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A T-count Zero Protocol for Heralded W -state Preparation via Clifford-only Gates

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Abstract

For quantum communication and computation protocols, preparing W states is a fundamental requirement. Traditionally, deterministic W -state generation requires non-Clifford rotations. In fault-tolerant architectures, the cost of T-gate synthesis and Magic State Distillation (MSD)[1] is very high. As a result, non-Clifford rotations for W state preparation introduce significant overhead. This research paper proposes a heralded, probabilistic circuit for the 3-qubit W -state[2] preparation using only Clifford gates[1] and one ancillary qubit. The quantum circuit utilizes the interaction between two Bell pairs[3], followed by post-selection, and achieves a success probability of **0.75**. The proposed protocol maintains a T-count of zero. It also reduces the circuit depth and error susceptibility compared to standard unitary methods. This paper provides details of the protocol and compares its gate-count efficiency with that of existing methods. The proposed protocol substitutes high-overhead non-Clifford gates for a manageable post-selection loss and thus, provides a resource-efficient and robust alternative for W -state preparation in near-term hardware and early fault-tolerant systems.

Keywords: W state, quantum computing, quantum circuit

1 Introduction

The W state is a 3-qubit entangled quantum state. Using the bra-ket notation, the W state can be represented in the following way:

$$|W\rangle = \frac{|001\rangle + |010\rangle + |100\rangle}{\sqrt{3}} \quad (1)$$

This state is remarkable for its high level of multipartite entanglement. Unlike the *GHZ* state, if one qubit of its 3 qubits is measured or lost, the remaining qubits stay entangled. Because of its unique property of robustness against qubit loss, the *W*-state has several applications in quantum information, quantum memories, and robust quantum communication protocols.

Figure 1 shows a standard quantum circuit for generating the 3-qubit *W* state deterministically[4].

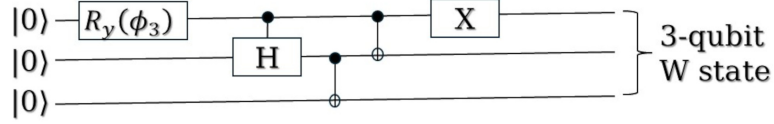


Fig. 1: Standard Quantum Circuit for Generation of the 3-qubit *W* State

This quantum circuit uses $R_y(\phi_3)$, Controlled-Hadamard (CH), CNOT, and X gates. Please note that the value of ϕ_3 is $2 \cos^{-1}(\frac{1}{\sqrt{3}})$.

The standard method of *W*-state preparation relies on non-Clifford gates. The CNOT gates are Clifford gates, and the controlled Hadamard and $R_y(\phi_3)$ are non-Clifford gates. In the context of Fault-Tolerant Quantum Computing (FTQC), non-Clifford gates must be decomposed into T-gates. These T-gates are significantly more expensive to implement than Clifford gates (H, CNOT, S). Non-Clifford operations, such as the T-gate, require complex Magic State Distillation (MSD) protocols. These protocols require substantial spatial overhead. They often require thousands of physical qubits to distill a single high-fidelity magic state.

The controlled-Hadamard (CH) gate has a T-count of 2. And the expected T-count for the $R_y(\phi_3)$ gate is 4 if we use a Repeat-Until-Success (RUS) circuit. In a standard deterministic circuit, its T-count is approximately $3 \log_2(\frac{1}{\epsilon})$, where ϵ is the

desired precision. Moreover, in the NISQ era, complex rotations often lead to higher decoherence and gate errors.

The proposed protocol uses a heralded protocol with a T-count of zero, thereby circumventing the requirements for T-gates and Magic State Distillation (MSD) and offering a significantly more efficient path for early fault-tolerant quantum processors.

Many existing protocols explore measurement-based approaches to circumvent deterministic gate synthesis. However, these protocols often require the pre-generation of complex, multi-qubit entangled resource states. These states themselves require non-Clifford logic to prepare. Other heralded protocols often rely on a high ratio of ancillary to data qubits. In contrast, our proposed protocol is initialized in the computational basis state $|0000\rangle$. The proposed protocol does not require any complex initial resources or high-depth ancillary logic. It also utilizes a minimal number of ancillary qubits to achieve high-fidelity preparation of the 3-qubit W state.

The rest of this paper is organized as follows: Section 2 describes the theoretical framework and the Clifford gate set; Section 3 presents the proposed circuit; Section 4 provides a comparative analysis of resource costs; and Section 5 concludes with future outlooks for scaling the protocol.

2 Clifford Gates and Fault-Tolerant Resource Costs

The Clifford gate set in quantum computing consists of gates that preserve the structure of the Pauli group. The key gates in this set are:

Hadamard gate(H) : Creates an equal superposition state from a given computational basis state.

Phasegate(S) : Applies a global phase to the input.

CNOTgate(CNOT) : Performs NOT operation on the target qubit based on the control qubit.

These gates are essential for constructing quantum circuits and are crucial for fault-tolerant quantum computation. The Clifford group is generated by these gates, and they can be efficiently simulated on classical computers as per the Gottesman-Knill theorem.

2.1 The Gottesman-Knill Theorem

The significance of this Clifford gate set is established by the Gottesman-Knill Theorem. It states that any circuit consisting entirely of Clifford gates, starting from a computational basis state and ending with Pauli measurements, can be efficiently simulated on a classical computer in polynomial time. Consequently, Clifford circuits alone cannot provide a quantum advantage for computation. To achieve universality, the Clifford set must be augmented with at least one non-Clifford gate, typically the T-gate or a Toffoli gate.

2.2 The T-gate Bottleneck in Fault-Tolerance

In the context of Fault-Tolerant Quantum Computing (FTQC), there is a massive disparity in the resource cost of these gates. In most quantum error-correcting codes,

such as the Surface Code, Clifford gates are transversal or can be implemented via lattice surgery with relatively low overhead.

However, the T-gate is not transversal in these codes. Its implementation requires a high-overhead process known as Magic State Distillation (MSD). A single T-gate often requires:

1. Spatial Overhead : Dedicated factory regions on the quantum chip consisting of hundreds or thousands of physical qubits.

2. Temporal Overhead : Significant time cycles to perform multiple rounds of distillation to reach the required purity.

2.3 Significance of T-count Zero

T-gates are significantly more expensive than Clifford gates. In the era of fault tolerance, the T-count of a circuit is the primary metric for determining its feasibility. We restrict our protocol to a T-count of zero. Thus, the proposed protocol ensures that the W-state preparation can be executed using only the most native and high-fidelity operations available in a protected quantum processor.

In this paper, we move beyond the Gottesman-Knill limit by using post-selection, which allows the preparation of non-stabilizer states, such as the W-state. The operational complexity remains strictly within the low-overhead Clifford regime.

3 The Proposed Quantum Circuit

Figure 2 shows the proposed quantum circuit for heralded W-state preparation using only Clifford gates and one ancillary qubit.

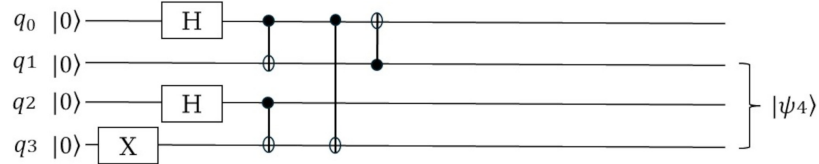


Fig. 2: The Proposed Quantum Circuit

The initial state of the quantum circuit is $|0000\rangle$. q_0 and q_1 generate the Bell state $\frac{(|00\rangle+|11\rangle)}{\sqrt{2}}$ while q_2 and q_3 generate the Bell state $\frac{(|01\rangle+|10\rangle)}{\sqrt{2}}$.

Therefore, the quantum state after generating the Bell pair becomes the following:

$$|\psi_1\rangle = \frac{(|01\rangle + |10\rangle)(|00\rangle + |11\rangle)}{2} = \frac{(|0100\rangle + |0111\rangle + |1000\rangle + |1011\rangle)}{2} \quad (2)$$

We apply a CNOT gate that uses q_0 as the control qubit and q_3 as the target qubit. Therefore, after applying the CNOT gate, the quantum state becomes the following:

$$|\psi_2\rangle = \frac{(|0100\rangle + |1111\rangle + |1000\rangle + |0011\rangle)}{2} \quad (3)$$

We use another CNOT gate that uses q_1 as the control qubit and q_0 as the target qubit. As a result, we get the following quantum state $|\psi_3\rangle$:

$$\begin{aligned} |\psi_3\rangle &= \frac{(|0100\rangle + |1110\rangle + |1000\rangle + |0010\rangle)}{2} \\ &= \frac{(|010\rangle + |111\rangle + |100\rangle + |001\rangle)|0\rangle}{2} \end{aligned} \quad (4)$$

If we measure q_3, q_2, q_1 at this point, we get the following quantum state $|\psi_4\rangle$:

$$|\psi_4\rangle = \frac{(|010\rangle + |111\rangle + |100\rangle + |001\rangle)}{2} \quad (5)$$

The outcome of measuring $|\psi_4\rangle$ can be $|010\rangle, |111\rangle, |100\rangle$, or $|001\rangle$ with equal probability. We use post-selection with two ancillary qubits and two CNOT gates to detect whether the two most significant qubits of the measurement are 1s. If we obtain $|111\rangle$ after the measurement, we discard the result and repeat the process.

3.1 State Distillation and Renormalization

Please note that $|\psi_4\rangle$ may result in $|111\rangle, |100\rangle, |010\rangle$, or $|001\rangle$ after measuring the state and each with a probability $\frac{1}{4}$. If the outcome is $|111\rangle$ after the measurement, we discard the result.

$$|\psi_4\rangle = \frac{(|010\rangle + |111\rangle + |100\rangle + |001\rangle)}{2} \quad (6)$$

Therefore, the output will be discarded with a probability $\frac{1}{4}$.

$$|\psi_{collapsed}\rangle = \frac{(|010\rangle + |100\rangle + |001\rangle)}{2} \quad (7)$$

$$P_{success} = \langle\psi_{collapsed}|\psi_{collapsed}\rangle = (1/2)^2 + (1/2)^2 + (1/2)^2 = 3/4 \quad (8)$$

$|\psi_{collapsed}\rangle$ is not normalized. To obtain the final state, we need to renormalize $|\psi_{collapsed}\rangle$.

$$\begin{aligned}
|\psi_{final}\rangle &= \frac{1}{\sqrt{P_{success}}} |\psi_{collapsed}\rangle \\
&= \frac{1}{\sqrt{\frac{3}{4}}} \times \frac{(|010\rangle + |100\rangle + |001\rangle)}{2} \\
&= \frac{(|010\rangle + |100\rangle + |001\rangle)}{\sqrt{3}}
\end{aligned} \tag{9}$$

4 Comparative Analysis of Resource Costs

To evaluate the efficiency of the proposed heralded Clifford protocol, we compare it against two primary benchmarks: the deterministic cascaded circuit and the logarithmic-depth unitary synthesis method proposed by Cruz et al. [5]. We focus on three critical metrics: T-count, CNOT-depth, and Sampling Overhead.

4.1 T-gate and Non-Clifford Overhead

The most significant advantage of our protocol is the elimination of non-Clifford gates. Deterministic state preparation requires implementing rotations by $\theta = 2 \cos^{-1}(\frac{1}{\sqrt{3}})$. As shown in Table 1, while standard decomposition methods require approximately 30 to 100 T-gates (depending on the required precision ϵ) to approximate these rotations, our circuit achieves a T-count of zero.

In the Fault-Tolerant Quantum Computing (FTQC) regime, T-gates require expensive Magic State Distillation (MSD). This reduction of T-gates represents a multi-order-of-magnitude saving in both spatial (physical qubit) and temporal resources. Even compared to Repeat-Until-Success (RUS) synthesis, which has an expected T-count of 4, our Clifford-only approach offers the lowest possible overhead for protected architectures.

4.2 CNOT Count for NISQ Devices

For Near-term Intermediate-Scale Quantum (NISQ) devices, the CNOT gate is the primary source of decoherence. Our protocol utilizes only 4 CNOT gates to prepare the 3-qubit W state.

Linear Methods : Often require gates with a strictly serial depth.

Our Method : By utilizing the interaction between two independent Bell pairs, we maintain a low depth that is resilient to T_1 and T_2 relaxation times.

4.3 Success Probability and Sampling Overhead

We acknowledge the trade-off inherent in a heralded protocol. Our circuit possesses a success probability of $P_{success} = 0.75$. Mathematically, the expected number of trials required to obtain a successful state is $\frac{1}{P_{success}} \approx 1.33$.

In high-fidelity regimes, this 25% post-selection overhead is significantly less burdensome than the cumulative error introduced by noisy T-gate synthesis or complex

rotations. We argue that for current hardware with gate error rates exceeding 10^{-3} , a heralded protocol that filters out incorrect states provides a higher Circuit Fidelity than a deterministic protocol that accumulates errors throughout its execution.

Table 1: Comparison of Resource Costs

Metric 1	Deterministic(R_y)	Cruz et al. (Log-depth)	This Work (Heralded)
Success Logic	Deterministic	Deterministic	Heralded ($P_{success} = 0.75$)
Gate Set	Universal	Universal	Clifford-only
T Count	$\approx 30+$	$2 \times (n - 1)$	0
Ancilla Qubits	0	0	1
Fault-Tolerant Cost	Very High (Uses MSD)	Moderate (Uses CH gate)	Minimal

5 Conclusion and Future Outlooks

This work demonstrates a heralded, Clifford-only protocol for preparing a 3-qubit W state. By leveraging the interaction between Bell pairs and a minimal number of ancillas, the proposed protocol bypasses the T-gate overhead typically associated with deterministic W -state synthesis. This protocol offers a success rate of 75% which is a favorable trade-off for early fault-tolerant systems where non-Clifford resources are the primary bottleneck.

5.1 Scaling to N-qubit W States

A natural extension of this research is the generalization of the Bell-pair interaction to N-qubit W states. Future work could investigate a recursive fusion architecture where W_n states are built by interacting W_{n-1} states with additional Bell pairs. While the success probability may decrease with n , the implementation of intermediate heralding—where success is confirmed at each fusion step—could mitigate this loss. Investigating the minimum T-count required for a hybrid approach for large N remains a compelling open question.

5.2 Application in Modular Quantum Architectures

The protocol’s reliance on Bell pairs makes it particularly suited for Modular Quantum Computing (MQC). In modular systems, qubits are grouped into small clusters with high internal connectivity but limited inter-cluster links. Since our protocol relies on bipartite entanglement as a starting resource, it can be implemented in a distributed fashion. Bell pairs can be generated across different modules and fused via heralded measurements to create a global state.

5.3 Integration with Error Correction

Finally, we suggest that the heralding mechanism itself can be integrated into Quantum Error Detection (QED) frameworks. Because the ancilla measurement flags the presence of certain multi-qubit errors, this protocol could be evolved into a fault-tolerant preparation routine. Future studies should perform a full density matrix simulation to quantify the fidelity improvement of this heralded approach over noisy deterministic circuits in the presence of noise and cross-talk.

To conclude, as the field transitions from NISQ to fault-tolerant hardware, protocols that prioritize T-count reduction through measurement-based trade-offs will be essential. This work provides a foundational step toward resource-efficient entanglement generation in this new era of quantum computation.

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