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Mathematical Modeling of Engineering Problems and Solutions via Polynomial Systems Using Custom-Designed Software

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Research Article

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Abstract: Engineering disciplines fundamentally rely on the mathematical interpretation of physical phenomena. This paper explores the integration of linear algebra and polynomial modeling in solving multifaceted engineering challenges, ranging from resource allocation and financial engineering to aerospace kinematics. A primary focus is placed on systems of linear equations (SLE) and their role in modeling real-world scenarios, such as the atmospheric reentry of spacecraft and equilibrium state analysis in networked systems. Due to the computational intensity of classical methods like Cramer's Rule in high-dimensional systems, this study presents a specialized software solution developed to optimize these calculations. The software employs high-precision algorithms to minimize rounding errors and enhance decision-making efficiency. Five distinct case studies are analyzed: industrial resource management, interest rate optimization, price equilibrium in transport networks, constant acceleration kinematics, and cubic polynomial trajectory planning for spacecraft landing. The results demonstrate that the synergy between robust mathematical modeling and algorithmic automation significantly improves the accuracy and speed of engineering analyses.

Keywords: Mathematical modeling, Engineering software, Cramer's Rule, Polynomial trajectories, Spacecraft landing simulation, Linear algebra.

Polynomials in Engineering Problems: From Theory to Application

Engineering is the art of creating practical solutions by expressing the laws of nature in mathematical language (Pollak, 2012). One of the most powerful and universal elements of this language is the polynomial. Although they may seem simple at first glance, polynomials are indispensable for modeling complex physical processes, organizing vast datasets, and predicting the behavior of engineering systems (Blanchard & Fabrycky, 2011) . Whether it is the deflection of a bridge under a load, the digital processing of a signal, or the calculation of a spacecraft's atmospheric reentry trajectory – polynomial equations lie at the heart of all these processes. In this article, we will explore the critical role of polynomials across various engineering disciplines and how they transform complex challenges into manageable solutions.

Computational Implementation of Linear Systems in Engineering

"A significant number of engineering problems ultimately necessitate the solution of systems of linear equations (SLE). One of the most fundamental methods for addressing such systems is **Cramer's Rule**; however, applying this method manually to high-dimensional systems entails extensive and laborious calculations **(Kurosh, 1971; Tyutin, 1977)**. that are prone to human error. To overcome these computational challenges, the integration of specialized software solutions becomes indispensable **(Ministry of Education of Azerbaijan, 2018)**.

The software presented in this study automates the execution of the Cramer algorithm, effectively bridging the gap between theoretical mathematical models and practical engineering applications **(Ministry of Education of Azerbaijan, 2018)**. By utilizing this computational tool,

complex systems—ranging from industrial resource allocation to the dynamic trajectory modeling of spacecraft—can be solved with high precision and significantly reduced processing time. This automation not only enhances efficiency but also ensures the reliability of results in critical engineering decision-making processes."

Algorithm and Computational Complexity

"The developed software is specifically optimized for high-order determinants, where manual calculation becomes impractical. For a system of degree n = 5, the computation of a single determinant involves n! (120) permutations if using the Leibniz formula. The software automates this process by applying recursive expansion or row reduction methods, ensuring that 4th and 5th-order systems are solved instantaneously.

Below is the conceptual logic (Pseudo-code) used in the program to handle these high-order systems:"

Section 1. Solve the equarin sistems:

$$\begin{cases} x_1 + 2x_2 - x_3 + x_4 = -3 \\ 2x_1 - x_2 + x_3 - x_4 = -4 \\ 3x_1 + 2x_2 + 3x_3 + 2x_4 = -1 \\ -2x_1 + 3x_2 - x_3 - x_4 = -2. \end{cases}$$

The general layout and navigation of the developed tool are shown in Figure 1.



Figure 1. The graphical user interface and functional modules of the software.**Section 2.** Solve the sistems:

$$\begin{cases} x_1 + x_2 + x_3 + x_4 + x_5 = 2 \\ 2x_1 + x_2 - x_3 - x_4 + 2x_5 = 9 \\ 3x_1 - 2x_2 - 3x_3 + 2x_4 - x_5 = 0 \\ -x_1 + x_2 - 2x_3 - 3x_4 + 2x_5 = 9 \\ -2x_1 - 2x_2 + 2x_3 + x_4 + 3x_5 = -1 \end{cases}$$

The calculated values are presented in Figure 2.



Figure 2. Numerical calculation of polynomial systems and input parameters.

Section 3: Case Studies and Numerical Results

In this section, five distinct engineering and economic scenarios are modeled and solved using the custom-developed software based on Cramer's Rule.

Case Study 1: Industrial Resource Management

Three types of products are manufactured using three different raw materials with total capacities of 800 tons, 1200 tons, and 1400 tons, respectively. Given the consumption matrix A, we determine the exact amount of each raw material required for full utilization of resources.

Mathematical Model:

The system is represented as:

$$\begin{cases} 2x_1 + 3x_2 + 2x_3 = 1200 \\ 2x_1 + 3x_2 + x_3 = 1000 \\ 3x_1 + 2x_2 + 3x_3 = 1400 \end{cases}.$$

$$x_1 = \frac{\Delta_1}{\Delta}, x_2 = \frac{\Delta_2}{\Delta}, x_3 = \frac{\Delta_3}{\Delta}.$$

The calculated values are presented in Figure 3.



Figure 3. Display of the computed results and data output.

$x_1 = 160, x_2 = \frac{\Delta_2}{\Delta} = 160, x_3 = \frac{\Delta_3}{\Delta} = 200.$

The software processes the input matrix and provides the optimized resource distribution.

Case Study 2: Financial Interest Optimization

A total investment of 18,000 AZN is distributed across three banks with different interest rates. Based on the total returns observed after one year under different allocation scenarios, the specific interest rate for each bank is calculated.

Mathematical Model:

The system is modeled by varying the proportions $\frac{1}{2}$-; $\frac{1}{3}$; $\frac{1}{6}$ to match the observed returns of 21,750 AZN, 21,600 AZN, and 21,000 AZN. The software calculates the interest percentages effectively. $\begin{cases} 900x_1 + 600x_2 + 300x_3 = 21750 \\ 1200x_1 + 300x_2 + 600x_3 = 21600. \\ 300x_1 + 900x_2 + 600x_3 = 21000 \end{cases}$

The calculated values are presented in Figure 4.

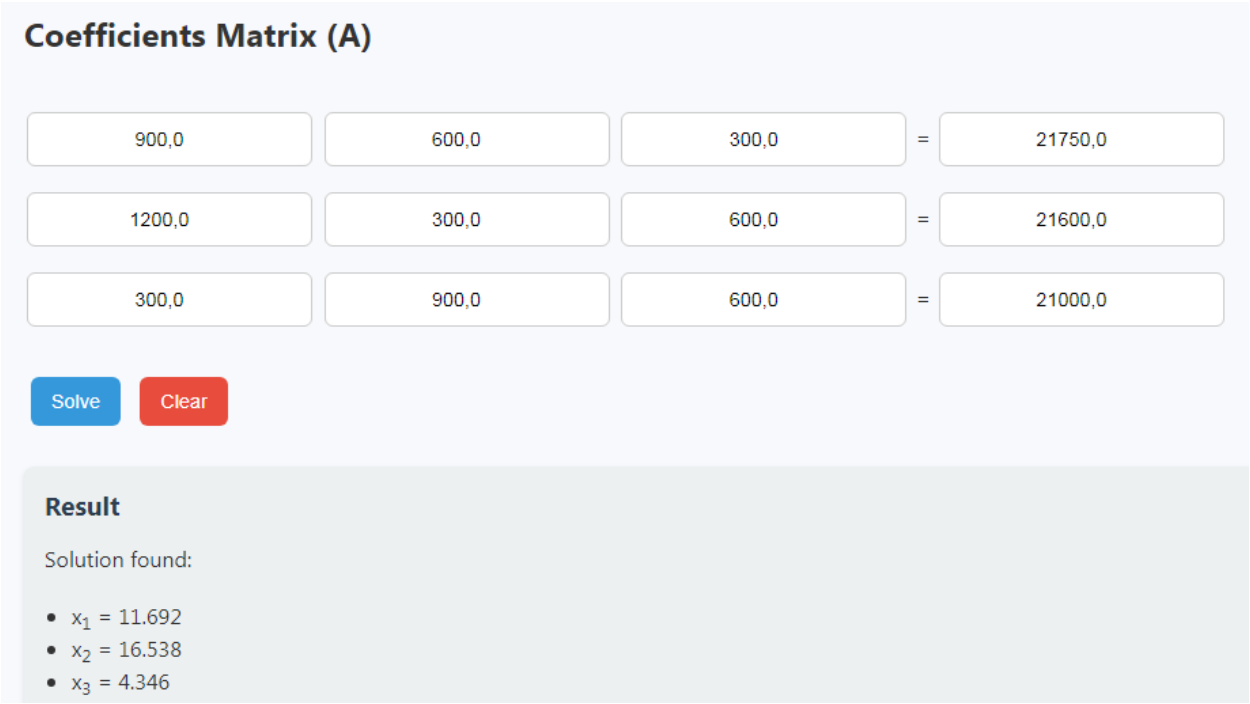


Figure 4. Final numerical output and data tables for the engineering model.

Case Study 3: Network Equilibrium in Transport Systems

Six cities are interconnected via a road network. Cities A_1, A_2, A_3 produce goods at specific prices (200, 150, 180 AZN). In cities B_1, B_2, B_3 where the product is not manufactured, the price is defined as the arithmetic mean of the prices in directly connected cities.

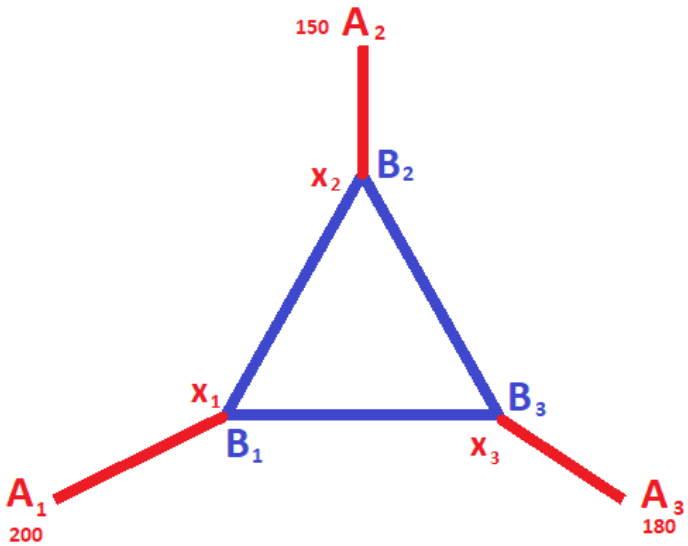


Figure 5. City map

Mathematical Model:

The equilibrium prices x_1, x_2, x_3 for the non-producing cities are solved using the following linear system:

$$\begin{cases} x_1 = \frac{1}{3}(x_2 + x_3 + 200) \\ x_2 = \frac{1}{3}(x_1 + x_3 + 150) \\ x_3 = \frac{1}{3}(x_1 + x_2 + 180) \end{cases} \text{ or, } \begin{cases} 3x_1 - x_2 - x_3 = 200 \\ 3x_2 - x_1 - x_3 = 150. \\ 3x_3 - x_1 - x_2 = 180 \end{cases}$$

The calculated values are presented in Figure 6.



Figure 6. Velocity and acceleration components analysis.

This represents a steady-state distribution within a supply chain network.

Case Study 4: Kinematics of Constant Acceleration

An automobile moves with constant acceleration **(Halliday et al., 2014)**. Sensors record the distance covered at specific intervals: 6m at 2s, 20m at 4s, and 42m at 6s. The goal is to determine the initial position, initial velocity, and acceleration.

Mathematical Model:

Using the quadratic displacement formula $s(t) = s_0 + v_0(t) + \frac{at^2}{2}$ the software solves the system for the variables $\begin{cases} s_0 + 2v_0 + 2a = 6 \\ s_0 + 4v_0 + 8a = 20 \\ s_0 + 6v_0 + 18a = 42 \end{cases}$.

The calculated values are presented in Figure 7.



Figure 7. The movement of cars

The resulting motion equation is: $s_0 = 0, v_0 = 1, a = 2,$

$s = t^2 + t.$

Case Study 5: Spacecraft Landing Trajectory with Variable Acceleration

During the descent phase, a spacecraft experiences time-dependent acceleration due to engine pulses and atmospheric resistance. The displacement is modeled using a third-degree (cubic) polynomial**(Meriam & Kraige, 2018; Hibbeler, 2016)**. Measurements show 600 km at 5 min,

2000 km at 10 min, and 3000 km at 15 min.

Mathematical Analysis:

1. **Model:** $s(t) = at^3 + bt^2 + ct.$
2. **Coefficients:** The software computes $\begin{cases} 125a + 25b + 5c = 600 \\ 1000a + 100b + 10c = 2000 \\ 3375a + 225b + 15c = 3000. \end{cases}$ Or

   $\begin{cases} 25a + 5b + c = 120 \\ 100a + 10b + c = 200 \\ 225a + 15b + c = 200. \end{cases}$

   The calculated values are presented in Figure 8.



Figure 8. Trajectory and velocity analysis for the cosmic flight model.

Velocity Equation: $a = -1{,}6; b = 40; c = -40,\ s(t) = -1{,}6t^3 + 40t^2 - 40t.$

$v(t) = s'(t) = -4{,}8t^2 + 80t - 40.$

Full Stop Prediction: Setting $3t^2 - 500t + 25 = 0; t \approx 16{,}17$. We find that the spacecraft reaches a full stop at 16.17 minutes. This precise calculation is vital for "soft landing" protocols in aerospace missions.

Section 4: Conclusion

The research demonstrates that the integration of custom-designed software with classical mathematical methods, such as Cramer's Rule, significantly enhances the efficiency of solving complex engineering problems. The following key conclusions can be drawn from this study:

- **Computational Efficiency:** The developed software successfully automates the solution of high-order linear systems, overcoming the labor-intensive nature of manual calculations and minimizing the risk of human error.
- **Modeling Versatility:** By applying the software to diverse scenarios—ranging from industrial resource allocation to aerospace kinematics—the study proves that polynomial modeling is a universal tool for engineering analysis.
- **Aerospace Precision:** The use of a third-degree (cubic) polynomial in Case Study 5 provided a highly accurate prediction for a spacecraft's landing trajectory, identifying the full-stop time at approximately 16.17 minutes.
- **Practical Utility:** The software serves as a reliable decision-support tool for both educational and professional engineering contexts, bridging the gap between theoretical linear algebra and real-world technological applications.

Declarations

- **Funding:** No funding was received for this study.

- **Conflict of Interest:** The author declares no conflict of interest.
- **Clinical Trial Registration:** Not applicable.
- **Consent to Publish declaration:** Not applicable.
- **Ethics and Consent to Participate declarations:** Not applicable.
- **Data Availability:** The data supporting the findings of this study are available from the corresponding author upon reasonable request.

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