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Effect of Initial System-Environment Correlations On Work Extraction From Open Quantum Systems

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ABSTRACT

In many standard analyses of quantum thermodynamics, the system and its environment are assumed to be initially independent and free of any correlations. However, in many physical scenarios, initial system-environment correlations are unavoidable. In this work, we investigate the role of initial system-environment correlations in work extraction from open quantum systems. Furthermore, we examine the effects of these correlations and the nature of environmental dynamics on the evolution of coherence, coherence-based ergotropy, and total ergotropy. In both Markovian and non-Markovian regimes, we show that initial correlations directly influence the amount of ergotropy. We also demonstrate that in the non-Markovian regime, reducing the initial correlation enables optimal utilization of memory effects and maintains work extraction efficiency over longer time intervals. In contrast, in the Markovian regime, such correlations only accelerate the decay process without producing a revival of quantum resources; therefore, work extraction must be carried out within very short time intervals.

Introduction

Quantum thermodynamics explores how the macroscopic principles of thermodynamics emerge and operate within systems governed by quantum mechanics. It provides a theoretical framework for identifying and understanding the fundamental energetic constraints that determine the operation and performance of quantum technologies. In this context, ergotropy is defined as the maximum amount of work extractable from a quantum system via unitary operations[1]. Over the past decade, the role of quantum resources in thermodynamic processes has attracted considerable attention[2–34]. Among these resources, quantum coherence stands out as the most fundamental nonclassical feature. Consequently, numerous studies have focused on investigating the impact of quantum coherence on thermodynamic processes[35–51]. Particular attention has been devoted to its role in ergotropy[47]. It has been shown that ergotropy can be decomposed into two distinct contributions — coherent and incoherent — and that the coherent component can be enhanced by the presence of quantum coherence. In recent decades, there has been growing theoretical interest in the dynamics of open quantum systems[52]. Considerable attention has been devoted to extracting work from open quantum systems interacting with their environment[53–59]. Many of these investigations assume that the system and its environment are initially uncorrelated. However, in many physical systems, especially at the nano or quantum scale, such initial correlations are unavoidable and play a pivotal role in the way a system exchanges energy with its environment[60–73]. It is still an open problem to understand the relationship between the structure of the initial system-environment states and the nature of the resulting dynamics in the situation where the system and the environment are initially correlated[72,73]. This observation motivates us to study work extraction from quantum systems in the presence of initial system-environment correlations. In this work, considering both the coherent and incoherent contributions to ergotropy, we investigate the dynamics of ergotropy, coherence, and coherence-based ergotropy in both Markovian and non-Markovian regimes. We aim to show how the nature of the environment dynamics determines the depletion or revival of quantum work resources. In the non-Markovian regime, we expect that the presence of memory in the environment will cause a portion of the information and coherence transferred to the environment to return to the system after a certain time. This return will lead to the emergence of a revival of coherence, followed by a temporary increase in ergotropy. We examine how the intensity of this revival depends on the initial correlation between the system and the environment. Specifically, we will demonstrate that a weaker initial correlation ensures a stronger backflow of information and higher quantum resources at intermediate and long times, whereas a stronger initial correlation causes a severe weakening of the revival. In the Markovian regime, we demonstrate that all quantities — whether coherence, coherence-based ergotropy, or total ergotropy — experience a monotonic decrease without revival, reaching very small values within short times. Increasing the strength of the initial correlation in this regime will only accelerate the decay process without offering any advantage for work extraction at later times. In general, by comparing the two regimes, we show that non-Markovian effects can act as a potential resource for the temporary recovery of quantum resources and an increase in work extraction efficiency, provided that the initial system-environment correlation is

small and the environmental memory conditions are strong. In Markovian dynamics, such recovery does not exist, and the entire process proceeds towards an irreversible reduction of resources. From this perspective, tuning the amount of initial correlation and selecting the appropriate dynamical regime (through environment engineering) are two key tools for controlling the stability and productivity of quantum resources in thermodynamic and work-oriented processes.

Ergotropy: Its Coherent and Incoherent Components

Ergotropy is defined as the maximum energy that can be extracted from a quantum system through a cyclic unitary process[1]:

$$\mathscr{E}(\rho) = \max_{V\in\mathscr{U}}\{U(\rho) - U(V\rho V^{\dagger})\}, \tag{1}$$

where $U(\rho) = \mathrm{tr}[\rho H]$ represents the internal energy, H and ρ are the Hamiltonian and the density matrix, respectively, and $\mathscr{U}$ is the set of all cyclic unitary transformations. These transformations must be cyclic with respect to H. Assuming the spectral decomposition of the density matrix ρ and the Hamiltonian H as

$$\rho = \sum_n r_n \mid r_n >< r_n \mid, \tag{2}$$

$$H = \sum_n \varepsilon_n \mid \varepsilon_n >< \varepsilon_n \mid, \tag{3}$$

with the eigenvectors ordered such that $r_n \geq r_{n+1}$ and $\varepsilon_n \leq \varepsilon_{n+1}$, a closed-form expression for ergotropy can be given in terms of the passive state. The passive state is defined as follows:

$$\rho_\pi = \sum_n r_n \mid \varepsilon_n >< \varepsilon_n \mid, \tag{4}$$

and consequently, the ergotropy takes the following form:

$$\mathscr{E}(\rho) = U(\rho) - U(\rho_\pi). \tag{5}$$

The incoherent ergotropy, i.e., the maximum amount of energy that can be extracted from ρ without altering its quantum coherence, is written as:

$$\mathscr{E}_I(\rho) = \mathscr{E}(\rho^{\mathscr{D}}), \tag{6}$$

where

$$\rho^{\mathscr{D}} = \sum_n \langle\varepsilon_n|\rho|\varepsilon_n\rangle|\varepsilon_n\rangle\langle\varepsilon_n|, \tag{7}$$

represents the dephased state of ρ. Having defined the incoherent component of ergotropy, one can define the coherent part of ergotropy as follows:

$$\mathscr{E}_C(\rho) = \mathscr{E}(\rho) - \mathscr{E}_I(\rho). \tag{8}$$

To investigate the role of quantum coherence, let us consider the l_1-norm of coherence, which is defined as[74,75]:

$$C_{l_1}(\rho) = \min_{\delta\in I}|\rho - \delta|_{l_1}, \tag{9}$$

where I denotes the set of all incoherent states (i.e., diagonal states) in the basis $\{|\varepsilon_n\rangle\}$. Performing the minimization leads to

$$C_{l_1}(\rho) = \sum_{\varepsilon_n\neq\varepsilon_{n'}} |\rho_{nn'}|, \tag{10}$$

which is the sum of the absolute values of all off-diagonal elements of the density matrix in the basis $\{|\varepsilon_n\rangle\}$.

Description of The Model

The model that we consider in this work describes a two-level system (a qubit Q) coupled to its environment B. We focus on the scenario where energy dissipation is neglected and pure dephasing is the only mechanism responsible for decoherence of the qubit. The system's dynamics are represented by the Hamiltonian

$$H = H_Q \otimes I_B + I_Q \otimes H_B + H_I. \tag{11}$$

Each of the Hamiltonian terms is defined as follows

$$H_Q = \varepsilon S_z, \quad H_B = \int_0^\infty d\omega\, h(\omega) a^\dagger(\omega) a(\omega), \tag{12}$$

$$H_I = S_z \otimes \int_0^\infty d\omega \left[g^*(\omega) a(\omega) + g(\omega) a^\dagger(\omega) \right], \tag{13}$$

in which S_z, the z-component of the spin operator, is represented by the following diagonal matrix:

$$S_z = \mathrm{diag}[1, -1], \tag{14}$$

and, the parameter ε represents the qubit energy splitting, I_Q and I_B denote the identity operators for the Hilbert spaces of the qubit Q and the environment B, respectively. The bosonic creation and annihilation operators are denoted by $a^\dagger(\omega)$ and $a(\omega)$ respectively. The environment is characterized by the real spectral function $h(\omega)$, and the coupling is described by the function $g(\omega)$, whose complex conjugate is $g^*(\omega)$. One can rewrite the Hamiltonian 11 into a block-diagonal structure[77,78],

$$H = \mathrm{diag}[H_+, H_-], \quad H_\pm = H_B \pm H_I \pm \varepsilon I_B. \tag{15}$$

Following the approach in reference[79], we consider an initial correlated state for the qubit-environment system, such that:

$$|\Psi(0)\rangle = b_+ |1\rangle \otimes |\Omega_0\rangle \; + \; b_- |-1\rangle \otimes |\Omega_\lambda\rangle, \tag{16}$$

where the states $|1\rangle$ and $|-1\rangle$ represent the excited and ground states of the qubit, respectively. The non-zero complex numbers b_+ and b_- satisfy the following condition:

$$|b_+|^2 + |b_-|^2 = 1. \tag{17}$$

The states $|\Omega_0\rangle$ and $|\Omega_\lambda\rangle$ represent the states of the environment. For example, we assume $|\Omega_0\rangle$ to be the ground state of the environment, and

$$|\Omega_\lambda\rangle = C_\lambda^{-1} \left[(1-\lambda)|\Omega_0\rangle + \lambda |\Omega_f\rangle \right], \tag{18}$$

is a linear combination of the ground state $|\Omega_0\rangle$ and the coherent state $|\Omega_f\rangle = D(f)|\Omega_0\rangle$, where the displacement operator $D(f)$ is defined as[80]:

$$D(f) = \exp\left[\int_0^\infty d\omega \left(f(\omega) a^\dagger(\omega) - f^*(\omega) a(\omega) \right) \right], \tag{19}$$

for an arbitrary function f that is square-integrable. The constant C_λ normalizes the state $|\Omega_\lambda\rangle$ and is given by:

$$C_\lambda^2 = (1-\lambda)^2 + \lambda^2 + 2\lambda(1-\lambda)\,\mathrm{Re}\langle \Omega_0 | \Omega_f \rangle, \tag{20}$$

where Re denotes the real part of the inner product $\langle \Omega_0 | \Omega_f \rangle$ between the two states in the environment Hilbert space. The parameter $\lambda \in [0,1]$ controls the initial entanglement between the qubit and the environment. For $\lambda = 0$, there is no initial entanglement; the qubit and environment are uncorrelated. Conversely, at $\lambda = 1$, the entanglement reaches its maximum for a specific class of initial states. It is important to note, however, that states with $\lambda = 1$ do not represent maximal entanglement in the conventional sense. This is due to the non-orthogonality of coherent states, which constitute an overcomplete set. At a time $t > 0$, the total state of the system can be expressed in the form:

$$|\Psi(t)\rangle = b_+ |1\rangle \otimes |\psi_+(t)\rangle \; + \; b_- |-1\rangle \otimes |\psi_-(t)\rangle, \tag{21}$$

where

$$|\psi_+(t)\rangle = e^{-iH_+t}|\Omega_0\rangle, \quad |\psi_-(t)\rangle = e^{-iH_-t}|\Omega_\lambda\rangle. \tag{22}$$

To obtain the qubit's density matrix $\rho_\lambda(t)$, we use the relation:

$$\rho_\lambda(t) = \mathrm{Tr}_B\{|\Psi(t)\rangle\langle\Psi(t)|\}, \tag{23}$$

where Tr_B is the partial trace over the environment. The explicit form of it is

$$\rho_\lambda(t) = \begin{pmatrix} |b_+|^2 & b_+b_-^*A_\lambda(t) \\ b_+^*b_-A_\lambda^*(t) & |b_-|^2 \end{pmatrix}, \tag{24}$$

where

$$A_\lambda(t) = C_\lambda^{-1}e^{-2i\varepsilon t}e^{-r(t)}\left[1-\lambda+\lambda e^{-2i\Phi(t)}e^{s(t)}\right], \tag{25}$$

and according to[77,78] we have

$$r(t) = 4\int_0^\infty d\omega\, g_h^2(\omega)\,[1-\cos(\omega t)], \tag{26}$$

$$s(t) = 2\int_0^\infty d\omega\, g_h(\omega)f(\omega)\,[1-\cos(\omega t)] - \frac{1}{2}\int_0^\infty d\omega\, f^2(\omega), \tag{27}$$

where

$$g_h(\omega) = \frac{g(\omega)}{h(\omega)}, \tag{28}$$

and

$$\Phi(t) = \int_0^\infty d\omega\, g_h(\omega)f(\omega)\,\sin(\omega t). \tag{29}$$

We have assumed $g(\omega)$ and $f(\omega)$ to be real functions for simplicity. We model the qubit's coupling to the environment using the spectral density function

$$g_h^2(\omega) = \eta\,\omega^{\mu-1}\exp\left(-\frac{\omega}{\omega_c}\right), \tag{30}$$

where $\eta > 0$ represents the qubit-environment coupling strength, $\mu > -1$ shows the ohmicity parameters. For $-1 < \mu < 0$ the environment is sub-ohmic, $\mu = 0$ shows the ohmic environments and $\mu > 0$ denotes super-ohmic environment. To determine the coherent state $|\Omega_f\rangle$, we use the function:

$$f^2(\omega) = \omega^{\nu-1}\exp\left(-\frac{\omega}{\omega_c}\right), \tag{31}$$

where ν is an arbitrary parameter. This choice, while arbitrary, is convenient as it permits the derivation of explicit analytical formulas for

$$r(t) = 4\,\omega_c^\mu\,\eta\,\Gamma(\mu)\,\{1-\frac{\cos[\mu\,\arctan(\omega_c t)]}{(1+\omega_c^2t^2)^{\mu/2}}\}, \tag{32}$$

$$s(t) = 2\,\omega_c^\kappa\sqrt{\eta}\,\Gamma(\kappa)\,\{1-\frac{\cos[\kappa\,\arctan(\omega_c t)]}{(1+\omega_c^2t^2)^{\kappa/2}}\} - \frac{\omega_c^\nu}{2}\Gamma(\nu), \tag{33}$$

$$\Phi(t) = \omega_c^\kappa\sqrt{\eta}\,\Gamma(\kappa)\,\frac{\sin[\kappa\,\arctan(\omega_c t)]}{(1+\omega_c^2t^2)^{\kappa/2}}, \tag{34}$$

where $\Gamma(.)$ is the Euler gamma function, and κ is

$$\kappa = \frac{\mu+\nu}{2}. \tag{35}$$

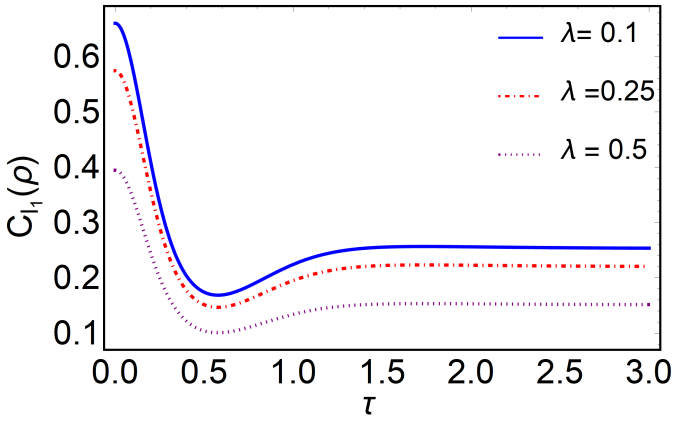


Figure 1. Coherence as a function of dimensionless time $\tau = \omega_c t$ in the non-Markovian regime for different values of the initial system-environment correlation, with parameters $\varepsilon = 1$, $\mu = 5$, $\eta = \nu = 0.01$, and $r_x = r_y = r_z = 0.5$.

The Roles of Initial System-Environment Correlations on Ergotropy and Coherence

In this section, we consider the initial state of the qubit to be a mixed state with Bloch vector $\mathbf{r} = (r_x, r_y, r_z)$. The corresponding density is

$$\rho(0) = \frac{1}{2}\left(I + r_x\sigma_x + r_y\sigma_y + r_z\sigma_z\right) = \begin{pmatrix} \frac{1+r_z}{2} & \frac{r_x - ir_y}{2} \\ \frac{r_x + ir_y}{2} & \frac{1-r_z}{2} \end{pmatrix}, \tag{36}$$

where σ_i , with $i \in x, y, z$ are Pauli operators. In the pure dephasing model introduced earlier, the diagonal elements remain constant, and the off-diagonal elements are multiplied by the factor $A_\lambda(t)$ (obtained from the environment dynamics). Therefore, the time evolution of the mixed state can be obtained as

$$\rho_\lambda(t) = \begin{pmatrix} \frac{1+r_z}{2} & \frac{r_x - ir_y}{2} A_\lambda(t) \\ \frac{r_x + ir_y}{2} A^*_\lambda(t) & \frac{1-r_z}{2} \end{pmatrix}. \tag{37}$$

According to Eq. (37) the l_1-norm of coherence can be obtained as

$$C_{l_1}(\rho) = \sqrt{r_x^2 + r_y^2}\,|A_\lambda(t)|. \tag{38}$$

Also, considering relations 5, 6, and 8 for this mixed state, the ergotropies will be as follows

$$\mathscr{E}(\rho) = \frac{1}{2}\left(r_z + \sqrt{|A_\lambda(t)|^2(r_x^2 + r_y^2) + r_z^2}\right), \tag{39}$$

$$\mathscr{E}_I(\rho) = \max(0, r_z), \tag{40}$$

$$\mathscr{E}_C(\rho) = \begin{cases} \frac{1}{2}\left(\sqrt{|A_\lambda(t)|^2(r_x^2 + r_y^2) + r_z^2} - r_z\right), & \text{if } r_z \geq 0, \\ \frac{1}{2}\left(\sqrt{|A_\lambda(t)|^2(r_x^2 + r_y^2) + r_z^2} + r_z\right), & \text{if } r_z < 0. \end{cases} \tag{41}$$

We are now in a position to investigate the effect of the initial correlation between the system and the environment on work extraction, in both the Markovian and non-Markovian regimes. In Ref.[81], the authors have quantified the degree of non-Markovianity for the environment parameters. We use the results of that reference to examine work extraction in both the Markovian and non-Markovian regimes. To perform the calculations, we choose the parameter values as follows:

$$\varepsilon = 1, \quad \eta = 0.01, \quad \nu = 0.01, \quad \mu = 5 \text{ (for the non-Markovian regime)}, \quad \mu = 6 \text{ (for the Markovian regime)}. \tag{42}$$

In Fig. 1, the l_1-norm of coherence is plotted as a function of the dimensionless parameter $\tau = \omega_c t$ in the non-Markovian regime for different values of the initial correlation between the system and the environment. As can be seen, in the initial moments, the system's coherence decreases rapidly a behavior arising from the initial interaction with the environment and the immediate transfer of a portion of quantum information to it. This initial drop is observed for all values of the initial

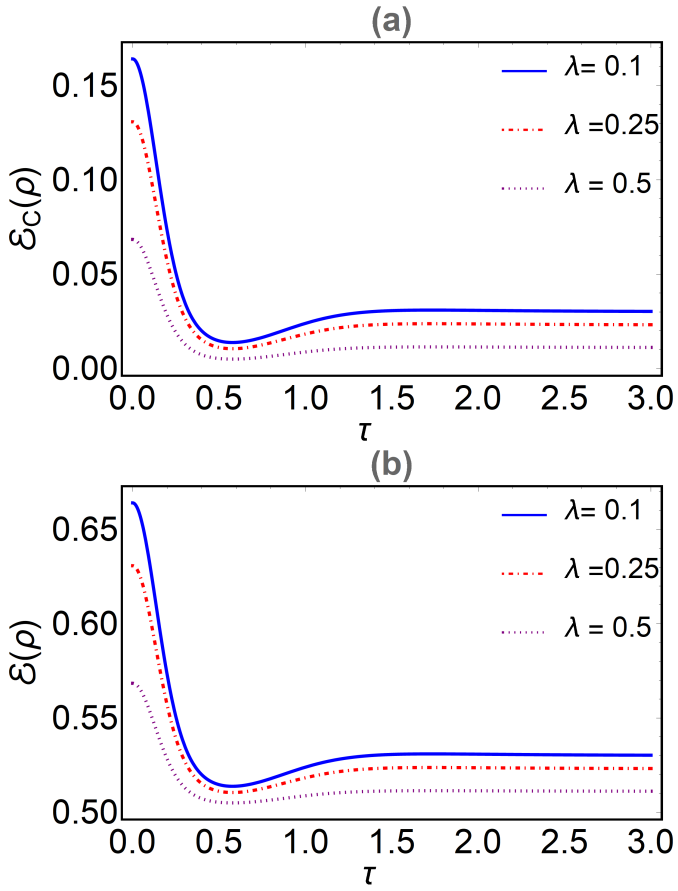


Figure 2. Time evolution of (a) $\mathscr{E}_C(\rho)$ and (b) $\mathscr{E}(\rho)$ as a function of dimensionless time $\tau = \omega_c t$ in the non-Markovian regime ($\mu = 5$) for three values of the initial system-environment correlation $\lambda = 0.1, 0.25, 0.5$.

correlation λ. After this initial drop, some of the information returns from the environment to the system, and a relative revival of coherence occurs. This return indicates the presence of memory in the environment dynamics. However, increasing the value of λ causes this coherence revival to be weaker, because a stronger initial interaction with the environment leads to a more widespread distribution of phases and amplitudes in the environment's state, making their complete recovery more difficult. At long times, the coherence converges to a steady-state value that decreases as λ increases. From a practical perspective, if the goal is to preserve or exploit coherence (for example, in quantum work extraction), choosing a smaller λ is advantageous, because both the extent of coherence revival over time and the final coherence value at longer times will be greater.

Fig. 2(a) shows the time evolution of coherence-based ergotropy, $\mathscr{E}_C(\rho)$, in the non-Markovian regime for different values of the initial correlation λ. This quantity describes the part of the extractable work from the system that arises exclusively from the presence of quantum coherence in the state. It can be observed that, in the initial moments, the value of $\mathscr{E}_C(\rho)$ decreases rapidly for all values of λ, which is due to the loss of coherence caused by the initial interaction with the environment. In subsequent time intervals, for smaller λ, a relative revival in $\mathscr{E}_C(\rho)$ is observed, indicating the return of a portion of coherence to the system due to the environment's memory effects (non-Markovianity). However, for larger λ, this revival is weakened or becomes very small, because the phases and amplitudes of the environment state, which were disturbed during the initial interaction, return to the system less constructively. At long times, $\mathscr{E}_C(\rho)$ converges to a steady-state value that decreases as λ increases. This means that the presence of a smaller initial correlation not only helps preserve the coherence contribution to the extractable work but also enables optimal exploitation of non-Markovian effects.

Figure 3 shows the time evolution of coherence $C_{l_1}(\rho)$ as a function of dimensionless time τ in the Markovian regime ($\mu = 6$) for three different values of the initial correlation between the system and the environment ($\lambda = 0.1,\ 0.25,\ 0.5$). The other parameters are $\varepsilon = 1$, $\eta = \nu = 0.01$, and $r_x = r_y = r_z = 0.5$. In this regime, the decrease in coherence exhibits a monotonic behavior without revival. This is consistent with the nature of Markovian dynamics: because the environment lacks memory, the information or coherence transferred to it does not return. Therefore, after the interaction begins, coherence decreases uniformly and exponentially, tending towards a value close to zero at long times. Comparison of the curves shows

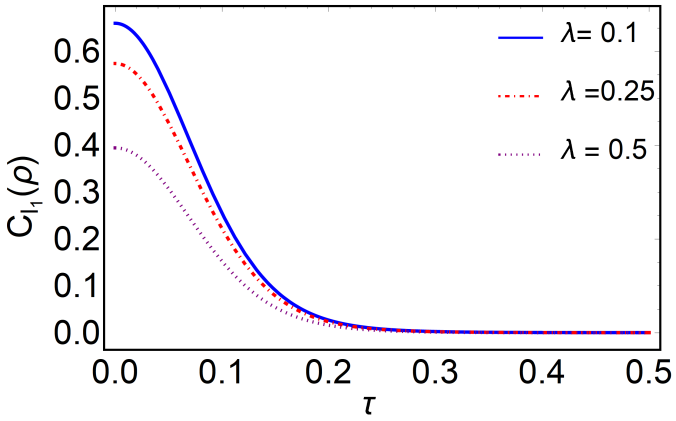


Figure 3. Coherence as a function of dimensionless time $\tau = \omega_c t$ in the Markovian regime for different values of the initial system-environment correlation, with parameters $\varepsilon = 1$, $\mu = 6$, $\eta = \nu = 0.01$, and $r_x = r_y = r_z = 0.5$.

that the initial value of coherence for smaller λ is greater; however, the final value of coherence is the same for all λ. Therefore, for applications such as quantum work extraction that depend on coherence, reducing λ as much as possible is essential in this regime.

Figure 4(a) shows the evolution of coherence-based ergotropy in the Markovian regime. This quantity, which represents the share of extractable work arising from quantum coherence, decreases uniformly and without revival immediately after the interaction with the environment begins, reaching a value close to zero within short time intervals. The absence of any peaks or revivals in the curve is a signature of memoryless dynamics, meaning that the coherence transferred to the environment does not return to the system. As the initial correlation between the system and the environment increases, the initial amount of coherence-based ergotropy becomes smaller.

Figure 4(b) shows the evolution of total ergotropy. The qualitative behavior of this curve is similar to that of Figure 4(a), and its decrease is also uniform and without revival, with the difference that its value is greater than the purely coherence-based ergotropy at all times, because the population part persists for a longer period and its complete decay occurs more slowly. As the initial correlation increases, this curve also shifts downward, and both the depth of the initial drop and the reduction of the long-term steady-state value become more pronounced. This indicates that the initial correlation not only weakens the quantum resource for work (coherence) but also gradually eliminates the advantage arising from population heterogeneity. In general, both quantities in the Markovian regime evolve unidirectionally towards small values without any return, and there is no sign of information backflow. Therefore, work extraction must be carried out very early. Furthermore, coherence-based ergotropy is more sensitive to changes in λ, meaning that increasing the initial correlation affects the quantum resource for work the most. Hence, to maximize the extractable work in this regime, it is necessary to reduce the initial correlation as much as possible and to implement the extraction protocol as soon as possible after the interaction begins, before the coherence and population heterogeneity have decreased appreciably.

Conclusion

Our analysis has revealed that the nature of the environment dynamics plays a decisive role in how quantum work resources are depleted or revived. In the non-Markovian regime, the presence of memory in the environment causes a portion of the information and coherence transferred to the environment in the early stages of the interaction to return to the system after a certain time. This return leads to the emergence of the coherence revival phenomenon, followed by a temporary increase in ergotropy. The intensity of this revival is found to depend on λ: smaller values of λ ensure a stronger backflow of information, while larger values of λ cause a severe weakening of the revival. Therefore, in the non-Markovian regime, reducing the initial correlation can be an effective strategy for maximizing work extraction efficiency.

In contrast, in the Markovian regime, the information and coherence transferred to the environment do not return to the system. Under these conditions, all quantities exhibit a monotonic decrease without revival. Increasing λ in this regime accelerates the decay process without offering any advantage for work extraction at later times. Hence, in the Markovian regime, keeping the initial correlation at the lowest possible level and implementing work extraction protocols within very short time intervals are essential.

In general, comparing the two regimes shows that non-Markovian effects can act as a potential resource for the temporary recovery of quantum resources, provided that λ is small and the environmental memory conditions are strong. In Markovian

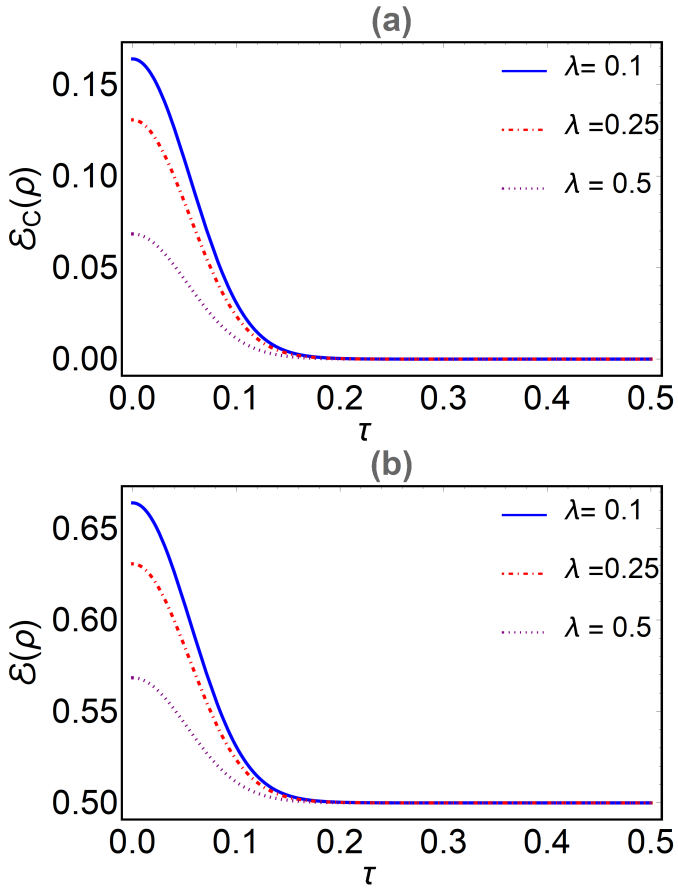


Figure 4. Time evolution of (a) $\mathscr{E}_C(\rho)$ and (b) $\mathscr{E}(\rho)$ as a function of dimensionless time $\tau = \omega_c t$ in the Markovian regime ($\mu = 6$) for three values of the initial system-environment correlation $\lambda = 0.1,\ ,0.25,\ ,0.5$.

dynamics, such recovery does not exist, and the entire process proceeds towards an irreversible reduction of resources. Tuning the amount of initial correlation and selecting the appropriate dynamical regime are two key tools for controlling the stability and productivity of quantum resources. This conclusion highlights the importance of carefully designing initial conditions and understanding the environment's characteristics in optimizing quantum systems.

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Data availability

The datasets used and/or analysed during the current study are available from the corresponding author on reasonable request.

Additional information

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