Environmental Fluid Mechanics

Supporting Information for

Incipient Motion of Microplastic Particles: A Rigorous Test of the Shields Diagram under Its Original Experimental Premises

Minwoo Son[1], Wonha Shin[1]*

[1] Department of Civil Engineering, Chungnam National University, Daejeon 34134, Republic of Korea

Corresponding author : Wonha Shin, wonha@o.cnu.ac.kr

Contents:

Number of pages: 5

Number of texts: 1

Number of tables: 1

Table of Contents

Text S1 Detailed Derivation of the Mean Tangential Velocity (u_{av}) in an Annular Flume

To analyze the internal flow of the annular flume, Booij [S1] introduced the universal law of the wall, which simulates the logarithmic velocity profile of the fluid near the wall, as follows:

$$\frac{u - u_w}{u_*} = \frac{1}{\kappa}\ln\left(\frac{a}{a_0}\right) \tag{S1}$$

Where u is the local flow velocity at a perpendicular distance a from the wall, u_w is the velocity of the wall itself, u_* is the friction velocity, κ is the von Kármán constant (approximately 0.4), and a_0 is the roughness length of the wall.

Turbulent flow within an annular flume is physically characterized by a steep velocity gradient within the wall boundary layer, beyond which a uniform flow with negligible velocity variation is maintained in the central region. The turbulent boundary layer thickness (δ_t) serves as the precise physical boundary where the wall's frictional influence ceases and the flow reaches u_{av}. By applying this boundary condition, specifically substituting $a = \delta_t$ and $u = u_{av}$ into Eq. (S1), the following relationship is obtained:

$$\frac{u_{av} - u_w}{u_*} = \frac{1}{\kappa}\ln\left(\frac{\delta_t}{a_0}\right) = c_t \tag{S2}$$

Since δ_t is relatively large compared to the roughness length (a_0) and varies minimally across different flow conditions, coupled with the nature of the logarithmic function, its variation exerts a negligible effect. Therefore, the entire right-hand side can be treated as a constant, c_t.

As the flow accelerates and ultimately reaches a dynamic equilibrium state, the net moment of the shear forces acting on the fluid becomes zero. Consequently, the driving shear force exerted by the rotating top lid is counterbalanced by the resisting shear force from the bottom flume (bed and sidewalls). Given that shear force is the product of shear stress (i.e., $\tau = \rho_w u_*^2$) and the corresponding contact area, the momentum balance equation is constructed as follows:

$$\rho_w \left(\frac{(u_{av} - \omega_t r)}{c_t}\right)^2 A_l = \rho_w \left(\frac{u_{av} - \omega_b r}{c_t}\right)^2 A_c \tag{S3}$$

Where ω_t and ω_b are the rotational angular velocities of the top lid and the bottom flume, respectively. r is the centerline radius of the flume. A_l represents the contact area of the top lid with the fluid, while A_c is the wetted area of the main bottom flume. Since both areas share the same mean circumference ($2\pi r$), they are directly proportional to their cross-sectional contact lengths: the flume width (b) for the lid, and the wetted perimeter ($b + 2h$, where h is the water depth). Assuming similar surface roughness characteristics of both components, the common variables ρ_w and c_t cancel out. Rearranging the balance equation using the area ratio ($\beta = \frac{A_c}{A_l} = (b + 2h)/b$) gives:

$$\frac{(u_{av} - \omega_t r)^2}{(u_{av} - \omega_b r)^2} = \beta \qquad \text{(S4)}$$

Solving this equilibrium equation for u_{av} derives the theoretical mean velocity at the center of the flume.

$$u_{av} = \left(\omega_t \frac{r}{1 + \sqrt{\frac{b+2h}{b}}} + \omega_b \frac{r\sqrt{\frac{b+2h}{b}}}{1 + \sqrt{\frac{b+2h}{b}}} \right) - w_b r \qquad \text{(S5)}$$

This expression serves as the basis for calculating the friction velocity and bed shear stress in the main manuscript.

Table S1 Physical properties, settling velocities, and critical incipient motion conditions for individual microplastics on a smooth bed (n = 0.011)

Material	Particle diameter, d (mm)	Particle density, ρ_s (kg/m^3)	Settling velocity, w_s (m/s)	Critical friction velocity, $u_{*,cr}$ (m/s)	Critical shear stress, τ_{cr} (Pa)	Critical movability number, $\Lambda_c = \frac{u_{*,\mathrm{cr}}}{w_s}$
PLA marble	2.0	1220	0.0636	0.0030	0.0088	0.047
PLA marble	3.0	1220	0.0841	0.0033	0.0109	0.039
PLA marble	4.0	1220	0.1005	0.0031	0.0093	0.030
PLA marble	5.0	1220	0.1145	0.0035	0.0120	0.030
PLA Silk	3.0	1270	0.0940	0.0040	0.0157	0.042
PLA Silk	4.0	1270	0.1120	0.0036	0.0132	0.032
PLA Silk	5.0	1270	0.1274	0.0041	0.0170	0.032
ABS	2.0	1050	0.0258	0.0030	0.0088	0.115
ABS	3.0	1050	0.0364	0.0026	0.0070	0.072
ABS	4.0	1050	0.0449	0.0027	0.0071	0.059
ABS	5.0	1050	0.0521	0.0026	0.0066	0.049

References for Supplementary Material

S1. Booij R (1994) Measurements of the flow field in a rotating annular flume. Report No. 94-2, Delft University of Technology, Delft, The Netherlands