 -Invariance): $u_\theta(g' \cdot \mathbf{x}) = u_\theta(\mathbf{x}) \quad \forall g' \in D_4, \quad \forall \theta \in \Theta$ • Guaranteed at machine precision ($\sim 10^{-8}$) for every forward pass