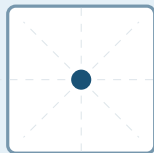


D_4 -PINN — Reynolds-Averaged Group-Equivariant Architecture

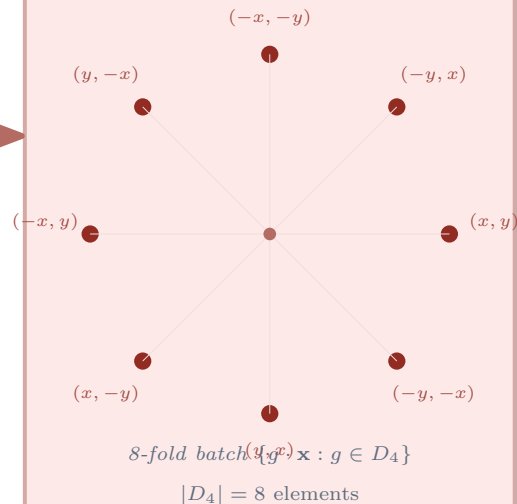
Input Space

$\Omega = (-1, 1)^2$
square domain



$\mathbf{x} = (x, y)$

D_4 Orbit Expansion

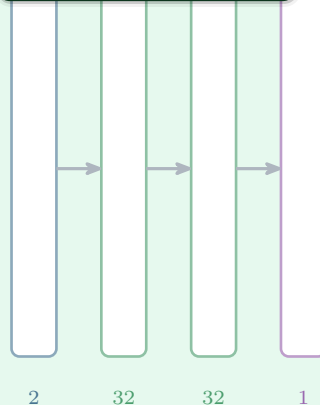


Reynolds Average \mathcal{R}_{D_4}

$$\frac{1}{|D_4|} \sum_{g \in D_4} f_\theta(g \cdot \mathbf{x})$$

Shared MLP f_θ

Activation: $\sigma = \tanh$



Batched $\times 8$ (einsum)
1185 parameters

Output

$u_\theta(\mathbf{x})$
 D_4 -invariant
solution

✓ Sum $\approx 10^{-8}$

Proposition 1 (Architectural Invariance): $\forall \theta \in \Theta, \forall \mathbf{x} \in \mathbb{R}^2, u_\theta(g' \cdot \mathbf{x}) = u_\theta(\mathbf{x})$ for all $g' \in D_4$ (the Reynolds average makes every forward pass exactly D_4 -invariant at machine precision)

Input Space

D_4 Orbit Expansion

Shared MLP f_θ

Reynolds Average

Output