Appendix

Proof of Lemma 1. We perform a comparative statics analysis on the optimal offset rate $\delta_{wc}(E, c', c'', Q, m, \alpha)$ by taking its partial derivatives with respect to each parameter.

The partial derivative with respect to the initial industry emissions E is:

$$\frac{\partial\delta_{wc}(E,c',c'',Q,m,\alpha)}{\partial E} = \frac{1}{n_1c''Q(1/n_1c''+1/n_2m+\alpha/\gamma)} > 0 \quad \text{(A.1)}$$

The partial derivative with respect to the average initial marginal abatement cost c' is:

$$\frac{\partial\delta_{wc}(E,c',c'',Q,m,\alpha)}{\partial c'} = \frac{1}{Q(1/n_1c''+1/n_2m+\alpha/\gamma)} > 0 \quad \text{(A.2)}$$

The partial derivative with respect to the average abatement technological efficiency parameter c'' is:

$$\frac{\partial\delta_{wc}(E,c',c'',Q,m,\alpha)}{\partial c''} = \frac{c'-(E-Q)/n_2m-E\alpha/\gamma}{nc''^2Q(1/n_1c''+1/n_2m+\alpha/\gamma)^2} \quad \text{(A.3)}$$

The sign of (A.3) is not immediately apparent. However, under the plausible condition that initial baseline emissions are sufficiently large ($E > 2Q$), and using the equilibrium condition $c' \le (2Q-E)/n_1c'' + Q/n_2m + 2Q\alpha/\gamma$ from our model, we can establish the following upper bound for the derivative:

$$\frac{\partial\delta_{wc}(E,c',c'',Q,m,\alpha)}{\partial c''} \le -\frac{E-2Q}{n_1c''^2Q(1/n_1c''+1/n_2m+\alpha/\gamma)} \quad \text{(A.4)}$$

Since the right-hand side of the inequality is negative, it follows that $\partial\delta_{wc}/\partial c''$ is negative under these conditions.

The partial derivative with respect to the initial volume of free carbon allowances Q is:

$$\frac{\partial\delta_{wc}(E,c',c'',Q,m,\alpha)}{\partial Q} = \frac{-(E+n_1c'c'')/n_1c''Q^2}{1/n_1c''+1/n_2m+\alpha/\gamma} < 0 \quad \text{(A.5)}$$

The partial derivative with respect to the average marginal cost of CCER supply m is:

$$\frac{\partial\delta_{wc}(E,c',c'',Q,m,\alpha)}{\partial m} = \frac{[E-Q+n_1c'c''/n_1c''Q-\alpha/\gamma]}{n_2m^2(1/n_1c''+1/n_2m+\alpha/\gamma)^2} > 0 \quad \text{(A.6)}$$

Finally, the partial derivative with respect to the government's preference for environmental outcomes α is:

$$\frac{\partial\delta_{wc}(E,c',c'',Q,m,\alpha)}{\partial\alpha} = -\frac{1/n_1c''+\alpha/\gamma+1/n_2m+(E-Q+n_1c'c'')/n_1c''-\alpha Q/\gamma}{\gamma(1/n_1c''+1/n_2m+\alpha/\gamma)^2} < 0 \quad \text{(A.7)}$$

Proof of Proposition 1. The optimal threshold E^* is determined by the first-order condition derived from the central government's objective function. This condition is:

$$\delta'_{LG,np}(E^*)\int_{E^*}^{\overline{E}}\left[\frac{\delta_{LG,np}(E^*)Q}{n_2m} - \frac{E-\delta_{LG,np}(E^*)Q-Q+n_1c'c''}{n_1c''} + \alpha_{CG}\frac{(1+\delta_{LG,na}(E^*))Q}{\gamma}\right]Q\,dF(E) = 0 \quad \text{(A.8)}$$

We note that at the threshold E^*, the LG's optimal rate $\delta_{LG,np}(E^*)$ must satisfy its own optimization condition, which implies:

$$\frac{\delta_{LG,np}(E^*)Q}{n_2m} - \frac{E-\delta_{LG,np}(E^*)Q-Q+n_1c'c''}{n_1c''} + \alpha_{LG}\frac{(1+\delta_{LG,na}(E^*))Q}{\gamma} = 0 \quad \text{(A.9)}$$

Substituting this relationship into the first-order condition (A.8) allows us to simplify the integrand. After substitution, equation (A.8) becomes:

$$\delta'_{LG,np}(E^*)\left\{\left[\frac{E^*}{n_1c''} + \frac{(\alpha_{CG}-\alpha_{LG})(1+\delta_{LG,np}(E^*))Q}{\gamma}\right](1-F(E^*)) - \frac{\int_{E^*}^{\overline{E}}E\,dF(E)}{n_1c''}\right\} = 0 \quad \text{(A.10)}$$

Rearranging this final expression yields the condition as stated in Proposition 1.

Proof of Proposition 2. To simplify the exposition, we focus our analysis on the separating equilibrium. The equations of motion for the state variable for rent, $\mu(E)$, and the co-state variable, $\lambda(E)$, must satisfy the following conditions:

$$\mu'(E) = -\frac{\partial L_{A,wp}(E,\delta,R,\mu,\lambda)}{\partial R} = \lambda(E) - \beta^{-1}f(E) \quad \text{(A.11)}$$

$$R'(E) = \frac{\partial L_{A,wp}(E,\delta,R,\mu,\lambda)}{\partial \mu} = \frac{(\delta-\delta_{LG,wp})Q}{n_i c''} \tag{A.12}$$

The analysis also requires the following Karush-Kuhn-Tucker (KKT) conditions for the multiplier $\lambda(E)$:

$$\frac{\partial L_{A,wp}(E,\delta,R,\mu,\lambda)}{\partial \lambda} = -R(E) \leq 0 \tag{A.13}$$

$$\lambda(E) \geq 0 \tag{A.14}$$

$$\lambda(E) \cdot \frac{\partial L_{A,wp}(E,\delta,R,\mu,\lambda)}{\partial \lambda} = 0 \tag{A.15}$$

Since upward distortion (over-reporting) is the dominant incentive problem, the principal would determine information rent of the lowest-type agent, $\underline{E}$. This establishes the transversality condition at the lower boundary:

$$\mu(\underline{E}) = 0 \tag{A.16}$$

The information rent for an arbitrary type E is then found by integrating the equation of motion (A11) from the lower bound $\underline{E}$ upwards. Applying the boundary condition $\mu(\underline{E}) = 0$ and the property $F(\underline{E}) = 0$ yields:

$$\mu(E) = -\beta^{-1}F(E) + \int_{\underline{E}}^{E} \lambda(\widetilde{E})d\widetilde{E} \tag{A.17}$$

We define $\mu_a(E)$ as the value of the rent when the participation constraint for type E is binding, meaning $R(E) = 0$ and $\lambda(E) > 0$. Conversely, if the constraint is slack ($R(E) > 0$), then $\lambda(E) = 0$. The optimal path for the co-state variable $\mu(E)$ is therefore characterized as follows:

$$\mu(E) = \begin{cases} -\beta^{-1}F(E) & if \quad \mu_a(E) \leq -\beta^{-1}F(E) \\ \mu_a(E) & if \quad \mu_a(E) > -\beta^{-1}F(E) \end{cases} \tag{A.18}$$

Note that if the participation constraint is binding for an interval of E, then $R(E) = 0$ and thus $R'(E) = 0$ within that interval. From Eq.(43), we observe that this implies $\delta_{A,wp} = \delta_{LG,wp}$. This allows us to derive the expression for $\mu_a(E)$:

$$\mu_a(E) = \frac{-(E+n_1c'c''+n_1c''Q/n_2m)(\alpha_{CG}-\alpha_{LG})/\gamma}{1/n_1c''+1/n_2m+\alpha_{LG}/\gamma} f(E) < 0 \tag{A.19}$$

Finally, clarifying the underlying logic is crucial for the monotonicity results in Proposition 2. Since the CG has a stronger preference for environmental outcomes ($\alpha_{CG} > \alpha_{LG}$), it follows that $\mu_a(E) < 0$. This implies that when $E \to \underline{E}$, there would exist $R(E) > 0$, which means $\mu(\underline{E}) = 0$. It also implies the incentive to over-report emissions is the dominant concern for the principal. To counteract this incentive, the information rent R(E) must be decreasing in the agent's type E. This requires $R'(E) < 0$. Assuming a uniform distribution for $F(E)$, we obtain the results stated in Proposition 2.