Supplemental Material for

Screened Copy-Time Transport at Cosmic Dawn: A Microscopic Onsager Mechanism for Early Galaxy Compaction and Black-Hole Seed Growth

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1 Fisher projection and the Schur complement

Let $g_A(Y)$ denote the score functions of the variables already evolved by a baseline radiation-hydrodynamic closure. The Fisher covariance matrix and the cross-covariance with the record score are

$$F_{AB} = \langle g_A g_B \rangle, \qquad b_A = \langle s_Q g_A \rangle. \tag{1}$$

On the retained score space,

$$s_Q^{\parallel} = g_A (F^{-1})^{AB} b_B, \qquad s_Q^{\perp} = s_Q - s_Q^{\parallel}. \tag{2}$$

The residual Fisher rate is therefore

$$\dot{\mathcal{I}}_Q^{\perp} = \dot{\mathcal{I}}_Q^{\mathrm{raw}} - b_A (F^{-1})^{AB} b_B. \tag{3}$$

This is the Schur complement of the full Fisher matrix and is nonnegative. In the analytical benchmark used in the main text, Eq. (3) is not evaluated as a dense operation at each time step. Its channel-level result is precomputed from the Poisson rate expansion and the covariance integral of the filtered power spectrum.

2 Variational origin of the logarithmic scalar

Let $P_0(Y)$ be the projected standard RHD record measure. Among measures absolutely continuous with respect to P_0, maximizing relative entropy at fixed mean copied charge gives

$$P_\psi(Y) = \exp[q_Q N_b \psi - \Lambda(\psi)] P_0(Y). \tag{4}$$

The intensive conjugate variable is the log-likelihood ratio per copied baryon,

$$\psi = \frac{1}{q_Q N_b} \log \frac{\mathrm{d}P_\psi}{\mathrm{d}P_0} + \frac{\Lambda(\psi)}{q_Q N_b}. \tag{5}$$

When record channels are independent, likelihood ratios multiply and their logarithms add. This is the physical reason for using a logarithm rather than a linear Fisher excess. The regularized expression in the main text,

$$\psi_\varepsilon = \log \frac{\dot{\mathcal{I}}_Q^{\parallel} + \dot{\mathcal{I}}_Q^{\perp} + \varepsilon \dot{\mathcal{I}}_*}{\dot{\mathcal{I}}_Q^{\parallel} + \varepsilon \dot{\mathcal{I}}_*}, \tag{6}$$

keeps the same likelihood-ratio meaning while encoding the finite shot-noise floor of a real receiver.

The entropy contribution follows from the Radon-Nikodym derivative of the coarse-grained measure,

$$\log \frac{\mathrm{d}\Gamma_\psi}{\mathrm{d}\Gamma_0} = \int q_Q n_b \psi \, \mathrm{d}^3 x - \Lambda[\psi]. \tag{7}$$

Terms independent of n_b do not enter the baryonic thermodynamic force. Hence

$$\frac{\delta S}{\delta n_b} = -\frac{\mu_B}{T} + q_Q \psi. \tag{8}$$

3 Finite receiver floor

The one-count receiver floor is the finite Fisher rate associated with one statistically distinguishable record during the causal cell-crossing time,

$$\dot{\mathcal{I}}_* = \frac{c}{\ell}. \tag{9}$$

For $\ell = 100$ pc, this gives $\ell/c = 326.16$ yr and $\dot{\mathcal{I}}_* = 9.72 \times 10^{-11}\,\mathrm{s}^{-1}$. At $z = 10$, the same cell contains 9.78×10^{57} baryons at the cosmic mean density and 1.96×10^{60} baryons at $200\bar{\rho}_b$. The regularization therefore has a definite finite-count meaning and is not a fitted amplitude.

4 General p-body coefficient

For a p-body channel

$$R = K \int_{V_\ell} n_1 n_2 \cdots n_p \,\mathrm{d}^3x, \tag{10}$$

write $n_a = \bar{n}_a(1 + \delta_a)$. At fixed means,

$$\frac{R}{R_0} = 1 + \sum_{a<b} C_{ab} + \sum_{a<b<c} C_{abc} + \cdots\ , \tag{11}$$

where $C_{ab} = \langle \delta_a \delta_b \rangle$. If

$$C_{ab} = \chi_{ab,2}\delta_L^2 + O(\delta_L^3), \tag{12}$$

then

$$\frac{\partial R^\perp}{\partial \delta_L} = 2R_0 \left(\sum_{a<b} \chi_{ab,2} \right) \delta_L + O(\delta_L^2). \tag{13}$$

The Poisson identity $\dot{\mathcal{I}} = (\partial R)^2/R$ gives

$$A_2 = 4R_0 \left(\sum_{a<b} \chi_{ab,2} \right)^2 . \tag{14}$$

For equal pair response $\chi_{ab,2} = \chi_2$,

$$A_2 = [p(p-1)\chi_2]^2 R_0. \tag{15}$$

For nonlinear chemistry the resolved covariance is supplemented by an unresolved covariance integral,

$$C^{\mathrm{sub}}_{ab,\ell} = \int_{k_{\mathrm{Ny}}(\ell)}^{k_{\mathrm{diss}}} \frac{\mathrm{d}k}{k}\, \Delta^2_{ab,\mathrm{sub}}(k). \tag{16}$$

A lognormal unresolved density closure gives the exact moment relation

$$\frac{\langle n^p \rangle}{\langle n \rangle^p} = \exp\left[\frac{1}{2} p(p-1)\sigma_s^2 \right]. \tag{17}$$

The effective rate and Fisher coefficient are therefore

$$R_0^{\mathrm{eff}} = \mathcal{C}_p R_0, \qquad A_2^{\mathrm{eff}} = \left[p(p-1)\chi_2^{\mathrm{eff}} \right]^2 R_0^{\mathrm{eff}}. \tag{18}$$

The table `subgrid_clumping_correction.csv` reports conservative multipliers for $p = 2$ and $p = 3$ channels as a function of turbulent Mach number and forcing parameter. These multipliers are independent of the JWST and X-ray observables and provide an error-control envelope for recombination, line cooling and three-body molecular formation.

5 Analytical $P(k)$ and mobility

The benchmark uses a CDM transfer function normalized to σ_8. With k in $h\,\mathrm{Mpc}^{-1}$,

$$P(k,z) = Ak^{n_s}T^2(k)D^2(z), \tag{19}$$

where A is fixed by

$$\sigma_8^2 = \int_0^\infty \frac{\mathrm{d}k}{k}\,\frac{k^3P(k,0)}{2\pi^2}W_{\mathrm{TH}}^2(8h^{-1}\mathrm{Mpc}\,k). \tag{20}$$

The filtered velocity power is

$$\Delta_u^2(k,z) = \left(\frac{aH(z)f(z)}{k}\right)^2\Delta_\delta^2(k,z). \tag{21}$$

The mobility is

$$D_B^{(\ell)} = \frac{1}{3}\int\frac{\mathrm{d}k}{k}\,\Delta_u^2(k,z)|W_\ell(k)|^2\tau(k,z), \tag{22}$$

with the decorrelation time bounded by the minimum of the Hubble, eddy and cooling times. This is the analytical replacement for treating unresolved turbulence as a free parameter.

6 Construction of $P(\mathcal{K})$

The halo-spin distribution is taken to be lognormal,

$$P(\lambda) = \frac{1}{\lambda\sigma_\lambda\sqrt{2\pi}}\exp\left[-\frac{(\ln\lambda - \ln\lambda_0)^2}{2\sigma_\lambda^2}\right], \tag{23}$$

with $\lambda_0 = 0.035$ and $\sigma_\lambda = 0.50$. The low-angular-momentum gas fraction is described by a beta density centered on

$$\bar{f}_{\mathrm{lowJ}}(\lambda) = \left[1 + \left(\frac{\lambda}{0.035}\right)^{1.8}\right]^{-1}. \tag{24}$$

The receiver-band factor is

$$W_{\mathrm{rec}}(x) = \frac{x^2}{(1+x^2)^2}, \qquad x = (M_h/M_{\mathrm{peak}})^{1/3}. \tag{25}$$

The compaction-feeding kernel used in the quadrature is

$$\mathcal{K} = f_{\mathrm{lowJ}}W_{\mathrm{rec}}\frac{D_B^{(\ell)}t_{\mathrm{dyn}}}{LL_\psi}\left[1 + \left(\frac{D_B^{(\ell)}}{L_\psi v_{\max}}\right)^2\right]^{-1}. \tag{26}$$

The probability density $P(\mathcal{K} \mid z)$ is then obtained by deterministic quadrature over M_h, λ and f_{lowJ}. The output files `pkappa_summary.csv` and `jwst_xray_predictions.csv` are generated by the script included in the package.

7 Observable scalings

The central reservoir equation over one local dynamical time is

$$\frac{\mathrm{d}M_c}{\mathrm{d}t} = \frac{\mathcal{K}}{t_{\mathrm{dyn}}} M_c. \tag{27}$$

It gives

$$M_c^{\mathrm{CT}}/M_c^0 = \dot{M}_{\mathrm{supply}}^{\mathrm{CT}}/\dot{M}_{\mathrm{supply}}^0 = N_H^{\mathrm{CT}}/N_H^0 = \mathrm{e}^{\mathcal{K}}. \tag{28}$$

If projected luminosity is conserved over the compaction step, the half-light radius scales as

$$R_e^{\mathrm{CT}}/R_e^0 = \mathrm{e}^{-\mathcal{K}/2}. \tag{29}$$

These relations are deliberately simple: they expose the prediction directly and make it easy to reject if the inferred population does not show the predicted joint shift in compactness, column density and feeding rate.

8 Lyapunov estimate

For the hyperbolic transport system

$$\partial_t n + \nabla \cdot J = 0, \tag{30}$$

$$\tau \partial_t J + J = -D\nabla n + qDn\nabla\psi, \tag{31}$$

with periodic boundaries, define

$$\mathcal{F} = \int \left[n \log \frac{n}{\bar{n}} - n + \bar{n} + \frac{\tau}{2nD}|J|^2 \right] \mathrm{d}^3x. \tag{32}$$

Differentiating in time, using the equations of motion, and integrating by parts gives

$$\frac{\mathrm{d}\mathcal{F}}{\mathrm{d}t} = -\int \frac{1}{nD}|J - qDn\nabla\psi|^2 \mathrm{d}^3x + \mathcal{R}_\psi, \tag{33}$$

where $\mathcal{R}_\psi$ contains terms with $\partial_t\psi$ and second derivatives of ψ. If ψ is fixed, $\mathcal{R}_\psi = 0$. If ψ varies slowly, $|\mathcal{R}_\psi| \leq C_\psi \mathcal{F}$ by boundedness of ψ and standard convexity estimates. This proves the stability statement in the main text. The 1D finite-volume test included with the package verifies mass conservation and decay of $\mathcal{F}$ for a nonlinear density profile.

The linear self-gravitating dispersion relation is obtained by combining continuity, Euler, Poisson and Maxwell-Cattaneo relaxation. With $\omega_J^2 = c_s^2 k^2 - 4\pi G\rho_0$, one obtains

$$\tau s^3 + s^2 + (\tau\omega_J^2 + Dk^2)s + \omega_J^2 = 0. \tag{34}$$

For $\omega_J^2 > 0$, all coefficients are positive and the Routh-Hurwitz inequality reduces to $Dk^2 > 0$, so Jeans-stable modes remain linearly stable. For $\omega_J^2 < 0$, the ordinary Jeans mode remains and is not a new CT instability. The table `jeans_ct_stability_grid.csv` gives the maximum real root over the displayed parameter domain.

9 Numerical feasibility

The analytical projection reduces the algorithm to local operations:

1. evaluate channel rates $R_r(n, T, x_e, Z)$;
2. interpolate the precomputed structural coefficients $A_{2,r}$;
3. compute ψ_ε and its face gradients;
4. update the relaxation current;
5. add mass, momentum and enthalpy fluxes conservatively.

The operation count scales as $O(N_{\rm cell}N_r)$. No global Fisher matrix is inverted during a timestep. For N_r retained channels, a conservative memory model is

$$B_{\rm cell} \simeq 8(18 + 6N_r)\ \text{bytes}. \tag{35}$$

For $N_r = 7$, this is 480 bytes per cell. The vectorized CPU benchmark records 2.1×10^7 updates s^{-1}; a conservative 1 TB s^{-1} accelerator roofline at 15 percent efficiency gives 3.1×10^8 updates s^{-1}. The latter is a feasibility roofline, not a hardware measurement. The point needed for the theory is that the projected Fisher operation is precomputed into local tables, so the timestep update has the same locality class as standard finite-volume thermochemistry.

10 Population closure and finite-band angular-momentum turnover

For a required multiplicative increase $F_{\rm req}$ in central gas, supply rate or obscuring column, the analytical distribution gives

$$f_{\rm cap}(F_{\rm req}, z) = \int_{\log F_{\rm req}}^{\infty} P(\mathcal{K} \mid z)\, \mathrm{d}\mathcal{K}. \tag{36}$$

This expression is the population-level closure used in the main text. It is independent of the observed abundance of compact sources. The file `population_closure_thresholds.csv` tabulates this fraction for $F_{\rm req} = 1.25, 1.5, 1.75, 2, 2.5, 3$ at $z = 8, 10, 14$.

The spin discriminant follows from separating the standard low-angular-momentum reservoir from the finite-band receiver overlap. A monotone supply model scales approximately as

$$S_{\rm std}(\lambda) \propto f_{\rm lowJ}(\lambda), \tag{37}$$

whereas the CT contribution scales as

$$S_{\rm CT}(\lambda) \propto f_{\rm lowJ}(\lambda) \frac{x_\lambda^2}{(1 + x_\lambda^2)^2}, \qquad x_\lambda = \frac{\lambda_{\rm peak}}{\lambda}. \tag{38}$$

The second factor forces a peak at finite spin; the tabulated curve is included with the supplementary data. A purely monotone trend in compactness and feeding with decreasing spin would therefore not be the predicted CT signature.

11 Closed-form black-hole feeding response

Let the unresolved accretion closure be

$$\dot{M}_{\rm BH} = \min[\dot{M}_{\rm Edd}, \epsilon_{\rm cap}\dot{M}_{\rm supply}]. \tag{39}$$

Since CT changes the supply by $\dot{M}^{\rm CT}_{\rm supply} = \dot{M}^{0}_{\rm supply} e^{\mathcal{K}}$, the exact response is

$$\frac{\dot{M}^{\rm CT}_{\rm BH}}{\dot{M}^{0}_{\rm BH}} = \frac{\min[\dot{M}_{\rm Edd}, \epsilon_{\rm cap}\dot{M}^{0}_{\rm supply} e^{\mathcal{K}}]}{\min[\dot{M}_{\rm Edd}, \epsilon_{\rm cap}\dot{M}^{0}_{\rm supply}]}. \tag{40}$$

Thus the enhancement is $e^{\mathcal{K}}$ only while the source is supply-limited. When it is already Eddington-limited, the instantaneous accretion rate saturates, but the obscuring column and central reservoir are still increased. This distinction prevents the mechanism from being interpreted as an imposed super-Eddington law.

The file `semi_analytic_baseline_vs_ct.csv` applies this closure to a baseline supply-limited Eddington ratio $f_{\rm Edd} = 0.32$, a $10^5\,M_\odot$ seed at $z = 20$, and a 45 Myr Salpeter time. The CT curve modifies only the supply-limited duty cycle through $\exp \mathcal{K}(z)$ and is capped at Eddington. By $z = 8$, the CT history is 8.6 times larger than the baseline while remaining governed by the imposed Eddington cap whenever the cap is active.

12 Coupled conservative update

In a finite-volume discretization, the CT mass flux through face $i + 1/2$ is added to the usual advective flux. The corresponding momentum and enthalpy increments are

$$F^{\rm CT}_{\rho} = J^{\rm CT}_{i+1/2}, \tag{41}$$

$$F^{\rm CT}_{\rho u} = J^{\rm CT}_{i+1/2} u_{i+1/2}, \tag{42}$$

$$F^{\rm CT}_{E} = J^{\rm CT}_{i+1/2} \left(h_{i+1/2} + \frac{1}{2} u^2_{i+1/2} \right). \tag{43}$$

After the density update the gravitational potential is recomputed from Poisson's equation. The CT operator therefore redistributes baryons and their carried momentum/enthalpy; it does not create a new mass or energy source.

13 Reduced finite-volume benchmark and catalogue comparison files

The reduced benchmark in the main text is generated by `scripts/run_reduced_benchmark_and_catalog.py`. It solves the conservative radial update

$$M^{n+1}_i = M^n_i - \Delta t (F^n_{i+1/2} - F^n_{i-1/2}) \tag{44}$$

with upwind face fluxes and a bounded inward CT drift. The output files are

- `data/reduced_simulation/reduced_1d_history.csv`;
- `data/reduced_simulation/reduced_1d_summary_ratios.csv`;
- `data/reduced_simulation/reduced_1d_parameters.json`.

The benchmark is a local finite-reservoir calculation that demonstrates early central delivery and strict conservation in a controlled setting. The published comparison table is stored in `data/published_compariso`
It records the observational quantities used in Table 3 of the main text and appends the CT upper-tail factors evaluated from the analytical kernel at the same redshift.

A second reduced benchmark is generated by `scripts/run_2d_selfgravity_and_catalog_stats.py`. It evolves a two-dimensional thin gas sheet with periodic boundaries. Poisson's equation is solved spectrally at every step,

$$\nabla^2_\perp \Phi = 4\pi G(\Sigma - \bar{\Sigma})/H, \tag{45}$$

and the continuity equation is updated with upwind finite-volume fluxes, pressure regularization, a bounded CT drift and a central conservative sink. The script writes

- `data/reduced_2d_selfgravity/reduced_2d_selfgravity_history.csv`;
- `data/reduced_2d_selfgravity/reduced_2d_selfgravity_summary_ratios.csv`;
- `data/reduced_2d_selfgravity/reduced_2d_selfgravity_parameters.json`;
- `data/published_comparison/published_jwst_xray_statistical_catalogue.csv`.

The catalogue likelihood is a reproducible directional statistic. Its purpose is to test whether the analytical CT displacement moves compactness, obscuration, and black-hole-mass entries in the observed direction under a fixed set of coefficients.

The figure files `fig14`–`fig22` are generated from these tables. They provide reproducible one-dimensional and two-dimensional reduced simulations and a systematic catalogue comparison using the same theoretical coefficients throughout.

14 Standard simulation baseline overlay

The main text treats BlueTides, THESAN and FLARES as published baselines rather than as calibration data. The file

`data/standard_simulation_baselines/blue_tides_thesan_flares_ct_overlay.csv`

records the numerical residuals applied to those baselines. For the fiducial upper-tail point at $z \simeq 8$, the overlay uses $\Delta \log_{10} N_H = \Delta \log_{10} \dot{M}_{\rm supply} = 0.333$ and $\Delta \log_{10} R_e = -0.166$, obtained directly from $\mathcal{K}_{90} = 0.766$ in the main text. The comparison is therefore reproducible from the analytical prediction and does not require retuning the published simulation sub-grid models.

15 Reproducibility files

The submitted package contains the manuscript, this supplement, figures, analytical tables, one-dimensional and two-dimensional reduced-simulation outputs, published JWST/X-ray comparison tables, catalogue statistics, and reproducibility scripts:

- `code/analysis/analytical_pk_benchmark.py`;
- `code/analysis/complete_solution_extensions.py`.

The tables include the analytical power spectrum and receiver window, the distribution $P(\mathcal{K} \mid z)$, observable scaling factors, the JWST/X-ray prediction table, the reduced one-dimensional and two-dimensional baseline-versus-CT histories, the published-catalogue comparison tables, the catalogue likelihood summary, the nonlinear stability test, the feasibility benchmark, the population-closure thresholds and the finite-band spin-turnover signature.