Supplementary Information for: Large-Scale Air-Quality Sensor Placement via Joint Optimization under Regional Constraints

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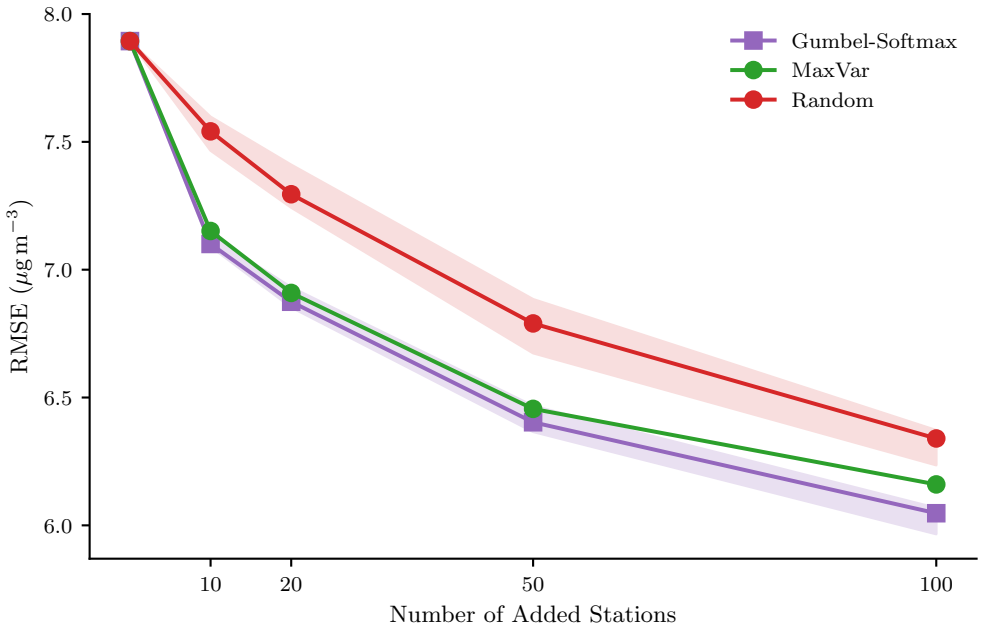

Supplementary Figure S1 The plot shows the RMSE (↓) on the validation set as a function of the number of new stations added to the existing CPCB network across India. The shaded region represents the interquartile range (25th to 75th percentile) in RMSE over 5 independent runs. Our proposed Gumbel-Softmax method consistently outperforms the MaxVar and Random baselines.

State	Budget	Random	MaxVar	GSM Indep	GSM Joint	Winner
Andhra Pradesh	3	5.46 ± 0.22	5.06 ± 0.00	5.07 ± 0.09	**4.85 ± 0.10**	**GSM Joint**
Arunachal Pradesh	1	5.94 ± 0.17	6.00 ± 0.00	5.95 ± 0.02	**5.86 ± 0.19**	**GSM Joint**
Assam	3	7.69 ± 0.11	7.74 ± 0.00	7.92 ± 0.12	**7.55 ± 0.11**	**GSM Joint**
Bihar	7	9.35 ± 0.23	9.08 ± 0.00	9.09 ± 0.13	**9.06 ± 0.07**	**GSM Joint**
Chhattisgarh	2	9.71 ± 0.46	8.88 ± 0.00	9.24 ± 0.31	**8.15 ± 0.15**	**GSM Joint**
Daman and Diu and Dadra and Nagar Haveli	1	6.71 ± 0.50	7.28 ± 0.00	7.31 ± 0.03	**6.32 ± 0.15**	**GSM Joint**
Delhi	1	17.02 ± 0.04	16.76 ± 0.00	16.86 ± 0.08	**16.66 ± 0.13**	**GSM Joint**
Goa	1	4.93 ± 0.74	4.72 ± 0.00	5.30 ± 0.02	**4.34 ± 0.02**	**GSM Joint**
Gujarat	5	10.04 ± 0.54	9.61 ± 0.00	9.92 ± 0.23	**8.74 ± 0.22**	**GSM Joint**
Haryana	2	7.20 ± 0.06	7.16 ± 0.00	**7.02 ± 0.07**	7.35 ± 0.04	**GSM Indep**
Himachal Pradesh	1	5.53 ± 0.18	**5.14 ± 0.00**	5.53 ± 0.16	5.43 ± 0.04	**MaxVar**
Jammu and Kashmir	2	7.66 ± 0.68	**6.75 ± 0.00**	8.90 ± 1.11	7.09 ± 0.60	**MaxVar**
Jharkhand	3	11.10 ± 0.28	11.26 ± 0.00	10.60 ± 0.14	**10.28 ± 0.43**	**GSM Joint**
Karnataka	5	**3.29 ± 0.04**	3.33 ± 0.00	3.33 ± 0.04	3.33 ± 0.05	**Random**
Kerala	3	4.57 ± 0.19	4.55 ± 0.00	4.60 ± 0.05	**4.30 ± 0.06**	**GSM Joint**
Madhya Pradesh	8	7.13 ± 0.27	7.30 ± 0.00	6.81 ± 0.18	**6.56 ± 0.13**	**GSM Joint**
Maharashtra	7	5.88 ± 0.07	**5.54 ± 0.00**	5.67 ± 0.12	5.71 ± 0.07	**MaxVar**
Manipur	1	6.21 ± 0.37	6.36 ± 0.00	**6.11 ± 0.06**	6.55 ± 0.04	**GSM Indep**
Meghalaya	1	**9.28 ± 0.34**	10.03 ± 0.00	10.22 ± 0.28	9.29 ± 0.93	**Random**
Mizoram	1	5.51 ± 0.86	**4.48 ± 0.00**	4.98 ± 0.02	4.83 ± 0.03	**MaxVar**
Nagaland	1	5.02 ± 0.24	**4.29 ± 0.00**	5.40 ± 0.12	5.07 ± 0.06	**MaxVar**
Odisha	4	8.90 ± 0.34	8.86 ± 0.00	**8.56 ± 0.21**	8.93 ± 0.18	**GSM Indep**
Puducherry	1	3.80 ± 0.32	3.59 ± 0.00	3.50 ± 0.04	**3.47 ± 0.04**	**GSM Joint**
Punjab	3	8.23 ± 0.41	8.72 ± 0.00	**8.09 ± 0.53**	8.42 ± 0.05	**GSM Indep**
Rajasthan	5	7.50 ± 0.16	7.24 ± 0.00	**7.18 ± 0.19**	7.18 ± 0.11	**GSM Indep**
Sikkim	1	6.61 ± 0.38	6.61 ± 0.00	**6.07 ± 0.00**	6.48 ± 0.05	**GSM Indep**
Tamilnadu	6	**3.84 ± 0.01**	3.93 ± 0.00	3.85 ± 0.03	3.89 ± 0.05	**Random**
Telengana	2	5.90 ± 0.32	6.09 ± 0.00	**5.69 ± 0.05**	5.82 ± 0.20	**GSM Indep**
Tripura	1	8.58 ± 0.72	8.73 ± 0.00	**8.39 ± 0.06**	8.52 ± 0.12	**GSM Indep**
Uttar Pradesh	15	9.05 ± 0.37	8.43 ± 0.00	**8.19 ± 0.17**	8.47 ± 0.12	**GSM Indep**
Uttarakhand	1	7.80 ± 0.22	7.65 ± 0.00	**7.51 ± 0.08**	7.58 ± 0.18	**GSM Indep**
West Bengal	5	12.75 ± 0.30	12.26 ± 0.00	**12.12 ± 0.26**	12.80 ± 0.27	**GSM Indep**

Table S1 State-Level comparison of RMSE after a pilot expansion of $N = 103$ monitoring stations under fixed state-level budgets (as listed). Our Joint GSM framework achieves the lowest RMSE in 13 out of 32 administrative regions, outperforming the independently constrained GSM variant (11 regions), the greedy Max-Variance baseline (5 regions), and random placement (3 regions). These results indicate that coordinated national-scale optimization provides broader and more reliable accuracy improvements than region-wise or purely heuristic deployment strategies. Results report the mean ± standard deviation over 5 independent optimization seeds.