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Vikas Kumar Sharma

Veer Bahadur Singh Purvanchal University

Deepak Kumar Maurya

deepak893395@gmail.com

Veer Bahadur Singh Purvanchal University

Pankaj Kumar Maurya

Prof. Rajendra Singh (Rajju Bhaiya) University

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Magnetohydrodynamic Behavior of Casson–Newtonian Composite Flows: Modeling Blood-like Fluids in Rotational Motion

Vikas Kumar Sharma¹, Deepak Kumar Maurya¹ and Pankaj Kumar Maurya²

¹*Department of Mathematics, Prof. Rajendra Singh (Rajju Bhaiya) Institute of Physical Sciences for Study and Research, V.B.S. Purvanchal University, Jaunpur, 222001, Uttar Pradesh, India*

²*Department of Mathematics, Prof. Rajendra Singh (Rajju Bhaiya) University, Prayagraj, 211010, Uttar Pradesh, India*

Emails: vs.vbspu@gmail.com (Vikas Kumar Sharma); deepak893395@gmail.com (Corresponding Author) and pkjmaurya88@gmail.com (Pankaj Kumar Maurya)

Abstract

In the proposed physical model, we have investigated the magnetohydrodynamic behavior of a composite cylindrical system comprising concentric layers of Newtonian and Casson fluids surrounding a solid core and both regions are subjected to rotational motion with distinct angular velocities, while an imposed axial magnetic field governs the coupled transport phenomena. Analytical modeling in cylindrical coordinates is employed to derive velocity distributions, shear stress profiles, and the influence of Casson rheology under magnetic field constraints. A comparative numerical study among the Newtonian-Casson model, the Newtonian-Newtonian model, and the Newtonian-Stokes model is made for velocity profiles and shear stresses under variations of certain flow parameters. The study highlights the interplay between blood-like fluid characteristics and magnetic field effects, offering insights into biomedical and industrial applications.

Keywords: Newtonian Fluid; Casson fluid; Modified Bessel functions; Shear Stress; Permeability

Nomenclature

Subscripts

$i = 1$	Newtonian fluid
$i = 2$	Casson fluid
C_j	Arbitrary parameters for $j = 1, 2, 3, 4$

Symbols	Description(Unit)	Dimension
(r^*, θ, z^*)	Cylindrical polar coordinates (m, c, m)	$([L], [L^0], [L])$
$(v_r^*, v_\theta^*, v_z^*)$	Components of fluid velocity ($m.s^{-1}, c, m.s^{-1}$)	$([LT^{-1}], [L^0T^0], [LT^{-1}])$
ρ^*	Fluid density ($kg.m^{-3}$)	$[ML^{-3}]$
p^*	Fluid pressure (Pa)	$[ML^{-1}T^{-2}]$
$\mathbf{v}_i^*$	Fluid velocity vector ($m.s^{-1}$)	$[LT^{-1}]$
τ	Stress tensor (Pa)	$[ML^{-1}T^{-2}]$
a^*	Radius of inner cylinder (m)	$[L]$
b^*	Radius of middle cylinder (m)	$[L]$
c^*	Radius of outer cylinder (m)	$[L]$
$\mathbf{B}^*$	External magnetic field (T)	$[MT^{-2}A^{-1}]$
μ_i^*	Viscosity of the fluid ($Pa.s$)	$[ML^{-1}T^{-1}]$
σ_i^*	Electric conductivity (<i>Siemens</i>)	$[M^{-1}L^{-3}T^3A^2]$
ω_i^*	Angular velocity of cylindrical shell (s^{-1})	$[T^{-1}]$
k_i^*	Permeability of the porous cylindrical shell (m^2)	$[L^2]$

Notation

β	Casson parameter
ϕ	Porosity of the medium
H_i	Hartmann number corresponding to the fluid
λ	Viscosity ratio
Λ	Electric conductivity ratio
$\mathbb{R}$	Set of real numbers
ω	Ratio of angular velocities
$I_1(\cdot)$	Modified Bessel's function of first kind
$K_1(\cdot)$	Modified Bessel's function of second kind

1 Introduction

Fluid flow in concentric cylindrical geometries under hydromagnetic forces has attracted a lot of attention because of its many industrial and technical uses. Systems with a Newtonian fluid enclosed in a solid core and a Casson fluid flowing via rotating porous concentric cylinders provide unique problems that are essential for optimizing efficiency in biomedical devices, equipment, and polymer processing. To estimate sedimentation, flow resistance, and viscosity in collections of spherical particles, the free-surface model has been developed and successfully applied to the situation of flow relative to cylinders. It is demonstrated to be in excellent accord with the data already available. Blood, chocolate, and polymer suspensions are examples of non-Newtonian materials that are frequently modeled using Casson fluids, which are distinguished by their yield stress and shear-thinning characteristics [1]. In cylindrical polar coordinates, Deo et al. [2] developed a general stream function solution of the Brinkman equation. The solution provided covers a thorough framework for analyzing axisymmetric flows in porous materials. A model for the Casson fluid flow past a non-linearly stretched sheet under the influence of a magnetic field has been investigated by Mustafa and Khan [3]. This work provides substantial understanding into heat and mass transport processes by addressing the combined impacts of nonlinear stretching, magnetic forces, and fluid dynamics. Tagg et al. [4] studied the fluid flow between rotating concentric cylinders has gained significant attention due to its applications in engineering and industrial processes. fluids have emerged as effective heat transfer enhancers in heat exchangers [5], further advancing thermal management technologies. The stream function solution of the Brinkman equation in parabolic cylindrical coordinates has been identified by Maurya and Deo [6]. With the integration of viscous and Darcy effects for improved modeling of various flow scenarios this system gives an established basis for examining fluid flow in porous media with parabolic forms. The flow of a Casson fluid between two rotating cylinders has been studied by Batra and Das [7], whose work investigates the dynamics of a non-Newtonian Casson fluid confined between rotating cylindrical surfaces.

The behavior of micropolar fluid under Stokes flow scenarios in a porous cylinder was analyzed by Maurya et al. [8], in their analysis of Stokes flow of micropolar fluid through a porous cylinder, the study gives helpful details concerning how porosity and micro-rotation affect fluid dynamics. Maurya and Deo [9] have discussed how magnetic fields

impact fluid flow dynamics in composite porous media is obtained from research on the effects of MHD on micropolar-Newtonian fluid flow via a composite porous channel. Effect of thermal radiation on Powell-Eyring fluid MHD Flow, the effect of thermal radiation and magnetic fields on Powell-Eyring fluids, and this was induced by a stretching cylinder has been studied by Hayat et al. [10], gives significant understanding into heat transfer and flow dynamics. Deo et al. [11] have investigated the impact of a magnetic field on micropolar fluid flow in a cylindrical tube with an impermeable core covered in a porous layer. The intricate relationships between fluid dynamics and magnetic forces are studied in this study. The hydromagnetic flow of Casson fluid along a stretching plane through a porous material has been studied by Maurya et al. [12]. Venkatesan et al. [13] studied extended this framework to complex fluids, such as Casson fluid, which is particularly relevant in biomedical applications like blood flow modeling. The behavior of Casson fluid under torsional effects has been explored in various geometries, including concentric cylinders in porous media have been studied by Jat et al. [14]. Kameshwaran et al. [15] investigated the dual solutions for Casson fluid flow over stretching or shrinking sheets further demonstrate its nonlinearity and complex boundary interactions. The influence of magnetic fields on fluid flow has been studied in multi-layered systems, highlighting their role in controlling fluid motion within porous cylindrical shells studied by Maurya and Deo [16]. Maurya and Sharma [17] reported the analytical solution of the Brinkman equation in tangent sphere and cardioid coordinate systems.

2 Physical Model of the Problem and Solution

Let us consider a steady, incompressible, electrically conducting, Newtonian and Casson fluids, flowing through the rotating co-axial porous cylinders. The uniform magnetic field is taken in the direction perpendicular to the axis of co-axial cylinders. Assuming inner solid core and outer cylindrical shells are rotating with angular rotation ω_1^* and ω_2^* respectively, and middle cylindrical shell is at rest. The present research model exhibits the sandwiching of Newtonian fluid between the solid core and Casson fluid through two porous co-axial cylindrical shells (Fig. 1).

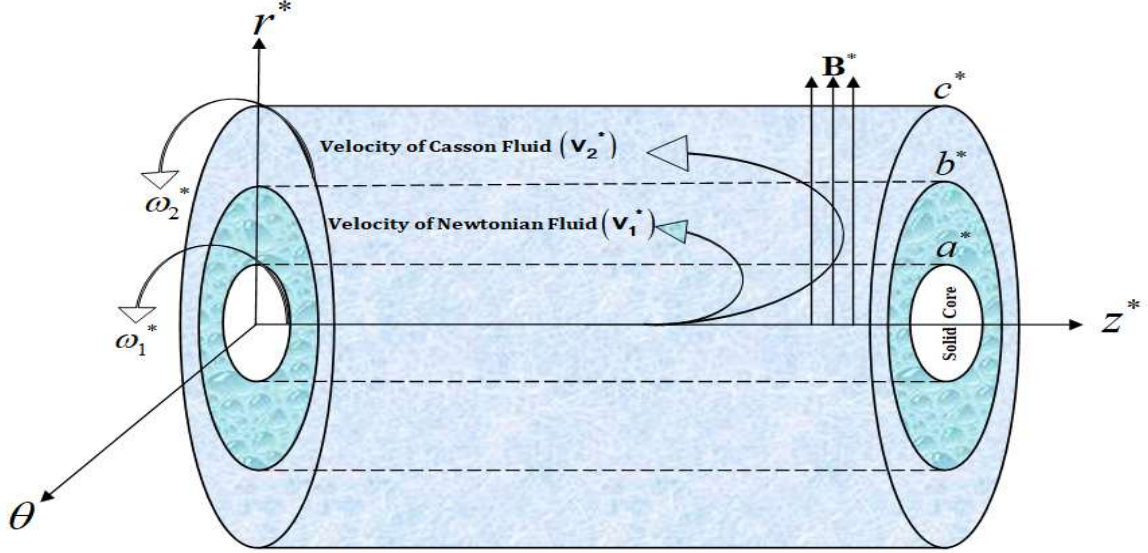


Fig. 1: Representation of the Physical Model

In the presence of uniform magnetic field, absence of body forces and body couples, governing equations of an incompressible steady Newtonian fluid flowing through the middle porous cylindrical shell ($a^* \leq r^* \leq b^*$) are given by

$$\nabla^* \cdot \mathbf{v}_1^* = 0, \quad (1)$$

$$\frac{\mu_1^*}{\phi} \nabla^2 \mathbf{v}_1^* - \left(\sigma_1^* B^{*2} + \frac{\mu_1^*}{k_1^*} \right) \mathbf{v}_1^* = 0. \quad (2)$$

The governing equations for the Casson fluid flowing through the outer porous cylindrical shell ($b^* \leq r^* \leq c^*$), the governing equations are given as follows

$$\nabla^* \cdot \mathbf{v}_2^* = 0, \quad (3)$$

$$\frac{\mu_2^*}{\phi} \left(1 + \frac{1}{\beta} \right) \nabla^2 \mathbf{v}_2^* - \left(\sigma_2^* B^{*2} + \frac{\mu_2^*}{k_2^*} \right) \mathbf{v}_2^* = 0. \quad (4)$$

Here, ϕ is porosity and k_1^* , k_2^* are permeability of the middle and outer porous cylindrical shells respectively, $\mathbf{v}_1^*$, $\mathbf{v}_2^*$ is fluid velocity vector of Newtonian fluid and Casson fluid respectively, p^* is fluid pressure, which is same for both fluid. where, $|\mathbf{B}^*| = B$, the magnitude of applied uniform magnetic field and taking pressure gradient zero.

The transformation of governing equations of the Newtonian fluid and Casson fluid flow problem into non-dimensional form, some non-dimensionalising variables are intro-

duced:

$$v_i = \frac{v_i^*}{b^* \omega_1^*}, \quad r = \frac{r^*}{b^*}, \quad l = \frac{a^*}{b^*}, \quad m = \frac{c^*}{b^*}, \quad \lambda = \frac{\mu_2^*}{\mu_1^*}, \quad k_i = \frac{k_i^*}{b^{2*}}, \quad \Lambda = \frac{\sigma_2^*}{\sigma_1^*}, \quad H_1 = b^* B^* \sqrt{\left(\frac{\sigma_1^*}{\mu_1^*}\right)},$$

$$H_2 = H_1 \sqrt{\Lambda}, \quad \omega = \frac{\omega_2^*}{\omega_1^*}, \quad \alpha = \phi \left(H_1^2 + \frac{1}{k_1} \right), \quad \gamma = \frac{\phi \beta}{\lambda(1 + \beta)} \left(H_1^2 \Lambda + \frac{\lambda}{k_2} \right). \quad (5)$$

3 Analytical Solution of the Problem

To find the analytical solution for Newtonian fluid flowing through the middle cylindrical shell, Due to rotation of co-axial horizontal cylinders about their common axis, fluid motion occurs about the common axis of cylinders. Since, the axial length of annulus is taken as infinite and fluid motion is taken as fully developed flow therefore, velocity components v_r and v_z along coordinate curves r and z , respectively will be absent. It means that fluid velocity component v_θ along coordinate curve θ will be present and flow field i.e. fluid velocity component v_θ will be independent of axial coordinate (z). Therefore, cross-radial component of velocity v_θ will depend only on the radius of cylinder. Using non-dimensional relations (5), Then the differential form of the governing equations for Newtonian fluid in non-dimensional form, can be written as

$$\frac{\partial(r v_\theta)}{\partial r} + \frac{\partial v_\theta}{\partial \theta} + r \frac{\partial v_\theta}{\partial \theta} = 0, \quad (6)$$

$$\frac{\partial^2 v_1}{\partial r^2} + \frac{1}{r} \frac{\partial v_1}{\partial r} - (1 + \alpha^2 r^2) v_1 = 0. \quad (7)$$

Again, using non-dimensional relations (5), the differential form of the governing equations, given in Eq. (4) for Casson fluid flow problem will be written as

$$\frac{\partial^2 v_2}{\partial r^2} + \frac{1}{r} \frac{\partial v_2}{\partial r} - (1 + \gamma^2 r^2) v_2 = 0. \quad (8)$$

In order to obtaining the analytical solution of the current problem for the Newtonian fluid and Casson fluid, analyze the Eqs. (7)-(8), which are the modified Bessel's equations of first order. therefore, the solution of Eq.(7) (For Newtonian fluid) will be of the form

$$v_1(r) = C_1 I_1(\alpha r) + C_2 K_1(\alpha r). \quad (9)$$

and the solution of Eq.(8) (for the Casson fluid) will be of the form

$$v_2(r) = C_3 I_1(\gamma r) + C_4 K_1(\gamma r). \quad (10)$$

3.1 Determination of the unknown parameters

Using certain boundary conditions to determine the unknown parameters. Due to analyticity (i.e., continuity) condition inside the porous cylinder at $r = l$, $r = 1$ and $r = m$. Since there are four unknown parameters to be determine, therefore, there are four boundary conditions are required, these are written as follows:

- Continuity of velocity at the porous interface i.e. $v_1 = v_2$ at $r = 1$.
- Angular rotation at outer surface i.e. $v_2 = m\omega$ at $r = m$.
- Angular rotation at solid core i.e. $v_1 = l$ at $r = l$.
- Continuity of the shear stresses at the porous interface i.e. $\tau_1(1) = \tau_2(1)$.

4 Result and Discussion

The authors presented a graphical explanation of the velocity profile's deviation for a range of flow parameters, including the Casson parameter, the angular rotation parameter, viscosity ratio, Hartmann number, porosity parameter and electric conductivity ratio. For a range of Casson parameter values, the fluid velocity and stresses are shown in figures 2 and 3. We have shown the comparison of fluid velocity and stresses. Figures 4, 6, 8, 10, and 12 are sketched the velocity profile with the Casson parameter, while figures 5, 7, 9, 11, and 13 are plotted without the Casson parameter. The comparison graphs of stresses are shown in figures 14, 15, 16, 17 and 18. Solid and dashed lines, respectively, are used for stresses with and without the Casson parameter. In these situations, there are two approaches to represent the velocity profile for the different values of the Casson parameter, flow field. First, we can see the velocity graph with various Casson parameter values. subsequently we can see the velocity graph against the radial coordinate.

In figure 2, region-I (Newtonian fluid), where velocity decreases non-linearly, and Region-II (Casson fluid), where velocity decreases linearly, the graph shows how velocity varies with radial position. Smaller values of $\beta = 0.1$ cause a more significant decrease in velocity, whereas larger values of $\beta = 0.7$ decrease the reduction in velocity, according to the impact of the Casson parameter. The vertical dashed line at indicates the change from Newtonian to Casson fluid behavior, with Region-II being more affected.

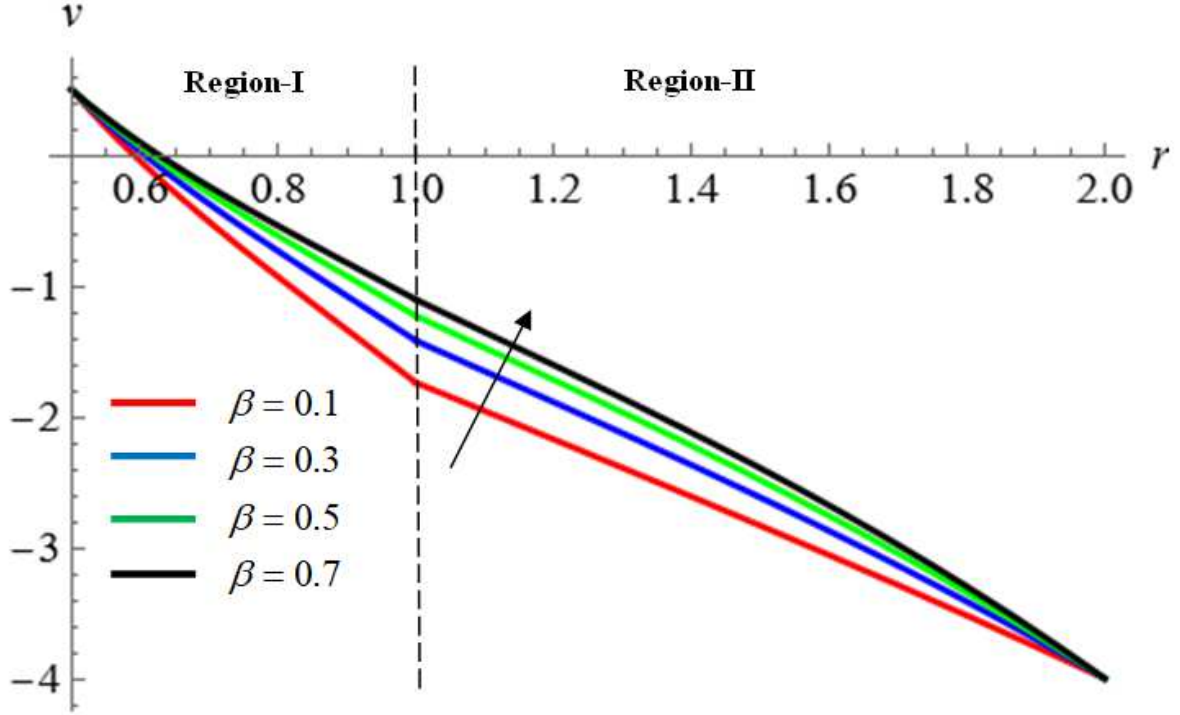


Fig. 2: Velocity profiles for Newtonian-Casson model when $\Lambda = 0.5$, $k_i = 0.5$, $H_1 = 1.5$, $\lambda = 0.5$, $\phi = 0.5$

In figure 3, region-I (Newtonian fluid), where stress increases non-linearly, and region-II (Casson fluid), where stress eventually saturates with almost linear behavior, the graph shows how stress varies with radial location. Lower stress corresponds with smaller Casson parameter values $\beta = 0.1$, whereas greater stress is associated with bigger values $\beta = 0.7$. The crossover between the Newtonian and Casson fluid regions is shown at $r = 1.0$, and the impact of Casson parameter is more noticeable in region-I than in region-II.

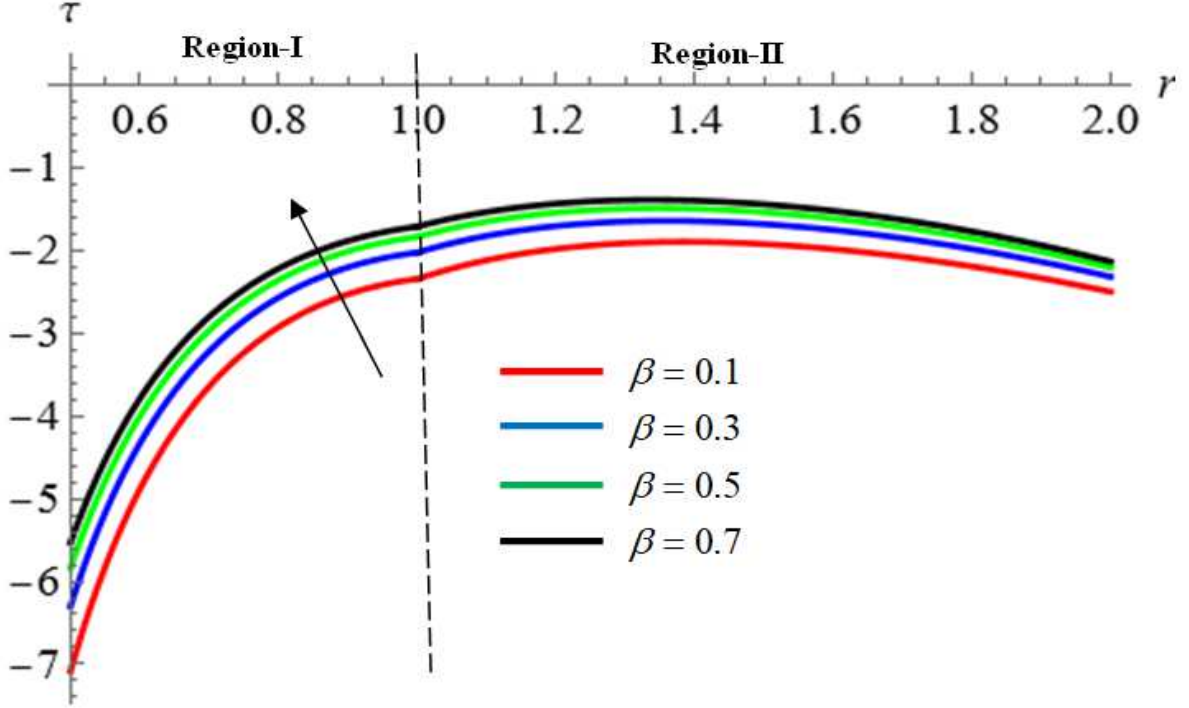


Fig. 3: Stress for Newtonian-Casson model when $\Lambda = 0.5$, $k_i = 0.5$, $H_1 = 1.5$, $\lambda = 0.5$, $\phi = 0.5$

4.1 Comparison of Velocity Distributions

Figures 4 and 5 show the comparison graphs of velocity profile as a function of radial position for different angular rotations in two cases, with the Casson parameter (figure 4) and without the Casson parameter (figure 5). The velocity in region-II (Casson fluid) with the Casson parameter shows a clear linear behavior, and the increasing slopes of the curves indicate that larger angular rotation has a greater impact. There is a progressive rise in velocity and decreased sensitivity to ω in region-I (Newtonian fluid). However, figure 5 without the Casson parameter shows nonlinear behavior in both areas, with velocity growing more noticeably over the radial position, particularly in region-II, where a greater ω causes the curves to separate more pronouncedly. Figure 4 illustrates the moderating influence of the Casson parameter on fluid flow by showing that its absence results in greater velocities overall.

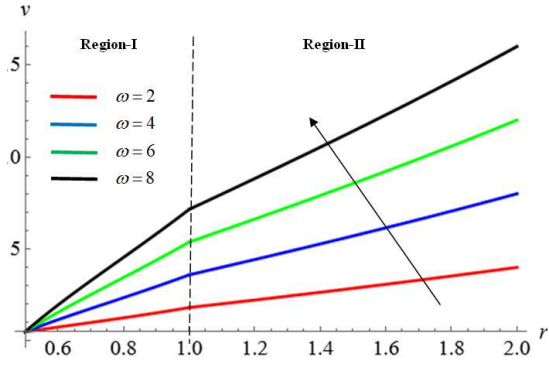


Fig. 4: Velocity profiles for Newtonian-Casson model when $\beta = 0.1$, $\Lambda = 0.5$, $k_i = 0.5$, $H_1 = 1.5$, $\lambda = 0.5$, $\phi = 0.5$

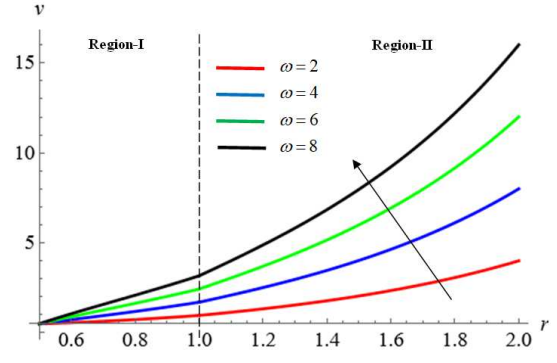


Fig. 5: Velocity profiles for Newtonian-Newtonian model [18] when $\Lambda = 0.5$, $k_i = 0.5$, $H_1 = 1.5$, $\lambda = 0.5$, $\phi = 0.5$

A comparison between figures 6 and 7 show how the Casson parameter affects velocity profiles for varying viscosity ratio values. The velocity profiles with the Casson parameter are shown in figure 6, but the velocity profiles in figure 7 without Casson parameter. In both graphs, velocity is shown on a vertical axis, whereas velocity is displayed on a horizontal axis from 0.6 to 2.0. The velocity profiles for λ in region-II in figure 6 show a notable decrease in velocity profiles as λ rises. The existence of the Casson parameter, which also shows how it influences the flow characteristics, clarifies this drop. However, figure 6 shows velocity profiles for the same λ values without the Casson parameter. When the Casson parameter is absent, the velocity reduction is less pronounced than in figure 6, suggesting a less severe effect on flow dynamics.

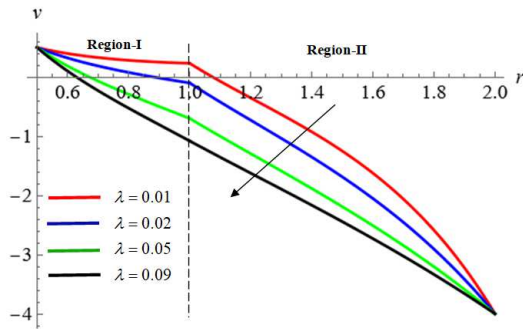


Fig. 6: Velocity profiles for Newtonian-Casson model when $\beta = 0.1$, $\Lambda = 0.5$, $k_i = 0.5$, $H_1 = 1.5$, $\omega = -2$, $\phi = 0.5$

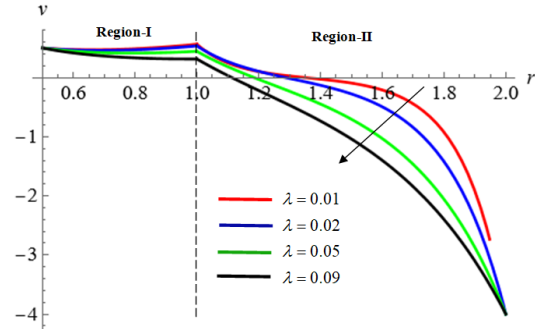


Fig. 7: Velocity profiles for Newtonian-Newtonian model [18] when $\Lambda = 0.5$, $k_i = 0.5$, $H_1 = 1.5$, $\omega = -2$, $\phi = 0.5$

Figures 8 and 9 show how different levels of the porosity parameter change velocity profiles in two separate scenarios, one with the Casson parameter figure 8 and the other without figure 9. In both graphs, the radial location on the horizontal axis is plotted against the velocity on the vertical axis. Figure 8 with Casson parameter, the velocity profiles converge and show less variation in region-II in comparison to region-I as the porosity parameter increases. This indicates how the Casson parameter improves the velocity distribution by smoothing it out. While in figure 9, without Casson parameter, the velocity profiles also converge as the porosity parameter increases, but without the Casson parameter, there is a more pronounced difference in region-II compared to region-I. This suggests that the absence of the Casson parameter leads to greater variability in the velocity distribution. Velocity profiles for various values of the porosity parameter and the impact of the Casson parameter. The velocity profiles in region-II show reduced volatility in figure 8, which include the Casson parameter, suggesting a stabilizing influence. On the other hand, the velocity profiles in region-II exhibit more notable variations in figure 9, which does not include the Casson parameter. This emphasizes how the Casson parameter helps to moderate the effects of the porosity parameter, which leads to a more even distribution of velocity and improves the fluid flow's overall stability.

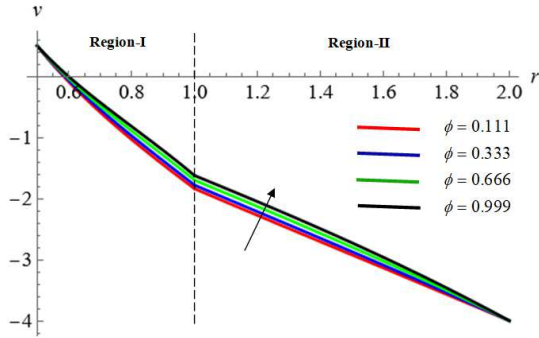


Fig. 8: Velocity profiles for Newtonian-Casson model when $\beta = 0.1$, $\Lambda = 0.5$, $k_i = 0.5$, $H_1 = 1.5$, $\lambda = 0.5$, $\omega = -2$

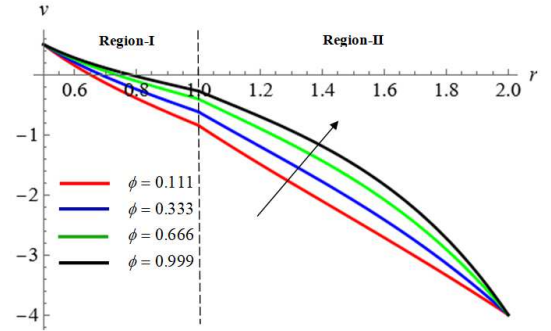


Fig. 9: Velocity profiles for Newtonian-Newtonian model [18] when $\Lambda = 0.5$, $k_i = 0.5$, $H_1 = 1.5$, $\lambda = 0.5$, $\omega = -2$

Figures 10 and 11 show the comparison graph of velocity profiles for different values of the Hartman number under two conditions, with Casson parameter (figure 10) and without Casson parameter (figure 11). In figure 10, the profiles are plotted for H_1 values of 1.5, 2.5, 3.5, and 4.5, with a clear trend of decreasing velocity as H_1 increases, especially in

region-II. The velocity decreases more steadily and smoothly when the Casson parameter is present. On the other hand, figure 11, which displays the velocity profiles for the identical H_1 values without the Casson parameter, likewise demonstrates a decreasing pattern in velocity, but region-II shows less stability and more noticeable variations. This comparison demonstrates how the Casson parameter significantly reduces the impact of the Hartman number on the velocity profiles, resulting in a more balanced and smoother flow pattern when it is present.

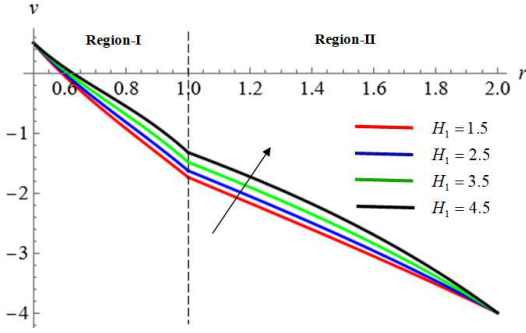


Fig. 10: Velocity profiles for Newtonian-Casson fluid when $\beta = 0.1$, $\Lambda = 0.5$, $k_i = 0.5$, $\omega = -2$, $\lambda = 0.5$, $\phi = 0.5$

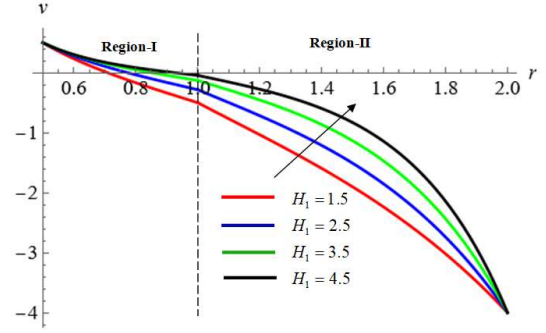


Fig. 11: Velocity profiles for Newtonian-Newtonian model [18] when $\Lambda = 0.5$, $k_i = 0.5$, $\omega = -2$, $\lambda = 0.5$, $\phi = 0.5$

The velocity profiles for various values of ratio of electric conductivity are compared in figures 12 and 13 under two different scenarios, one with the Casson parameter (figure 12) and the other without Casson parameter (figure 13). The profiles for Λ values of 0.5, 1.5, 2.5, and 4.5 are presented in figure 12, which also contains the Casson parameter. According to these profiles, the velocity decreases more sharply in region-I than in region-II, with a sharper reduction caused by the Casson parameter. On the other hand, figure 13, which shows the velocity profiles without the Casson parameter for the identical Λ values, shows a less sudden drop in velocity between region-I and region-II and a smoother transition. This comparison shows how the Casson parameter significantly contributes to the reduction of velocity for different values of ratio of electric conductivity, resulting in a more pronounced separation of velocity profiles within region-I, whereas the profiles move between the regions more slowly when it is absent.

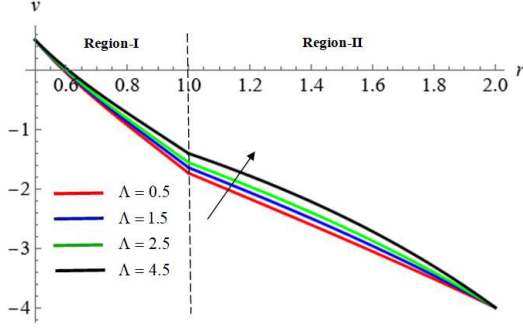


Fig. 12: Velocity profiles for Newtonian-Casson model when $\beta = 0.1$, $\omega = -2$, $k_i = 0.5$, $H_1 = 1.5$, $\lambda = 0.5$, $\phi = 0.5$

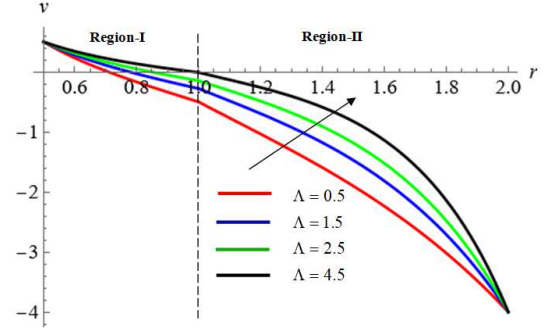


Fig. 13: Velocity profiles for Newtonian-Newtonian model [18] when $\omega = -2$, $k_i = 0.5$, $H_1 = 1.5$, $\lambda = 0.5$, $\phi = 0.5$

4.2 Comparison of Shear Stresses

The comparison of stresses across various amounts of angular rotation is shown in figure 14, where solid lines indicate fluids with the Casson parameter and dashed lines represent fluids without the Casson parameter. The graph shows the stresses in two different regions (region-I and region-II) plotted against the radial position. Initially, the Casson parameter indicates greater stress levels than without it. The solid lines, which correspond to ω values of 2 (red) and 6 (blue), show a quicker drop in stresses in region-I and a rise in region-II. For Newtonian fluids without the Casson parameter, the dashed lines, which show the identical values, show a less noticeable increase in region-II and a more gradual fall in region-I. This comparison demonstrates the Casson parameter's substantial impact on the stress behavior, which leads to greater and more unpredictable stress levels, especially at higher angular rotations.

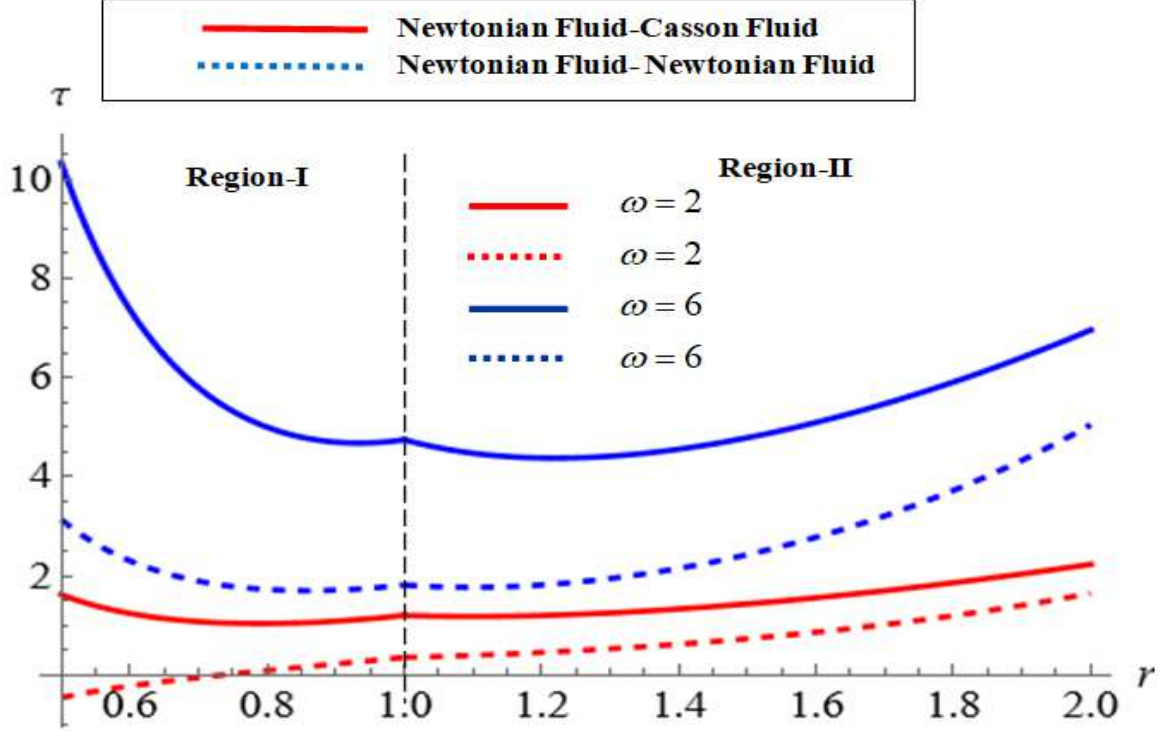


Fig. 14: Stresses for angular rotation (ω) when $\beta = 0.1$, $\Lambda = 0.5$, $k_i = 0.5$, $H_1 = 1.5$, $\lambda = 0.5$, $\phi = 0.5$

Figure 15 shows the comparison graph, fluids with and without the Casson parameter can be compared with solid lines representing stresses with respect to radial position for varying values of the ratio of viscosity throughout two regions (region-I and region-II). The solid red and blue lines for fluids with the Casson parameter, which represent λ values of 0.01 and 0.05, indicate that stress values are greater in region-I and sharply decrease as they go into region-II. The fluids without the Casson parameter, shown by the dashed red and blue lines, on the other hand, indicate a more progressive decrease in stresses in both regions. This comparison highlights how the Casson parameter significantly increases the stresses, particularly in region-I, causing a sharper decrease into region-II. However, the stress profiles are less diverse and more consistent when the Casson parameter is not present.

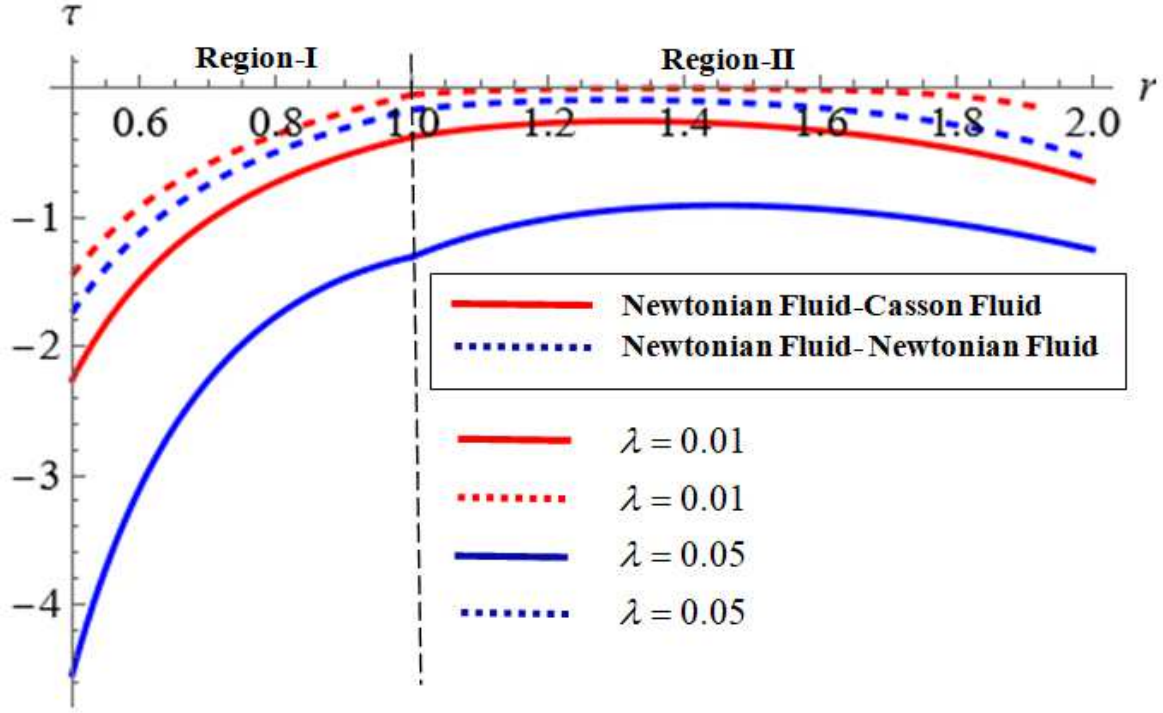


Fig. 15: Stresses for viscosity ratio (λ) when $\beta = 0.1$, $\Lambda = 0.5$, $k_i = 0.5$, $H_1 = 1.5$, $\omega = -2$, $\phi = 0.5$

For various values of the porosity parameter, the comparison graph shows the stresses versus the radial coordinate in two regions, region-I ($0.6 \leq r \leq 1.0$) and region-II ($1.0 \leq r \leq 2.0$). Fluids with the Casson parameter are shown by solid lines, whereas fluids without the Casson parameter are represented by dashed lines. In comparison to the dashed lines, the solid lines indicate a less sharp drop in stresses in region-I and a more mild increase in region-II for ϕ values of 0.333 (red) and 0.999 (blue). This suggests that a smoother stress distribution is the outcome of the Casson parameter stabilizing the stress behavior. The dashed lines, on the other hand, show a more noticeable shift in region-II and a greater fall in region-I in the absence of the Casson parameter, underscoring the greater variability in the distribution of stress.

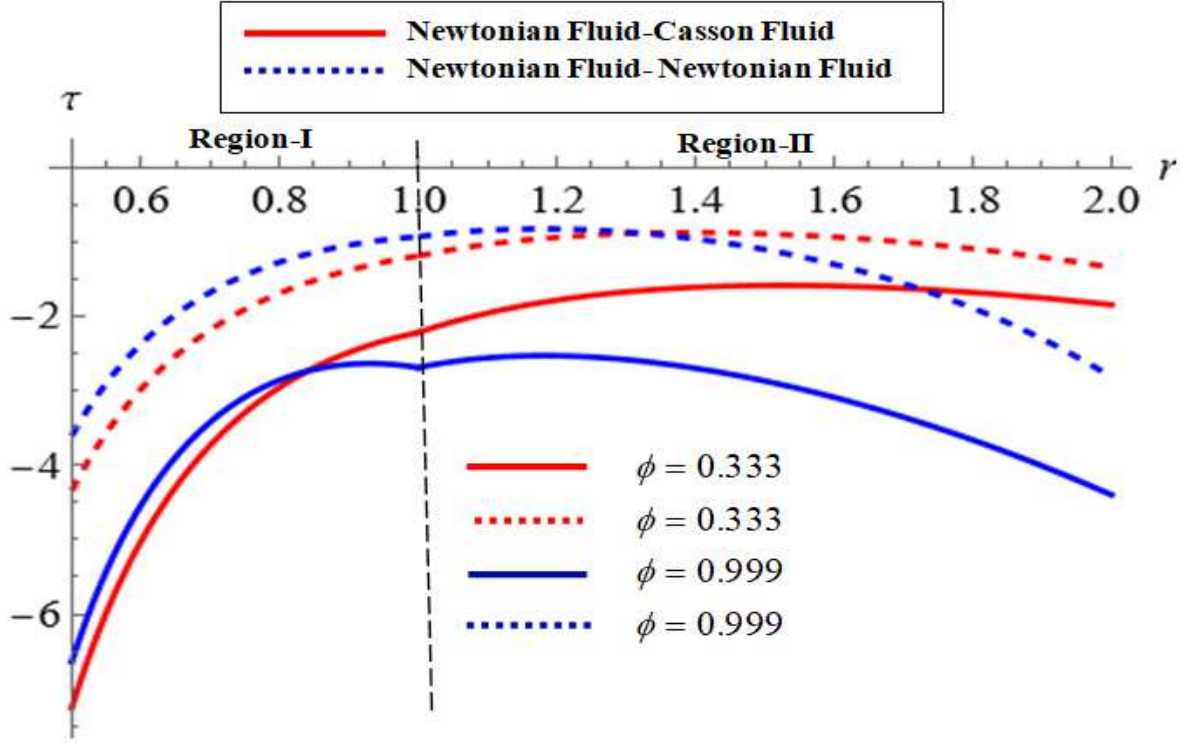


Fig. 16: Stresses for porosity parameter (ϕ) when $\beta = 0.1$, $\Lambda = 0.5$, $k_i = 0.5$, $H_1 = 1.5$, $\lambda = 0.5$, $\omega = -2$

The comparison graph depicts stresses across radial positions for different Hartman number in two regions (region-I and region-II) with the Casson parameter (solid lines) and without it (dashed lines). For H_1 values of 1.5 (red) and 2.5 (blue), the solid lines, representing fluids with the Casson parameter, display greater stress values in region-I and a moderated increase in stress in region-II. On the other hand, the dashed lines i.e. without the Casson parameter, illustrate a sharper decline in region-I and a more significant variation in region-II. This comparison underscores the Casson parameter's influence in stabilizing and moderating stress distribution, reducing variability and ensuring a smoother transition between regions, particularly at higher Hartman numbers. This makes the Casson parameter crucial for accurately modeling fluid flows in magneto-hydrodynamic scenarios.

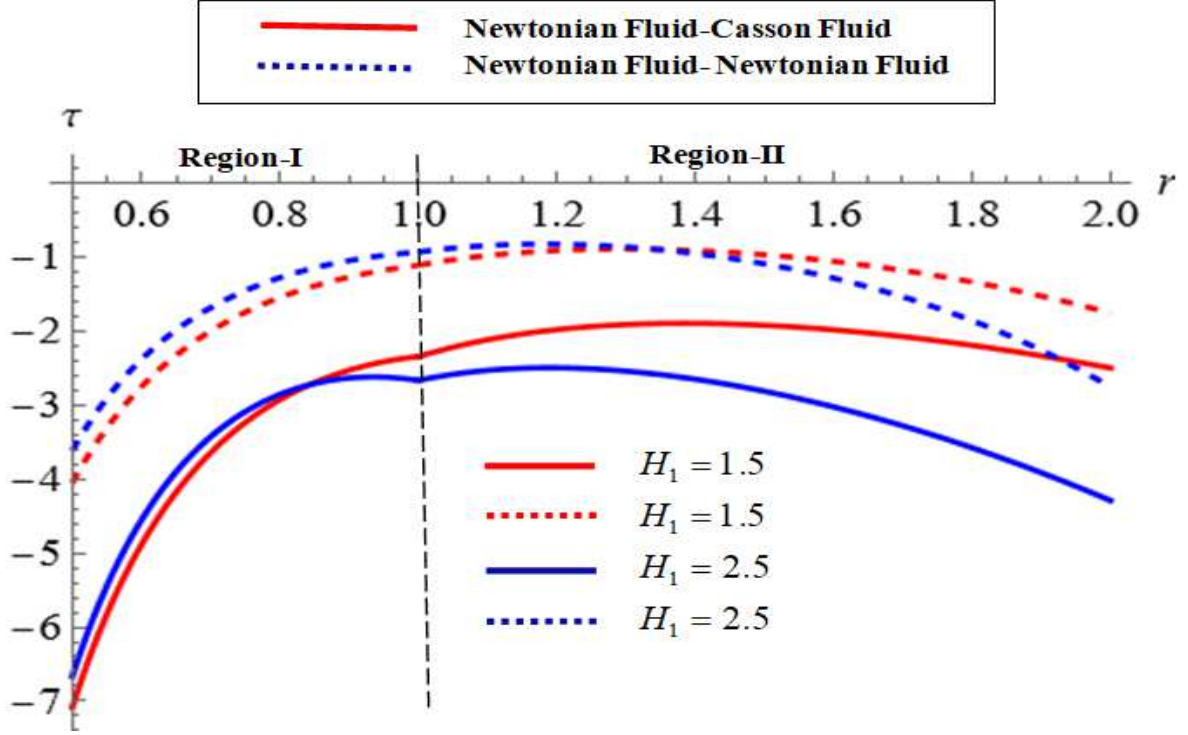


Fig. 17: Stresses for Hartman number (H_1) when $\beta = 0.1$, $\Lambda = 0.5$, $k_i = 0.5$, $\omega = -2$, $\lambda = 0.5$, $\phi = 0.5$

The graph compares stresses across radial positions for different values of the ratio of electric conductivity in two regions, region-I ($0.6 \leq r \leq 1.0$) and region-II ($1.0 \leq r \leq 2.0$). Solid lines represent fluids with the Casson parameter, while dashed lines represent fluids without Casson parameter. Without the Casson parameter, the dashed lines show a sharper increase in region-I and a steeper decline in region-II, indicating higher variability in stress distribution. This comparison underscores the stabilizing effect of the Casson parameter, resulting in smoother and more consistent stress profiles across varying the ratio of electric conductivity .

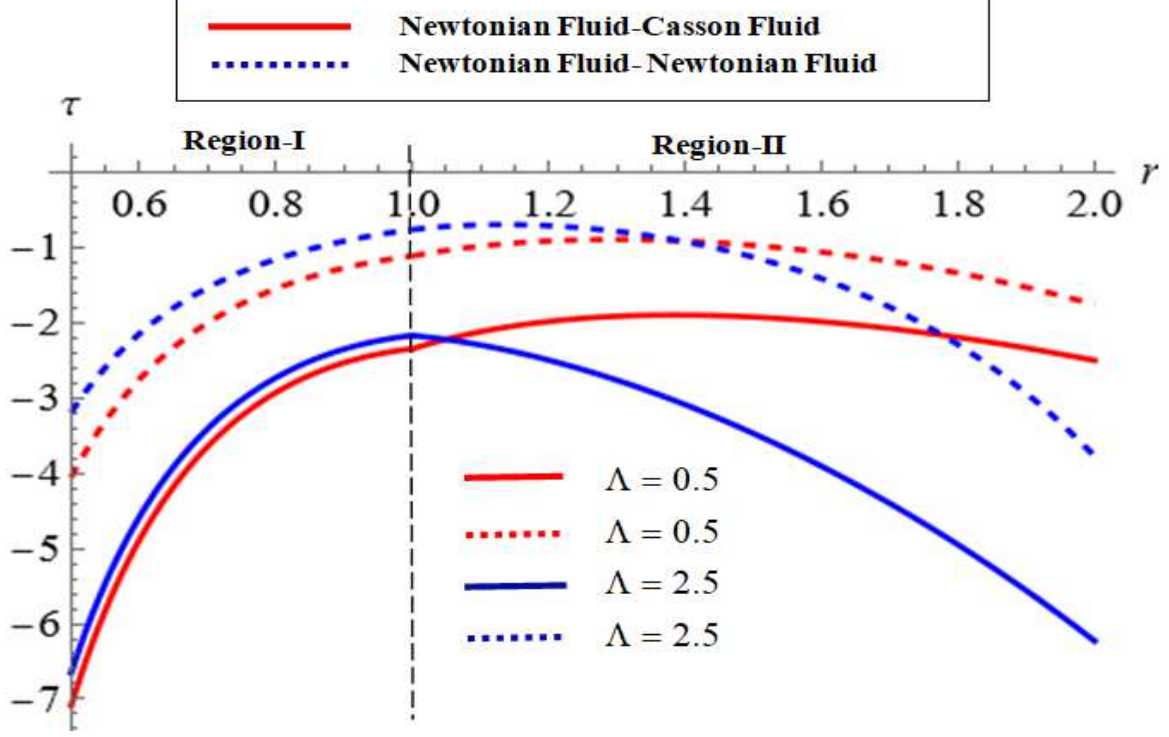


Fig. 18: Stresses for electric conductivity ratio (Λ) when $\beta = 0.1$, $\omega = -2$, $k_i = 0.5$, $H_1 = 1.5$, $\lambda = 0.5$, $\phi = 0.5$

The velocity profiles and stresses at a porous interface for various angular rotation values are compared in the table 1 between the Newtonian-Casson, Newtonian-Newtonian and Newtonian-Stokes models. Here, ω is taken into consideration for values of 2, 4, 6, and 8. In the Newtonian-Casson model, the stresses ($\tau_1 = \tau_2$) and velocity profiles ($v_1 = v_2$) show a steady rise as angular rotation increases. Likewise, the Newtonian-Newtonian model shows a rising pattern in stresses ($\tau_1 = \tau_2$) and velocity profiles ($v_1 = v_2$) as ω rises. In the Newtonian-Stokes model, the velocity profiles ($v_1 = v_2$) and stresses ($\tau_1 = \tau_2$) similarly show rising values as ω increases. This detailed comparison highlights how angular rotation significantly impacts fluid behavior across different models, with the Newtonian-Casson model showing the most pronounced changes in both velocity profiles and stresses.

Table 1: Numerical values of velocity profile and stress at porous interface for different values of angular rotation

Parameter	Newtonian-Casson model		Newtonian-Newtonian model [18]		Newtonian-Stokes model [19]	
ω	$v_1 = v_2$	$\tau_1 = \tau_2$	$v_1 = v_2$	$\tau_1 = \tau_2$	$v_1 = v_2$	$\tau_1 = \tau_2$
2	1.81578	1.19707	0.96610	0.35041	1.14033	0.52401
4	3.59328	2.96826	1.69919	1.08089	2.03534	1.41585
6	5.37077	4.73945	2.43227	1.81137	2.93036	2.30770
8	7.14827	6.51064	3.16535	2.54186	3.82538	3.19954

Comparative analysis of velocity profiles and stresses across a porous interface for different viscosity ratios across three models Newtonian-Stokes, Newtonian-Casson, and Newtonian-Newtonian—is given in table 2. Considered are λ values of 0.01, 0.02, 0.05, and 0.09. As λ rises, the velocity profiles ($v_1 = v_2$) and stresses ($\tau_1 = \tau_2$) for the Newtonian-Casson model show a decreasing pattern, indicating increased flow resistance. In the same way, the Newtonian-Newtonian model exhibits a decrease in stresses ($\tau_1 = \tau_2$) and velocity profiles ($v_1 = v_2$) with increasing λ , demonstrating a regular pattern of increased stress with larger viscosity ratios. Additionally, the Newtonian-Stokes model shows that when λ increases, the velocity profiles ($v_1 = v_2$) and stresses ($\tau_1 = \tau_2$) decrease, highlighting the connection between fluid dynamics and viscosity ratios. This detailed comparison across different models and viscosity ratios illustrates how increased viscosity impacts both velocity and stress at the porous interface, providing valuable insights into fluid behavior in porous media.

Table 2: Numerical values of velocity profile and stress at porous interface for different values of viscosity ratio

Parameter	Newtonian-Casson model		Newtonian-Newtonian model [18]		Newtonian-Stokes model [19]	
λ	$v_1 = v_2$	$\tau_1 = \tau_2$	$v_1 = v_2$	$\tau_1 = \tau_2$	$v_1 = v_2$	$\tau_1 = \tau_2$
0.01	0.23234	-0.38074	0.562578	-0.051686	0.562896	-0.051369
0.02	-0.10010	-0.71201	0.534431	-0.079733	0.534930	-0.079236
0.05	-0.69938	-1.30917	0.443327	-0.170514	0.440831	-0.173002
0.09	-1.07986	-1.68830	0.312987	-0.300392	0.300153	-0.313181

A detailed analysis of numerical values for stresses and velocity profiles across a porous interface for varying porosity values across three fluid models—Newtonian, Newtonian-Casson, and Newtonian-Stokes—is shown in table 3. Porosity parameter values of 0.111, 0.333, 0.666, and 0.999 are studied. The velocity profiles ($v_1 = v_2$) and stresses ($\tau_1 = \tau_2$) for the Newtonian-Casson model both drop as ϕ increases, suggesting that larger porosity reduces flow resistance. Similarly, when ϕ rises, the Newtonian-Newtonian model shows decreasing patterns of stresses ($\tau_1 = \tau_2$) and velocity profiles ($v_1 = v_2$), indicating consistent behavior throughout more pore volume values. The relationship between porosity and fluid dynamics is further shown by the Newtonian-Stokes model, which shows that velocity profiles ($v_1 = v_2$) and stresses ($\tau_1 = \tau_2$) drop as ϕ increases. This detailed comparison presents significant information into how various fluid models react to changing porosity parameters and highlights the major impact of porosity on both velocity and stress distributions at the porous interface.

Table 3: Numerical values of velocity profile and stress at porous interface for different values of porosity parameter

Parameter	Newtonian-Casson model		Newtonian-Newtonian model [18]		Newtonian-Stokes model [19]	
ϕ	$v_1 = v_2$	$\tau_1 = \tau_2$	$v_1 = v_2$	$\tau_1 = \tau_2$	$v_1 = v_2$	$\tau_1 = \tau_2$
0.111	-1.83918	-2.01824	-0.846681	-1.28205	-0.902196	-1.32323
0.333	-1.78094	-2.21239	-0.621298	-1.18186	-0.743958	-1.29086
0.666	-1.69939	-2.46726	-0.407082	-1.04392	-0.571754	-1.22529
0.999	-1.62407	-2.68438	-0.275414	-0.92350	-0.450044	-1.15151

The table 4 compares velocity profiles and stresses at a porous interface for varying Hartman number across three models, Newtonian-Casson, Newtonian-Newtonian, and Newtonian-Stokes. The Hartman numbers examined are 1.5, 2.5, 3.5, and 4.5. The velocity profiles ($v_1 = v_2$) and stresses ($\tau_1 = \tau_2$) for the Newtonian-Casson model drop as H_1 rises, indicating increased electromagnetic resistance. The Newtonian-Newtonian model also shows consistent behavior across these values, with velocity profiles ($v_1 = v_2$) and stresses ($\tau_1 = \tau_2$) declining with increasing H_1 . The influence of the magnetic

field strength on fluid dynamics is further demonstrated by the Newtonian-Stokes model, which shows a drop in velocity profiles ($v_1 = v_2$) and stresses ($\tau_1 = \tau_2$) with rising H_1 . This detailed comparison of various models and Hartman numbers presents important insights into fluid dynamics under the effect of magnetic fields by showing how increased electromagnetic influence impacts both velocity and tension at the porous interface.

Table 4: Numerical values of velocity profile and stress at porous interface for different values of Hartman number

Parameter	Newtonian-Casson model		Newtonian-Newtonian model [18]		Newtonian-Stokes model [19]	
H_1	$v_1 = v_2$	$\tau_1 = \tau_2$	$v_1 = v_2$	$\tau_1 = \tau_2$	$v_1 = v_2$	$\tau_1 = \tau_2$
1.5	-1.73922	-2.34532	-0.500054	-1.110550	-0.649706	-1.259670
2.5	-1.63027	-2.66719	-0.284498	-0.933103	-0.364082	-1.035650
3.5	-1.48720	-3.03151	-0.130361	-0.727606	-0.167555	-0.790762
4.5	-1.32645	-3.35550	-0.044782	-0.537153	-0.061241	-0.573348

Numerical velocity profiles and stresses across a porous interface with varying electric conductivity ratios are examined across three models (Newtonian-Stokes, Newtonian-Casson, and Newtonian-Newtonian) in table 5. The following values are analyzed: 0.5, 1.5, 2.5, and 5.5. The Newtonian-Casson model indicates that when Λ increases, the velocity profiles ($v_1 = v_2$) and stresses ($\tau_1 = \tau_2$) exhibit a decreasing pattern. Comparably, the Newtonian-Newtonian model shows consistent behavior over the range, showing a reduction in both velocity profiles ($v_1 = v_2$) and stresses ($\tau_1 = \tau_2$) with larger Λ values. As Λ rises, the Newtonian-Stokes model similarly shows a reduction in velocity profiles ($v_1 = v_2$) and a less noticeable decline in stresses ($\tau_1 = \tau_2$). Thorough comparison explains how changes in Λ affect fluid dynamics and highlights the important role of the electric conductivity ratio on velocity profiles and stresses at the porous interface.

Table 5: Numerical values of velocity profile and stress at porous interface for different values of electric conductivity ratio

Parameter	Newtonian-Casson model		Newtonian-Newtonian model [18]		Newtonian-Stokes model [19]	
Λ	$v_1 = v_2$	$\tau_1 = \tau_2$	$v_1 = v_2$	$\tau_1 = \tau_2$	$v_1 = v_2$	$\tau_1 = \tau_2$
0.5	-1.73922	-2.34532	-0.500054	-1.110550	-0.649706	-1.259670
1.5	-1.64677	-2.25320	-0.278531	-0.889812	-0.361933	-0.972918
2.5	-1.56130	-2.16803	-0.147372	-0.759118	-0.197922	-0.809489
5.5	-1.40854	-2.01581	-0.010194	-0.622427	0.0120332	-0.600278

Novelty of the Study

The hydromagnetic flow behavior of a Newtonian-Casson fluid system confined between rotating concentric cylinders—a configuration that has not gotten much attention in the literature is investigated in this work in a unique way. Unlike most studies that consider either Newtonian or non-Newtonian fluids separately, this research examines a two-fluid interface, providing new insights into the interplay between Newtonian and Casson fluid characteristics under rotational and magnetic effects. A more thorough comprehension of how these variables affect velocity and stress distributions is made possible by the addition of important parameters such the viscosity ratio, electric conductivity ratio, and porosity impacts in addition to the Casson and Hartmann numbers. Although Casson fluid behavior has been studied separately in the past, the results' practical consequences are increased by applying this hybrid model to lubrication, blood flow in biomedical equipment, and MHD energy systems.

Physical Significance and its Applications

This work is important for improving our knowledge of hydromagnetic flows in composite fluid systems, particularly for applications in biomedicine and engineering. It offers information on how to improve geophysical fluid models, increase the efficiency of biomedical devices, and optimize lubricated bearings by examining velocity and stress distributions

in Newtonian and Casson fluids. The results contribute to improved performance in material transport and industrial fluid systems by helping to construct electrically conducting fluid energy systems and magnetohydrodynamic (MHD) devices. These results are useful for both scientific study and real-world engineering solutions since they also have implications for energy conversion technologies and porous medium applications.

The findings have significant applications in industrial and engineering processes involving composite fluid systems, such as lubricated bearings, blood flow modeling in biomedical devices, and geophysical fluid dynamics. Furthermore, the study offers valuable information for improving material transport systems, Magnetohydrodynamic (MHD) device design, and electrically conducting fluid energy generating system performance.

5 Conclusion

We examined the flow of Newtonian fluid under the hydromagnetic effect via a porous rotating cylindrical shell that is surrounded by a solid core and Casson fluid. Moreover, specific situations that involve the velocity profile and stresses are investigated for both fluid types. For a range of parameter values, such as the Casson number, angular rotation, the viscosity ratio, the porosity parameter, the Hartman number, and the electric conductivity ratio, fluid velocity and stress are graphically displayed in two regions. In the presence of the Casson parameter, the flow field and stresses are also compared using both numerical and graphical ways. The following are an overview of the findings from this study:

- The graph shows velocity variation with radial position in Newtonian and Casson fluids. Velocity decreases non-linearly for Newtonian fluid and linearly for Casson fluid. Smaller Casson parameter values $\beta = 0.1$ result in a greater velocity reduction, while larger values $\beta = 0.7$ reduce the effect.
- The stress almost linear for Casson fluid transforms from the non-linear stress for Newtonian fluid. Lower Casson parameter values $\beta = 0.1$ show less stress, whereas higher values $\beta = 0.7$ show more stress.
- The velocity profiles for different angular rotations, with and without the Casson

parameter, are identified over radial locations. While the Casson parameter reduces sensitivity in region-I and reduces fluid flow with linear behavior in region II, its absence leads to higher, nonlinear velocities overall..

- The velocity profiles are compared with and without the Casson parameter for different viscosity ratio values. As λ increases, the velocity significantly drops, indicating the effect of the Casson parameter. In its absence, velocity decrease is less noticeable, suggesting a less significant impact on flow dynamics.
- Velocity profiles for different porosity levels in region-II is more stable and smoother when the Casson parameter is applied. In its absence, region-II shows more variances, highlighting the function of the Casson parameter in regulating fluid flow and decreasing variations in velocity.
- Velocity profiles for various H_1 values, which show that velocity decreases as H_1 rises, particularly in region-II. Whereas differences are more obvious when the Casson parameter is absent, velocity decreases slowly when it is included. By decreasing the effect of the Hartman number, the Casson parameter stabilizes flow.
- Fluids with the Casson parameter have higher beginning stress and more significant variations than those without it, according to the comparison of stress over various angular rotations. Stress decreases more quickly in region-I and increases significantly in region-II, showing the Casson parameter's major influence on stress behavior.
- In the Newtonian-Casson model, the stress exhibits a fast reduction in region-II and a larger stress in region-I, whereas in the Newtonian-Newtonian model, its reduction is more slow, corresponding to various values of viscosity ratio. This shows how the Casson parameter increases stress and produces increased regional variances.
- In the Newtonian-Casson model, the stress distribution is smoother, with a less sharp decrease in region-I and a more gentle increase in region-II, over a range of porosity parameter values. With a significant shift in region-II and a larger fall in region-I, the Newtonian-Newtonian model shows higher variation in stress.

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Conflict of Interest

Authors of the manuscript declare that they have no conflict of interest.

Author contributions

Vikas Kumar Sharma formulated and solved the problem. Deepak Kumar Maurya plotted the graphs and helped in the validation of numerical and graphical results. Pankaj Kumar Maurya helped in preparation of manuscript by typing and editing.

Data Availability and Reproducibility Statement

The present investigation is not generating or analyzing datasets, therefore sharing information is not applicable. Additional data related to this article can be accessed on request from the corresponding author.

Ethics approval and Consent to participate

Not applicable

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Not applicable

References

- [1] D. Kumar, K. Ramesh and S. Chandok, Bull. Braz. Math. Soc. **50**, 515-531 (2019).
- [2] S. Deo and D. K. Maurya, Spec. Top. Rev. Porous Media. **10**, 421-428 (2019).
- [3] M. Mustafa and J. A. Khan, AIP Adv. **5**, 077148 (2015).

- [4] Tagg, Hawkins and Danby, J. Appl. Engng. Math. **10**, 1-5 (2023).
- [5] H. M. Ali, A. Hassan and A. Wahab, *Nanofluids for Heat Exchangers* (Springer, 2023).
- [6] D. K. Maurya and S. Deo, Int. J. Appl. Comput. Math. **6**, 1-10 (2020).
- [7] R. L. Batra and B. Das, Fluid Dyn. **9**, 133-141 (1992).
- [8] D. K. Maurya, S. Deo and D. Y. Khanukaeva, Math. Meth. Appl. Sci. **44**, 6647-6665 (2021).
- [9] S. Deo and D. K. Maurya, Microfluid. Nanofluid. **26**, (2022).
- [10] T. Hayat, N. Gull, M. Farooq and B. Ahmad, J. Aerosp. Engng. **29**, 04015011 (2016).
- [11] S. Deo, D. K. Maurya and A. N. Filippov, Colloid J. **82**, 649-660 (2020).
- [12] D. K. Maurya, S. Deo and P. K. Maurya, Z. Angew. Math. Mech. **104**, e202300675 (2024).
- [13] J. D. Venkatesan, S. Sankar, K. Hemalatha and Y. Yatim, J. Appl. Math. **2013**, 583809 (2013).
- [14] K. Jat, K. Sharma, P. Choudhary, R. Thamizharasi, M. Abbas and V. R. Devi, Alex. Engng. J. **108**, 400-414 (2024).
- [15] P. K. Kameshwaran, S. Shaw and P. Sibanda, Sadhana. **39**, 1573-1583 (2014).
- [16] D. K. Maurya and S. Deo, Spec. Top. Rev. Porous Media. **13**, 75-92 (2022).
- [17] D. K. Maurya and V. K. Sharma, Jnanabha **54**, 329-337 (2024).
- [18] P. K. Yadav, A. Tiwari, S. Deo, M. K. Yadav, A. Filippov, S. Vasin and E. Sherysheva, Colloid J. **4**, 473-482 (2013).
- [19] S. Deo, P. K. Yadav and A. Tiwari, Appl. Math. Model. **34**, 1329-1343 (2010).