Research Square

Preprints are preliminary reports that have not undergone peer review.
They should not be considered conclusive, used to inform clinical practice,
or referenced by the media as validated information.

A Simple Two-branch Linear Structure in β^--decay Endpoint Energies Along Isotopic Chains for Z < 47

Tolga Yarman

tyarman@gmail.com

Istanbul Okan University

Nimet Zaim

Alexander Kholmetskii

Belarus State University

Ozan Yarman

Istanbul University

Faruk Yarman

Article

Keywords: β-decay, Nuclear Systematics, Radioactive Isotopes, Decay Energies, Isotopic Chains

Posted Date: June 18th, 2026

DOI: https://doi.org/10.21203/rs.3.rs-9655024/v1

License: This work is licensed under a Creative Commons Attribution 4.0 International License.
Read Full License

Additional Declarations: No competing interests reported.

A SIMPLE TWO-BRANCH LINEAR STRUCTURE IN β^--DECAY ENDPOINT ENERGIES ALONG ISOTOPIC CHAINS FOR $Z < 47$

Tolga Yarman,[a,*] Nimet Zaim,[b] Alexander Kholmetskii,[c] Ozan Yarman,[d] Faruk Yarman[e]

[a] *Istanbul Okan University, Istanbul, Türkiye (Corresponding Author: tyarman@gmail.com)*
[b] *Trakya University, Edirne, Türkiye (Retired on 22 April 2024)*
[c] *Belarusian State University, Minsk, Belarus*
[d] *Istanbul University, Istanbul, Türkiye*
[e] *Savronik, Çankaya, Ankara, Türkiye*

Abstract We investigate the systematics of β^--decay endpoint *(maximum)* energies along isotopic chains with fixed proton number ($Z < 47$). Using evaluated nuclear data, we find that, for each Z, the dependence on mass number A separates into two distinct linear trends corresponding to *even-A* and *odd-A nuclei.* This yields a simple two-branch structure characterized by *slope* and *intercept* parameters that can be systematically tabulated. The fits show excellent agreement, with coefficients of determination typically exceeding 0.99 across a wide range of elements. The emergence of such a simple linear structure in a complex many-body nuclear system provides a compact parametrization of decay energetics and offers a practical framework for analyzing trends and making preliminary estimates along isotopic chains. The empirical regularity is established directly from data; its broader implications are discussed.

Keywords: β-decay, Nuclear Systematics, Radioactive Isotopes, Decay Energies, Isotopic Chains.

1. INTRODUCTION

The systematics of β^--decay energies across isotopic chains remain an important aspect of nuclear structure, as they reflect the underlying evolution of nuclear binding with neutron number. While global trends are generally smooth, simple and compact parametrizations at the level of individual isotopic chains are still lacking.

The systematics of β-decay energetics *(whether through positron or electron emission),* constitute an important subject of extensive tabulations and theoretical treatments, including semi-empirical mass formulae,[1,2,3,4,5] shell-model approaches,[6,7,8,9,10] gross-theory parameterizations,[11,12,13,14] and modern microscopic methods such as QRPA.[15,16,17,18,19]

Despite this extensive literature, a particularly simple empirical regularity that tackles all the radioisotopes in elemental groups does not appear to have been previously presented in a compact systematic form. In the present work, we show that, for each fixed proton number within the range $Z<47$, the maximum β^--decay energy E depends linearly on the mass number A when even-A and odd-A isotopes are treated separately. Thus, each isotopic chain is described by two straight lines, one for even-A nuclei and one for odd-A nuclei, with excellent agreement across all cases.

This result provides a compact and transparent parameterization of β^--decay energetics along isotopic chains. The resulting slope and intercept parameters can be tabulated element by element, and may serve as a practical tool for characterizing decay-energy evolution as well as for making preliminary estimates about varying disintegration behavior.

The present investigation was initially motivated by broader considerations, more specifically the Universal Matter Architecture (UMA) [20,21,22,23,24,25,26,27,28] but is presented here as an empirical study based directly on nuclear data.

Under such a framework, binding-energy variations along an isotopic chain suggested that the related β^--decay energies should exhibit a basic linearity with increasing A at fixed Z.[29,30] Since we choose to present this article primarily as a study for showing how the available empirical findings match the predictions of a simple formalism, we do not rely on the UMA framework that originally motivated this investigation

Thus, in this work, we show that the β^--decay endpoint energy along isotopic chains exhibits a remarkable linear dependence on the mass number A, leading to a simple two-line structure *(even-A and odd-A)* for each fixed Z. This provides a compact parametrization of decay energetics and offers a new perspective on their systematic behavior.

We emphasize that the central result of this work is the emergence of a simple two-branch linear structure governing β^--decay energetics along isotopic chains.

We organize our presentation in the following manner: Section 2 lays out the formalistic core of our study; thus, for each element with Z<47, we demonstrate that the dependence of a set of corresponding β^--decay energies E on the mass number A is highly linear, along with separate straight-line trends for *even-A* and *odd-A* isotopes. In Section 3, we explain the corresponding slope and intercept parameters for these lines. Finally, we delve into our conclusions in Section 4.

2. THE LINEAR DEPENDENCE OF THE ENDPOINT DECAY ENERGY *E* ON THE MASS NUMBER *A* FOR β^--DECAYING RADIOISOTOPES OF THE SAME ELEMENT

For each element in the range Z<47, we extracted the evaluated β^--decay maximum energies E from the National Nuclear Data Center (NNDC)[31] and Brookhaven databases.[32,33]

Thus, the analysis is based on evaluated nuclear data tables of β^--decay endpoint energies.

The data are compiled in Table 1. For each fixed proton number Z, the dependence of the endpoint energy on the mass number A was analyzed using linear least-squares fits.

When plotted as a function of the mass number A along an isotopic chain of any given fixed Z, the decay energies reveal a particularly simple and systematic pattern; *e.g.*, each chain is accurately described by two linear trends: *one for even-A isotopes* and *one for odd-A isotopes.* Thus, the analysis was performed separately for even-A and odd-A isotopic chains.

The small correction due to the electron rest mass (0.511 MeV) is negligible compared to typical endpoint energies, which often exceed 10 MeV.

The resulting linear trends are illustrated in Figs. 1–4. In all the cases examined, an excellent linear fit, when applied separately to even and odd mass numbers, provides an excellent description of the isotopic evolution of the decay energy.

The fitted parameters p (intercept) and q (slope) for all elements with Z<47 are compiled in Table 2. The quality of the fits was quantified using the coefficient of determination* (R^2), which was found to be systematically close to unity.

Thus, across all isotopic chains, we have the following:

- R^2 values typically exceed 0.99;
- they rarely fall below 0.95;
- they all reflect the high experimental precision of the evaluated decay-energy data *(usually at the 0.1% level or better).*

The observed linearity holds across a wide range of isotopic chains, and is robust with respect to the selection of the data sets of concern.

* The coefficient of determination R^2 quantifies the goodness of the linear fit; values close to unity indicate minimal deviation between the fitted line and the data.

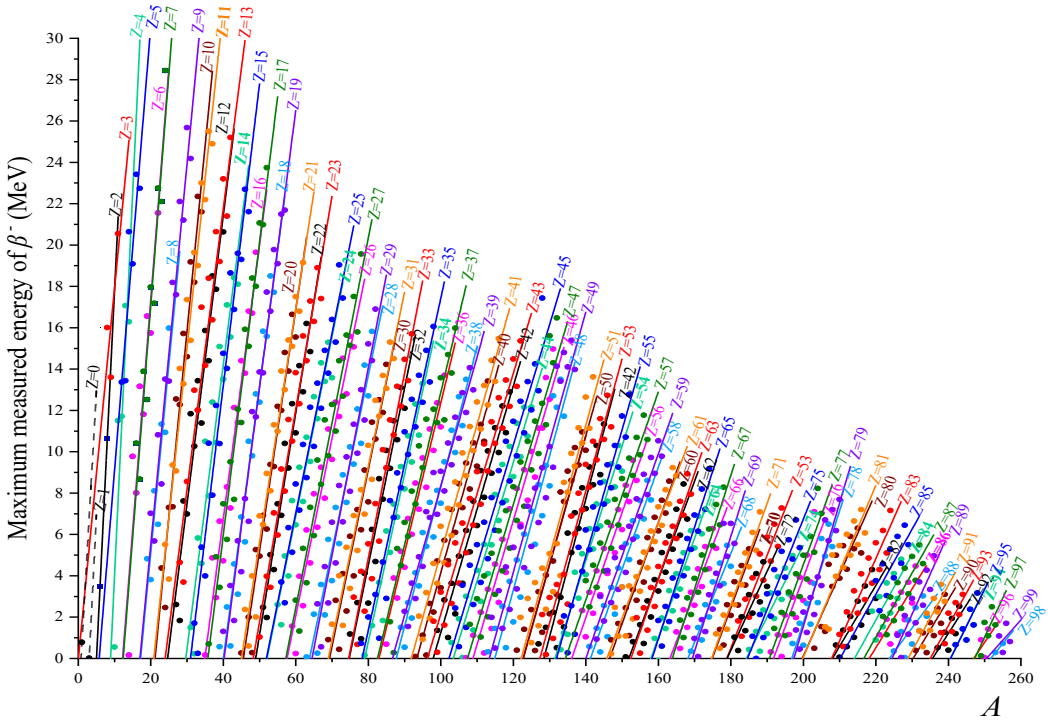


Figure 1 The clean linear dependence, for radioisotopes of the same element per a given Z, of the β^--decay endpoint energy (maximum energy) E on the mass number A across all available Z values ranging from 0 to 46. We include the range Z>47 here for completeness, although it is outside the scope of the current contribution. To avoid overcrowding this panorama, we did not mark the nuclei in the display with their corresponding symbols *(for which, see further below for details)*. The two dashed black lines on the far left indicate Z=0 and Z=1, where we have only one datapoint for each, and where we have deliberately plotted their lines more or less parallel to the neighboring line at Z=2 for visual clarity.

The aim is not merely to obtain numerical fits, but to identify a simple structural regularity in the evolution of decay energies. This approach effectively reduces the description of each isotopic chain to two linear parameters.

Next, we focus on nuclei whose Z values lie in-between Z=0 and Z=17 (Fig. 2), where we immediately note that the straight lines passing by *even A's (contiguous lines)* and *odd A's (dashed lines)* fit the data better separately.

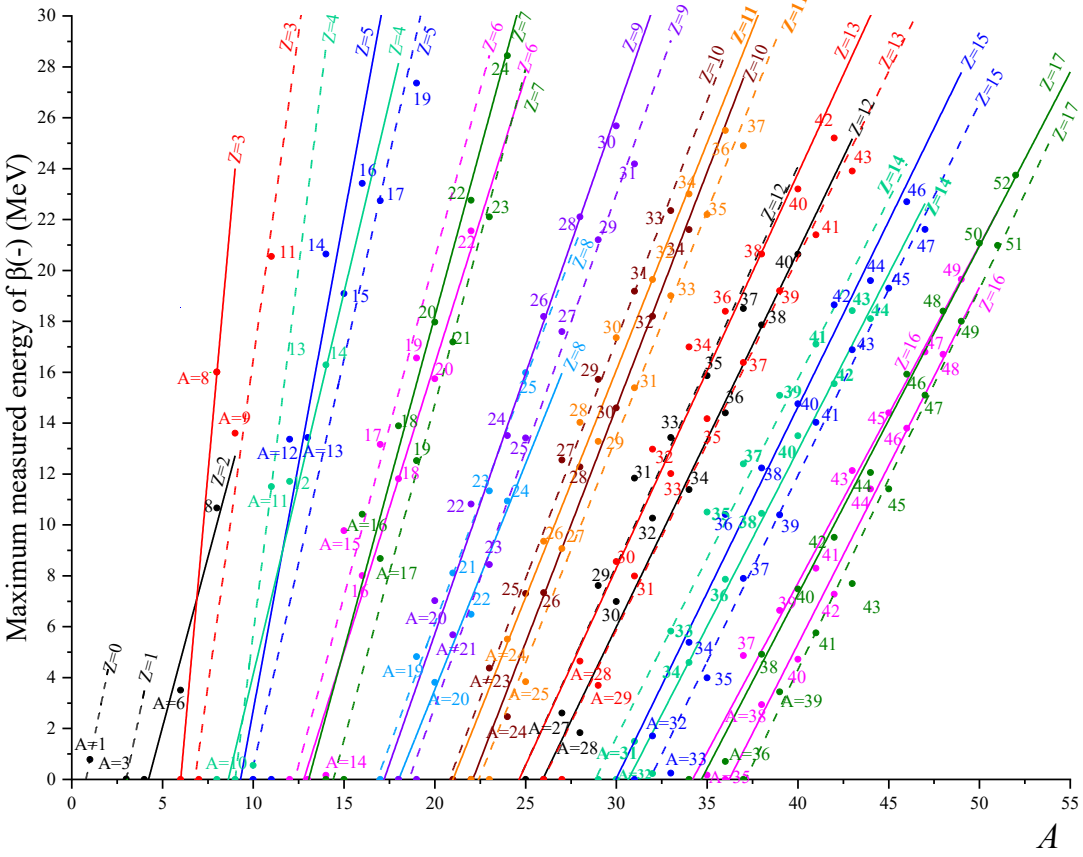


Figure 2 β^--decay endpoint energy (maximum energy) E versus the mass number A for radioisotopes of the same element per a given Z throughout $Z<18$. The contiguous lines trace the *even A* nuclei datapoints, whereas the dashed lines trace the *odd A* nuclei datapoints. The separation between even-A and odd-A chains is clearly visible. Wherever we have just a single nucleus at hand, that is, with respect to $Z=0$ (neutron) and $Z=1$ (Tritium), we have deliberately drawn a line intersecting the single datapoint in question practically parallel to the neighboring straight line at $Z=3$.

This behavior admits a natural explanation:

The vertical offset between the *even-A* and *odd-A* trends reflects the well-known *pairing contribution* to nuclear binding. *Even-A* nuclei being more stable, their decay energies come to be noticeably smaller than those of odd-*A* nuclei within the body of the same element.

In what follows, we plot the maximum decay energy E versus A for Z values between $Z=18$ and $Z=33$ (Fig. 3).

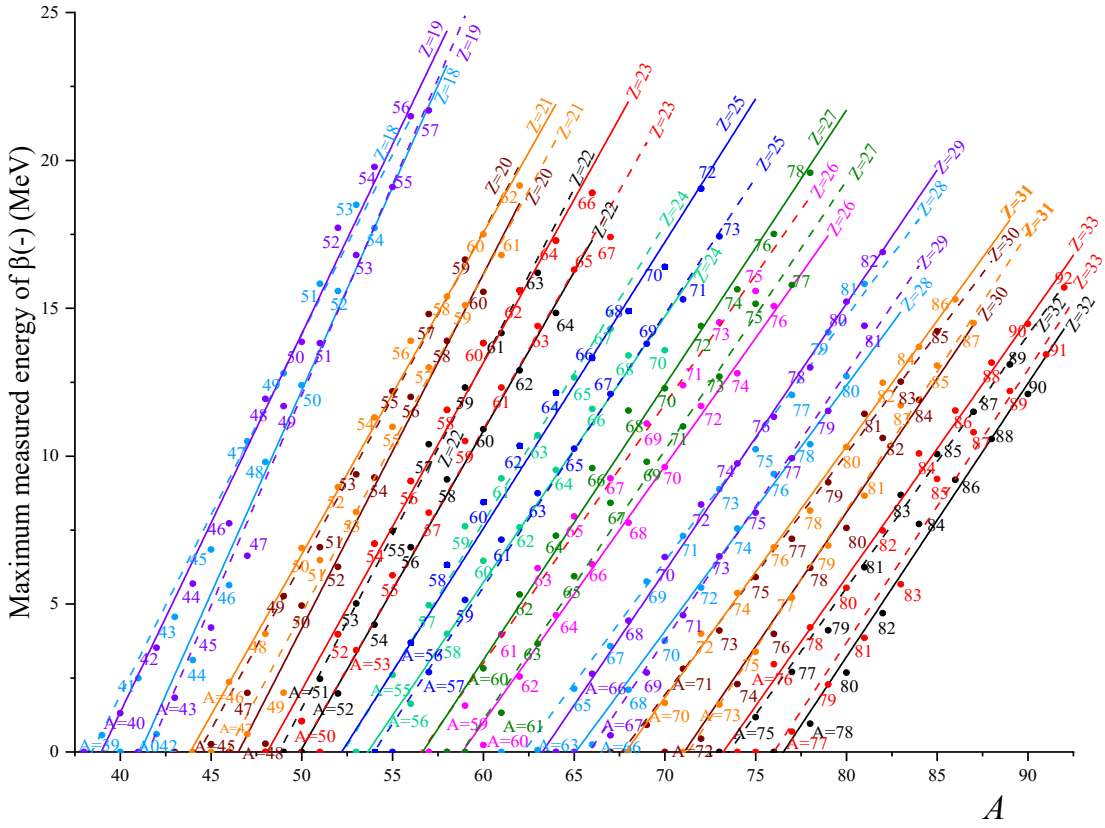


Figure 3 The β^--decay endpoint energy (maximum energy) E versus the mass number A for radioisotopes of the same element in the case of $17<Z<34$. The contiguous straight lines trace the *even A* nuclei datapoints, whereas the dashed lines trace the *odd A* nuclei datapoints. The separation between even-A and odd-A chains is clearly visible.

Proceeding forward, we plot the maximum decay energy E versus A for Z values between Z=35 and Z=46 (Fig. 4); and this is where we stop—*viz.*, at Z=46—for, this presentation is already as voluminous as would be necessary at this point.

Besides, for *higher Z values* , the straight lines of concern become splintered, making it a requisite to draw quadruple lines per any radioisotope ensemble of the same element belonging to a given Z beyond Z=46.

Such an endeavor deserves further attention though, which we propose to tackle in a concomitant article.

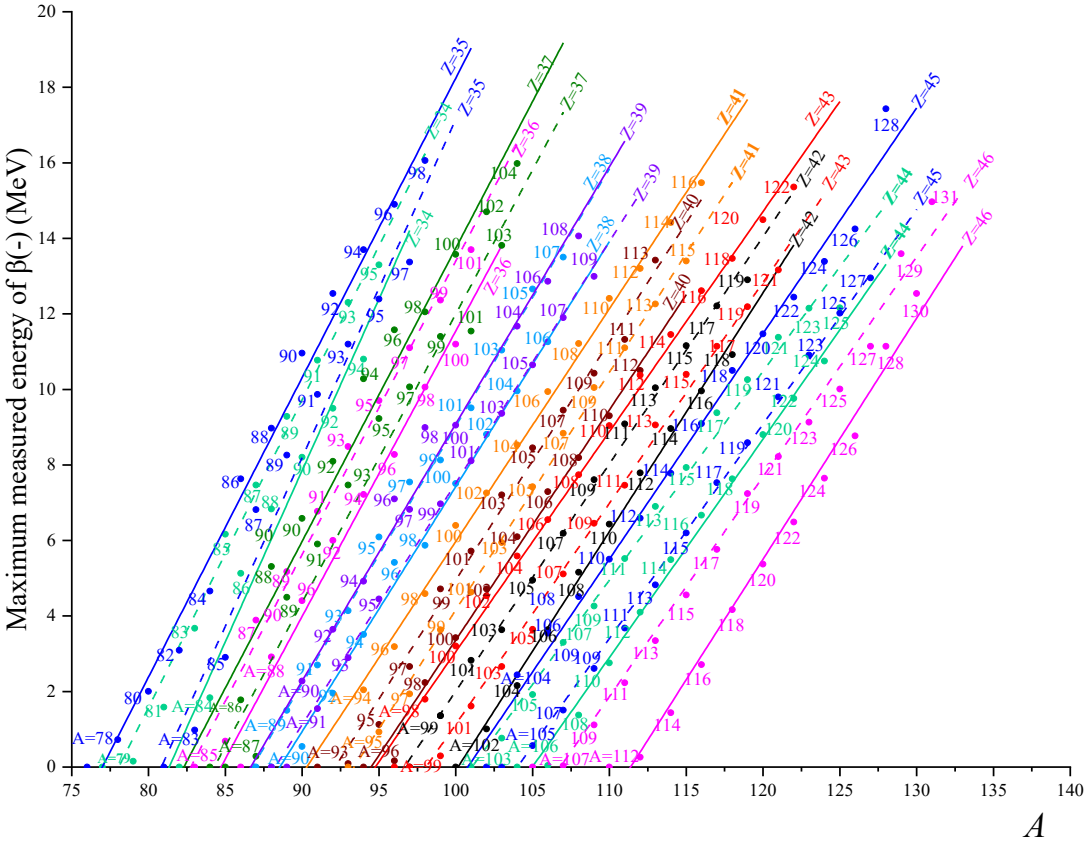


Figure 4 The β^--decay endpoint energy (maximum energy) E versus the mass number A for radioisotopes of the same element in the case of $34<Z<47$. The contiguous straight lines trace the *even A* nuclei datapoints, whereas the dashed lines trace the *odd A* nuclei datapoints. The separation between even-A and odd-A chains is clearly visible.

Provided at this juncture, right below, is Table 1 (see Supplementary Material), listing all of the experimentally gathered half-life and decay-energy data pertaining to the radioisotopes encompassed by this study.

The simple representation we have provided, may facilitate rapid estimates of β^--decay endpoint energies in regions where experimental data are sparse.

3. PARAMETERS OF THE DRAWN STRAIGHT LINES

As we just demonstrated, the results reveal amazingly simple and systematic structure in the β^--decay energetics along isotopic chains. For each fixed Z, the data separate visibly into two distinct linear trends corresponding to *even-A and odd-A isotopes.*

We now propose to write the maximum energy E of the β^--decay under consideration as

$$E = p + qA \,, \tag{1}$$

where p and q are parameters to be determined from the plots of the straight lines in Figs. 2-4; and we provide them in Table 2 (see Supplementary Material).

For each isotopic chain, the quality of the linear fits is quantified by the *coefficient of determination* R^2. Thus, across all elements with $Z<47$, the R^2 values cluster near unity, typically exceed 0.99 and never fall below ≈0.94.

This regularity is observed consistently across a wide range of elements ($Z < 47$).

It is all the more so, since the E values used in the figures we presented above are available with a precision of about 0.1%, if not even better.

No systematic offset is hence observed in the residuals, which indicates the absence of higher-order dependence on A within the examined range.

Note that, while sophisticated mass models reproduce β-decay energy values globally, they do not single out or emphasize any strict linear dependence on A at fixed Z as an organizing principle.

The slope and intercept parameters p and q provide a compact characterization of the decay energetics for each isotopic chain.

The emergence of such a simple linear structure in a complex many-body nuclear system is noteworthy.

It would certainly be interesting if we could formalize the relationship between the parameters p and q the way presented in Table 2 above. For one thing, as we will show forthrightly, they depend on each other.

To achieve this, we rewrite Eq. (4) for a given Z as

$$E = p + qA + qA_{stable} - qA_{stable} \,, \tag{2}$$

or as

$$E = p + q(A - A_{stable}) + qA_{stable} \,. \tag{3}$$

Let us pose:

$$\Delta A = A - A_{stable} \,, \tag{4}$$

as a positive quantity.

Let us furthermore transcribe

$$Q = p + qA_{stable}. \tag{5}$$

This leads to

$$E = q\Delta A + Q. \tag{6}$$

It is evident that, for $\Delta A = 0$, meaning for the stable nucleus, E will be 0. Therefore,

$$Q = 0 \,, \tag{7}$$

or, the same,

$$-\frac{p}{q} = A_{stable}\,. \tag{8}$$

We prepared Fig. 5 below to numerically cross-check this *correspondence,* where *-p/q* values from Table 2 are plotted against the corresponding A_{stable}, which we expect to trace the bisector of the coordinate system; and it remarkably does!

Thus,

$$E = q\Delta A\,, \tag{9}$$

for a given *Z*.[†]

This is an unexpectedly elegant relationship in full conformity with the Figs. 1, 2, 3 and 4 presented above.

We are thereby left with just one parameter—*i.e., q*—which is easy to determine; and it is already furnished in the last column of Table 2 above.

It so happens that, in the case of radioisotopes of the same element, knowing just one β^--decay energy *E* for a given *A* of elongation ΔA to the stable nucleus A_{stable}, one can write

$$q = \frac{E}{\Delta A}\,. \tag{10}$$

To corroborate this result, we plot in Figs. 6a, 6b and 6c the $E_{measured}$ versus $E_{calculated} = q\Delta A$ quantities for the elements Cobalt, Gallium and Molybdenum that pertain, respectively, to *Z*=27, *Z*=31 and *Z*=42.

[†] The Universal Matter Architecture (UMA) holds valid for the entire body of quantum mechanics; it allows us to perceive how different parameters—*i.e., mass, period of time, size and energy*—associated with a given entity are interrelated with each other, and this can be shown to be true without the need to solve the cumbersome quantum mechanical description of the said entity. One other thing is that we can check every aspect of UMA on the Hydrogen (H) atom as the simplest baseline. While we referred to UMA in several places so far just to recall the fact that it was basically the tool which motivated us to draw the linear dependence of *E* on *A* (cf. Figs 1-4), the corresponding mathematical details lie far beyond the content of this article. Still, it will be enriching to check Eq. (9), on the basis of H atom. Thus, we consider an outmost and practically stable energy level of the H atom determined by the quantum number *n*. We can now express the stability of this level with respect to an even upper energy level close to the previous one as determined by the quantum number $n+\Delta n$ via writing the Bohr decay-energy $E_{k\to N}=(2\pi^2e^4m/h^2)[1/n^2-1/(n+\Delta n)^2]$, where *e* is the electron charge, *m* the electron mass, and *h* the Planck Constant. If *n is* sufficiently big, this can be written as $E_{k\to N}=2C[\Delta n/n^3]$, where we have $C=2\pi^2e^4m/h^2$. Via defining a new constant, $C'=2C/n^3$, we thus recover the cast of Eq. (9) finally written as $E_{k\to N}=C'\Delta n$. The analogy we depicted is solely intended to illustrate how linear quantization can naturally emerge in bound quantum systems without implying a full physical equivalence between atomic and nuclear dynamics. All the same, the present nuclear results provide an empirical arena by which the broader applicability of the UMA can be meaningfully addressed.

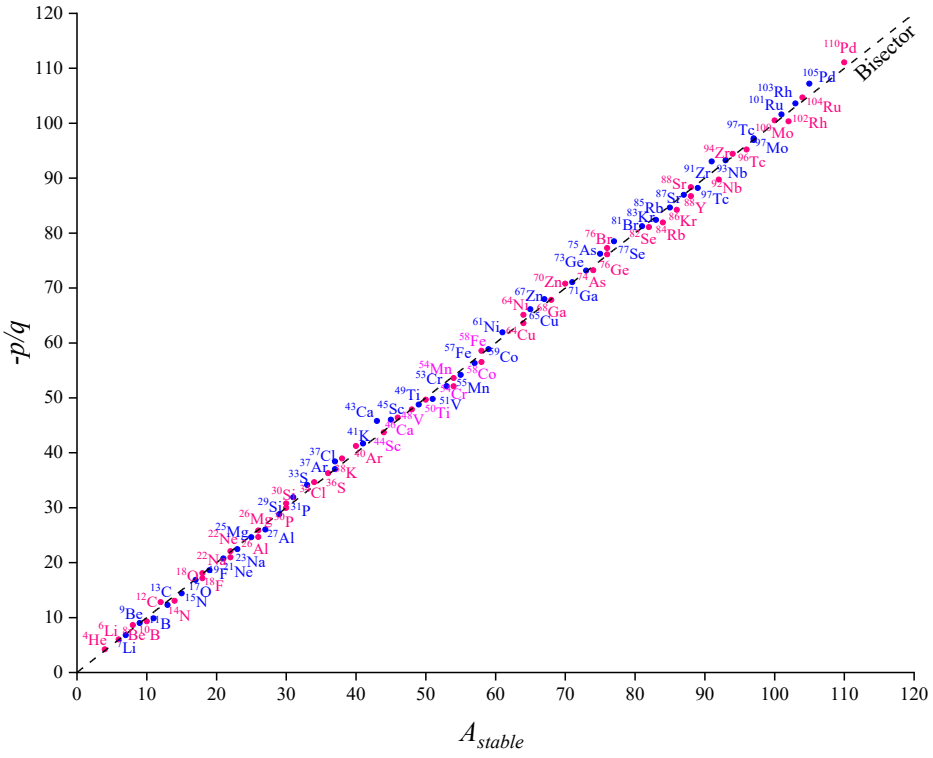


Figure 5 The $-p/q$ relationship from $E=p+qA$ (cf. Figs. 2, 3 and 4) versus A_{stable} in accordance with Eq. (10). When it is possible to find more than one stable nucleus for a given element, the stable nucleus nearest to the β^--decaying nuclei set is considered. All nuclei are then expected to be situated on the bisector of the graph which, very satisfactorily, is indeed the case.

The drawings are based on data from Tables 3a, 3b and 3c in succession; and the fact that the final outcome yields a neat alignment with the bisector every time is a witness to the explanatory and predictive power of the UMA scaffolding.

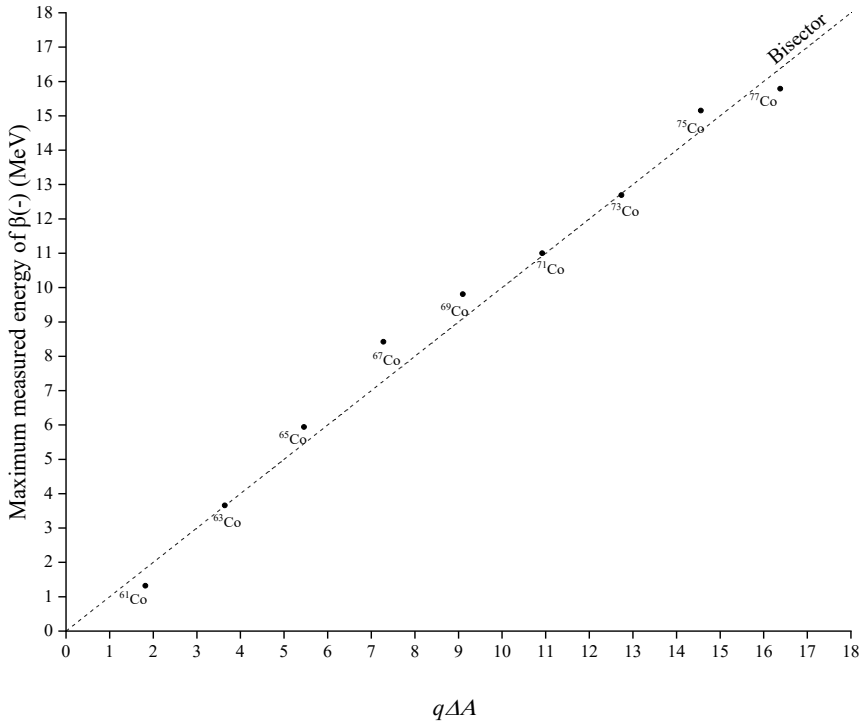


Figure 6a $E_{measured}$ versus $E_{calculated}=q\Delta A$ for Cobalt.

Table 3a $E_{measured}$ versus $E_{calculated}=q\Delta A$ for Cobalt, where $A_{stable}=59$ and $q=0.91$.

Radioisotopes	*ΔA*	*qΔA*	*E (MeV)*
^{61}Co	2	1.82	1.32
^{63}Co	4	3.64	3.66
^{65}Co	6	5.46	5.94
^{67}Co	8	7.28	8.42
^{69}Co	10	9.10	9.81
^{71}Co	12	10.92	11.0
^{73}Co	14	12.74	12.69
^{75}Co	16	14.56	15.15
^{77}Co	18	16.38	15.79

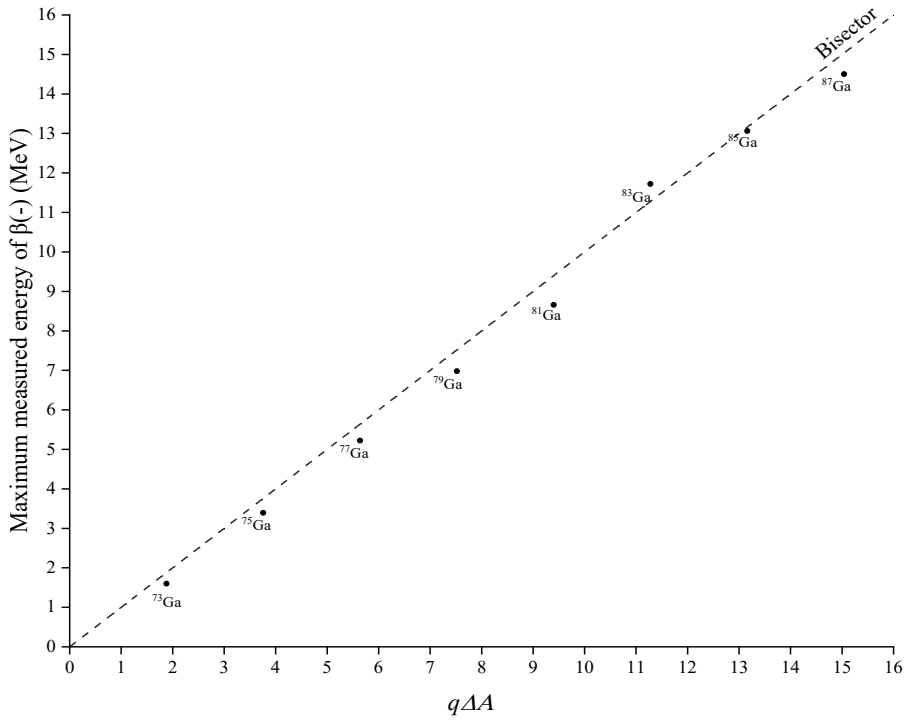


Figure 6b $E_{measured}$ versus $E_{calculated}=q\Delta A$ for Gallium.

Table 3b $E_{measured}$ versus $E_{calculated}=q\Delta A$ for Gallium, where $A_{stable}=71$ and $q=0.94$.

Radioisotopes	*ΔA*	*qΔA*	*E (MeV)*
^{73}Ga	2	1.88	1.6
^{75}Ga	4	3.76	3.39
^{77}Ga	6	5.64	5.22
^{79}Ga	8	7.52	6.98
^{81}Ga	10	9.40	8.66
^{83}Ga	12	11.28	11.72
^{85}Ga	14	13.16	13.06
^{87}Ga	16	15.04	14.50

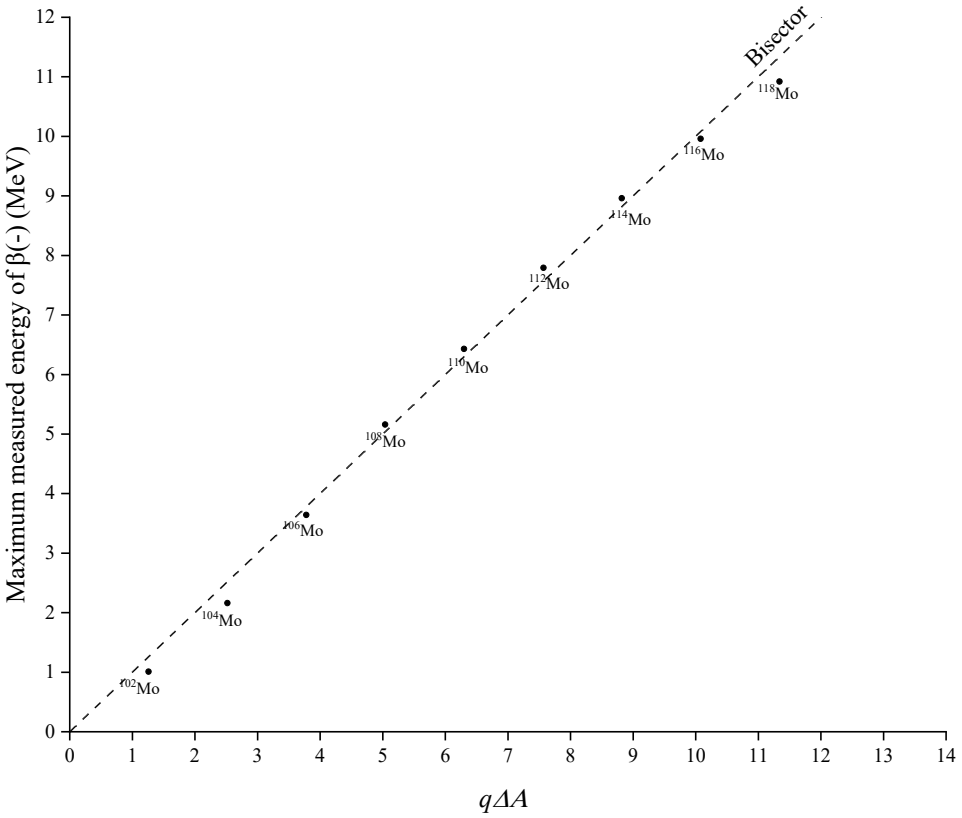


Figure 6c $E_{measured}$ versus $E_{calculated}=q\Delta A$ for Molybdenum.

Table 3c $E_{measured}$ versus $E_{calculated}=q\Delta A$ for Molybdenum, where $A_{stable}=100$ and $q=0.60$.

Radioisotope	ΔA	$E_{calculated}=q\Delta A$ *(MeV)*	$E_{measured}$ *(MeV)*
^{102}Mo	2	1.26	1.01
^{104}Mo	4	2.52	2.16
^{106}Mo	6	3.78	3.64
^{108}Mo	8	5.04	5.16
^{110}Mo	10	6.30	6.43
^{112}Mo	12	7.57	7.79
^{114}Mo	14	8.82	8.96
^{116}Mo	16	10.08	9.96
^{118}Mo	18	11.34	10.92

As the above Figs. 6a, 6b and 6c witness, Eq. (9) confirms the generic quantization of β^--decay maximum energy E of the nucleus A within the given element Z. This approach thus uniquely frames how *β^--decay nuclear instability ought to be quantized.*

One key point to be emphasized is that, had we endeavored to express E as a function of A, this function, owing to the UMA, *and hence in the light of the preceding discussion,* would be linear. It is because, as discussed above, the behavior in question appears to be a quantum mechanical necessity.

More specifically, the *linearity* under consideration quantifies the *increase in the instability* of β^--decay as one jumps from A to A+1 within the given element. In different terms, what we have here is *a unique quantization of nuclear instability.*

Besides this, note that, in the UMA framework, the β^--decay lifetime is governed by the effective energy release E through the generic scaling [29, 30]

$$T \sim \sqrt{\frac{1}{E}} \quad . \tag{11}$$

We would like to accentuate that this scaling, in relation to decay half-lives, serves here only as a phenomenological guide for interpreting the observed linear energy E with respect A for nuclei configured practically alike.

4. CONCLUSION

In this work, we have demonstrated that the maximum β^--decay energy E exhibits a remarkably simple and robust linear dependence on the mass number A with respect to any particular isotopic chain of fixed proton number Z in the range Z<47 encompassed by this study, provided that *even-A and odd-A nuclei* are treated separately. This simple linear structure provides a compact and potentially useful parametrization of β^--decay energetics.

The emergence of a simple two-branch linear structure in a complex many-body nuclear system suggests that the evolution of β^--decay energetics is governed by underlying regularities not explicitly captured in standard tabulations. This provides a compact and potentially predictive framework for describing isotopic chains, reducing each chain to two parameters and enabling rapid estimation of decay properties.

Thus, across all the elements examined herein, the curated nuclear data are matched with excellent accuracy by two straight-line trends that are characterized by the slope and intercept parameters in the way we have systematically tabulated so far.

The quality of these linear relations, which are reflected in the coefficients of determination typically exceeding 0.99, indicates that the observed behavior is not incidental, but represents a genuine and previously unrecognized regularity in nuclear systematics.

While approximate linear trends may be expected from smooth mass formulas, the present work demonstrates a remarkably precise and systematic organization of the data into linear relations across a wide range of isotopic chains, as predicted in effect by UMA *(cf. the footnote below Eq. (9)).*

The simplicity of our proposed description stands in contrast to the complexity of existing models, and provides a compact framework for organizing β^--decay energetics along isotopic chains.

Beyond its conceptual interest to the scientific community, the present approach lends itself as a practical tool for analyzing decay-energy trends and for making preliminary estimates of β^--decay properties in regions where data are sparse or incomplete. It may also serve as a useful benchmark for testing and constraining more detailed nuclear-structure models.

The broader theoretical considerations that originally motivated the search for the disclosed regularity lie outside the scope of the present contribution. The linear dependence reported here can be assessed independently on the basis of the experimental data alone.

Be that as it may, in a subsequent paper, we aim to present a similar study for β^--decaying nuclei of the radioisotopes of the same element in the range Z>46.

ACKNOWLEDGEMENT

We extend our gratitude to Savronik for their support, and continuous encouragement.

DATA AVAILABILITY

All data generated or analysed during this study are included in this published article, and its supplementary information file.

FUNDING

The authors declare that no funding was received for this study.

REFERENCES

[1] C.F.v. Weizsäcker, 1935, *Z. Physik*, **96**.
[2] H. A. Bethe and R. F. Bacher, 1936, *Rev. Mod. Phys.*, **8** (82).
[3] W. D. Myers and W. J. Swiatecki, 1966, *Nucl. Phys.*, **81** (1).
[4] P. Möller *et al.*, 1995, *At. Data Nucl. Data Tables*, **59** (2).
[5] G. Audi *et al.*, 2017, *Chin. Phys. C*, **41** (3), 030001.
[6] M. G. Mayer, 1949, *Phys. Rev.*, **75**.
[7] O. Haxel, J. H. D. Jensen and H. E. Suess, 1949, *Phys. Rev.*, **75**.
[8] B.A. Brown and B.H. Wildenthal, 1988, *Annu. Rev. Nucl. Part. Sci.*, **38**.
[9] E. Caurier *et al.*, 2005, *Rev. Mod. Phys.*, **77**.
[10] B. A. Brown, 2022, *Physics*, **4** (2).
[11] K. Takahashi and M. Yamada, 1969, *Prog. Theor. Phys.*, **41** (6).
[12] T. Tachibana, M. Yamada and Y. Yoshida, 1990, *Prog. Theor. Phys.*, **84** (4).
[13] H. Koura and S. Chiba, 2017, *Phys. Rev. C*, **95**, 064304.
[14] M. Gorchtein and C.-Y. Seng, 2023, *Universe*, **9** (9).
[15] J. Suhonen and O. Civitarese, 1994, *Phys. Rev. C*, **49** (6).
[16] T. Marketin, D. Vretenar and P. Ring, 2007, *Phys. Rev. C*, **75** (2), 024304.
[17] I. Daoutidis and P. Ring, 2011, *Phys. Rev. C.*, **83**, 044303.
[18] R. Oktova, 2020, *J. Phys.: Conf. Ser.*, **1428**, 012013.
[19] F. Minato, Z. Niu and H. Liang, 2022, *Phys. Rev. C*, **106** (2), 024306.
[20] T. Yarman, 2004, *Opt. Spectrosc. Mol. Spectrosc*., **97** (683).
[21] T. Yarman, 2004, *Opt. Spectrosc. Mol. Spectrosc*., **97** (691).
[22] T. Yarman, 2013, *Phys. Essays*, **26** (473).
[23] T. Yarman *et al.*, 2011, *Sci. Res. Essays*, **6**, 4478.
[24] T. Yarman *et al.*, 2021, *Mol. Phys.*, **119**, 1871087.
[25] T. Yarman *et al.*, 2016, *Eur. Phys. J. A*, **52** (140).
[26] T. Yarman *et al.*, 2017, *Eur. Phys. J. A*, **53** (1).
[27] T. Yarman *et al.*, 2017, *Int. J. Mod. Phys. E*, **26**, 1750068.
[28] C.B. Marchal *et al.*, *Ann. Phys*., **454** (2023) 169346.
[29] T. Yarman *et al.*, 2024, *Int. J. Mod. Phys. E*, **33** (8), 2450032.
[30] T. Yarman *et al.*, 2025, *Int. J. Mod. Phys. E*, **34** (6), 2550023.
[31] Brookhaven National Laboratory, National Nuclear Data Center, NuDat 3.0 Nuclear Structure and Decay Data Navigator (http://www.nndc.bnl.gov/chart).
[32] J. K. Tuli *(date is current),* Nuclear Wallet Cards. National Nuclear Data Center, *Brookhaven National Laboratory, New York.*

[33] D.H. Wilkinson, 1989, *Nucl. Instrum. Methods Phys. Res. A*, **275** (103).

Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- UMANSE1NatureSRSupplementary3June2026.docx