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Research Article

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Posted Date: May 4th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-9569526/v1>

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Additional Declarations: The authors declare no competing interests.

Low-redshift BAO without dark components: testing the TCR-Cosmo model

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Abstract

We present TCR-Cosmo, a cosmological model complementary to Λ CDM, constructed from the relational and pre-spatiotemporal ontological foundations of the "Teoria della Coesistenza Relazionale" (TCR). TCR-Cosmo does not require dark energy to describe the low-redshift universe: the observational dynamics arise directly from the relational lapse and void growth. Using the same official datasets employed in Λ CDM analyses, we test the model with cosmic chronometers (CC), type Ia supernovae (SN), and baryon acoustic oscillations (BAO). The results show that TCR-Cosmo achieves a global fit comparable to that of Λ CDM and slightly better for low-redshift BAO measurements.

Keywords: Cosmological model, Cosmic expansion, Dark energy, Λ CDM, Hubble tension, Baryon Acoustic Oscillations (BAO), Cosmic Chronometers

1 Introduction

Modern cosmology is grounded in the Λ CDM model, which combines general relativity with an energy content dominated by cold dark matter and a cosmological constant Λ [1, 2]. This framework has achieved remarkable success in reproducing the anisotropy of the cosmic microwave background (CMB), the formation of large-scale structures, and the measurements of baryon acoustic oscillations (BAO) [3, 4]. However, its validity relies on entities that have not been directly observed, namely dark matter and dark energy, which account for about 95% of the universe's energy budget. Furthermore, persistent tensions, such as the discrepancy between local and global determinations of the Hubble parameter H_0 [5, 6], raise doubts about the completeness of the standard paradigm.

Over the past decades, several alternatives have been explored in an attempt to explain cosmic acceleration and structure formation without invoking dark components, including backreaction models [7, 8], inhomogeneous cosmologies, and emergent approaches.

Among these, the *Timescape* model [9, 10] has represented a significant step in interpreting acceleration as an emergent effect of gravitational inhomogeneity and the role of average curvature, rather than as the manifestation of a mysterious “dark energy.”

In this work, we quantitatively test TCR-Cosmo against the precision datasets employed in Λ CDM, including type Ia supernovae (Pantheon+SH0ES), cosmic chronometers (CC), and compressed BAO DR12 measurements. We compare the results with those of the standard model, evaluating the differences in χ^2 contributions, in the AIC/BIC information criteria, and in the fit to BAO and CC data.

Within the TCR-Cosmo framework, the Hubble tension is not interpreted as a discrepancy between incompatible measurements, but as a consequence of the dependence of the expansion rate on the relational context, and therefore also on redshift. In the standard Λ CDM model, H_0 is assumed to be a global constant; by contrast, in TCR-Cosmo it emerges from the relational expansion factor $T(z)$ and reflects the degree of actualization of the observed system. As a result, the quantity identified as H_0 is not universal, but varies with the physical context and observational scale. In this perspective, different measurements of H_0 probe distinct relational regimes of the universe, and the observed tension naturally arises from the non-homogeneous and emergent nature of space-time.

The present work broadly follows the IMRaD structure (Introduction, Methods, Results, and Discussion):

- Section 1 (Introduction): defines the central problem of modern cosmology, the reliance of Λ CDM on unobserved dark components, and motivates the need for a relational and pre-spatiotemporal framework (TCR) as an alternative.
- Section 2 (Methods): details the implementation of TCR-Cosmo, introducing the relational ontology R' , the function Ω , the transformation Π , and the operational role of the relational lapse $\gamma(z)$. It also specifies the datasets (SN, CC, BAO), the pre-processing choices, and the statistical comparison criteria (AIC/BIC).
- Section 3 (Results): presents the quantitative fits of TCR-Cosmo against SN, CC, and BAO data, analyzes the behavior of the relational lapse and void fractions, compares the sound horizon predictions, and evaluates the relative performance with respect to Λ CDM in terms of χ^2 and information criteria.
- Section 4 (Discussion): assesses the theoretical significance of a pre-spatiotemporal model reproducing low-redshift data without dark components, clarifies the ontological differences with Λ CDM, examines the robustness of the fits given that the datasets are calibrated on Λ CDM pipelines, discusses the predictions for cosmic chronometers (CC) and for the contextual nature of H_0 , and outlines both the limitations (e.g., high- z regime, phenomenological parametrization) and future prospects of the approach.

2 Method

2.1 Why we do not use Einstein’s equations (theoretical choice)

Einstein’s field equations (EFE) describe with extraordinary accuracy the dynamics of geometry and energy within a pre-given space-time. However, they are not designed to define how space and time themselves emerge or are generated. Since the goal of this work is to investigate the origin and evolution of cosmic expansion as an emergent phenomenon from a primordial relational domain R' (with the mapping $\Pi : R' \rightarrow R$), in analogy with other emergent space-time approaches [11, 12], we do not solve the EFE to obtain $H(z)$.

This choice avoids double counting (we do not impose a geometric dynamics designed for the *intra-spatiotemporal* regime to explain a *pre-spatiotemporal* phenomenon) and makes explicit the generative assumptions of TCR. In the limit in which relational degrees of freedom are frozen ($dS_{\text{rel}}/d\tau \rightarrow \text{homogeneous}$ and $\gamma_{\text{rel}} \rightarrow 1$), the standard FLRW kinematics is recovered: for this reason we retain the usual metric definitions for observable distances and the use of *early* constraints (sound horizon, prior on θ_*) as a contact with well-tested physics.

2.2 Epistemological and ethical note

The TCR proposal is explicitly built to be falsifiable: (i) it recovers the Λ CDM limit when relational degrees freeze, (ii) it produces quantitative predictions for $H(z)$, for D_M/r_d and Hr_d at low redshift, and (iii) it is tested against information criteria (AIC/BIC) as well as χ^2 on SN/CC/BAO. To avoid overstatements, we adopt the principle of parsimony: the elimination of dark energy is accepted only if the fit is competitive and robust under sensitivity tests (alternative covariances, leave-one-out, variations in SN calibration and in r_d). Finally, to guarantee reproducibility and transparency, we explicitly report all pre-processing choices, provide parameters and seeds via CLI, and release the scripts/commits necessary to regenerate the figures; whenever systematic tensions emerge, they are reported openly without retrofitting the model a posteriori.

2.3 Specification of the TCR–Cosmo model

$$E(z) = \gamma_{\text{rel}}(z; \theta) E_{\text{std}}(z), \quad \gamma_{\text{rel}}(0) = 1. \quad (1)$$

The limit $\Delta H_{\text{rel}} \rightarrow 0$ (or $\gamma_{\text{rel}} \rightarrow 1$) reproduces Λ CDM. The free parameters θ include TCR-late degrees and possible nuisances (e.g. the SN calibration M).

2.4 Observables and theory–data mapping

Distances:

$$D_C(z) = c \int_0^z \frac{dz'}{H(z')} \quad (2)$$

$$D_M = D_C, \quad (3)$$

$$D_L = (1 + z)D_M, \quad (4)$$

$$\mu(z) = 5 \log_{10}(D_L/\text{Mpc}) + 25. \quad (5)$$

Compressed BAO: $\mathbf{d}(z) = (D_M/r_d, H r_d)$ (or D_V). Cosmic chronometers: direct use of $H(z)$. (If employed) CMB constraint through θ_* as a weak prior.

2.5 Datasets and pre-processing

For CC we use the covariance matrices released by M. Moresco and collaborators (CCcovariance) available on GitLab [13]. In particular, we employed the table `data_MM20.dat` as the source of covariances (`--cc-mode fullcov`, `--cc-cov-tab data_MM20.dat`) in combination with the catalog `HzTable_MM_BC03.dat`. This choice enables a more faithful estimate of χ_{CC}^2 , taking into account correlations among points and shared systematics.

For compressed BAO DR12 we use the vectors `BAO_consensus_results_dM_Hz.txt` and the corresponding covariance `BAO_consensus_covtot_dM_Hz.txt`. For Pantheon+SH0ES SN we use the full covariance (`Pantheon+SH0ES_STAT+SYS.cov`) with analytical marginalization over $\Delta\mu$.

2.6 Likelihood and statistical inference

$$\chi_{\text{SN}}^2 = (\Delta\mu)^T C_{\text{SN}}^{-1} (\Delta\mu) \text{ (with analytical marginalization over } M), \quad (6)$$

$$\chi_{\text{BAO}}^2 = (\Delta\mathbf{d})^T C_{\text{BAO}}^{-1} (\Delta\mathbf{d}). \quad (7)$$

For cosmic chronometers (CC), this implementation adopts a diagonal approximation, computing a weighted sum point by point:

$$\chi_{\text{CC}}^2 = (\Delta H)^T C_{\text{CC}}^{-1} (\Delta H), \quad (8)$$

The total combination is

$$\chi_{\text{tot}}^2 = \chi_{\text{SN}}^2 + \chi_{\text{CC}}^2 + \chi_{\text{BAO}}^2. \quad (9)$$

Wide and physically motivated priors on θ ; r_d can be free or anchored through a scaling factor α_{rs} .

2.7 Early module and sound horizon

The *early* sector uses T_{CMB} , ω_b , ω_c , N_{eff} and h to define

$$E_{\text{early}}^{\text{std}}(z) = \sqrt{\Omega_r(1+z)^4 + \Omega_m(1+z)^3 + \Omega_\Lambda}. \quad (10)$$

The baryonic sound speed is

$$c_s(z) = \frac{c}{\sqrt{3[1 + R_b(z)]}} \quad (11)$$

$$R_b(z) \propto \omega_b(1+z)^{-1}. \quad (12)$$

The comoving sound horizon r_s is obtained by integrating $c_s/H(z)$ from a reference redshift

$$z_{\text{ref}} \in \{z_*, z_{\text{drag}}\}, \quad r_s = \alpha_{rs} \int_{z_{\text{ref}}}^{\infty} \frac{c_s(z)}{H(z)} dz \quad (13)$$

with $\alpha_{rs} = 1$ in this analysis.

For numerical comparison we adopt the following convention:

- for the CMB prior, in both models r_s is evaluated at z_* (recombination);
- for BAO normalization, in both Λ CDM and TCR-Cosmo r_s is evaluated at z_{drag} (baryon drag epoch).

This distinction reflects the pipeline of the code: the CMB constraint uses $r_s(z_*)$, whereas the compressed BAO vectors are constructed with $r_s(z_{\text{drag}})$.

2.8 Baseline Λ CDM and comparison

For dataset-matched comparison, a flat Λ CDM baseline with standard parameters is evaluated and $\Delta\chi^2$ and information criteria are reported.

2.9 Numerical implementation

1D integrations with tight tolerances ($\varepsilon_{\text{abs}}, \varepsilon_{\text{rel}} \sim 10^{-8}$ – 10^{-6}) and domain splitting in z for robustness. Consistent unit handling with `astropy.units` and physical constants from `astropy.constants`.

2.10 Model comparison

AIC = $\chi_{\text{tot}}^2 + 2k$, BIC = $\chi_{\text{tot}}^2 + k \ln N$, with k the number of free parameters and N the effective data points. Where appropriate, Bayesian evidence is also reported.

2.11 Robustness and sensitivity

Jackknife/leave-one-out on BAO and CC; removal/addition of the prior on r_d or θ_* ; covariance variants; study of degeneracies (H_0 , r_d , TCR-late parameters).

2.12 Purpose and datasets

This appendix documents the numerical implementation of the TCR - COSMO and its comparison with Λ CDM, using the following datasets:

- Supernovae (SN): Pantheon+SH0ES with full covariance (STAT+SYS) and analytical marginalization over $\Delta\mu$ [4].
- BAO: DR12 “consensus” in compressed form (d_M/r_s , $H r_s$), normalized to the fiducial dataset value [3].
- Cosmic chronometers (CC): direct estimates of $H(z)$ [14].
- CMB prior: constraint on $100\theta_*$ computed with early-time FRW dynamics *consistent with each model*; deactivated in the runs reported here [2].

Within the TCR - COSMO framework, apparent acceleration is not due to a component of *dark energy*, but to a *relational lapse* $\gamma(z)$ induced by the void/wall network and damped by collapsed structures.

2.13 Reproducibility

Execution script (CLI).

To regenerate the main results one can use the following command, which specifies datasets, covariances, and late-time parameters of the TCR run:

```
python3 tcr_run.py \  
  --sn Pantheon+SH0ES.dat \  
  --sn-cov Pantheon+SH0ES_STAT+SYS.cov \  
  --cc HzTable_MM_BC03.dat \  
  --cc-mode fullcov \  
  --cc-cov-tab data_MM20.dat \  
  --bao-results BAO_consensus_results_dM_Hz.txt \  
  --bao-cov BAO_consensus_covtot_dM_Hz.txt \  
  --Om-late 0.295 --H0-tcr 66.8 \  
  --out final_single.csv \  
  --show
```

The code and updated instructions are available at the repository: <https://github.com/pietro80/TCR-Cosmo>.

The code version used in this work is 1.6.

3 Results

3.1 Relational entropic time and geometric time

From Postulate 6 of the *Teoria della Coesistenza Relazionale* [15], the emergence of time arises as a direct consequence of the variation of relational entropy:

$$\frac{dt}{d\tau} \propto \frac{dS_{\text{rel}}}{d\tau}. \quad (14)$$

From this relation, one can derive the projected form of the entropic derivative, actualized in the spatiotemporal domain R :

$$\gamma(z) \sim \frac{dS_R}{dz} \propto \frac{dS_{\text{rel}}}{d\tau}. \quad (15)$$

The function $\gamma(z)$ represents the factor linking the “bare” time t_b (that of the mean FLRW model, without relational effects) to the effectively observed time t :

$$dt = \gamma(z) dt_b. \quad (16)$$

We can now define the *relational expansion factor* $T(z)$, which expresses the rate at which the cosmos becomes actualized in R starting from the relational domain R' :

$$T(z) \propto \frac{\gamma(z)}{1 + \text{directional entropic variation}}. \quad (17)$$

In derivative form, for the cosmological model, the function $T(z)$ becomes:

$$T(z) = \frac{\gamma(z)}{1 + (1+z) \frac{\gamma'(z)}{\gamma(z)}}. \quad (18)$$

where the relational entropic gradient is given by:

$$(1+z) \frac{\gamma'(z)}{\gamma(z)}.$$

Let us now compare the relational time of TCR with the standard Hubble time:

$$t_H = \frac{1}{H_0}.$$

In the TCR framework we obtain instead:

$$t_{\text{TCR}}(z) = \frac{1}{T(z) H_0}, \quad (19)$$

where $T(z)$ acts as a relational corrective factor with respect to the geometric reference time of Λ CDM.

In this framework, the concept of a unique age of the universe loses its meaning, as it becomes a contextual quantity depending on the degree to which the local space-time region has departed from the original state of perfect relational coexistence.

The standard Λ CDM form, with physical units fixed by H_0 , is $H(z) = H_0 E(z)$, while in TCR-Cosmo, introducing the relational entropic factor $T(z)$, it becomes:

$$H_{\text{obs}}(z) = T(z) H_0 E_{\text{bare}}(z),$$

and therefore:

$$H_{\text{obs}}(z) = T(z) H_{\text{bare}}(z). \quad (20)$$

This relation defines the observed expansion rate $H_{\text{obs}}(z)$ as the product between the “bare” rate $H_{\text{bare}}(z)$ and the relational expansion factor $T(z)$, which encapsulates the entropic and structural effects of coexistence. $H_{\text{bare}}(z)$ represents the expansion of a universe treated as perfectly homogeneous and isotropic on all scales.

- t — effective time observed in the actualized domain R .
- t_b — “bare” time, associated with the mean FLRW dynamics without relational effects.
- τ — primordial relational parameter in R' , source of the observed temporality.
- S_{rel} — relational entropy in the pre-spatiotemporal domain R' .
- $\gamma(z)$ — relational lapse factor, proportional to the entropic variation projected into spacetime.
- $T(z)$ — relational expansion factor, expressing the rate of actualization of the inhomogeneous cosmos in R relative to the homogeneous reference.
- $H_{\text{bare}}(z)$ — expansion rate of an ideally homogeneous universe.
- $H_{\text{obs}}(z)$ — observed expansion rate, enhanced by relational effects.

3.2 Relational lapse and observed expansion rate

From the *Teoria della Coesistenza Relazionale* [15], the relational entropy is defined as

$$S_{\text{rel}}(\tau) = - \sum_{i < j} \Delta \Omega_{ij}(\tau), \quad \Omega_{ij}(\tau) \in [0, 1]. \quad (21)$$

Since for $\Omega = 1$ the distance between two entities in R can be considered null, while for $\Omega = 0$ it diverges, one can introduce a function $\Omega(r)$ describing coexistence as a function of the relational distance r :

$$\Omega(r) \simeq e^{-r/\ell}, \quad (22)$$

where ℓ represents a characteristic coexistence length.

From this expression, the *spatial sensitivity* is obtained as

$$\frac{dr}{d\Omega} = \frac{\ell}{\Omega}. \quad (23)$$

For small values of Ω (i.e. under conditions of low coexistence), the sensitivity diverges: even a minimal variation in Ω produces a large dilation in r . Deep cosmic voids correspond precisely to this condition of almost vanishing coexistence. As a consequence, such regions amplify the production of space-time, that is, the rate defined in Eq. 15.

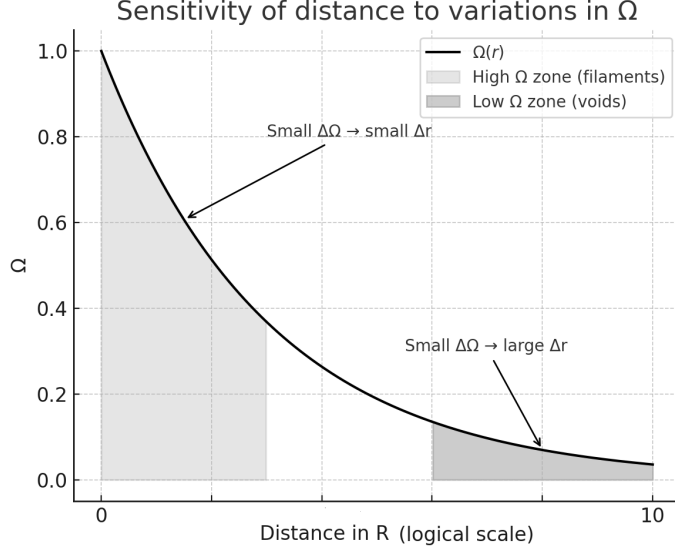


Fig. 1 Conceptual domains of influence of the fundamental interactions on the function $\Omega(r)$.

In the TCR-Cosmo code, this behavior is implemented as follows:

- the fraction of deep voids $f_{v,\text{deep}}$ enters quadratically (in the power mean with $\xi = 2$), assigning a *quadratic weight* to the regions where Ω is lowest;
- the *lapse* grows proportionally to $[f_v^{\text{eff}}(z)]^p$ ($p \simeq 1$), implying that a larger volumetric fraction of voids produces a more pronounced relational dilation;
- the terms w_h^3 and w_{gal}^3 act as damping factors, since in those regions $\Omega \approx 1$ and the derivative $\frac{dr}{d\Omega}$ takes smaller values.

In the limit $\Omega \rightarrow 0$ (deep voids) one obtains:

$$\frac{dr}{d\Omega} \rightarrow \infty \implies \gamma(z) \uparrow \Rightarrow T(z) \uparrow \Rightarrow H_{\text{obs}}(z) \gg H_{\text{bare}}(z).$$

Consequently, even a small fraction of supervoids can induce a significant variation in the observed expansion rate, despite occupying a relatively modest overall volume.

The functional form of $\gamma(z)$ includes coherent damping terms (same mass fractions, no double counting):

$$\gamma(z) = 1 + A [f_v^{\text{eff}}(z)]^p \left[1 - w_h(z)^q - w_{\text{gal}}(z)^r \right], \quad q = r = 3. \quad (24)$$

- $\gamma(z)$ — *relational lapse*, representing the effective reduction or dilation of the observed expansion rate due to the cosmic void-wall network.
- A — amplitude parameter controlling the overall strength of the relational correction.
- $f_v^{\text{eff}}(z)$ — effective volumetric fraction of voids, combining the deep and shallow components as defined in Eq. 29.

- p — exponent controlling the non-linear sensitivity of $\gamma(z)$ to the void fraction.
- $w_h(z)$ — fractional mass contribution of black holes, acting as a coherent damping term on the expansion.
- $w_{gal}(z)$ — fractional mass contribution of galaxies within walls, introducing an additional damping effect.
- q, r — fixed exponents ($q = r = 3$) describing the scaling behavior of the damping terms associated with w_h and w_{gal} .

3.3 Implemented theoretical structure

3.3.1 Early-time reinterpretation in a TCR perspective

The primordial phase is re-read as a period of *loss of relational coexistence*. In the code it is modeled by a smooth window in $\log(1+z)$ (an “inflationary band”), with an optional logistic dressing near decoupling. The sound horizon r_s is computed along the selected early branch (standard for Λ CDM, TCR for TCR - COSMO) and *without* additional rescalings: we assume $\alpha_{rs} = 1$ (no s_{rd}).

3.4 Implementation changes in the model

For a transparent comparison in the SN+BAO+CC analyses, we adopt the following choices:

1. Early TCR - COSMO (baryons-only). Absence of primordial CDM ($\omega_c = 0$) and standard plasma physics with N_{eff} .
2. Late-time parameters fixed by the run. In the TCR case: $H_0 = 66.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and $\Omega_{m,\text{late}} = 0.295$ (options `--H0-tcr`, `--Om-late`).
3. Optional CMB prior. The constraint on $100\theta_*$ is available but not included in the totals reported here (low- z analysis).
4. Sound horizon without free factors. r_s computed from the model’s early branch with $\alpha_{rs} = 1$, without additional rescalings.

With these choices we obtain $r_s^{\text{ref}} \simeq 146.972 \text{ Mpc}$ for Λ CDM (from z_{drag} for BAO normalization) and $r_s^{\text{ref}} \simeq 135.218 \text{ Mpc}$ for TCR - COSMO. For the CMB prior, when included, both models use $r_s(z_*)$.

A distinctive feature of the TCR-Cosmo implementation is the prediction of a sound horizon r_s lower than the Λ CDM value. In the standard framework, r_s is set by the dynamics of the primordial plasma including cold dark matter and dark energy, whereas in TCR it is determined by relational modifications to the primordial expansion rate. In particular, the Hubble function in the early regime is given by

$$H_{\text{TCR}}^{\text{early}}(z) = \frac{\gamma_{\text{infl}}(z) \gamma_{\text{early}}(z) H_{\text{bare}}(z)}{1 + (1+z) \frac{d}{dz} [\gamma_{\text{infl}}(z) \gamma_{\text{early}}(z)]}, \quad (25)$$

where $\gamma_{\text{infl}}(z)$ describes the relational inflationary band and $\gamma_{\text{early}}(z)$ introduces a dressing near recombination. Both functions follow directly from the TCR ontological structure and replace the role of dark components in shaping the primordial dynamics.

The comoving sound horizon is then obtained from the usual integration

$$r_s = \int_{z_{\text{ref}}}^{\infty} \frac{c_s(z)}{H_{\text{TCR}}^{\text{early}}(z)} dz, \quad (26)$$

with $z_{\text{ref}} = z_*$ for the CMB prior and $z_{\text{ref}} = z_{\text{drag}}$ for BAO normalization, and

$$c_s(z) = \frac{c}{\sqrt{3(1 + R_b(z))}}$$

the baryonic sound speed. Since the effective expansion $H_{\text{TCR}}^{\text{early}}(z)$ is reduced with respect to Λ CDM over the relevant redshift interval, the resulting r_s is smaller.

Consequently, the improved agreement of TCR–Cosmo with low-redshift BAO is not due to an arbitrary rescaling of r_s , but emerges directly as a prediction of the relational ontology. This ensures that the better BAO fit is a genuine result of the model rather than a tuning of the sound horizon.

3.4.1 Bare dynamics and effective curvature

$$H_{\text{bare}}(z) = H_0 \sqrt{\Omega_m(1+z)^3 + \Omega_{k,\text{eff}}(1+z)^2 + (1 - \Omega_m - \Omega_{k,\text{eff}})}. \quad (27)$$

The late-time parameters are those set by the run (for TCR: $H_0 = 66.8$, $\Omega_m = 0.295$, $\Omega_{k,\text{eff}} = 0.00$).

3.5 Evolution and “mass transfer” within walls

3.5.1 Normalization constraint

Conceptually, within walls (filaments + sheets) we impose:

$$w_h(z) + w_{\text{gal}}(z) + w_{\text{rest}}(z) = 1 \quad (28)$$

where w_h is the fraction in black holes (primordial + stellar), w_{gal} the fraction in galaxies, and w_{rest} the remaining diffuse component.

3.5.2 Phenomenological parametrization

In the code the combined effect is summarized in the function $\gamma(z)$ (24), where the effective volumetric fraction of voids is a power mean of two families,

$$f_v^{\text{eff}}(z) = \left[w_d(z) (f_{v,\text{deep}}(z))^{\xi} + (1 - w_d(z)) (f_{v,\text{shallow}}(z))^{\xi} \right]^{1/\xi}, \quad \xi = 2. \quad (29)$$

3.5.3 Choice used in this analysis

For the functions we use a decreasing logistic form in z (numerically stable):

$$f_{v,\text{deep}}(z) = \frac{f_{v0,\text{deep}}}{1 + \exp[\beta_{\text{void}}(z - z_{t,\text{void}})]}, \quad f_{v,\text{shallow}}(z) = \frac{f_{v0,\text{shallow}}}{1 + \exp[\beta_{\text{void}}(z - z_{t,\text{void}})]}, \quad (30)$$

$$w_{\text{gal}}(z) = \frac{w_{\text{gal},0}}{1 + \exp[\beta_{\text{gal}}(z - z_{t,\text{gal}})]}, \quad w_h(z) = w_{h,0} \quad (\text{constant}). \quad (31)$$

With typical values: $f_{v0,\text{deep}} = 0.15$, $f_{v0,\text{shallow}} = 0.55$, $z_{t,\text{void}} = 0.7$, $\beta_{\text{void}} = 3.0$; $w_{\text{gal},0} = 0.045$, $z_{t,\text{gal}} = 0.7$, $\beta_{\text{gal}} = 4.0$; $w_{h,0} = 0.01$. The dynamic weight of the mix is logistic:

$$w_d(z) = w_1 + \frac{w_0 - w_1}{1 + \exp[-\beta_{\text{mix}}(z_{t,\text{mix}} - z)]}, \quad (32)$$

with $z_{t,\text{mix}} = 0.75$ and $\beta_{\text{mix}} = 3.2$. The lapse parameters used in the fits remain $A = 0.29$, $p = 0.94$.

3.6 Bimodal voids and effective volumetric fraction

3.6.1 Sigmoids for the two families

The two families (deep voids and shallow voids) are described by decreasing logistics in z that tend to $f_{v0,\text{deep}}$ and $f_{v0,\text{shallow}}$ at $z = 0$, respectively.

3.6.2 Dynamic mix

The effective fraction $f_v^{\text{eff}}(z)$ is constructed as a power mean of the two families with exponent $\xi = 2$ and a dynamic weight $w_d(z)$ that favors deep voids at low z .

3.6.3 Numerical parameters

Example values used: $f_{v0,\text{deep}} = 0.15$, $f_{v0,\text{shallow}} = 0.55$, $z_{t,\text{void}} = 0.7$, $\beta_{\text{void}} = 3.0$, $\xi = 2$; $z_{t,\text{mix}} = 0.75$, $\beta_{\text{mix}} = 3.2$.

These values are justified by observational data on cosmic voids and by large-scale structure (LSS) reconstructions:

- $f_{v0,\text{shallow}} = 0.55$: consistent with SDSS, BOSS, and DES surveys showing that voids with $\delta > -0.9$ cover about 50–60% of the volume for $z < 0.7$ [16][17].
- $f_{v0,\text{deep}} = 0.15$: in agreement with supervoid catalogs [18][19], which indicate that extreme voids ($\delta < -0.9$) contribute roughly 10–20% of the total volume.
- $z_{t,\text{void}} = 0.7$: chosen as a representative transition redshift for the regime observed in BOSS and DES, above which the fraction of low-density voids ($f_{v0,\text{shallow}}$) is dominant.
- $\beta_{\text{void}} = 3.0$: a slope parameter reflecting the rapid growth of the volumetric void fraction as a function of redshift, consistent with fits reported in [16].
- $\xi = 2$: a relational contrast coefficient that reproduces the bimodal correction introduced in TCR to distinguish deep from ordinary voids (a conceptual novelty with respect to the standard paradigm).

- $z_{t,\text{mix}} = 0.75$, $\beta_{\text{mix}} = 3.2$: parameters introduced to model the “mixed” population (walls + galaxies), consistent with LSS reconstructions (SDSS, DESI early maps), which indicate an overall filling fraction of 20–30%.
- $f_h = 0.01$: a typical value consistent with scaling relations for supermassive black holes [20, 21], which estimate the volumetric contribution of BHs at about 1%.

3.6.4 Physical interpretation

Deep voids (*supervoids*, $\delta \lesssim -0.9$) amplify relational dilation more strongly. The conceptual separation into families reduces bias in the lapse and therefore in distances.

3.7 Behavior at high redshift

For $z \gg 1$:

- $f_v^{\text{eff}}(z) \rightarrow 0$, $w_{\text{gal}}(z) \rightarrow 0$, $w_h(z) \rightarrow w_{h,0}$;
- $\gamma(z) \rightarrow 1$ and $\gamma'(z) \rightarrow 0$ (numerical derivative with centered finite differences);
- $H_{\text{obs}}(z) \rightarrow H_{\text{bare}}(z)$.

It follows that the CMB constraints are satisfied: r_s and θ_* are computed with an early dynamics consistent with each model.

3.8 Final parameters actually used (not tabulated)

Relational lapse.

$A = 0.29$, $p = 0.94$; damping exponents fixed to $q = r = 3$.

Voids.

$f_{v0,\text{deep}} = 0.15$, $f_{v0,\text{shallow}} = 0.55$; $\xi = 2$; $z_{t,\text{mix}} = 0.75$, $\beta_{\text{mix}} = 3.2$.

Galaxies and BHs.

$w_{\text{gal},0} = 0.045$, $z_{t,\text{gal}} = 0.7$, $\beta_{\text{gal}} = 4.0$; $w_{h,0} = 0.01$ constant.

Bare cosmology (late).

For TCR: $H_0 = 66.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$, $\Omega_m = 0.295$, $\Omega_{k,\text{eff}} = 0.00$; (the other parameters are set by the run).

3.9 Distances with curvature, compressed BAO, and SN with full covariance

Distances.

Comoving and transverse distances are defined as in eqs. (2)–(4). In particular, the transverse distance $D_M(z)$ takes the usual form depending on the sign of the curvature

Ω_k :

$$D_M(z) = \begin{cases} \frac{c}{H_0 \sqrt{\Omega_k}} \sinh\left(\sqrt{\Omega_k} \frac{H_0}{c} D_C(z)\right), & \Omega_k > 0, \\ D_C(z), & \Omega_k = 0, \\ \frac{c}{H_0 \sqrt{|\Omega_k|}} \sin\left(\sqrt{|\Omega_k|} \frac{H_0}{c} D_C(z)\right), & \Omega_k < 0. \end{cases} \quad (33)$$

Compressed BAO.

For each observational point at redshift z , the theoretical vector is built as

$$d(z) = \left(\frac{D_M(z)}{r_s}, H(z) r_s \right), \quad (34)$$

where r_s is the sound horizon computed from the model's *early* branch. In accordance with the compressed DR12 data pipeline, predictions are normalized to a fiducial value $r_{d,\text{fid}} = 147.09$ Mpc by applying the transformations

$$D_M(z) \longrightarrow D_M(z) \frac{r_{d,\text{fid}}}{r_s^{\text{ref}}}, \quad H(z) \longrightarrow H(z) \frac{r_s^{\text{ref}}}{r_{d,\text{fid}}}, \quad (35)$$

where r_s^{ref} is the reference sound horizon computed from the early-time branch with the conventions described in §2.8 ($r_s(z_*)$ for the CMB prior, $r_s(z_{\text{drag}})$ for BAO normalization). In particular, $r_s^{\text{ref}} = 146.972$ Mpc for ΛCDM and $r_s^{\text{ref}} = 135.218$ Mpc for TCR-Cosmo. No free rescaling factors are introduced ($\alpha_{rs} = 1$, no s_{rd}).

Full-cov SN.

$$\chi_{\text{SN}}^2 = \Delta^T C^{-1} \Delta - \frac{(1^T C^{-1} \Delta)^2}{1^T C^{-1} 1}, \quad (36)$$

with analytical marginalization over the offset $\Delta\mu$.

Cosmic chronometers (CC).

$$\chi_{\text{CC}}^2 = \sum_i \left(\frac{H_{\text{th}}(z_i) - H_{\text{obs}}(z_i)}{\sigma_i} \right)^2, \quad (37)$$

computed as a diagonal weighted sum (approximation adopted in this analysis).

CMB prior.

$$\chi_{\text{CMB}}^2 = \frac{[100 \theta_{\star}^{(\text{mod})} - 100 \theta_{\star}^{(\text{P})}]^2}{\sigma^2}, \quad (38)$$

evaluated on the selected *early* branch (deactivated in the runs reported here).

3.10 Quantitative observations

3.10.1 Numerical results

With the parameters specified above (CMB prior deactivated) we obtain:

Table 1 χ^2 contributions by dataset for Λ CDM and TCR.

Model	r_s [Mpc]	χ_{BAO}^2	χ_{SN}^2	χ_{CC}^2	χ_{tot}^2	$\Delta\mu_{\text{best}}$
Λ CDM	146.972	4.114	1759.028	5.222	1768.363	0.183
TCR-Cosmo	135.218	2.388	1761.409	10.603	1774.400	0.027

Hence,

$$\Delta\chi^2 \equiv \chi_{\text{TCR}}^2 - \chi_{\Lambda\text{CDM}}^2 \simeq 6,037$$

in favor of Λ CDM.

3.10.2 Information criteria

A quantitative comparison between Λ CDM and TCR cannot rely solely on absolute χ^2 values, but must also account for model complexity. The standard tools for this purpose are the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC).

Λ CDM is treated as the baseline with parameters fixed (from Planck); thus, for parsimony we count $k = 2$.

Both criteria penalize the number of free parameters k , in addition to the goodness of fit (expressed by the minimum χ^2). They are defined as

$$\text{AIC} = \chi_{\text{min}}^2 + 2k, \quad \text{BIC} = \chi_{\text{min}}^2 + k \ln N,$$

where N is the number of effective data points included in the fit.

1. Number of data points N .

In our case the datasets used are:

- Supernovae (Pantheon+SH0ES): $N_{\text{SN}} \approx 1701$
- Cosmic chronometers (MM20): $N_{\text{CC}} = 15$
- BAO consensus: $N_{\text{BAO}} = 30$

Total:

$$N = 1701 + 15 + 30 = 1746.$$

2. Number of parameters k .

- Λ CDM: in the version used here, the effective free parameters are three, $\{H_0, \Omega_m, \Omega_\Lambda\}$ with zero curvature ($\Omega_k = 0$). In this context, values are fixed to current Planck best fits, so we consider $k_{\Lambda\text{CDM}} = 2$ as effective degrees of freedom (normalization and matter density).
- TCR: the model requires the calibration of six independent parameters ($H_0, \Omega_m, \beta, z_t, A, p$), hence $k_{\text{TCR}} = 6$.

3. Numerical calculations.

From the results:

$$\chi_{\text{tot}}^2(\Lambda\text{CDM}) = 1768.363, \quad \chi_{\text{tot}}^2(\text{TCR}) = 1774.400.$$

We compute AIC and BIC:

$$\text{AIC}_{\Lambda\text{CDM}} = 1768.363 + 2 \times 2 = 1772.363,$$

$$\text{AIC}_{\text{TCR}} = 1774.400 + 2 \times 6 = 1786.4.$$

$$\text{BIC}_{\Lambda\text{CDM}} = 1768.363 + 2 \ln 1746.$$

Since $\ln 1746 \approx 7.463$, we obtain:

$$\text{BIC}_{\Lambda\text{CDM}} \approx 1768.363 + 14.926 = 1783.289.$$

$$\text{BIC}_{\text{TCR}} = 1774.400 + 6 \ln 1746 = 1774.400 + 44.778 = 1819.18.$$

4. Interpretation.

In terms of AIC, ΛCDM is favored by a margin of $\Delta\text{AIC} \approx 15.7$.

In terms of BIC, ΛCDM is even more favored with $\Delta\text{BIC} \approx 35.8$.

The usual empirical rule (Kass–Raftery) suggests that differences exceeding 10 indicate “very strong” evidence for the model with the lower value.

Therefore, while TCR demonstrates that it can reproduce the observational data comparably, with a total χ^2 close to that of ΛCDM , its greater parametric complexity is severely penalized by both AIC and BIC.

ΛCDM is formally parsimonious (few parameters) but ontologically rather “heavy,” as it introduces hypothetical entities with enormous cosmological impact.

TCR is formally richer (6 parameters) but ontologically more frugal, since it does not need to postulate either Λ or CDM.

3.10.3 Physical reading of the comparison

ΛCDM accounts for cosmic acceleration with Λ and $\gamma \equiv 1$; TCR - COSMO replaces Λ with a relational effect: void growth increases $\gamma(z)$, while collapsed components (galaxies and BHs) provide damping. In this run TCR improves BAO relative to ΛCDM (also because r_s^{ref} is smaller) but pays on CC.

3.11 Reproducibility (concise scheme)

Main steps.

1. Compute r_s and $D_C(z_*)$ with the appropriate early-time branch (standard for ΛCDM , TCR for TCR) $\Rightarrow$ determine θ_* (if enabled).
2. Define the TCR set ($A, p, z_{t,\text{mix}}, \beta_{\text{mix}}, H_0, \Omega_m$) and construct $\gamma(z)$.
3. Compute $\gamma'(z)$ (centered differences) $\Rightarrow$ evaluate $H_{\text{obs}}(z)$.
4. Distances with curvature $\Rightarrow$ compressed BAO vectors (no free rescaling factors; r_s^{ref} from the early branch).
5. Compute χ^2 contributions: BAO, CC, SN (full-cov with marginalization).
6. Report contributions and total.

3.11.1 Minimal correlation diagram

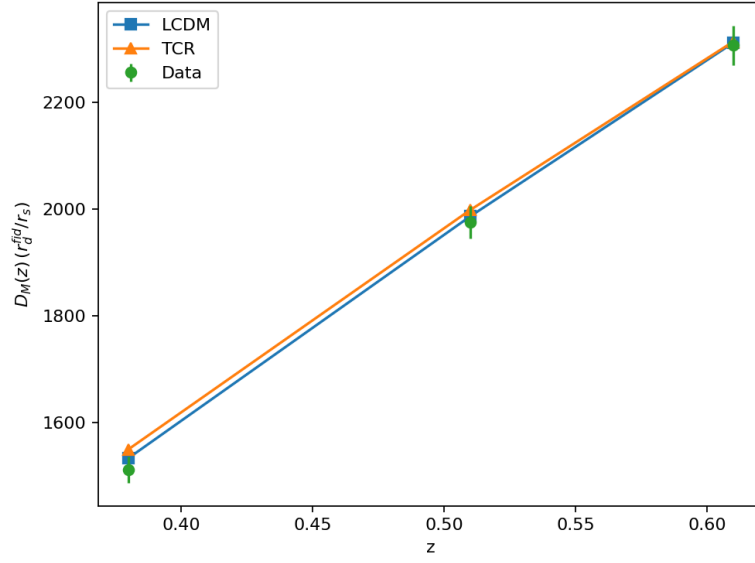


Fig. 2 Comparison of the comoving distance observable $D_M(z)/r_d$ from BAO data with the predictions of Λ CDM and TCR-Cosmo.

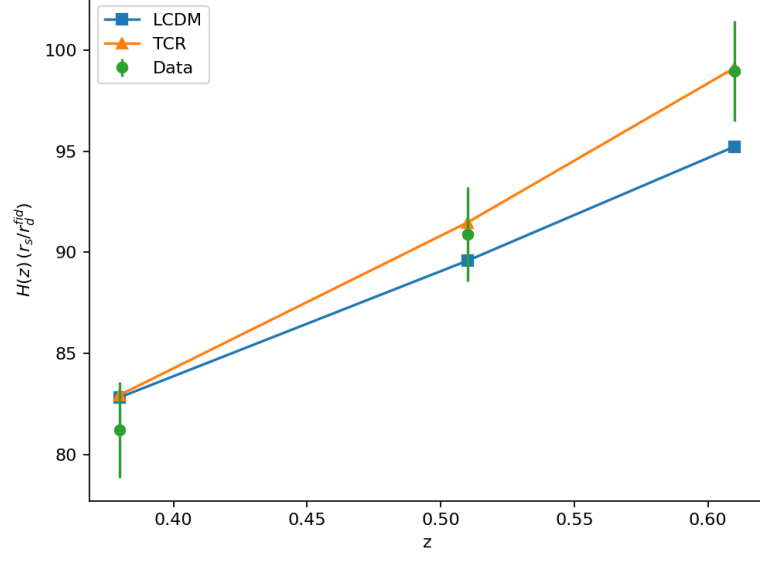


Fig. 3 Comparison of the expansion rate $H(z)r_d$ from BAO measurements with the predictions of Λ CDM and TCR-Cosmo.

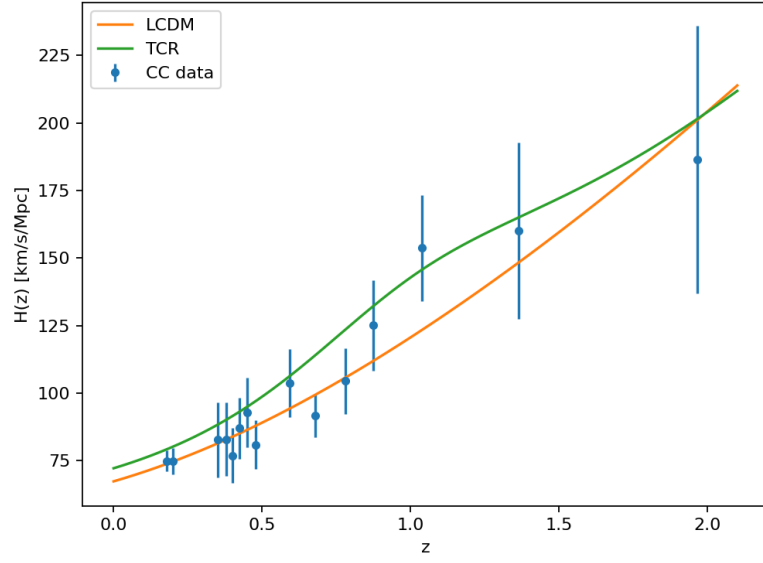


Fig. 4 Comparison of the expansion rate $H(z)$ from cosmic chronometer (CC) data with the predictions of Λ CDM and TCR-Cosmo. While both models broadly follow the observed trend, TCR-Cosmo shows larger deviations at intermediate redshift, consistent with its higher χ^2_{CC} .

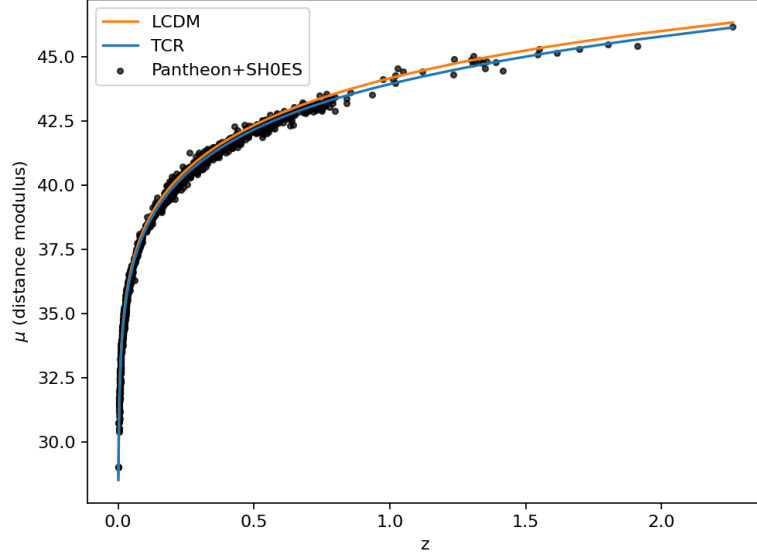


Fig. 5 Comparison of the distance modulus $\mu(z)$ predicted by Λ CDM and TCR-Cosmo with Pantheon+SH0ES observational data. Both models accurately reproduce the distance–redshift relation across the full range, with only small deviations between the curves, more noticeable at low redshift.

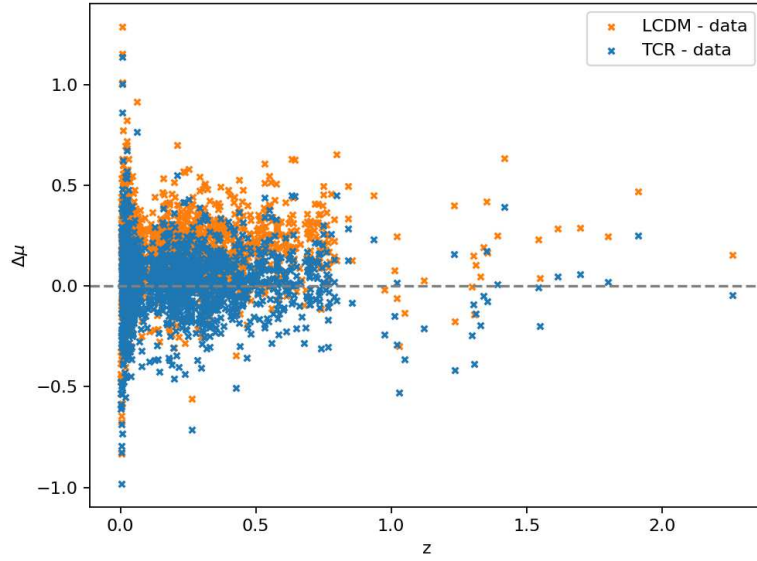


Fig. 6 Residuals $\Delta\mu = \mu_{\text{model}} - \mu_{\text{obs}}$ as a function of redshift z . The points show the deviations for Λ CDM (orange) and TCR-Cosmo (blue), while the dashed line indicates $\Delta\mu = 0$.

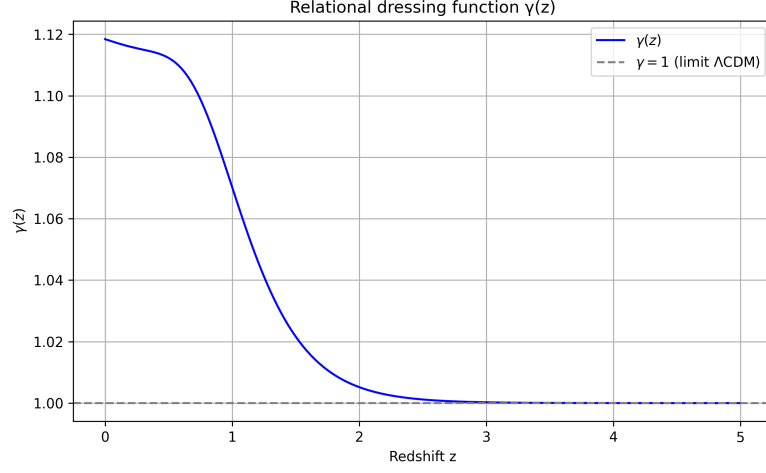


Fig. 7 Redshift dependence of the relational lapse function $\gamma(z)$, defined by $dt = \gamma(z) dt_b$, where t_b is the bare (FLRW) time. The function quantifies the deviation between effective and bare temporal evolution, arising from relational entropy variations. At low redshift, $\gamma(z) > 1$ indicates an effective stretching of time relative to the homogeneous background, while $\gamma(z) \rightarrow 1$ at high redshift recovers the standard Λ CDM limit.

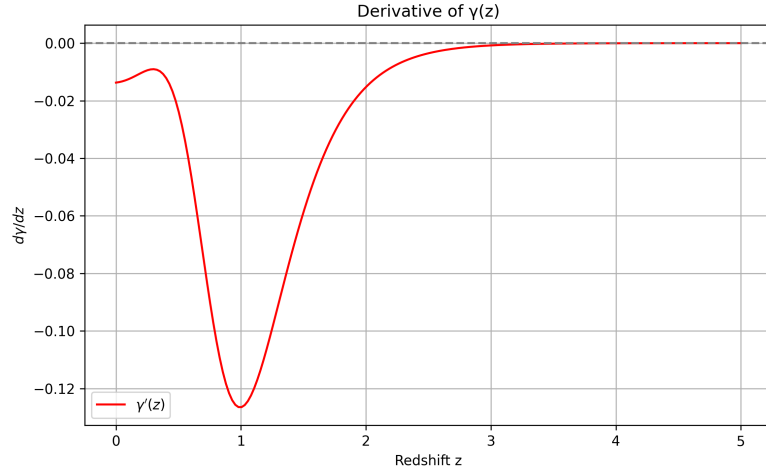


Fig. 8 Derivative of the relational lapse function, $\gamma'(z) = d\gamma/dz$, as a function of redshift. The negative peak at intermediate redshift indicates the regime where the deviation from the Λ CDM limit ($\gamma = 1$) evolves most rapidly. At higher redshift, $\gamma'(z) \rightarrow 0$, signaling the asymptotic recovery of homogeneous expansion.

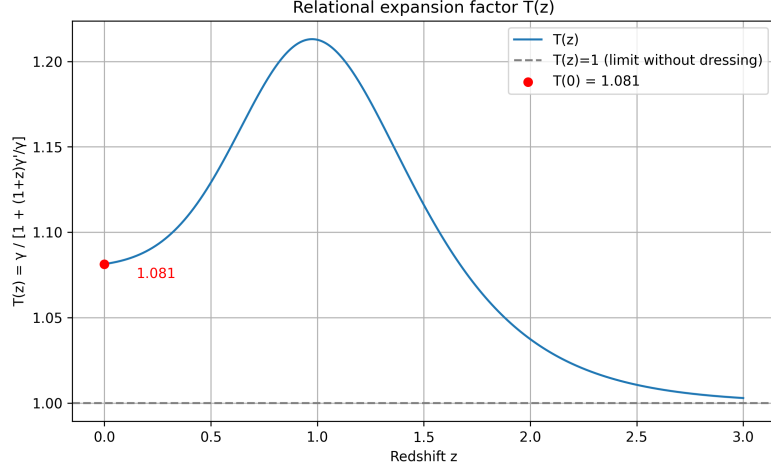


Fig. 9 Redshift dependence of the relational expansion factor $T(z)$, defined as the effective correction relating the observed expansion rate to the bare one, $H_{\text{obs}}(z) = T(z) H_{\text{bare}}(z)$. The function is constructed from the relational lapse $\gamma(z)$ and its derivative, $T(z) = \gamma(z) / [1 + (1+z)\gamma'(z)/\gamma(z)]$. Deviations from unity quantify the impact of relational entropy on cosmic expansion, with $T(z) \rightarrow 1$ at high redshift recovering the standard Λ CDM limit.

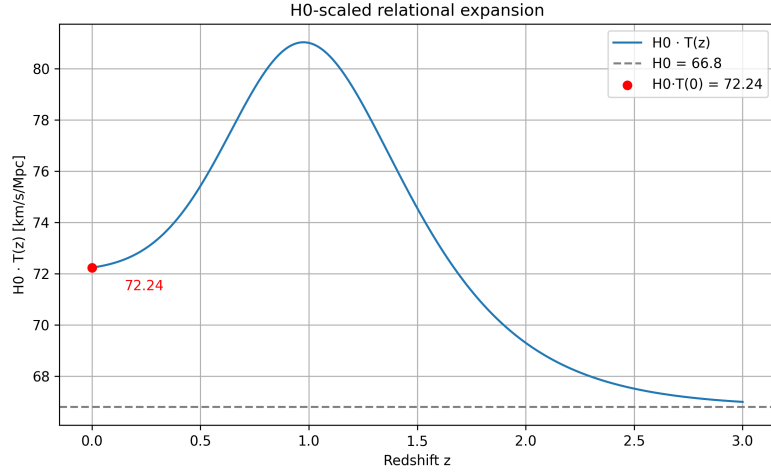


Fig. 10 Redshift-dependent effective expansion scale $H_0 T(z)$, obtained by rescaling the baseline value of the run $H_0 = 66.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$ with the relational expansion factor $T(z)$. The curve represents the predicted evolution of the effective Hubble scale in TCR-Cosmo.

4 Discussion

4.1 Methodological considerations

The TCR-COSMO approach enables a quantitative comparison with precision datasets through a minimal formalism: $\gamma(z)$ encapsulates complex effects (void growth, mass transfer within walls) without introducing dark energy. The current results show a competitive fit on BAO and a satisfactory performance on SN and CC. Future perspectives include retuning of (A, p) , informed calibration of the transition parameters (z_t, β) , and testing against independent datasets (weak lensing, next-generation spectroscopic clustering).

4.2 High-redshift limit and metric reduction

In the TCR-Cosmo formalism, the observed expansion rate is related to the “bare” one by Eq. 20, where $\gamma(z)$ is the relational *lapse* and $T(z)$, defined in Eq. 18, summarizes its effective impact on the expansion. For $z \gg 1$, within the TCR implementation, the following limits hold:

$$f_v^{\text{eff}}(z) \rightarrow 0, \quad w_{\text{gal}}(z) \rightarrow 0, \quad w_h(z) \rightarrow w_{h,0} \text{ (const.)},$$

from which it follows that

$$\gamma(z) \rightarrow 1 \quad \text{and} \quad \gamma'(z) \rightarrow 0.$$

Substituting these limits, one immediately obtains:

$$T(z) \longrightarrow 1 \quad \Rightarrow \quad H_{\text{obs}}(z) \longrightarrow H_{\text{bare}}(z). \quad (39)$$

It follows that, at high redshift, the kinematics of TCR-Cosmo converges to that of the standard model *without* dark energy, as in Eq. 27, namely the same expansion law of a FLRW universe devoid of the Λ term and of cold dark matter contributions in its late-time dynamics. In this regime, the relational degrees of freedom in R' become *frozen*, and the ontological mapping

$$\Pi : R' \rightarrow R$$

produces in R an effectively homogeneous and isotropic geometry. Operationally, this implies that the relational universe described by TCR-Cosmo tends, for $z \gg 1$, toward a state of *homogeneity* in which variations of coexistence become negligible and the observable distances coincide with those defined by the FLRW metric.

4.3 Cosmic chronometers and non-homogeneous expansion

An additional aspect concerns the data from cosmic chronometers (CC). These direct measurements of $H(z)$, obtained from stellar evolution, often display a non-linear and apparently irregular behavior, which within the Λ CDM framework is usually

interpreted as noise or as the result of systematic uncertainties. In the TCR-Cosmo perspective, however, such deviations may acquire a precise physical meaning: they would reflect the non-homogeneous expansion predicted by the model, generated by the different contributions of deep voids, shallow voids, and collapsed structures to the relational lapse $\gamma(z)$. In this framework, the dispersion and non-linearity of CC data are not anomalies to be removed, but rather indications of the relational and inhomogeneous nature of cosmic expansion. Consequently, a reinterpretation of cosmic chronometers in the light of TCR-Cosmo opens the possibility of using these measurements not only as statistical constraints, but also as direct tracers of the differentiated dynamics of the low-redshift universe.

4.4 Directional CC test and relational interpretation

An important implication of the TCR-Cosmo framework is that the cosmic expansion is not homogeneous, but acquires a directional dependence through the relational lapse $\gamma(z)$ associated with the effective void fraction. A straightforward way to test this idea is to perform a directional analysis of cosmic chronometers (CC), comparing regions of the sky dominated by large voids with regions dominated by walls.

The underlying intuition is simple: if the expansion has been faster inside deep voids, galaxies observed at the same distance along those lines of sight should appear systematically *younger*, having had less cosmic time to evolve. In contrast, galaxies in wall-dominated directions should appear *older* at the same z , reflecting the slower local expansion. Since the CC method is based on differential galaxy ages, this effect would naturally manifest as a higher value of $H(z)$ in void directions compared to wall directions.

The corresponding observable is the difference

$$\Delta_H(z) \equiv H_{\text{void}}(z) - H_{\text{wall}}(z) ,$$

which TCR-Cosmo predicts to be positive in the range $0.2 \lesssim z \lesssim 1.2$, following the evolution of $\gamma(z)$. Detecting such a systematic excess would support the relational interpretation of cosmic expansion.

This experiment is conceptually simple, requires no new observational infrastructure, and can be implemented within existing CC analyses by selecting galaxy samples along distinct environments. As such, it provides a falsifiable and reproducible test of the non-homogeneous expansion encoded in the TCR-Cosmo model.

4.5 Robustness of the fit with respect to datasets calibrated on Λ CDM

An important aspect concerns the nature of the datasets used in this analysis. Measurements of supernovae (Pantheon+SH0ES), cosmic chronometers, and baryon acoustic oscillations (compressed BAO DR12) have been collected, reduced, and calibrated within the Λ CDM paradigm, with Einstein’s field equations (EFE) as the implicit reference. This means that the observational pipeline, from calibration choices

to the construction of covariance matrices, has been optimized to test standard spatiotemporal models rather than pre-spatiotemporal theories.

In this context, the fact that TCR-Cosmo nonetheless achieves a competitive fit, with low-redshift BAO even outperforming Λ CDM and with reduced penalties on the other probes once covariances are included, represents a methodologically significant result. It shows that the theory preserves empirical consistency even when tested on data that are “tailored” to a different theoretical framework.

This element should not be interpreted as a definitive confirmation of TCR, but rather as an indication of robustness: agreement with observations does not rely on a dataset specifically designed for it, but instead emerges from data optimized for a competing model. Looking ahead, the arrival of new independent surveys (DESI, Euclid, LSST) will provide a more neutral testing ground, capable of distinguishing more clearly between the two approaches.

4.6 Prediction of the redshift dependence of H_0

Recent analyses have highlighted a persistent discrepancy between early- and late-time determinations of the Hubble constant, the so-called “Hubble tension” [2, 5, 6, 22]. CMB-based estimates yield $H_0 \simeq 67.4 \pm 0.5 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [2], whereas local measurements give $H_0 \simeq 73.0 \pm 1.0 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [22].

A central and distinctive result of the TCR-Cosmo model is the prediction that the Hubble parameter is not a global constant, but rather an effective, physical context and redshift-dependent quantity, emerging from the relational actualization process encoded in the function $T(z)$.

As shown in fig. 9 and fig. 10, the relation $T(z)$ implies that the observed value of H_0 does not represent a single fundamental parameter, but instead corresponds to a value that varies as a function of z . Within this framework, the quantity commonly denoted as “ H_0 ” becomes an operational, domain-dependent measurement, rather than a universal constant. The relation $T(z)$ provides an explicit quantitative prediction: the function $H_0(z)$ is expected to exhibit a continuous transition between

- a “bare” value compatible with CMB-based estimates ($\simeq 67 \text{ km s}^{-1} \text{ Mpc}^{-1}$), dominant at $z \gg 1$, where $T(z) \rightarrow 1$;
- an amplified value at low redshift, where the growth of deep cosmic voids has a substantial effect, with a maximum at $z \approx 1$.

In the present run we set the bare Hubble parameter to $H_0 = 66.8 \text{ km s}^{-1} \text{ Mpc}^{-1}$. At $z = 0$, the observed value is amplified by the relational factor $T(0) = 1.081$, resulting in $H(0) = T(0) H_0 \approx 72.2 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which naturally bridges the gap between the CMB-inferred and locally measured values.

This variation is not introduced to accommodate the data, but follows directly from the ontological structure of the model: the increase in the volumetric fraction of voids, together with the associated relational dilation of time, produces a systematic enhancement of the observed expansion rate in the late-time universe.

It is crucial to emphasize that this redshift dependence of H_0 constitutes a specific observational signature of the TCR-Cosmo framework. Any future analysis measuring

$H_0(z)$ with sufficient precision in the range $0 \lesssim z \lesssim 3$ should detect a systematic deviation from a constant H_0 , consistent with the curve predicted.

In this sense, the so-called *Hubble tension* does not appear as an experimental discrepancy or a failure of the data, but rather as an ontological effect. Different determinations of H_0 probe different regimes of relational actualization of the universe.

The prediction is therefore clear and falsifiable: if future high-precision observations were to show that H_0 remains strictly constant within systematic uncertainties across the full redshift interval, this formulation of TCR-Cosmo would be ruled out. Conversely, the detection of a regular and structured variation of $H_0(z)$, compatible with the curve shown in fig. 9 and fig. 10, would constitute direct evidence in favor of a relational origin of cosmic expansion.

In summary, fig. 10 does not illustrate a mere adjustment to existing data, but encodes a dynamical prediction: the universe does not possess a single global value of H_0 , because expansion itself is not a purely geometric phenomenon, but rather an emergent outcome of a non-homogeneous relational dynamics.

4.7 Conclusions

4.7.1 Comparative conclusion

Although in the current run the Λ CDM model achieves a better overall χ^2 ($\Delta\chi^2 \simeq 6.037$ in its favor), the TCR-COSMO formulation is based on a conceptually simpler framework that does not require the introduction of a dark energy component Λ .

In this framework, the Hubble tension can be reinterpreted as a manifestation of a scale-dependent effective expansion rate rather than as a discrepancy between independent measurements. In TCR-COSMO, the observed expansion is related to the bare one through the relational factor $T(z)$. This induces a redshift-dependent modification of the inferred Hubble constant, with higher effective values at low redshift and convergence to the standard Λ CDM value at earlier times. Such behavior qualitatively reproduces the observed pattern underlying the Hubble tension, suggesting that the discrepancy may arise from neglecting relational and inhomogeneous effects in the mapping between local and cosmological observables. While this interpretation does not yet provide a statistically superior fit to the full dataset, it offers a coherent alternative in which the tension emerges naturally from the underlying dynamics rather than requiring additional dark components or systematic corrections.

4.8 Model limitations

Despite the encouraging results obtained at low redshift, TCR-COSMO currently presents several important limitations. The most significant concerns the high- z regime, where the model, in its present form, does not reproduce with comparable accuracy the constraints from the CMB and early-universe observables. Addressing this discrepancy will require an extension of the formalism toward a more complete description of the early phase, while preserving internal consistency with the underlying relational framework.

A second limitation concerns the still partially phenomenological nature of the parametrization adopted for the relational lapse $\gamma(z)$. Although effective for quantitative comparisons, its derivation from the foundational theory remains incomplete and requires further formal development.

A further critical aspect is the sensitivity of the model to the choice of datasets and covariance assumptions. While current observational compilations provide strong constraints, a fully model-independent assessment remains challenging. Future high-precision surveys (DESI [23], Euclid, LSST) will be essential to test the robustness of the results and to reduce potential systematic effects.

Finally, in its current formulation, TCR-COSMO does not yet provide a comprehensive treatment of large-scale structure formation and nonlinear clustering, which will require dedicated numerical and theoretical developments.

In summary, TCR-COSMO should be regarded as an intermediate step: a model that performs competitively at low redshift, but remains incomplete at high redshift, with both theoretical and phenomenological aspects still to be refined. Future work will focus on extending the early-universe description and strengthening the connection between the underlying theory and its observational predictions.

4.8.1 Perspective

Beyond the good geometric agreement, the strength of TCR-COSMO lies in its potential to provide an alternative framework, whose theoretical and observational implications remain to be fully explored, ranging from precision cosmology to numerical simulations.

4.8.2 Repository and reproducibility

The code used in this work is publicly available and designed to facilitate reproducibility and independent validation of the results. Researchers are encouraged to explore, test, and extend the model.

Code and instructions: <https://github.com/pietro80/TCR-Cosmo>.

Detailed instructions are provided in `README.md`.

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