From Representation to Abstraction: Supporting Preschool Children's Advanced Mathematical Thinking

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Abstract

Learning mathematics progresses from concrete experiences to abstract representations, and carefully sequenced instruction is essential for preventing conceptual gaps. This mixed-methods study examined the effectiveness of a Concrete–Representational–Abstract (CRA)-based Mathematics Program on preschool children's early numeracy and advanced mathematical skills. Participants were 32 children aged 60–72 months (experimental group n = 16; control group n = 16) and their teachers. Quantitative data were collected using the Early Numeracy Test–Revised and the Advanced Mathematics Skills Test for Preschool Children administered before and after the intervention. Qualitative data were obtained through teacher interviews and researcher diaries. Results indicated statistically significant improvements in both early numeracy and advanced mathematical skills among children in the experimental group compared with the control group. Qualitative findings supported these results, revealing increased use of mathematical language, representations, and symbolic reasoning. Overall, the findings suggest that systematically structured CRA-based instruction effectively supports preschool children's mathematical development by fostering both foundational skills and higher-order mathematical thinking.

1. Introduction

Mathematics serves as a fundamental tool for enhancing individuals' problem-solving abilities, understanding the world, and conducting analyses. Early childhood mathematics education is crucial for establishing a strong foundation for children's future learning and supporting their academic success (Clements & Sarama, 2021; Hsieh & McCollum, 2018). As children progress through education, they encounter increasingly complex mathematical ideas that build upon earlier skills and understandings. Representations connect prior knowledge with new knowledge (Gilmore et al., 2018; Uttal et al., 2009), and recent research highlights their role in supporting early mathematical environments (Björklund & Palmér, 2022; Sterner et al., 2020). Further investigation is needed to understand how specific types of representations influence the development of mathematical skills and how teachers can strategically select and implement them (Björklund & Palmér, 2022; Mainali, 2021).

The transition between concrete and abstract thinking can be difficult (Dijk et al., 2004; van Oers & Poland, 2012). Visual representations such as number lines help children develop mental models of number order and magnitude (Woods et al., 2018). Flexibly moving between forms of representation is essential for deep conceptual understanding (Duval, 2006; Mainali, 2021; Otsuka & Jay, 2017). Progressing from concrete to abstract representations enables children to develop stronger understanding of mathematical ideas (Lesh et al., 1987; Mainali, 2021; NCTM, 2000; Sterner et al., 2020). Although theory highlights their importance, few structured instructional models guide teachers in sequencing these learning experiences.

1.1 Representation in Early Mathematics

Representations establish connections between real-world and mathematical ideas through formulae, tables, graphs, numbers, equations, or materials (Post & Cramer, 1989). Instruction should support transitions between concrete, semi-concrete, and abstract stages. The Concrete–Representational–Abstract (CRA) approach offers a systematic progression from tangible experiences to symbolic reasoning. Representations also support problem-solving, reasoning, and communication (Mainali, 2021). Children who encounter various forms of representation achieve more permanent learning (Mainali, 2021; NAEYC & NCTM, 2010).

Theoretical foundations draw on Bruner (1966), Lesh (1981), and Duval (2006). Bruner categorized representational forms as actions, images, and symbols. Lesh emphasized movement across real-world situations, manipulatives, pictures, verbal symbols, and written symbols. Duval focused on transitions between semiotic systems (Björklund & Palmér, 2022). Together, these frameworks support the use of the CRA sequence in instructional design. Goldin and Kaput (2013) distinguished internal mental representations from external expressions such as drawings and verbal explanations. External representations enable educators to infer children's internal mathematical structures (Lesh et al., 1987; Mainali, 2021). Representational transitions are considered essential for the development of mathematical proficiency (Ayyıldız & Aktaş, 2022; Björklund & Palmér, 2022; Sterner et al., 2020). Despite this, empirical studies examining structured CRA applications in early childhood remain limited.

1.2 Advancing Mathematical Thinking through Representations in Early Childhood

Advanced mathematical thinking in early childhood emerges through structured representational experiences that guide learners from concrete actions toward abstract reasoning. CRA is a three-stage model that supports this progression using concrete materials, visuals, and symbols (Witzel, 2005; Witzel et al., 2008).

- **Concrete Stage:** Children experience concepts through physical materials.
- **Representational Stage:** Children express ideas through drawings and visual models (Janvier, 1987).
- **Abstract Stage:** Children work with symbols and notations (Tzekaki & Papadopoulou, 2017).

Children who integrate symbolic meaning into their drawings demonstrate greater representational flexibility (Cartwright, 2023). Recent research shows that symbolic development, abstraction, problem solving, and spatial thinking emerge earlier than previously assumed (Björklund et al., 2020; Donlan, 2020; Gilmore et al., 2018; McCluskey et al., 2023; Sterner et al., 2020; Worthington et al., 2019). Mathematical development follows systematic, sequential progressions (Cahoon et al., 2021), highlighting the importance of instructional approaches that support smooth transitions across CRA stages.

Empirical evidence demonstrates that varied representational experiences strengthen symbolic reasoning and support durable learning (Li et al., 2018; Lira et al., 2017; Ching & Wu, 2019; Scalise & Ramani, 2021). Despite strong theoretical support, structured classroom-based CRA applications in early childhood are still limited. Therefore, this study examines a CRA-based mathematics program designed to support preschool children's early mathematical thinking and explores how children experience and express mathematics throughout the program together with teacher perspectives.

Therefore, there is a need for research that not only tests CRA-based instruction in real classroom contexts but also investigates how young children experience and express mathematics as they transition across representational forms. This study addresses this gap by examining a CRA-based mathematics program designed to support preschool children's early mathematical thinking. The aim of this study is to examine the effectiveness of a Concrete-Representational-Abstract (CRA) based Mathematics Program on preschool children's early mathematical thinking and to explore how children experience and express mathematics throughout the program, together with teacher perspectives.

Accordingly, the present study addressed the following questions:

1. Does participation in the CRA-based Mathematics Program lead to significant improvements in preschool children's Early Numeracy Test–Revised and Advanced Mathematical Skills Test scores compared to the control group?
2. How do teachers perceive the impact of the CRA-based Mathematics Program on children's learning and engagement in mathematics?
3. In what ways do children's approaches to experiencing and expressing mathematics change after participating in the program?

2. Methods

This study adopted a mixed-methods research design integrating quantitative and qualitative data to comprehensively examine the effects of the CRA-based Mathematics Program. In mixed-methods research, different data types are combined to address the research purpose and to complement one another (Cohen et al., 2017; Creswell & Plano Clark, 2020).

For the quantitative component, a quasi-experimental pretest–posttest control group design was employed, as random assignment was not feasible due to the use of intact preschool classrooms (Büyüköztürk et al., 2017). Children's mathematical skills were assessed before and after the intervention using the Early Numeracy Test–Revised and the Advanced Mathematics Skills Test for Preschool Children.

The qualitative component included semi-structured teacher interviews, researcher diary notes, and children's drawings, which provided in-depth insights into the learning process and supported triangulation of findings.

2.1 Participants

The participants consisted of 32 children aged 60–72 months (16 in the experimental group and 16 in the control group) and their classroom teachers from preschools in the **** provincial center during the 2022–2023 academic year. Criterion sampling was used, considering similar socio-economic backgrounds, class size, physical conditions, and the absence of prior structured early mathematics training.

In the experimental group, nine children were boys and seven were girls, with six having more than one year of preschool experience. In the control group, ten children were boys and six were girls, with four having more than one year of preschool experience. The experimental group teacher was 40 years old with 18 years of experience, while the control group teacher was 35 years old with 12 years of experience. Two comparable classrooms from the same institution were selected, and one was assigned as the experimental group based on administrative approval and teacher availability.

2.2 Data Collection Tools

Both quantitative and qualitative instruments were employed in line with the mixed-methods design. Quantitative data were collected using two standardized tests: the Early Numeracy Test–Revised and the Advanced Mathematics Skills Test for Preschool Children. Qualitative data were obtained through teacher interviews, a researcher diary, and children's drawings.

Early Numeracy Test–Revised: Developed by Van Luit and Van Rijt (2009) and adapted into Turkish by Kaçıra and Dağlıoğlu (2019), this test assesses early numeracy skills in children aged 3–8 across nine subdomains. Reliability coefficients were acceptable (KR-20 = .810 for Form A; .749 for Form B).

Advanced Mathematics Skills Test for Preschool Children: This instrument measures advanced skills such as matching, sequencing, spatial perception, and problem-solving (Authors, 2025). The scale demonstrated strong psychometric properties, with factor loadings ranging from 0.663 to 0.865 and Cronbach's alpha values between 0.89 and 0.94.

Teacher Interviews: Semi-structured interviews were conducted to explore teachers' views on early mathematics, the CRA-based program, and observed changes in children's engagement and understanding. This format ensured consistency while allowing flexibility (Merriam, 2015).

Researcher Diary: The diary included systematic observations of classroom interactions, children's discourse, and instructional practices recorded after each session, contributing to methodological triangulation.

Children's Drawings: Drawings produced during activities were analyzed as visual representations of children's mathematical thinking, focusing on symbolic, spatial, and representational features.

2.3 CRA-based Mathematics Program

The CRA-based Mathematics Program constituted the main intervention of the study and was grounded in constructivist learning theory, representation models, and sociocultural perspectives. The program aimed to enhance preschool children's mathematical thinking, problem-solving, and representational skills through a structured progression from concrete experiences to representational and symbolic understanding.

Program content addressed key early mathematics domains, including number sense, patterns, spatial reasoning, measurement, and problem-solving. Each session followed three stages: (1) hands-on activities with concrete materials, (2) representational activities such as drawings or visual models, and (3) symbolic representations using mathematical notation. Teachers received preparatory training and ongoing support to ensure fidelity to CRA principles.

A distinctive feature of the program was its emphasis on advanced mathematical reasoning rather than rote learning, encouraging children to communicate and externalize their mathematical thinking through multiple representations. For example, in the “Numbers Apartment” activity, children engaged with a real-life story context, used manipulatives to represent quantities, created visual models, and completed number lines and symbolic tasks.

Each activity systematically incorporated all three CRA stages, supporting children’s cognitive development and ensuring alignment with the CRA framework (Appendix A).

2.4. Data Collection

Data collection was conducted in three stages. Ethical approval was obtained from the **** University Senate Ethics Commission, and official permission was secured from the ***** Provincial Directorate of National Education. Written informed consent was obtained from teachers and parents, and age-appropriate assent was obtained from the children.

A pilot study was conducted prior to implementation to examine feasibility and refine teacher guidance procedures. Based on pilot feedback, minor revisions were made to activity instructions and support materials.

Before implementation, the experimental group teacher participated in four preparatory training sessions (Appendix B) focusing on early mathematics education, the CRA approach, lesson planning, and observation procedures. Weekly planning and feedback meetings were held during implementation. The researcher attended all sessions as a non-participant observer and maintained a structured research journal.

The experimental group received the CRA-based Mathematics Program twice a week for 12 weeks, with each session lasting approximately 50 minutes, between November 7, 2022, and January 24, 2023. The control group followed the regular preschool curriculum. Quantitative data were collected at pre-test, post-test, and four-week follow-up stages. A semi-structured interview was conducted with the experimental group teacher after post-testing, and the program was subsequently shared with the control group teacher.

2.7. Data Analysis

Pre-test, post-test, and follow-up data were analyzed using a statistical software package. As the number of children in each group was below 30, normality was examined using the Shapiro–Wilk test (Shapiro & Wilk, 1965). For the Early Numeracy Test–Revised, pre-test scores of the experimental group showed normal distribution, whereas post-test and follow-up scores did not ($p < .05$). In the control group, all measurements violated normality ($p < .05$). For the Advanced Mathematics Skills Test for Preschool Children, pre-test and follow-up scores in the experimental group, as well as all measurements in the control group, were non-normally distributed ($p < .05$). Additionally, skewness and kurtosis values exceeded the acceptable ±1 range for several variables, further supporting non-normality (Büyüköztürk, 2016). Therefore, non-parametric statistical methods were employed in accordance with sample size and data characteristics (Büyüköztürk, 2017).

The Mann–Whitney U test was used to compare pre-test scores between the experimental and control groups. Within-group comparisons of pre-test–post-test and post-test–follow-up scores were conducted using the Wilcoxon Signed-Rank Test. Differences in gain scores between groups were also analyzed using the Mann–Whitney U test.

Qualitative data were obtained through semi-structured interviews with the experimental group teacher and researcher diaries. Data were analyzed using content analysis to identify codes, categories, and themes (Creswell, 2016; Yıldırım & Şimşek, 2016). The analysis process involved organizing raw data, coding meaningful units, grouping similar codes into themes, and interpreting relationships among themes. Teacher statements and observation notes were coded systematically.

To ensure reliability, qualitative data were independently coded by two researchers and subsequently compared. Inter-coder reliability was calculated using the Miles and Huberman (1994) formula, yielding an agreement rate of 88.8%.

Validity was addressed in line with mixed-methods research principles. Both quantitative and qualitative validity criteria were considered, along with strategies supporting integration and consistency between data strands (Creswell & Plano Clark, 2020; Cohen et al., 2017). Particular attention was given to the coherence between quantitative and qualitative findings to strengthen overall research validity.

3. Findings

The findings are presented in alignment with the research questions. Quantitative and qualitative results are integrated to provide comprehensive insights into the effects of the CRA-based Mathematics Program.

RQ1. Does participation in the CRA-based Mathematics Program lead to significant improvements in preschool children’s early numeracy and advanced mathematics skills compared to the control group?

The Early Numeracy Test–Revised was first used to examine group equivalence. Pre-test Mann–Whitney U analyses showed no statistically significant differences between the experimental and control groups on any subdimension or the total score ($p > .05$), indicating comparable initial numeracy levels (Table 1). After the intervention, post-test results revealed statistically significant advantages for the experimental group on almost all subdimensions, and follow-up data collected one month later showed that these differences were largely maintained, again favouring the experimental group (Table 1).

Table 1 *Mann-Whitney U Test Results for Early Arithmetic Test Scores of Experimental and Control Groups*

	Group	n	Rank Average	U	Z	p
Pre-Test						
Comparison	Experiment	16	14.63	98.000	1.183	0.237
	Control	16	18.38			
Classification	Experiment	16	15.50	112.000	0.616	0.538
	Control	16	17.50			
Merging	Experiment	16	17.03	119.500	0.327	0.744
	Control	16	15.97			
Ranking	Experiment	16	15.00	104.000	0.926	0.355
	Control	16	18.00			
Using Numbers	Experiment	16	13.72	83.500	1.756	0.079
	Control	16	19.28			
Simultaneous and Abbreviated Counting	Experiment	16	15.25	108.000	0.780	0.435
	Control	16	17.75			
Consequential Counting	Experiment	16	13.78	84.500	1.666	0.096
	Control	16	19.22			
Applying Number Knowledge	Experiment	16	15.81	117.000	0.424	0.671
	Control	16	17.19			
Estimate	Experiment	16	15.09	117.000	0.424	0.671
	Control	16	17.91			
Total	Experiment	16	14.78	100.500	1.038	0.299
	Control	16	18.22			
Post Test						
Comparison	Experiment	16	18.19	101.000	1.294	0.196
	Control	16	14.81			
Classification	Experiment	16	23.72	12.500	4.433	0.000*
	Control	16	9.28			
Merging	Experiment	16	22.25	36.000	3.637	0.000*
	Control	16	10.75			
Ranking	Experiment	16	21.63	46.000	3.174	0.002*
	Control	16	11.38			
Using Numbers	Experiment	16	20.69	61.000	2.587	0.010*
	Control	16	12.31			
Simultaneous and Abbreviated Counting	Experiment	16	22.59	30.500	3.744	0.000*
	Control	16	10.41			
Consequential Counting	Experiment	16	19.88	74.000	2.087	0.037*
	Control	16	13.13			
Applying Number Knowledge	Experiment	16	22.50	32.000	3.694	0.000*
	Control	16	10.50			

Estimate	Experiment	16	21.44	49.000	3.051	0.002*
	Control	16	11.56			
Total	Experiment	16	22.81	27.000	3.811	0.000*
	Control	16	10.19			
Follow-up Test						
Comparison	Experiment	16	19.69	77.000	2.357	0.018*
	Control	16	13.31			
Classification	Experiment	16	22.66	29.500	3.775	0.000*
	Control	16	10.34			
Merging	Experiment	16	21.34	50.500	3.060	0.002*
	Control	16	11.66			
Ranking	Experiment	16	21.72	44.500	3.212	0.001*
	Control	16	11.28			
Using Numbers	Experiment	16	19.34	82.500	1.751	0.080
	Control	16	13.66			
Simultaneous and Abbreviated Counting	Experiment	16	21.59	46.500	3.132	0.002*
	Control	16	11.41			
Consequential Counting	Experiment	16	20.34	66.500	2.364	0.018*
	Control	16	12.66			
Applying Number Knowledge	Experiment	16	22.06	39.000	3.433	0.001*
	Control	16	10.94			
Estimate	Experiment	16	20.78	59.500	2.621	0.009*
	Control	16	12.22			
Total	Experiment	16	22.31	35.000	3.511	0.000*
	Control	16	10.69			
*p<0.05						

Effect size values (r) for the Early Numeracy Test ranged from medium to large at post-test (r = .369–.784) and from small-to-medium to large at follow-up (r = .310–.667), indicating that gains were both statistically and educationally meaningful.

For the Advanced Mathematics Skills Test for Preschool Children, pre-test Mann–Whitney U results showed no significant group differences on any subdimension or the total score (p > .05), confirming comparable initial advanced mathematics performance. Post-test and follow-up comparisons, however, consistently favoured the experimental group on all indices, demonstrating strong and sustained effects of the CRA-based program (Table 4).

Table 2 *Mann-Whitney U Test Results for Pre-Test Scores on Advanced Mathematics Skills Test"*

		Group	n	Rank Average	U	Z	p	r
	Pre-Test							
	Basic Skills	Experiment	16	19.69	77.000	1.925	0.054	
		Control	16	13.31				
	Counting and Operation	Experiment	16	17.22	116.500	0.434	0.664	
		Control	16	15.78				
	Spatial	Experiment	16	17.00	120.000	0.304	0.761	
		Control	16	16.00				
	Problem Solving	Experiment	16	17.69	109.000	0.718	0.473	
		Control	16	15.31				
	Total	Experiment	16	17.66	109.500	0.697	0.486	
		Control	16	15.34				
	Post Test							
	Basic Skills	Experiment	16	23.41	17.500	4.171	0.000*	**0.737**
		Control	16	9.59				
	Counting and Operation	Experiment	16	21.72	44.500	3.153	0.002*	**0.557**
		Control	16	11.28				
	Spatial	Experiment	16	23.72	12.500	4.399	0.000*	**0.778**
		Control	16	9.28				**0.634**
	Problem Solving	Experiment	16	22.41	33.500	3.584	0.000*	
		Control	16	10.59				**0.697**
	Total	Experiment	16	23.03	23.500	3.940	0.000*	
	Follow-up Test							
Basic Skills		Experiment	16	24.22	4.500	4.683	0.000	**0.827**
		Control	16	8.78				
Counting and Operation		Experiment	16	23.25	20.000	4.103	0.000	**0.726**
		Control	16	9.75				
Spatial		Experiment	16	24.22	4.500	4.789	0.000	**0.846**
		Control	16	8.78				
Problem Solving		Experiment	16	23.97	8.500	4.545	0.000	**0.803**
		Control	16	9.03				
Total		Experiment	16	24.00	8.000	4.548	0.000	**0.804**
		Control	16	9.00				

*$p<0.05$

Post-test effect sizes in the experimental group ranged from medium to large (r = .557–.778), and follow-up effect sizes ranged from large to very large (r = .726–.846), showing that the program produced substantial and durable improvements in advanced mathematical skills.

Within-group change was examined using Friedman tests. For the experimental group, rank averages on all Early Numeracy Test-Revised subdimensions and the total score increased from pre-test to post-test and follow-up; all omnibus tests were significant ($p < .01$, $W = .46–.72$; Table 3). Bonferroni-adjusted pairwise comparisons ($α = .025$) confirmed significant gains from pre-test to both post-test and follow-up, with no significant decline between post-test and follow-up. For the control group, no consistent improvement pattern emerged.

Table 3 *Friedman F Test Results for the Experimental and Control Groups' Early Numeracy Test-Revised Scores*

Experimental group	n	Rank Average			Friedman Test	Degree of Freedom	p	
		Pre-Test (a)	Post Test (b)	Follow-up Test (c)				W (χ²
Comparison	16	1.28	2.34	2.38	19.366	2	0.000*	.61
Classification	16	1.19	2.47	2.34	17.322	2	0.000*	.54
Merging	16	1.22	2.47	2.31	21.111	2	0.000*	.66
Ranking	16	1.19	2.50	2.31	21.957	2	0.000*	.69
Using Numbers	16	1.31	2.44	2.25	16.174	2	0.000*	.51
Simultaneous and Abbreviated Counting	16	1.25	2.22	2.53	16.321	2	0.000*	.51
Consequential Counting	16	1.16	2.50	2.34	22.571	2	0.000*	.71
Applying Number Knowledge	16	1.25	2.31	2.44	19.818	2	0.000*	.62
Estimate	16	1.31	2.38	2.31	14.560	2	0.001*	.46
Total	16	1.06	2.41	2.53	23.017	2	0.000*	**.72**
Control Group								
Comparison	16	1.84	2.16	2.00	1.613	2	0.446	.05
Classification	16	2.63	1.50	1.88	14.933	2	0.001*	.47
Merging	16	1.78	2.06	2.16	1.773	2	0.412	.06
Ranking	16	2.16	1.97	1.88	0.977	2	0.614	.03
Using Numbers	16	2.38	1.59	2.03	6.408	2	0.041*	.20
Simultaneous and Abbreviated Counting	16	2.25	1.56	2.19	6.435	2	0.040*	.20
Consequential Counting	16	2.16	2.03	1.81	1.442	2	0.486	.05
Applying Number Knowledge	16	2.16	1.81	2.03	1.216	2	0.545	.04
Estimate	16	2.09	1.97	1.94	0.400	2	0.819	.01
Total	16	2.25	1.78	1.97	2.000	2	0.368	**.06**
*p<0.05								

For the Advanced Mathematics Skills Test, Friedman tests likewise indicated significant time effects for all subdimensions and the total score in the experimental group ($p < .001$, $W = .78–.92$; Table 4), with improvements from pre-test to post-test and follow-up remaining significant after Bonferroni correction and no deterioration between post-test and follow-up. In the control group, no significant time effects were observed for basic skills, counting and operations, problem-solving, or total score, and only a small effect emerged for spatial perception.

Table 4 *Friedman F Test Results for Experimental Group's Advanced Mathematics Skills Test Scores"*

Experimental Group	n	Rank Average			Friedman Test	Degree of Freedom	p	
		Pre-Test (a)	Post Test (b)	Follow-up Test (c)				W (χ²/32)
Basic Skills	16	1.13	1.94	2.94	27.226	2	0.000*	**.85**
Counting and Operation	16	1.09	2.00	2.91	28.033	2	0.000*	**.88**
Spatial	16	1.13	2.28	2.59	25.020	2	0.000*	**.78**
Problem Solving	16	1.03	2.16	2.81	29.158	2	0.000*	**.91**
Total	16	1.06	1.97	2.97	29.556	2	0.000*	**.92**
Control Group								
Basic Skills	16	1.75	2.19	2.06	1.733	2	0.545	.05
Counting and Operation	16	1.56	2.25	2.19	4.852	2	0.088	.15
Spatial	16	1.56	2.41	2.03	6.203	2	0.045*	.19
Problem Solving	16	1.91	2.28	1.81	2.000	2	0.368	.06
Total	16	1.56	2.38	2.06	5.375	2	0.068	.17
*p<0.05								

The impact of the *CRA-based mathematics Program* on teachers' professional development and children's mathematical and representational skills was also revealed by teacher interviews and researcher diary entries.

Qualitative Evidence Supporting Quantitative Results

Teacher observations validated the quantitative improvements. Teacher E noted substantial increases in attention, motivation, and mathematical engagement:

“Children's attention span increased with activities. They are very interested in mathematics and often engage in mathematical operations… they found it themselves and learned it themselves.”

The teacher emphasized transition to abstract thinking:

“Initially, the children struggled during semi-concrete activities… when we transitioned to the abstract, difficulties diminished. They no longer have difficulties with abstract concepts because they understand them.”

Researcher diary entries also confirmed gains during the program's progression.
During the concrete stage:

“All of the children wanted to touch the jars… They compared them with pictures and showed the jar at the same level.”

During representational activities:

“Ç8 started counting… from the middle of the pattern.”

During abstract stage:

“The number of drawings related to sequencing, patterns, and geometric shapes increased after the third week… Their math dialogue increased.”

Example student dialogue:

“Half… No, that is quarter… if it’s smaller than quarter, it’s a slice.”

Taken together, observational and interview data support statistically significant progress and internalization of mathematical reasoning.

RQ2. How do teachers perceive the impact of the CRA-based Mathematics Program on children’s learning and engagement in mathematics?

Teacher interviews indicated that the CRA-based Mathematics Program positively influenced children’s cognitive, social-emotional, linguistic, and mathematical development. According to the teacher, children showed increased attention, curiosity, and willingness to engage in mathematical tasks, along with improvements in abstract thinking, symbolic use, and awareness of mathematics in daily life. The systematic progression from concrete to abstract representations helped children overcome initial difficulties and facilitated readiness for abstract concepts:

“Children’s attention span increased… They are very interested in mathematics and often engage in mathematical operations… Their difficulties during the semi-concrete activities diminished as we transitioned to the abstract stage.”

The teacher also reported greater participation in mathematical activities, increased peer collaboration, and the extension of mathematical discourse into play and everyday classroom contexts. Children frequently referred to numbers and mathematical concepts during both indoor and outdoor activities.

Regarding teacher development, three main themes emerged: pedagogical content knowledge, attitudes toward mathematics, and the implementation process. The teacher reported increased confidence in planning and delivering developmentally appropriate mathematics instruction:

“Before the program, I didn’t know how to approach this age group, especially in mathematics.”

The program enhanced the teacher’s ability to connect concrete and abstract stages, design varied mathematical activities, and integrate mathematics more intentionally into daily routines:

“Through the CRA program, I gained awareness of how to engage with them… The children understand concepts more easily because they are familiar with them.”

Regular coaching and weekly meetings supported effective implementation and strengthened instructional practices, contributing to a more systematic and integrated approach to early mathematics teaching.

RQ3. In what ways do children demonstrate and express mathematical thinking during and after the program?

Qualitative observations supported the quantitative findings by revealing enriched mathematical expression, increased symbolic use, and more frequent child-initiated mathematical behaviors during and after the program. Researcher diaries documented children’s progression across representational levels.

At the concrete level, children explored mathematical ideas through manipulation, comparison, and physical interaction with materials:

“All of the children wanted to touch the jars… They compared the water level with the pictures and matched them correctly.”

At the representational level, children used drawings, mark-making, and visual models to express mathematical relationships:

“Ç8 started counting from the middle of the pattern sample hung on the clipboard.”

At the abstract level, children increasingly used mathematical language and symbols to reason about quantities and relationships:

“Half… No, that is quarter… If it's smaller than a quarter, it's a slice.”

Figure 1 illustrates the overall developmental progression from concrete engagement toward representational and symbolic reasoning across the implementation period, while also reflecting individual differences in learning pace.

Figure 1 illustrates the weekly progression of preschool children’s mathematical behaviors across the CRA stages during the 12-week intervention. The results show a clear developmental shift from concrete exploration toward increasingly representational and symbolic engagement. In the initial weeks, children predominantly interacted with concrete materials, with behaviors such as manipulating objects, grouping items, and solving problems in physical contexts being most frequent. Representational and symbolic expressions appeared only sporadically.

From Week 4 onward, representational behaviors increased noticeably, with children expressing mathematical ideas through drawings, mark-making, and visual models—indicating a transition from concrete experiences toward abstraction. Beginning in Weeks 6–7, symbolic behaviors became more prominent, including the use of number symbols, simple operations, and positional language. Peaks in symbolic engagement in Weeks 7, 9, and 12 reflect internalization of abstract concepts and evidence of cognitive transfer.

Although weekly fluctuations were observed-an expected pattern in early childhood learning-the overall trend indicates growing representational flexibility and strengthening symbolic reasoning. Some children continued to rely on concrete supports during abstract tasks, demonstrating individual differences in the pacing of conceptual development.

Figure 2 presents examples of children’s mathematical drawings. According to researcher notes, “the use of mathematical expressions by children increased throughout the educational program compared to the initial activities,” demonstrating a progression from concrete experience to symbolic representation and verbal expression.

4. Discussion

The findings of this study indicate that the CRA-based Mathematics Program was associated with notable improvements in children’s early numeracy and advanced mathematical skills, supported by converging quantitative and qualitative evidence. Children demonstrated increasingly sophisticated use of mathematical language, visual and symbolic representations, and problem-solving strategies across the intervention. These results are consistent with a broad body of research showing that structured early mathematics programs—such as the Concept Education Program, Early Number Development Program, Big Math for Little Kids, and the Early Arithmetic Program—effectively promote children’s mathematical competencies (Akuysal-Aydoğan & Şen, 2011; Çelik & Kandır, 2013; Karakuş, 2020Iıçkaya & Avcı, 2021; Nisan & İnal-Kızıltepe, 2019).

The CRA-based program was intentionally designed to support children's transition from concrete experiences to representational and abstract thinking. Although causal inference is limited by the quasi-experimental design, the developmental trajectory observed across concrete, semi-concrete, and abstract stages aligns with theoretical models emphasizing the role of representation in mathematical learning (Bruner, 1966; Lesh, 1979; Vygotsky, 1978). Children's growing representational fluency—evident in their drawings, symbolic notations, and verbal explanations—mirrors findings from studies showing that multi-representational instruction fosters conceptual understanding and symbolic reasoning (Sterner et al., 2020; Björklund & Palmér, 2022). The program's structure also parallels Montessori-based progressions from manipulation to abstraction (Laski et al., 2015), offering practical guidance for how such transitions can be operationalized in contemporary classroom contexts.

The abstract phase of the program strengthened children's ability to use mathematical symbols meaningfully, a process supported in the literature as foundational for developing mathematical thinking (Worthington et al., 2019; Cartwright, 2023). Children's drawings and verbalizations provided clear evidence of their ability to connect quantities with symbols, make comparisons, and use mathematical terminology appropriately. These qualitative indicators reinforced quantitative gains in advanced mathematical subskills, suggesting that representational and symbolic learning experiences contributed to deeper conceptual understanding.

Teacher and researcher observations further highlighted developmental shifts across the CRA stages. At the concrete level, children effectively used manipulatives to explore mathematical properties. At the representational stage, they engaged with diagrams and visual models to structure their thinking. At the abstract stage, they increasingly used numerical symbols, operations, and mathematical language to express ideas. This progression is consistent with literature emphasizing the importance of supporting children's use of multiple representation types—including dynamic, pictorial, symbolic, and written forms—in developing abstract reasoning (Otsuka & Jay, 2017). The observed ability to transfer ideas across representations suggests that the program strengthened children's representational flexibility, a hallmark of advanced mathematics learning.

Beyond mathematics skills, the program was also associated with gains in language, attention, curiosity, and social-emotional behaviors, as reported by the teacher. These findings align with research demonstrating that early mathematics engagement contributes to broader cognitive and linguistic development (Pappas et al., 2018; Watanabe, 2019; Yıldız & Kayılı, 2023). Children's increased use of mathematical discourse, both in structured activities and free play, supports longitudinal evidence linking language proficiency and working memory to mathematical growth (Viesel-Nordmeyer et al., 2022).

The program additionally contributed to the teacher's pedagogical knowledge and practice. The teacher reported increased competence in designing activities using different representation types, integrating mathematics across learning contexts, and providing individualized support. These outcomes are consistent with studies showing that participation in mathematics-focused professional development enhances teachers' beliefs, instructional strategies, and pedagogical content knowledge (Fishman et al., 2003; Chen & McCray, 2012). Weekly planning meetings and in-class coaching further supported the teacher's ability to guide children's learning, strengthen teacher-child interactions, and scaffold transitions toward abstraction—practices central to high-quality mathematics instruction (Clements & Sarama, 2021; Witzel et al., 2008).

The program's representational structure enriched the learning environment by providing concrete, visual, and symbolic experiences that addressed individual learning differences and supported the development of conceptual understanding (Duval, 2006; Worthington et al., 2019). Thus, the CRA-based Mathematics Program enhanced both children's mathematical thinking and the teacher's pedagogical development, offering a practical and theoretically grounded model for early childhood mathematics instruction.

5. Contributions and implications

The findings suggest that the CRA-based Mathematics Program is an effective instructional approach for supporting early numeracy and advanced mathematical skills in preschool children. The use of multiple representations facilitates meaningful learning and enhances children's problem-solving and mathematical expression. For practice, integrating CRA-oriented, application-based early mathematics courses into teacher education programs may strengthen pedagogical quality and instructional sustainability. For research, future studies should examine the long-term effects of the program through longitudinal designs, test its effectiveness across diverse socio-economic contexts, and explore its impact on children with varying levels of mathematical proficiency. Collaborative research with mathematics education specialists may further improve instructional strategies, particularly for abstract mathematical concepts.

Declarations

Author Contribution

F.A.B. and İ.U. contributed to the conception and design of the study. F.A.B. and İ.U. collected and analyzed the data. F.A.B. wrote the main manuscript text, and İ.U. critically reviewed and revised the manuscript for intellectual content. Both authors reviewed and approved the final version of the manuscript.

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Data Availability

The datasets analyzed during the current study are available from the corresponding author upon reasonable request.

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Figures

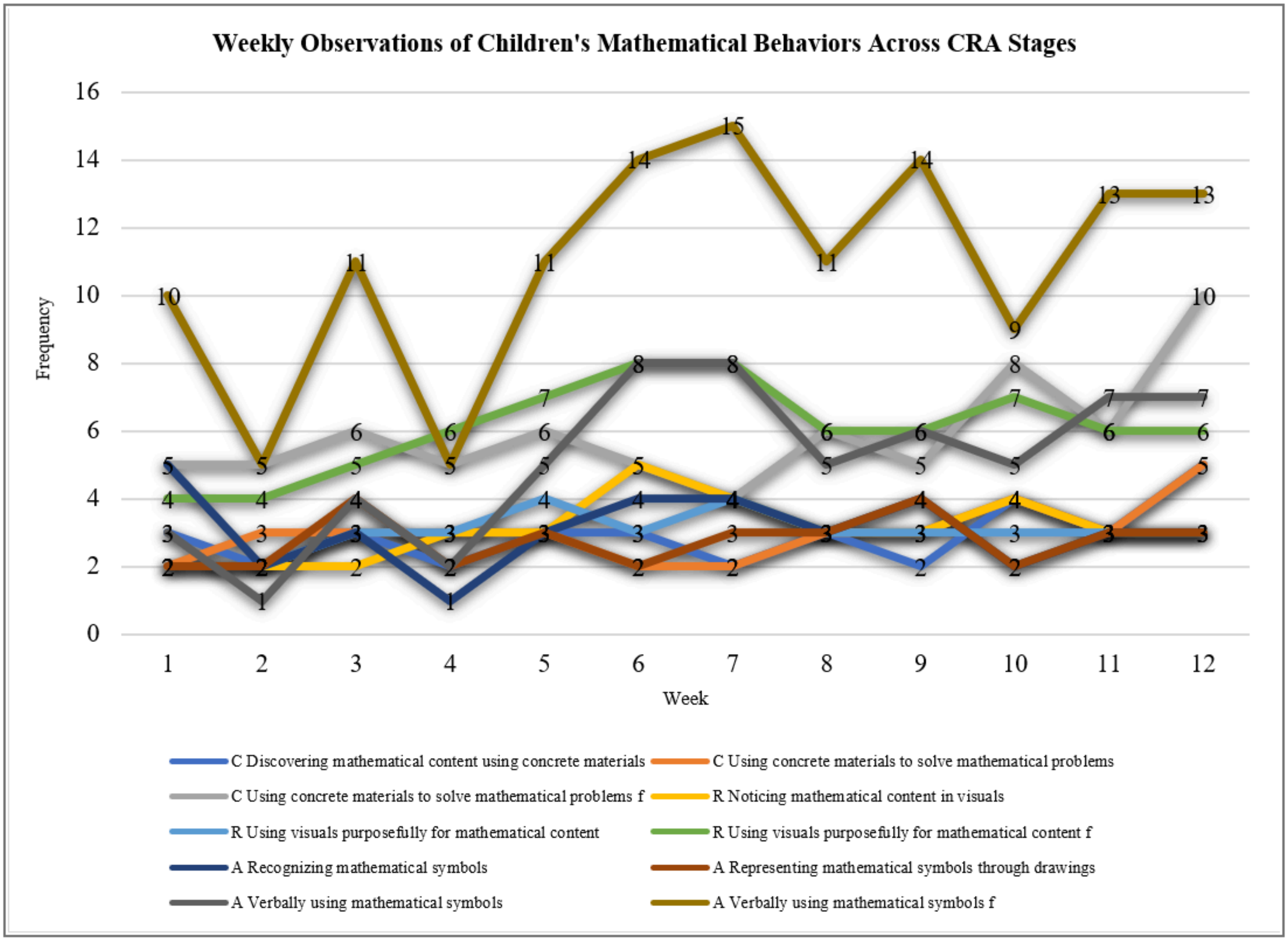
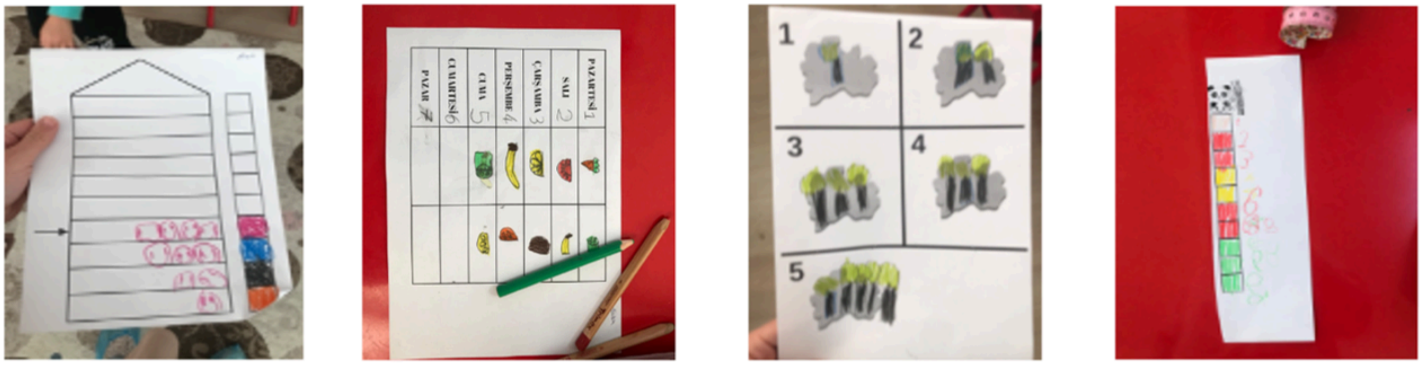


Figure 1

Weekly Progression of Preschool Children’s Mathematical Behaviors Across CRA Stages



Numbers Apartment Drawing:
A child visually represents the number of individuals living on each floor using a combination of symbols and drawings. This reflects their understanding of quantity matching and spatial placement within a structured template.

Part-Whole Representation through a Story Context:
Inspired by the story "The Chick's Diet Plan," children created visual representations using half and whole drawings. This activity supports understanding of part-whole relationships and develops early mathematical thinking through contextualized visuals.

Drawing According to Number (Child-Created Tree Representations):
The child was asked to draw anything they wanted based on the given number. They chose to draw trees, showing quantity through repeated visuals. This supports number sense and creativity together.

Colorful Number Line Work:
A number line is used to reinforce number sequencing and recognition. Children enhance this by coloring each segment and writing the corresponding numeral, promoting both fine motor and number sense skills.

Figure 2

Mathematical drawings made by children in the CRA-based Mathematics Program

Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- Appendix.docx