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Article

Keywords:

Posted Date: May 12th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-9586246/v1>

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Additional Declarations: There is **NO** Competing Interest.

Topological protection of photosynthetic quantum heat engines

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Abstract

Quantum coherence is increasingly recognized as a foundational resource in quantum thermodynamics, serving as a key driver of thermodynamic performance in quantum heat engines (QHEs). However, a central paradox remains unresolved: how do natural photosynthetic systems achieve exceptionally high and robust energy-conversion efficiencies, even within noisy, dissipative environments where quantum coherence typically decays rapidly? Here, we present a coherently driven four-level QHE model, inspired by photosystem II—the primary energy-harvesting complex in plants, to elucidate the mechanisms by which quantum coherence and system topology jointly enable extraordinary efficiency and resilience. By performing spectral analysis of the system’s non-

Hermitian Liouvillian superoperator, we reveal a topological phase transition characterized by a quantized change in the winding number. This transition separates two operational regimes: in the topologically trivial phase, stronger driving enhances coherence and boosts efficiency, whereas in the topologically nontrivial phase, topological protection preserves optimal efficiency even as coherence is largely destroyed. Our results establish a direct connection between topological effects and photosynthetic energy conversion, providing a new framework for understanding the resilience of biological energy harvesting and informing the design of robust, high-efficiency quantum energy conversion devices.

Efficient solar energy conversion remains a key challenge for sustainable energy, grounded in fundamental physical, chemical, and biological principles.^{1,2} Quantum heat engines (QHEs) provide a model for investigating energy conversion at the quantum level and for systematically exploring their underlying principles and theoretical limits.^{3–10} As the primary photochemical engine in plants, algae, and cyanobacteria, Photosystem II (PSII) embodies a paradigm of highly efficient solar-to-chemical energy conversion.^{11–16} Inspired by PSII, recent theoretical and experimental efforts have demonstrated that PSII-type QHEs can exploit quantum coherence to exceed classical efficiency bounds and minimize thermodynamic losses.^{16–19} These advances position PSII-inspired quantum devices as a unique platform to interrogate the fundamental interplay between quantum coherence^{20,21} and thermodynamic efficiency,^{22–25} opening new directions for quantum-enhanced energy conversion technologies.

As a prototype model, we explore a coherence-assisted PSII-type QHE system in Fig. 1(a) that comprises four quantum states: the ground state $|0\rangle$, the first excited state $|1\rangle$, the second excited state $|2\rangle$, and the third excited state $|3\rangle$, as shown in Fig. 1(b). The QHE cycle starts with photoexcitation from $|0\rangle$ to $|3\rangle$ as the donor step, driven by a high-temperature photon reservoir that simulates concentrated solar radiation. The system then undergoes electron transfer to $|2\rangle$, storing energy as the acceptor, which releases usable energy to $|1\rangle$. Finally, the system returns to $|0\rangle$ by coupling with a cold phonon reservoir, completing the cycle. These four steps are analogous to the heating, expansion, cooling, and compression processes in a classical heat engine.

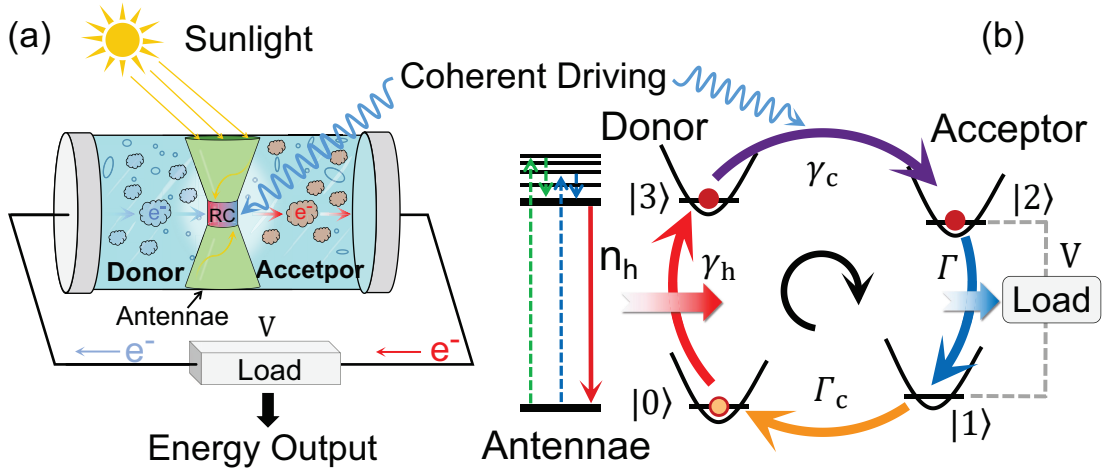


Figure 1: (a) Schematic illustration of a Photosystem II-type quantum heat engine under coherent driving. (b) Theoretical model comprising four quantum states: $|0\rangle$, $|1\rangle$, $|2\rangle$, and $|3\rangle$. Coherent driving couples the donor state $|3\rangle$ to the acceptor state $|2\rangle$, enhancing energy transfer to $|1\rangle$ with a load voltage V .

Since Scully and collaborators first introduced the concept of coherence-assisted quantum heat engines (QHEs) in 2003,²⁶ the PSII-type QHE model and its extensions have become fundamental for understanding quantum coherence in light absorption,^{27,28} energy conversion,^{29,30} and power generation.³¹ QHEs coupled to thermal reservoirs typically exhibit non-Hermitian dynamics.^{32,33} External driving in these open systems can induce exceptional points (EPs), where eigenvalues and eigenvectors merge, and trigger topological phase transitions (TPTs) that alter global response and transport.^{33–38} These phenomena significantly affect engine performance and resilience to environmental disturbances.^{39–41} Conventional models often link QHE efficiency to quantum coherence, implying vulnerability in noisy, dissipative environments. Whether natural systems can overcome this limitation by relying on robust topological structures rather than fragile coherence remains unclear. Addressing this issue requires insights from quantum thermodynamics,⁴² topological physics,^{43–45} open quantum systems,^{46–50} and macroscopic quantum phenomena.^{51–53} Here, we identify a TPT that enables a coherence switch between two operational regimes: a topologically trivial phase, where increasing coherence enhances QHE efficiency, and a topologically nontrivial phase, where efficiency approaches the theoretical limit even as coherence decreases. Our

findings establish a direct link between topological invariants and thermodynamic performance, demonstrating how robust topological structures protect energy conversion in photosynthetic QHE systems and suggesting new directions for designing resilient quantum energy devices.

Results

We consider a continuous-wave driving field to resonantly couple the donor and acceptor states depicted in Fig. 1. Under the rotating-wave approximation (RWA) and the Markovian limit, the system's time evolution is governed by the Lindblad master equation (atomic units are used unless stated otherwise):^{54–56}

$$\dot{\rho}(t) = -i[\hat{H}_I, \rho(t)] + \sum_{ij \in \mathcal{K}} \mathcal{D}_{ij} \rho(t), \quad (1)$$

where $\mathcal{K} = 03, 23, 01, 12$ indexes the four relevant transitions shown in Fig. 1(b). The Hamiltonian $\hat{H}_I = \Omega(S_{32} + S_{32}^\dagger)$ captures the coherent exchange between states, defined using the transition operators $S_{ij} = |i\rangle\langle j|$ and the Rabi frequency Ω . The dissipative dynamics are described by the standard dissipator $\mathcal{D}[S_{ij}]\rho = S_{ij}\rho S_{ij}^\dagger - \frac{1}{2}\{S_{ij}^\dagger S_{ij}, \rho\}$, such that the second term in Eq. (1) expands to

$$\begin{aligned} \mathcal{D}_{03}\rho &= \gamma_h((n_h + 1) \mathcal{D}[S_{03}] + n_h \mathcal{D}[S_{30}])\rho, \\ \mathcal{D}_{23}\rho &= \gamma_c((n_c + 1) \mathcal{D}[S_{23}] + n_c \mathcal{D}[S_{32}])\rho, \\ \mathcal{D}_{01}\rho &= \Gamma_c((N_c + 1) \mathcal{D}[S_{01}] + N_c \mathcal{D}[S_{10}])\rho, \\ \mathcal{D}_{12}\rho &= \Gamma \mathcal{D}[S_{12}]\rho, \end{aligned} \quad (2)$$

where n_h denotes the thermal photon occupation of the hot reservoir driving the $|0\rangle \leftrightarrow |3\rangle$ transition. The phonon occupation numbers for the transitions to the cold bath are $n_c = [\exp(\omega_{32}/k_B T_c) - 1]^{-1} \approx 0$ and $N_c = [\exp(\omega_{10}/k_B T_c) - 1]^{-1} \approx 0$, given the low environmental temperature T_c . The spontaneous relaxation rates γ_h and γ_c characterize the decay from $|3\rangle$ to $|0\rangle$ and $|2\rangle$, respectively,

74 while Γ_c describes the relaxation from $|1\rangle$ to $|0\rangle$, mediated by thermal reservoirs. In contrast,
 75 the extraction rate Γ from $|2\rangle$ to $|1\rangle$ is governed by the external load. This process, driven by
 76 the electron transport chain, depends on the applied voltage and energy-level alignment as $\Gamma =$
 77 $\Gamma_c \exp[-(V - \omega_{21})/(k_B T_c)]$, rather than thermal activation (Supplementary Note 1).

78 The stationary populations are derived analytically by solving Eq. (1) under the steady-state
 79 condition $\dot{\rho}(\infty) = 0$. The power output from the acceptor is given by $P = \Gamma \rho_{22} V$. To quantify
 80 performance relative to the thermodynamic limit, we introduce a dimensionless parameter η for
 81 power conversion following (Supplementary Note 1)

$$82 \quad \eta \equiv \frac{P}{\eta_C P_{\text{hot}}^{\text{max}}} = \frac{\Gamma_c \rho_{22} V}{\eta_C \omega_{30} n_h \gamma_h} e^{-\frac{V - \omega_{21}}{k_B T_c}}, \quad (3)$$

83 where $\eta_C = 1 - T_c/T_h$ is the Carnot efficiency and $P_{\text{hot}}^{\text{max}} = \omega_{30} \gamma_h n_h$ denotes the maximum heat
 84 flux from the hot bath. Note that η serves as a normalized power-conversion metric analogous to
 85 photovoltaic efficiency and does not represent the conventional thermodynamic efficiency. Since
 86 the acceptor population ρ_{22} is primarily built up via coherent exchange between states $|3\rangle$ and
 87 $|2\rangle$, we employ the ℓ_1 norm, $C_{\ell_1} \equiv 2|\rho_{32}|$, to quantify the quantum coherence between donor and
 88 acceptor. This directly measures the coherent transfer rate

$$89 \quad J_c \equiv 2\Omega \text{Im}[\rho_{32}] = \Omega C_{\ell_1}, \quad (4)$$

90 where ρ_{32} is purely imaginary in the steady state.

91 To examine how quantum coherence and topology affect power-conversion efficiency, we de-
 92 rive a minimal donor-acceptor transport model. Projecting the Lindblad master equation onto the
 93 no-jump subspace yields the effective non-Hermitian Hamiltonian^{57,58}

$$94 \quad \hat{H}_{\text{eff}} = \hat{H}_I - \sum_{(ij, \gamma_{ij}, n_{ij}) \in \mathcal{P}} \frac{i\gamma_{ij}}{2} [(n_{ij} + 1)S_{ij}^\dagger S_{ij} + n_{ij}S_{ji}^\dagger S_{ji}], \quad (5)$$

95 where $\mathcal{P} = (03, \gamma_h, n_h), (23, \gamma_c, n_c), (01, \Gamma_c, N_c), (12, \Gamma, 0)$ defines the relevant dissipative channels.

96 In the rotating frame at ω_{32} and the basis $|3\rangle, |2\rangle, |1\rangle, |0\rangle$, $\hat{H}_{\text{eff}}$ is block diagonal: $\hat{H}_{\text{eff}} = \text{diag}(\hat{H}_c, \hat{H}_{\text{dec}})$.
 97 $\hat{H}_{\text{dec}}$ describes incoherent processes, while the coherent donor-acceptor sector is governed by

$$98 \quad \hat{H}_c = \begin{pmatrix} -i\Gamma_{\text{eff}}/2 & \Omega \\ \Omega & -i\Gamma/2 \end{pmatrix}, \quad (6)$$

99 where $\Gamma_{\text{eff}} = \gamma_h(n_h + 1) + \gamma_c$ represents the effective linewidth of the donor state $|3\rangle$.

100 To clarify the topological properties of the non-Hermitian dynamics $\dot{\psi}(t) = -i\hat{H}_c\psi(t)$, we apply
 101 the Laplace transform $\hat{\Psi}(s) = \int_0^\infty \psi(t)e^{-st}dt$ with complex frequency $s = \sigma + i\omega$ ($\sigma > 0$). This
 102 Laplace transform maps time-domain evolution to the frequency domain, revealing the system's
 103 eigenmodes, spectral structure, and stability through the distribution of zeros and poles. Using the
 104 Nyquist stability criterion,^{59–62} we interpret the spectral response of the generator $s\hat{I} - \hat{A}$ (where
 105 $\hat{A} = -i\hat{H}_c$) as an analytic mapping from the right half-plane to itself. This approach leads to
 106 defining a winding number, based on the trajectory of the reflection coefficient, which serves as a
 107 topological indicator (Supplementary Note 1)

$$108 \quad \nu = \frac{1}{2\pi i} \oint_C \frac{dr(s)}{r(s)}, \quad (7)$$

109 where $r(s) = [s^2 + (\Gamma + \Gamma_{\text{eff}})s/2 + (\Gamma\Gamma_{\text{eff}}/4 - \Omega^2)]/[s^2 + (\Gamma + \Gamma_{\text{eff}})s/2 + (\Gamma\Gamma_{\text{eff}}/4 + \Omega^2)]$ depends
 110 only on $\hat{H}_c$. The contour C encloses the right half-plane, so ν is set by the zeros and poles of $r(s)$.
 111 The winding number $\nu = 0$ occurs when all roots are in the left half-plane, while $\nu = 1$ indicates
 112 roots in the right half-plane. Application of the residue theorem shows that the winding number ν
 113 undergoes a topological transition given by

$$114 \quad \nu = \begin{cases} 0, & \Omega < \Omega_T \quad \text{trivial phase,} \\ 1, & \Omega > \Omega_T \quad \text{nontrivial phase,} \end{cases} \quad (8)$$

115 where the critical Rabi frequency is given by $\Omega_T = \sqrt{\Gamma\Gamma_{\text{eff}}}/2$. It shows ν changes from 0 in the
 116 topological trivial phase to 1 in the topological nontrivial phase as Ω increases beyond this critical

117 value.

118 Using the identified topologically critical Rabi frequency Ω_T , the steady-state density-matrix
119 elements in the coherent subspace can be written in a compact form (Supplementary Note 1)

$$120 \quad \rho_{22}(\infty) = \frac{\zeta_1 \Omega_T^2 + \zeta_2 \Omega^2}{\zeta_3 \Omega_T^2 + \zeta_4 \Omega^2}, \quad \rho_{32}(\infty) = i \frac{\zeta_5 \Omega}{\zeta_3 \Omega_T^2 + \zeta_4 \Omega^2}. \quad (9)$$

121 where $\zeta_1 = \Gamma_c n_h \gamma_c \gamma_h (\Gamma + \gamma_c + n_h \gamma_h)$, $\zeta_2 = \Gamma \Gamma_c n_h \gamma_h (\gamma_c + n_h \gamma_h)$, $\zeta_3 = (\Gamma + \gamma_c + n_h \gamma_h) [\Gamma \Gamma_c (\gamma_c + 2n_h \gamma_h) +$
122 $(\Gamma + \Gamma_c) n_h \gamma_c \gamma_h]$, $\zeta_4 = \Gamma (\gamma_c + n_h \gamma_h) [\Gamma \Gamma_c + n_h \gamma_h (\Gamma + 3\Gamma_c)]$, and $\zeta_5 = \Gamma \Gamma_c n_h \gamma_h (\Gamma - \gamma_c) (\gamma_c + n_h \gamma_h) / 2$.
123 The ℓ_1 norm increases with driving strength in the trivial phase, peaks near Ω_T at the coherence-
124 optimization scale $\Omega_c = \sqrt{\zeta_3 / \zeta_4} \Omega_T$, and decreases in the topological phase. In contrast, J_c rises
125 rapidly in the topological trivial phase and then gradually saturates in the topological nontrivial
126 phase. This saturation reflects a competition in the topological regime: stronger driving suppresses
127 the coherence amplitude $|\rho_{32}|$, but the prefactor Ω in J_c offsets this reduction, maintaining the
128 coherent current. Since ρ_{22} determines the power-conversion efficiency η in Eq. (3) and shares
129 the same saturation-type dependence on Ω as J_c , the dependence of J_c on Ω closely follows the
130 behavior of η , thereby linking the coherence crossover at Ω_c to the enhancement and eventual
131 saturation of η (Supplementary Note 1).

132 By substituting $\rho_{22} = \zeta_1 / \zeta_3$ for $\Omega = 0$ and $\rho_{22} = \zeta_2 / \zeta_4$ for $\Omega \gg \Omega_T$ into Eq. (3), we obtain the
133 bounds for the power-conversion efficiency

$$134 \quad \eta_{\min} = \frac{\Gamma \Gamma_c V \gamma_c}{\omega_{30} \eta_C [\Gamma \Gamma_c (\gamma_c + 2n_h \gamma_h) + (\Gamma + \Gamma_c) n_h \gamma_c \gamma_h]}, \quad (10)$$

$$\eta_{\max} = \frac{\Gamma \Gamma_c V}{\omega_{30} \eta_C [\Gamma (\Gamma_c + n_h \gamma_h) + 3n_h \gamma_h \Gamma_c]}.$$

135 As Ω increases, the efficiency η ascends from its minimum $\eta_{\min}$ observed in the trivial phase to
136 its peak value $\eta_{\max}$ within the topological phase. This trend indicates that tuning the system near
137 the TPT optimizes quantum coherence, thereby potentially enhancing energy transfer efficiency.
138 Intriguingly, however, deeper exploration of the topological phase involves a fundamental trade-

off: coherence declines, yet the system achieves enhanced power-output efficiency, suggesting a mechanistic shift from coherence-dependent to topologically protected transport.

To validate our analytical predictions and precisely determine the location of the TPT in ef-

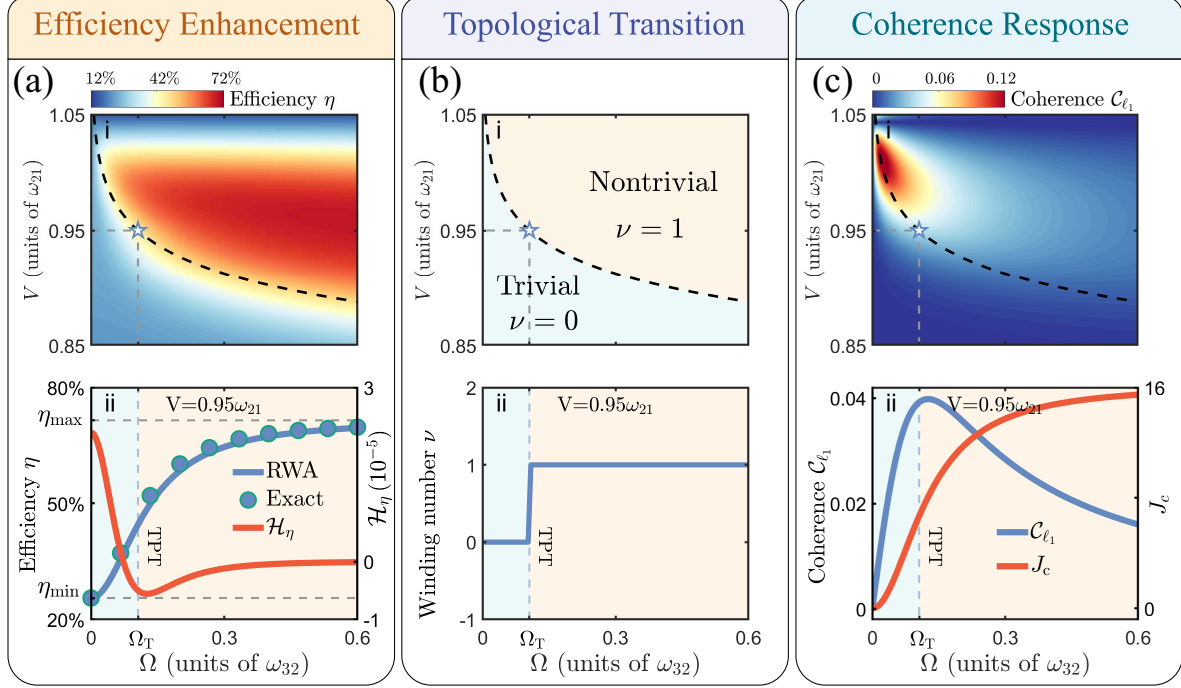


Figure 2: Efficiency enhancement, Topological transition, and Coherence response. (a) Power-output efficiency η , (b) winding number ν , and (c) ℓ_1 -norm coherence $\mathcal{C}_{\ell_1} = 2|\rho_{32}|$ as functions of the driving Rabi frequency Ω and load voltage V . Panels (a:ii), (b:ii), and (c:ii) show the respective cases (blue lines) for $V = 0.95\omega_{21}$. Dashed lines (black) indicate the topological phase transition (TPT) as determined by the critical Rabi frequency Ω_T and V ($\Omega_T = \sqrt{\Gamma\Gamma_{\text{eff}}}/2$ with stars at $V = 0.95\omega_{21}$). In (a:ii), the exact calculation of η without RWA (blue circles) is compared with that using RWA (blue line), together with the corresponding Hessian (orange line). In (c:ii), the coherent transfer rate $J_c = \Omega\mathcal{C}_{\ell_1}$ (orange line) is shown.

efficiency enhancement, we numerically solve the full Lindblad equation without employing RWA. For this, we adopt experimentally realistic parameters based on previous studies.^{18,27,31} The transition energies are set to $\omega_{30} = 1.82$ eV (i.e., 680 nm), $\omega_{32} = 0.20$ eV, and $\omega_{21} = 1.44$ eV. The decay rates are $\gamma_h = 2.23 \times 10^{-7}$ eV, $\gamma_c = 2.23 \times 10^{-3}$ eV, and $\Gamma_c = 0.0248$ eV. Reservoir temperatures are $T_h = 6000$ K and $T_c = 300$ K, with T_h characterizing the effective solar radiation spectrum driving the optical transition. To emulate strong solar pumping, we set the photon occupation number to $n_h = 1.0 \times 10^4$, representing a concentrated radiation field typical of antenna molecules in

photosynthesis, rather than the thermal population at T_h . The load voltage V is set near the output gap ω_{21} between $|2\rangle$ and $|1\rangle$, while the Rabi frequency Ω is varied from zero up to a high value of $0.6\omega_{32}$ within the effective bandwidth of the system.

Figure 2(a:i) shows the exact efficiency η as a function of Rabi frequency Ω and load voltage V . In this simulation, the steady-state population ρ_{22} , computed numerically without the RWA, is incorporated into Eq. (3). Convergence is achieved self-consistently by comparing density matrices over consecutive driving periods, and observables are averaged across an additional 20 periods to ensure accuracy. The efficiency η increases with Rabi frequency Ω , whereas it becomes insensitive to Ω as V approaches ω_{21} , consistent with the exponential decay in Eq. (3). Figure 2(a:i) shows that, across the two-dimensional parameter space, the efficiency η increases markedly from $\eta_{\min} = 13.54\%$ to $\eta_{\max} = 70.56\%$, representing a relative enhancement of $\eta_R = (\eta_{\max} - \eta_{\min})/\eta_{\min} = 421.09\%$. Figure 2(a:ii) presents η as a function of Ω at the optimal load voltage $V = 0.95\omega_{21}$. As Ω increases, η rises and eventually saturates at a high plateau in the strong-driving regime. This behavior aligns closely with the theoretical limits predicted by Eq. (10) within the RWA framework.

To reveal the topological features underlying the enhancement effect shown in Fig. 2(a:i), we present the phase diagram of the winding number ν as a function of Ω and V in Figs. 2(b:i). The winding number ν , defined by Eq. (7), describes how many times $r(s)$ encircles the origin. As Ω increases over the critical Rabi frequency Ω_T , the zero locations shift, changing ν from 0 to 1. Further simulations for $V = 0.95\omega_{21}$ in Fig. 2(b:ii) show the system remains in the trivial phase when $\Omega < \Omega_T$. The system undergoes a phase transition at the critical Rabi frequency $\Omega = \Omega_T$ and enters into the topological phase for $\Omega > \Omega_T$. The changes in efficiency enhancement across the dashed lines in Fig. 2(a:i) are associated with the winding-number (topological) transition occurring at $\Omega_T = \sqrt{\Gamma\Gamma_{\text{eff}}}/2$, which depends on the load voltage V .

Given the critical role of coherent driving in QHEs, we examine the ℓ_1 -norm coherence in coherent subspace by $C_{\ell_1} = 2|\rho_{32}|$ as a function of Ω and V in Fig. 2(c:i). In the topological trivial phase, C_{ℓ_1} increases with the Rabi frequency and peaks at $\Omega_c = \sqrt{\zeta_3/\zeta_4}\Omega_T$ near the critical line at

$\Omega = \Omega_T$. In the topological nontrivial phase, C_{ℓ_1} decreases as the Rabi frequency increases. These results show that coherence plays different roles in the trivial and topological phases. To quantify coherence in energy transfer, we evaluate the coherent transfer rate J_c . While the coherence C_{ℓ_1} peaks near Ω_T and then declines in the topological phase, J_c smoothly rises and becomes stable as Ω increases. Comparison of Figs. 2(c:ii) and (a:ii) shows that the increase in J_c with Ω mirrors the trend observed for η , consistent with our theoretical prediction. The rise in J_c with Ω directly increases the steady-state population ρ_{22} , and thus the output power $P = V\Gamma\rho_{22}$. In the topological nontrivial phase, coherent donor-acceptor transfer is the primary pathway for converting absorbed energy into electrical work (Supplementary Note 1).

The above analysis in Fig. 2 shows that TPTs in coherence can serve as markers for energy transfer. To provide an operational diagnostic of the TPT characterized by the winding number, Fig. 2(a:ii) presents the one-dimensional Hessian of the power-output efficiency η with respect to the driving amplitude Ω , defined as $\mathcal{H}_\eta \equiv \partial^2\eta/\partial\Omega^2$, evaluated at $V = 0.95\omega_{21}$. This analysis can be used to determine the concavity or convexity of the η curve (Supplementary Note 1). When $\mathcal{H}_\eta > 0$, the curve is concave up, whereas it becomes concave down for $\mathcal{H}_\eta < 0$. Its minimum at Ω_c corresponds to the point where the curve is least curved. We observe that the Hessian decreases from positive to negative values as Ω increases in the trivial phase, indicating that the system is not strongly driven and that η increases rapidly. In the topological phase, however, the Hessian rises toward zero when Ω goes beyond Ω_T , indicating efficiency stabilizes and further increases in Ω have a very small effect on η . A negative minimum occurs at Ω_c near Ω_T , marking an efficient QHE performance with the maximal coherence. As a result, the Hessian behavior shown in Fig. 2(a:ii) provides a clear operational signature of the TPT defined by the winding number, reflecting the change in concavity of the η curve across the transition.

Further analysis shows that both the Hessian of the analytical population ρ_{22} (Eq. (9)) and the coherence undergo a phase transition at the same critical point, Ω_c . This observation closely matches the numerical results in Fig. 2. Figure 3(a) shows the relative error $R_T = (\Omega_T - \Omega_c)/\Omega_c$ as a function of load voltage V . The model slightly underestimates Ω_T compared to Ω_c , but the

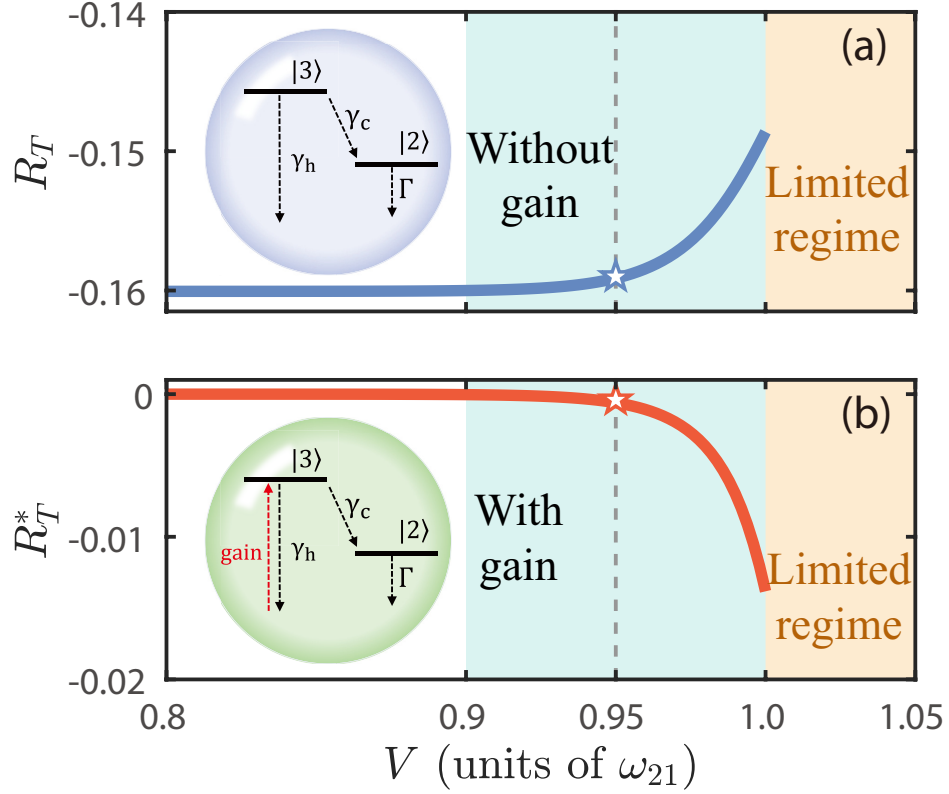


Figure 3: Feedback-renormalized topological transition and optimal load window. (a) Relative deviation $(\Omega_T - \Omega_c)/\Omega_c$ and (b) relative deviation $(\Omega_T^* - \Omega_c)/\Omega_c$ as functions of the output voltage V . Here Ω_T is the critical Rabi frequency derived without feedback, while Ω_T^* is its feedback-renormalized counterpart that incorporates the recycling pathway. Each panel also includes a schematic diagram illustrating the presence (b) or absence (a) of the feedback pathway. The blue shaded region marks the load voltages used in Fig. 2, and the yellow region indicates where the heat engine's current begins to decline sharply.

error remains below 16%. As load voltage increases, this discrepancy decreases. The difference between Ω_c and Ω_T occurs because Ω_T is based on the two-level non-Hermitian Hamiltonian $\hat{H}_c$, which does not include dissipative feedback from $|0\rangle$ to $|3\rangle$ (upper inset in Fig. 3(a)). To address this, we calculate a modified critical Rabi frequency, Ω_T^* , by including this feedback, as shown in the lower inset of Fig. 3. For $V < 0.9\omega_{21}$, Ω_T^* and Ω_c match exactly. At higher voltages, Ω_T^* is slightly less than Ω_c due to overcompensating feedback and underestimated effective dissipation, with the relative error $R_{T^*} = (\Omega_T^* - \Omega_c)/\Omega_c$ still under 2% (Supplementary Note 2 and Supplementary Figs. 2-3).

We now examine the eigenvalues and eigenstates of the non-Hermitian Hamiltonian $\hat{H}_c$

$$|\pm\rangle = \frac{1}{\mathcal{N}_\pm} \left(|3\rangle + \frac{\lambda_\pm + i\Gamma_{\text{eff}}/2}{\Omega} |2\rangle \right), \quad (11)$$

where the normalization factors are given by $\mathcal{N}_\pm = \sqrt{1 + |\lambda_\pm + i\Gamma_{\text{eff}}/2|^2/\Omega^2}$ with the eigenvalues $\lambda_\pm = -i(\Gamma_{\text{eff}} + \Gamma)/4 \pm \sqrt{16\Omega^2 - (\Gamma_{\text{eff}} - \Gamma)^2}/4$. The EP occurs at $\Omega_{\text{EP}} = |\Gamma_{\text{eff}} - \Gamma|/4 = |\Gamma_{\text{eff}} - \Gamma|/(2\sqrt{\Gamma\Gamma_{\text{eff}}})\Omega_T$, matching the EP of the four-level non-Hermitian Hamiltonian in Eq. (5), which divides the control space into broken and unbroken phase regimes. As V increases, Ω_c moves from the broken to the unbroken phase. Therefore, the phase transition at Ω_c , where coherence is maximized, does not correspond to the EP phase transition, but instead signifies the TPT (Supplementary Note 1).

Our topological analysis of the coherent-driven subspace identifies a rapid response regime near Ω_T that enhances performance and prevents overdriving. The efficiency improvement at Ω_T is calculated to yield a relative efficiency of $(\eta_T - \eta_{\text{min}})/\eta_{\text{min}} = 67.51\%$ at $V = 0.95\omega_{21}$. This is lower than the relative efficiency $(\eta_c - \eta_{\text{min}})/\eta_{\text{min}} = 80.87\%$ at Ω_c . When Ω exceeds Ω_T , the system enters into the topological nontrivial phase characterized by a nonzero winding number. For larger Ω within the considered parameter regime, the eigenstates $|\pm\rangle$ defined in Eq. (11) evolve toward nearly symmetric and antisymmetric superpositions, $|\pm\rangle \approx (|3\rangle \pm |2\rangle)/\sqrt{2}$. In this regime, coupling of the $|- \rangle$ state to the relevant transition channels is effectively suppressed. The QHE can then be

described by an effective cyclic three-state model with balanced gain and loss, and nearly equal populations in the donor and acceptor states (Supplementary Note 2 and Supplementary Fig. 4 and 5). As a result, the power output efficiency η saturates and remains stable despite variations in coherent coupling. This stability demonstrates topology-induced robustness through topological protection, as observed in non-Hermitian photonic and quantum systems.^{63–67}

Discussion

The TPT phenomenon, characterized by maximal coherence and strong topological protection, presents significant potential for coherently driven QHEs. Our analysis demonstrates that optimal performance occurs within a specific load window, $0.90\omega_{21} \lesssim V \lesssim 1.0\omega_{21}$ (Figs. 2 and 3 and Supplementary Fig. 1), where the critical Rabi frequency Ω_T is comparable to the intrinsic level spacing, specifically $\Omega_T \lesssim 0.1\omega_{32}$. This regime supports efficient donor-acceptor energy transfer. Outside this window, the engine cycle is disrupted: lower voltages cause excessive relaxation rates Γ that require impractically high driving amplitudes, while higher voltages restrict extraction from the excited state. Further analysis indicates that QHE efficiency can be improved without the need for ultrastrong driving fields by adjusting reservoir parameters. Reducing the hot-bath photon number and decay rate, accelerating reinitialization, and increasing the output energy gap together suppress dissipative broadening and stabilize the coherent transfer rate J_c . These results suggest that, instead of minimizing the energy gap, optimizing bath occupations and relaxation rates to reduce effective decoherence can also achieve TPT-enhanced performance under practical, low-power conditions. These findings open new avenues for investigating TPT in non-Hermitian quantum systems and for designing highly efficient, robust QHE devices based on bio-inspired energy conversion systems.

During the final stages of this work, we became aware of recent advancements in coherence-assisted QHEs by Scully and collaborators,⁶⁸ who investigated a photo-Carnot engine wherein

multilevel coherences primarily modulate the effective temperature of the cavity mode. In contrast, our study reveals a fundamentally distinct mechanism: a non-Hermitian topological effect in a photosystem II-inspired QHE. By leveraging a TPT, our work preserves high power-conversion efficiency despite significant reductions in donor-acceptor coherence—a resilience unattainable through thermal tuning alone. This distinction highlights the critical promise of topologically protected coherence as a controllable resource for enhancing quantum energy conversion.

Methods

To analyze the topological properties of donor-acceptor transport, we use the reduced non-Hermitian Hamiltonian $\hat{H}_c$ from Eq. (6). The spectral structure of this Hamiltonian determines steady-state observables by governing the system's long-term dynamics. By transforming to the Laplace domain with $s = \sigma + i\omega$ (where $\sigma \rightarrow 0^+$) and setting $\hat{A} = -i\hat{H}_c$, we recast the evolution equation as follows.

$$(s\hat{I} - \hat{A})\hat{\Psi}(s) = \psi(0), \quad (12)$$

where $\hat{\Psi}(s) = \int_0^\infty \psi(t)e^{-st} dt$. The characteristic polynomial is given by

$$\det(s\hat{I} - \hat{A}) = (s + \frac{\Gamma_{\text{eff}}}{2})(s + \frac{\Gamma}{2}) + \Omega^2 \equiv D(s) + \Omega^2, \quad (13)$$

This expression captures the balance between coherent exchange and dissipation, with the resolvent poles defined by $D(s) = -\Omega^2$.

Instead of analyzing these poles directly, we define a bounded scalar response function that integrates dissipative and coherent effects. Using the open-loop transfer function $L(s) = \Omega^2/D(s)$, we apply a Möbius transformation to define the response

$$r(s) = \frac{1 - L(s)}{1 + L(s)} = \frac{D(s) - \Omega^2}{D(s) + \Omega^2}. \quad (14)$$

Note that $r(s)$ maintains the analytic structure of the resolvent: its poles match those of $(s\hat{I} - \hat{A})^{-1}$, while its zeros, given by $D(s) = \Omega^2$, indicate coherence-dominated regimes. This approach offers three advantages: boundedness ($r(s) \rightarrow 1$ as $|s| \rightarrow \infty$, allowing closed Nyquist contours), unification (poles and zeros represent dissipation and coherence), and physical transparency ($r(s)$ acts as a reflection coefficient, with $r = 0$ marking a critical balance between coherent and dissipative rates). Thus, $r(s)$ provides a concise diagnostic for the interplay between coherence and dissipation.

The winding number ν , defined in Eq. (7), counts the zeros of $r(s)$ in the right-half-plane, which corresponds to the number of times the Nyquist contour $r(i\omega)$ encircles the origin. A TPT occurs when a zero crosses the imaginary axis. Setting $r(0) = 0$ gives the critical driving amplitude

$$\Omega_T = \frac{\sqrt{\Gamma\Gamma_{\text{eff}}}}{2}. \quad (15)$$

For $\Omega < \Omega_T$, all zeros are in $\text{Re}[s] < 0$, so $\nu = 0$, indicating a dissipation-dominated phase. For $\Omega > \Omega_T$, one zero moves to $\text{Re}[s] > 0$, resulting in $\nu = 1$, which marks a coherence-dominated phase with increased steady-state current and power. This Nyquist-based diagnostic connects microscopic spectral topology with macroscopic transport performance in the full Lindblad dynamics.

Data availability

The numerical source data generated in this study have been deposited through the integrated Figshare submission service and will be made publicly available upon publication.

Code availability

The Code used to generate the numerical data and reproduce the figures has been deposited through the integrated Code Ocean submission service and will be made publicly available upon publication.

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Acknowledgements

We are grateful to Ya-Ting Wei for his valuable suggestions. This work was supported by the National Key R&D Program under Grant No. 2024YFE0102400, the National Natural Science Foundation of China under Grants Nos. 12274470, 11935006, and 12421005, and the Hunan Major Sci-Tech Program under Grant No. 2023ZJ1010. H.Z. acknowledges support from the Fundamental Research Funds for the Central Universities of Central South University under Grant No. 1053320240539 and the Postgraduate Scientific Research Innovation Project of Hunan Province under Grant No. 2025ZZTS0117. The simulation was conducted using computing resources at the High Performance Computing Center of Central South University.

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C.C.S. and H.J. conceived and designed the project. H.Z. performed the analytical derivations and numerical simulations. Q.Q.H., X.X.J., and Y.X. discussed the results and commented on the manuscript. C.C.S. and H.Z. wrote the manuscript with input from all authors. C.C.S. supervised the project.

Competing interests

The authors declare no competing interests.

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