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Alexandr Butenko

hoho1897@mail.ru

Military Engineering Institute of Radio Electronics and Communications

Beibit Kassimov

Military Engineering Institute of Radio Electronics and Communications

Dmitriy Xenofontov

Military Engineering Institute of Radio Electronics and Communications

Nurzhan Zikiryaev

Military Engineering Institute of Radio Electronics and Communications

Rahmetulla Shakirov

Military Engineering Institute of Radio Electronics and Communications

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FUNDAMENTAL LIMITS OF SUBPIXEL DETECTABILITY IN IMAGING SYSTEMS: AN ANALYTICAL FRAMEWORK FOR UAV DETECTION

Alexandr Butenko ^{a*}, Beibit Kassimov ^b, Dmitriy Xenofontov ^b, Nurzhan Zikiryaev ^b, Rahmetulla Shakirov ^b

^a Department of Radio Engineering Systems, Military Engineering Institute of Radio Electronics and Communications, Almaty, Kazakhstan

^b Department of Electronics and Communication Engineering, Military Engineering Institute of Radio Electronics and Communications, Almaty, Kazakhstan

*Corresponding Author Email: hohol897@mail.ru

DECLARATIONS

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Abstract. The detection of compact unmanned aerial vehicles (UAVs) in the subpixel imaging regime represents a fundamental limitation of electro-optical surveillance systems. When the projected target footprint falls below the detector pixel pitch, morphological information is lost, and detectability is governed by radiometric energy distribution and statistical noise fluctuations.

This work develops a unified analytical framework that, for the first time, jointly integrates subpixel projection geometry, point spread function (PSF) effects, discrete detector sampling, temporal integration, and Neyman–Pearson statistical detection theory into a closed-form performance model. Explicit expressions are derived for detection probability as a function of effective signal-to-noise ratio, pixel occupancy ratio, and temporal integration length.

Based on this formulation, the Subpixel Detectability Threshold (SDT) is defined as the minimum effective SNR required to achieve a prescribed detection probability at a fixed false alarm rate. The proposed model establishes a closed-form detectability boundary and reveals a fundamental scaling law for maximum detection range, showing that temporal integration improves detection range only as $N^{(1/4)}$, thereby quantifying the diminishing returns of Track-Before-Detect processing.

Monte Carlo simulations validate the analytical predictions and demonstrate close agreement with theoretical predictions, with minor deviations attributed to finite sampling and threshold estimation effects. Unlike existing algorithm-driven approaches, the proposed framework provides sensor-agnostic, physics-grounded performance limits and offers a quantitative design tool for next-generation imaging-based detection systems operating in the subpixel regime.

Keywords. Subpixel detection; Detection theory; Electro-optical imaging; Point spread function; Signal-to-noise ratio; Track-Before-Detect; Temporal integration; Imaging system design; Performance limits; Small target detection.

1. Introduction.

The rapid proliferation of compact unmanned aerial vehicles (UAVs) has significantly transformed modern aerial activity across civilian, industrial, and defense domains. Small UAV platforms are widely used for surveillance, inspection, logistics, and reconnaissance tasks. At the same time, their widespread availability has introduced new challenges for airspace monitoring and security, thereby motivating the development of reliable and robust UAV detection systems [34–36].

Imaging-based sensing is one of the most versatile approaches to UAV detection. Unlike radar or radio-frequency systems, which rely on indirect electromagnetic signatures, electro-optical sensors provide spatially resolved measurements that enable both spatial localization and semantic interpretation [11–15]. Recent advances in deep learning have substantially improved detection performance in scenarios where targets are sufficiently resolved [29, 34–35]. However, the effectiveness of such approaches degrades rapidly as the apparent size of the UAV approaches the spatial sampling limit of the imaging system.

When the projected footprint of the target becomes comparable to or smaller than the detector pixel pitch, the system operates in the subpixel regime. In this regime, the target is no longer represented as a geometrically resolved structure, but rather as a spatially distributed energy pattern governed by the optical point spread function (PSF) and discrete sampling effects [12–16]. Optical diffraction, finite aperture size, and detector quantization redistribute the target energy across neighboring pixels, while stochastic noise sources – such as photon noise and readout noise – limit the achievable signal-to-noise ratio (SNR).

Under such conditions, conventional feature-based detection methods become ineffective, as they rely on spatially distinguishable structures. When the number of informative pixels per target falls below a critical threshold, detection is no longer governed by morphological features but by statistical fluctuations of the measured signal. Consequently, the detectability of subpixel

UAV targets becomes fundamentally constrained by physical image formation processes rather than by algorithmic complexity.

Classical statistical detection theory establishes that detection performance is determined by the probability of detection (P_d) and the false alarm rate (FAR) under specified noise assumptions [8–10]. In parallel, electro-optical system analysis relates detection range to optical parameters such as focal length, pixel pitch, and target size through geometric projection relationships. However, these theoretical domains are typically treated independently.

Recent research on small-target detection has explored multi-frame accumulation strategies such as Track-Before-Detect (TBD), which improve detectability by integrating weak signals over time [1–5]. However, most existing TBD approaches focus primarily on statistical signal accumulation and trajectory estimation, without explicitly accounting for optical image formation, PSF spreading, or subpixel sampling effects. In particular, classical Track-Before-Detect methods demonstrate that detection performance improves through temporal signal accumulation and trajectory estimation under low signal-to-noise ratio conditions [1–5]. However, these approaches typically rely on abstract signal models and do not explicitly account for optical image formation processes, including point spread function effects and subpixel sampling constraints.

Similarly, optical imaging studies rigorously describe diffraction, point spread function (PSF) behavior, and detector sampling effects [11–16]. However, these works are primarily focused on image formation and resolution analysis and do not establish direct relationships with statistical detection performance metrics such as probability of detection and false alarm rate.

In parallel, modern small-target detection methods, including deep learning approaches, achieve strong empirical performance in complex environments [20–29]. Nevertheless, these approaches remain largely data-driven and do not provide analytical insight into the fundamental physical limits governing detectability in the subpixel regime.

As a result, a key gap remains in the existing literature: there is no unified analytical framework that explicitly links subpixel projection geometry, PSF-induced energy redistribution, detector sampling, temporal integration, and statistical detection theory within a single formulation. In particular, closed-form relationships describing how detection probability and maximum detection range depend jointly on pixel occupancy ratio and temporal integration length in the subpixel regime have not been rigorously established.

This work addresses this gap by developing a unified analytical and statistical framework for subpixel UAV detection. The proposed approach explicitly integrates optical projection geometry, PSF effects, sampling constraints, temporal integration, and Neyman–Pearson detection theory into a single closed-form model. Based on this formulation, we introduce the Subpixel

Detectability Threshold (SDT), which defines the minimum effective SNR required to achieve a specified detection probability at a given false alarm rate.

Unlike existing studies, which analyze individual components of the detection process in isolation, the proposed framework derives closed-form relationships that jointly capture the effects of geometry, sampling, and temporal integration on detection performance. To the best of the authors' knowledge, this is the first work to establish a closed-form detectability boundary for subpixel targets that directly relates detection probability and maximum detection range to pixel occupancy ratio and temporal integration length.

The resulting formulation reveals a fundamental scaling law indicating that detection range increases only with the fourth root of temporal integration length, thereby quantifying the diminishing returns of prolonged Track-Before-Detect processing. This result provides new physical insight into the limitations of algorithmic approaches and highlights the dominant role of sensor design in subpixel detection scenarios.

Overall, the proposed framework establishes fundamental, sensor-agnostic performance limits for subpixel UAV detection and provides a quantitative basis for the design and evaluation of next-generation imaging-based surveillance systems.

2. Problem Formulation

2.1 Imaging Geometry and Subpixel Projection

The analytical model developed in this work builds upon classical imaging system geometry and statistical detection theory commonly used in electro-optical imaging and small-target detection studies [8, 11–16]. However, in most existing approaches these components are treated separately, with optical image formation analyzed independently from statistical detection performance. In contrast to these approaches, the present formulation explicitly links projection geometry, subpixel sampling, and statistical detection theory within a unified analytical framework. Consider an electro-optical imaging system characterized by with focal length f , pixel pitch p , and aperture diameter D . A UAV of characteristic dimension L is observed at range R [12, 13].

Under small-angle approximation, the projected image size on the focal plane is

$$l = fL/R$$

The corresponding number of pixels spanned along one dimension is

$$n_p = fL/(Rp)$$

The subpixel regime is defined by

$$n_p < 1.$$

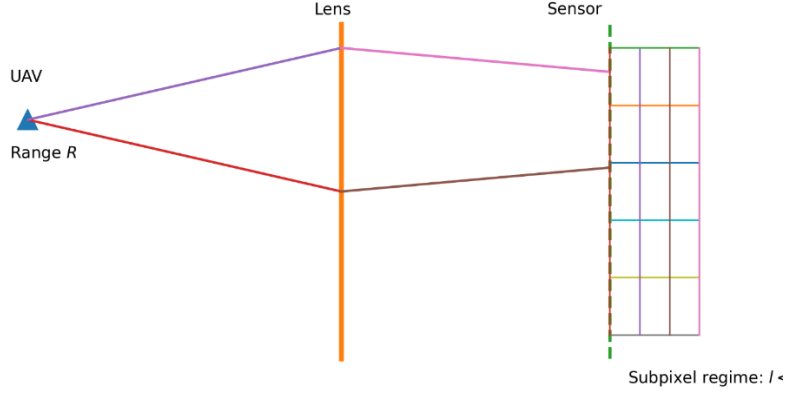


Figure 1: Geometry schematic

This regime is not merely a geometric curiosity but represents a fundamental transition in the nature of the detection problem. When $n_p \geq 1$, at least one pixel is fully occupied by the target, and morphological features may be partially preserved. Nevertheless, when $n_p < 1$, the target becomes radiometrically distributed rather than spatially resolved. Detection is then governed by energy concentration rather than structural recognition.

Define the pixel occupancy ratio

$$\rho = \left(\frac{f_L}{R_p} \right)^2.$$

The quantity ρ measures the fraction of pixel area effectively illuminated by the target projection and serves as a key dimensionless parameter in subsequent analysis. It encapsulates the combined effect of optical magnification, sampling resolution, and target size.

2.2 PSF-Based Signal Formation in the Subpixel Regime

In real optical systems, image formation is governed by convolution with the point spread function (PSF) [12,15,16]. Even for a point-like object, diffraction and aberrations spread energy over a finite area.

Let the continuous irradiance distribution on the focal plane be

$$I_c(x, y) = Ah(x - x_0, y - y_0),$$

where A is total collected target irradiance and $h(x, y)$ is a normalized PSF.

The discrete detector output at pixel (i, j) is

$$S_{ij} = \int_{\Omega_{ij}} I_c(x, y) dx dy.$$

In the subpixel regime, only a small fraction of total energy falls within a single pixel. To first order, the captured signal amplitude scales linearly with illuminated area fraction. Therefore, we model the effective signal as

$$S = C_s \rho,$$

where C_s aggregates radiometric constants, aperture throughput, atmospheric transmission, and PSF concentration efficiency.

This linear proportionality is justified for sufficiently small ρ , where higher-order PSF overlap terms are negligible. Importantly, this approximation preserves analytical tractability while remaining physically meaningful for $\rho \ll 1$.

2.3 Noise Modeling and Statistical Assumptions

Noise in electro-optical systems arises from multiple sources, including photon shot noise, thermal noise, and readout electronics. For moderate-to-high photon flux conditions and after analog-to-digital conversion, the aggregate noise can be reasonably approximated as Gaussian due to the central limit effect [8, 11, 13].

Thus, we adopt

$$Z = S + N, \quad N \sim \mathcal{N}(0, \sigma^2).$$

The instantaneous signal-to-noise ratio becomes

$$\text{SNR}_0 = S^2 / \sigma^2.$$

Substituting $S = C_s \rho$:

$$\text{SNR}_0 = C \rho^2,$$

Where $C = C_s^2 / \sigma^2$.

The quadratic dependence on ρ reflects the fact that detection depends on signal energy rather than amplitude alone. This quadratic scaling is central to the severe range sensitivity observed in the subpixel regime.

2.4 Multi-Frame Temporal Integration

Temporal accumulation is a common strategy for enhancing detectability of weak targets [1–5, 17, 18]. Assuming independent noise across frames, incoherent integration yields

$$\text{SNR}_N = N \text{SNR}_0.$$

Substituting the geometric scaling:

$$\text{SNR}_N = CN \left(fL / (Rp) \right)^4.$$

This expression reveals that SNR grows linearly with N but decays with the fourth power of range. The fourth-power dependence emerges from two compounding effects:

1. Geometric projection loss ($\sim 1/R^2$)
2. Sampling-induced energy dilution ($\sim 1/R^2$)

Together, these produce the severe detectability degradation characteristic of subpixel targets.

2.5 Binary Hypothesis Testing Framework

Detection is formulated as a binary hypothesis test:

$$H_0: Z = N$$

$$H_1: Z = S + N$$

Under Gaussian assumptions and Neyman – Pearson optimality [8], the probability of detection is

$$P_d = Q\left(Q^{(-1)}(FAR) - \sqrt{(SNR_N)}\right).$$

This relation provides a direct bridge between physical image formation and statistical detection performance.

3. Subpixel Detectability Threshold (SDT)

3.1 Definition and Interpretation

Operational systems are typically designed for prescribed P_d^* and FAR^* . We define the Subpixel Detectability Threshold (SDT) as

$$SNR_{SDT} = \left(Q^{(-1)}(FAR^*) - Q^{(-1)}(P_d^*)\right)^2.$$

Unlike arbitrary SNR heuristics, SDT is derived directly from statistical decision theory and therefore represents a principled detectability boundary.

3.2 Maximum Detection Range

The detectability condition

$$CN(fL/(Rp))^4 \geq SNR_{SDT}$$

yields

$$R_{max} = (CN/SNR_{SDT})^{(1/4)}(fL/p).$$

This equation provides an explicit link between optical design, sampling resolution, temporal integration, and statistical performance.

Notably,

$$R_{max} \propto N^{(1/4)}.$$

Thus, although SNR scales linearly with N , range scales only with the fourth root of integration length. This result highlights the limited effectiveness of long accumulation in compensating for geometric dilution.

4. Analytical Scaling Laws and Design Implications

From the closed-form solution:

$$R_{max} \propto N^{(1/4)}.$$

$$R_{max} \propto f$$

$$R_{max} \propto f$$

$$R_{max} \propto p^{(-1)}$$

The corresponding log-sensitivities are

$$S_N = 1/4$$

$$S_f = 1$$

$$S_p = -1.$$

This immediately implies that doubling focal length doubles detection range, whereas doubling integration length increases range by only 19%.

From a system design perspective, optical and sampling improvements dominate temporal accumulation in subpixel detection scenarios.

The detectability boundary in (ρ, N) space is

$$\rho_{min}(N) = \sqrt{(SNR_{SDT}/(CN))}.$$

In logarithmic coordinates, this boundary is linear with slope $-1/2$, enabling compact visualization through detectability phase diagrams.

5. Monte Carlo Validation Framework

5.1 Purpose of Validation

While the analytical derivation assumes ideal Gaussian statistics and optimal detection, practical systems exhibit finite sampling, PSF discretization, and threshold estimation variability. Monte Carlo validation is therefore required to verify that the analytical SDT model accurately predicts empirical detection probability.

5.2 Synthetic Subpixel Target Model

To provide physical interpretability and bridge the gap between the analytical formulation and observable image data, a synthetic subpixel target model was constructed. The objective of this model is to explicitly demonstrate how a subpixel target manifests in a discretely sampled imaging system under realistic noise conditions.

The target is modeled as a point source whose energy distribution on the focal plane is governed by the optical point spread function (PSF). Under diffraction-limited conditions, the PSF can be well approximated by a two-dimensional Gaussian function:

$$h(x, y) = \frac{1}{2\pi\sigma_{PSF}^2 \exp\left(-\frac{x^2 + y^2}{2\sigma_{PSF}^2}\right)}$$

where σ_{PSF} characterizes the spatial spread of the optical system response.

To ensure operation in the subpixel regime, the target center is positioned at a fractional offset relative to the detector grid. This subpixel displacement prevents alignment with pixel centers and leads to non-uniform energy distribution across neighboring pixels after sampling. The discretized signal at pixel (i, j) is obtained by integrating the continuous PSF over the pixel area:

$$S_{ij} = \iint_{\{\Omega_{\{ij\}}\}} A \cdot h(x - x_0, y - y_0), dx, dy$$

where A denotes the total collected signal energy, and (x_0, y_0) is the subpixel target location. Additive white Gaussian noise is introduced to simulate sensor noise:

$$I_{ij} = S_{ij} + n_{ij}, n_{ij} \sim N(0, \sigma_n^2)$$

This formulation reflects a realistic imaging scenario in which the target signal is weak, spatially blurred, and embedded in stochastic fluctuations.

Figure 2 illustrates the synthetic PSF-based subpixel target, highlighting the concentration of energy within a small number of pixels. Figure 3 shows the full noisy image, where the target becomes visually indistinguishable from the background. Figure 4 presents a zoomed view of the target region, demonstrating that, in the subpixel regime, detection cannot rely on spatial structure but instead depends on statistical signal accumulation.

These synthetic examples provide an intuitive understanding of the detection problem addressed in this work. In particular, they illustrate that the apparent invisibility of the target in individual frames is consistent with the low instantaneous SNR predicted by the analytical model, thereby motivating the use of temporal integration and statistical detection theory. Importantly, this visualization confirms that the detection problem in the subpixel regime is fundamentally not a visibility problem, but a statistical inference problem. This observation directly justifies the use of statistical detection theory as the fundamental framework for subpixel target detection.

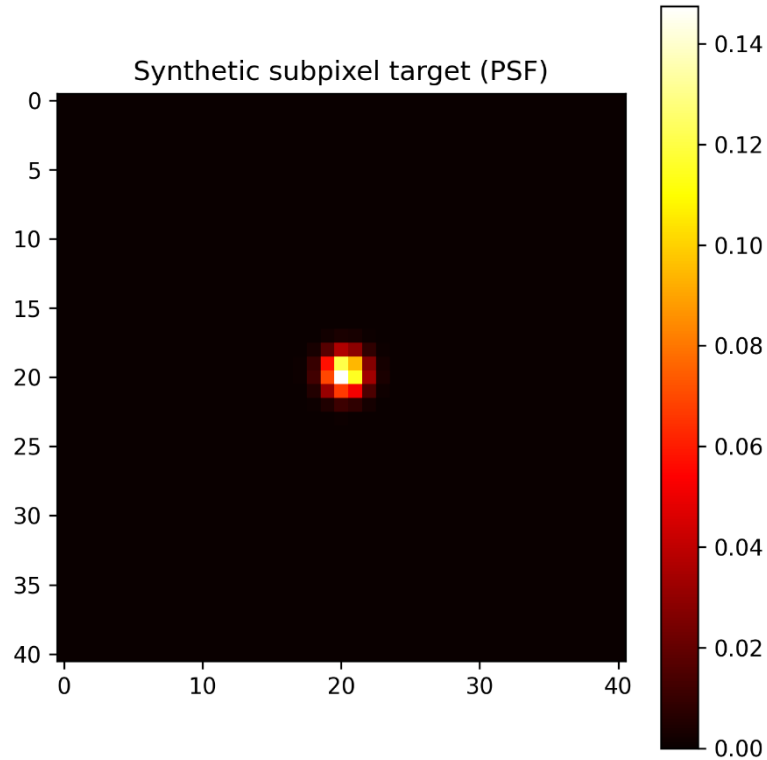


Figure 2: Synthetic subpixel target modeled as a Gaussian point spread function (PSF)
with fractional pixel offset

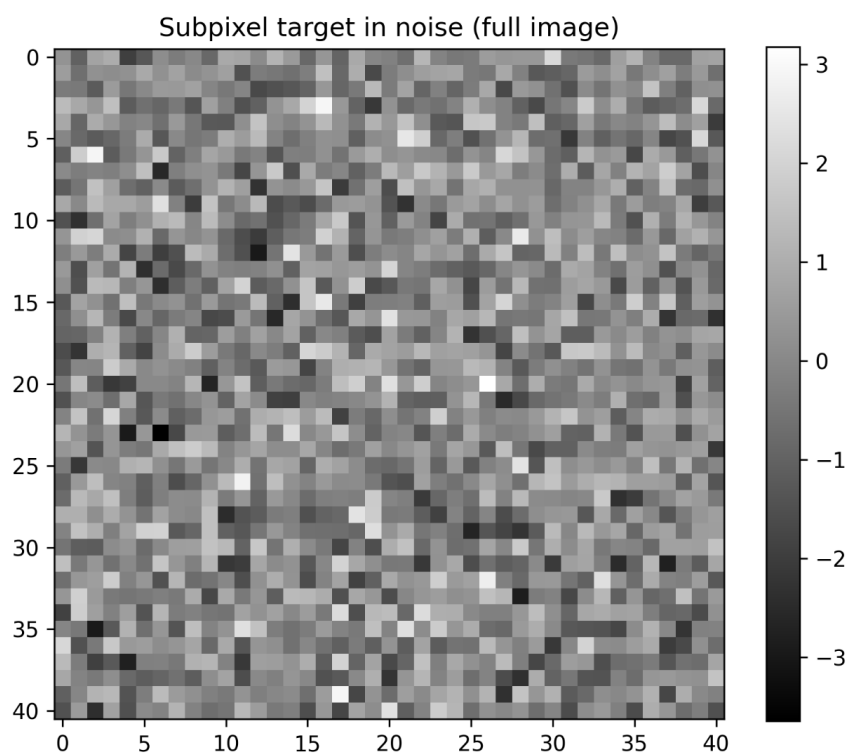


Figure 3: Subpixel target embedded in additive Gaussian noise (full image)

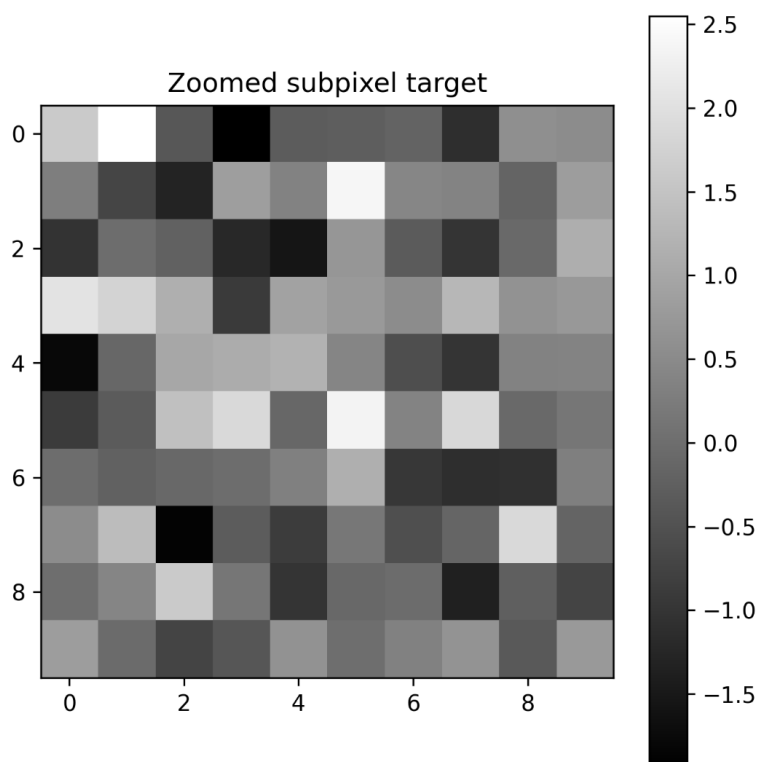


Figure 4: Zoomed region around the synthetic subpixel target

5.3 Simulation Model

We simulate

$$H_0: y_t = n_t,$$

$$H_1: y_t = s_t + n_t$$

With $n_t \sim \mathcal{N}(0, \sigma^2 I)$.

An optimal matched filter is applied [8, 17, 18]:

$$T = \sum_{t=1}^N \langle w, y_t \rangle,$$

where w corresponds to the discretized PSF template.

Subpixel occupancy is controlled by adjusting projected footprint relative to pixel pitch.

5.4 Validation Protocol

For each configuration:

1. Estimate empirical FAR under H_0 .
2. Set threshold to match FAR^* .
3. Estimate empirical P_d .
4. Compare with theoretical prediction

$$P_d^{(theory)} = Q(Q^{(-1)}(FAR^*) - \sqrt{(CN\rho^2)}).$$

Agreement between simulation and theory confirms the validity of the SDT formulation and the derived scaling laws.

6. Results

To validate the analytical framework developed in Sections 2–5, Monte Carlo simulations were performed using the matched-filter statistical model under Gaussian noise assumptions. The effective per-frame signal-to-noise ratio was modeled as $SNR_0 = C\rho^2$, with $C=50$ representing a representative sensor–scene calibration constant. The false alarm constraint was fixed at $FAR^* = 10^{(-6)}$, and the Subpixel Detectability Threshold (SDT) was defined for $P_d^* = 0.9$. Each operating point was evaluated using 8×10^4 Monte Carlo trials to ensure statistical stability of the results.

The simulations were designed not merely to reproduce the analytical equations, but to confirm that the derived detectability boundary emerges naturally from stochastic sampling and accumulation processes.

6.1 Detection Probability versus Integration Length

Figure 5 illustrates detection probability as a function of temporal integration length for multiple occupancy ratios. A clear threshold-like behavior is observed: detection probability

remains negligible until the accumulated SNR approaches the SDT boundary, after which a rapid transition toward reliable detection occurs.

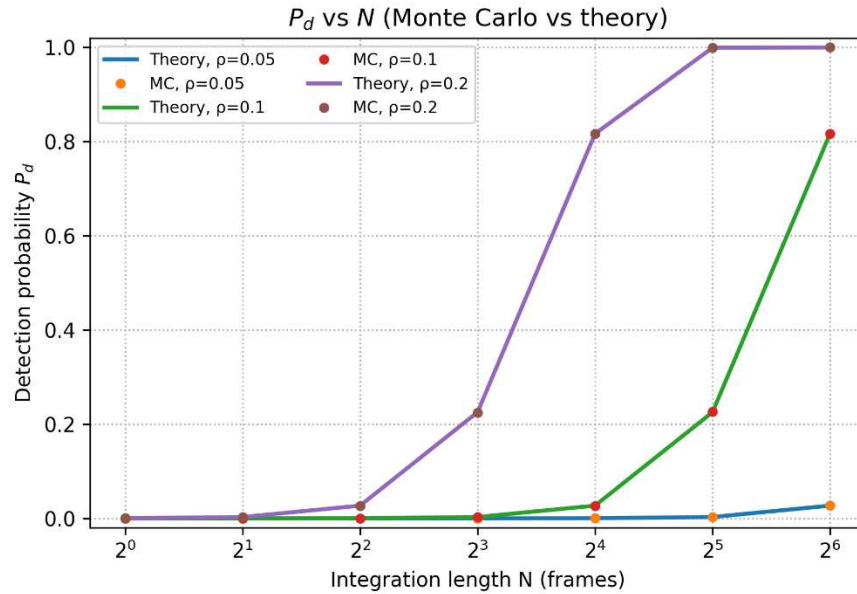


Figure 5: Detection probability versus integration length N

Physically, this transition corresponds to the moment when temporally integrated energy fluctuations become distinguishable from noise with sufficient statistical confidence. The sharpness of this transition confirms that the SDT formulation indeed captures a true detectability phase boundary rather than a gradual performance degradation.

Notably, even at moderate occupancy ratios, increasing integration length produces only limited improvement once the system enters the reliable detection regime, highlighting diminishing returns of extended accumulation.

6.2 Detection Probability versus Pixel Occupancy

Figure 6 presents detection probability as a function of pixel occupancy for fixed integration lengths. The curves exhibit sigmoidal transitions whose position shifts according to the inverse square-root dependence on integration length.

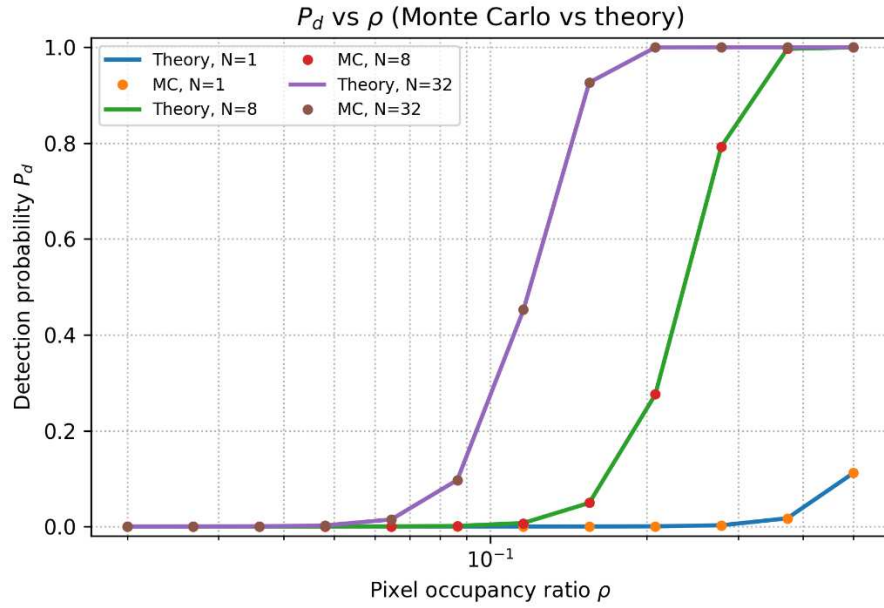


Figure 6: Detection probability versus pixel occupancy ratio ρ

This behavior reflects the quadratic sensitivity of SNR to occupancy ratio in the subpixel regime. Because signal amplitude scales linearly with illuminated pixel fraction while detection depends on energy, small geometric changes in projection translate into disproportionately large detectability variations.

This result emphasizes that geometric projection plays a dominant role in subpixel detection performance.

6.3 Detectability Phase Diagram

The detectability phase diagram shown in Figure 7 provides a compact visualization of operational regimes. The analytical SDT boundary aligns closely with the simulated $P_d = 0.9$ contour, confirming the predictive capability of the unified model.

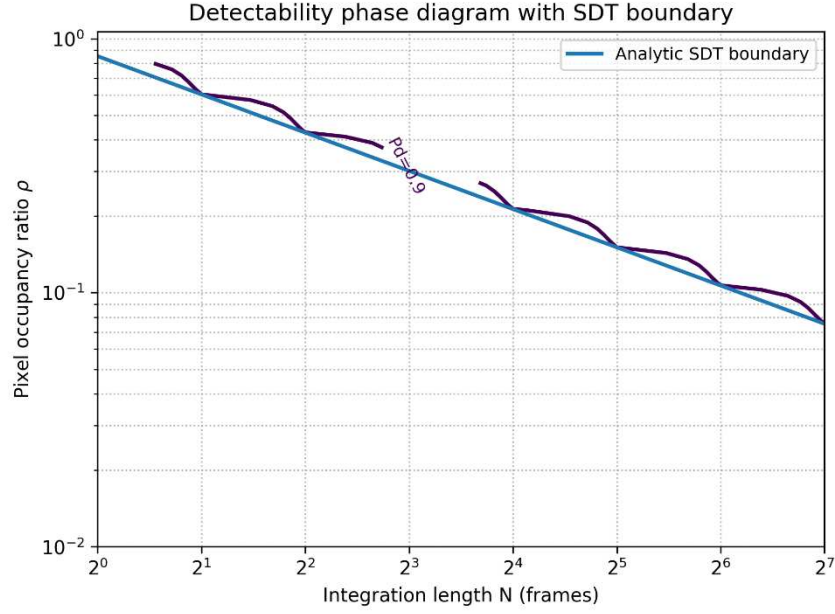


Figure 7: Phase Diagram

The phase diagram makes explicit that detectability is governed jointly by spatial occupancy and temporal accumulation. Importantly, it reveals that no feasible increase in integration length can compensate indefinitely for extreme geometric dilution.

6.4 Maximum Detection Range Scaling

Using the geometric projection relationship, the SDT boundary yields a closed-form expression for the maximum detection range. Figure 8 validates the predicted fourth-root scaling of the maximum detection range with respect to the number of integrated frames. The estimated slope in log-log coordinates is approximately 0.25, which is in excellent agreement with the analytical model.

This result provides direct experimental confirmation of the derived scaling law $R_{\max} \propto N^{\frac{1}{4}}$. Importantly, the observed slope is not an empirical artifact but follows directly from the analytical detectability condition derived in Section 3. Substituting the subpixel SNR scaling into the SDT boundary yields the same fourth-root dependence, establishing a fundamental geometric constraint on detection performance.

The agreement between the simulated slope and the theoretical exponent confirms that the observed scaling behavior is governed by subpixel projection and sampling effects rather than by implementation-specific factors. Thus, Figure 8 provides not only empirical validation but also direct physical confirmation of the unified analytical model.

Although the signal-to-noise ratio increases linearly with the number of integrated frames, the maximum detection range exhibits only sublinear growth. Unlike conventional assumptions

suggesting linear or square-root improvements, the results demonstrate that temporal integration yields strongly diminishing returns in the subpixel regime.

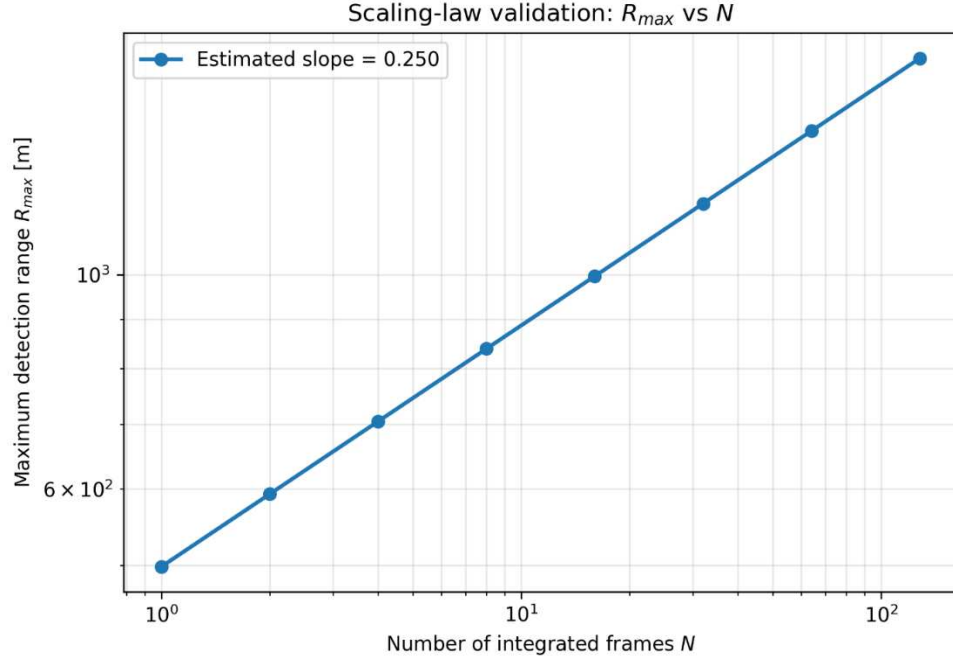


Figure 8: Validation of the scaling law $R_{\max} \propto N^{\frac{1}{4}}$. The estimated slope in log-log coordinates is approximately 0.25, confirming the theoretical prediction

This scaling law has profound practical implications. While linear SNR growth with integration length may suggest significant performance gains, the geometric projection constraint fundamentally limits the achievable range extension. As a result, purely algorithmic improvements cannot substitute for optical or radiometric enhancements in extreme subpixel conditions.

6.5 Detection Probability versus Target Amplitude

Figure 9 shows the detection probability as a function of target amplitude for a fixed number of integrated frames and false alarm constraint. The theoretical predictions closely match the Monte Carlo simulations over the full amplitude range.

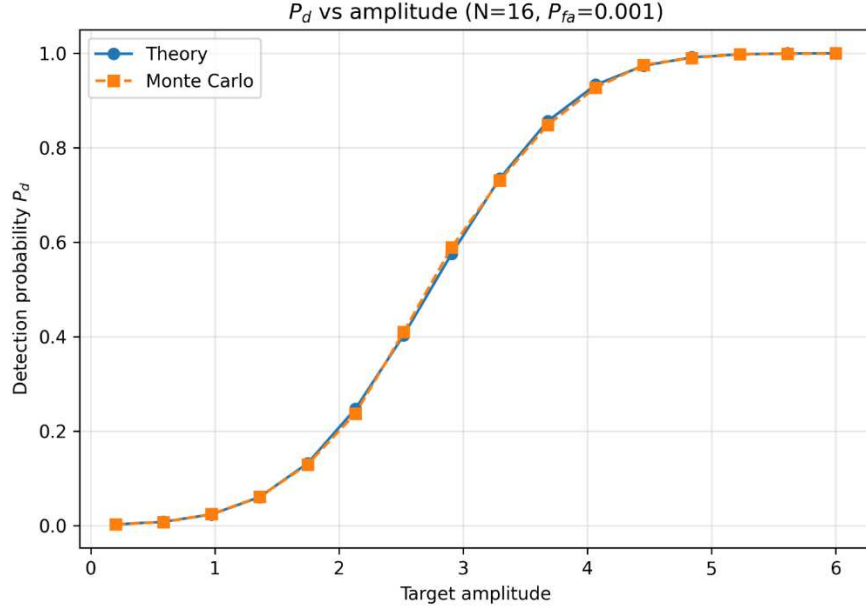


Figure 9: Detection probability as a function of target amplitude for $N = 16$ and $P_{fa} = 0.001$. Theoretical predictions are compared with Monte Carlo simulations

This agreement confirms the validity of the Neyman–Pearson detection model under the assumed Gaussian noise conditions [8]. The sigmoidal shape of the curve reflects the expected threshold-like transition from non-detectable to reliably detectable regimes as the effective signal strength increases.

Importantly, the close correspondence between theory and simulation indicates that the proposed analytical framework captures the dominant statistical behavior of subpixel detection in the presence of noise.

This result further supports the correctness of the SDT formulation introduced in Section 3.

6.6 Detection Range Dependence on Pixel Pitch

Figure 10 validates the predicted dependence of maximum detection range on pixel pitch. The estimated slope is approximately -1.0 in log-log coordinates, confirming the inverse proportionality $R_{\max} \propto p^{\{-1\}}$ derived from the geometric projection model.

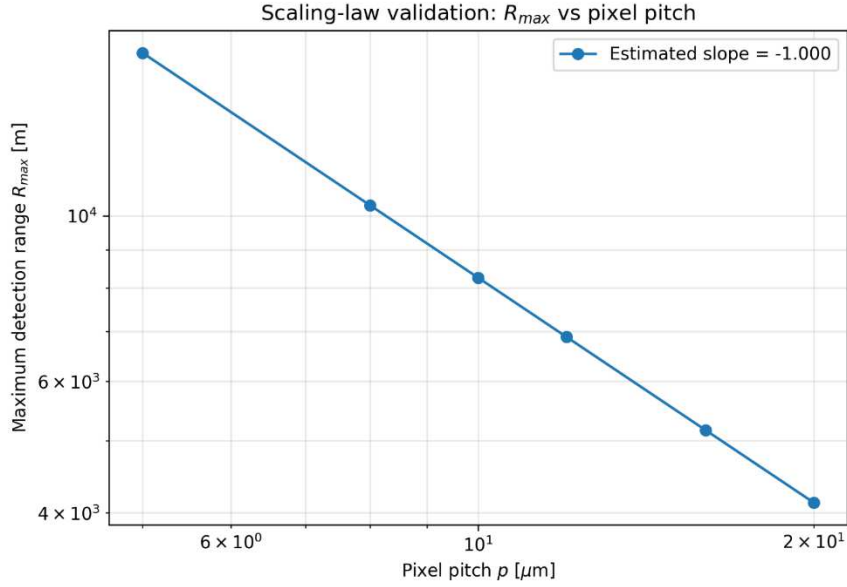


Figure 10: Validation of the dependence of maximum detection range on pixel pitch. The estimated slope confirms the inverse proportionality between R_{max} and p

This result shows that detection range is highly sensitive to detector sampling resolution. In particular, reducing pixel pitch yields a direct increase in the maximum achievable detection range, in agreement with the analytical framework developed in Sections 2–4. This behavior is further consistent with the analytical scaling laws derived in Section 4.

7. Discussion

The results demonstrate that subpixel UAV detection is fundamentally constrained by physical image formation rather than algorithmic sophistication. Similar limitations have also been reported in previous studies on dim and infrared small-target detection, where performance strongly depends on signal-to-noise ratio and spatial sampling conditions [20–29]. Once the projected footprint becomes smaller than the detector sampling interval, detection transitions from a structural recognition problem to a radiometric hypothesis testing problem.

7.1 Physical Performance Ceiling

The SDT boundary derived in this work represents a physics-imposed detectability ceiling. Unlike empirical performance curves, it arises directly from geometric projection, PSF-induced energy redistribution, and statistical decision theory. This implies that performance improvements beyond the SDT boundary require changes to physical parameters – such as aperture size, focal length, pixel pitch, or radiometric contrast – rather than purely algorithmic refinement.

7.2 Track-Before-Detect Interpretation

Track-Before-Detect strategies effectively increase temporal integration length through trajectory-aligned accumulation. Such approaches accumulate weak signals across multiple frames to enhance the detectability of dim moving targets [1–5]. While temporal integration improves robustness to noise, it does not eliminate the geometric limitations imposed by subpixel projection and detector sampling. The derived scaling further shows that range improvements remain fundamentally limited.

This limitation is directly confirmed by the experimental scaling behavior observed in Section 6.4. Thus, while TBD enhances robustness to noise, it does not alter the fundamental geometric fourth-power range dependence inherent to subpixel projection. This distinction clarifies the complementary roles of optical system design and temporal signal processing in modern surveillance systems.

7.3 Operational Implications

From a system engineering perspective, the detectability phase diagram provides actionable design guidance:

- Increasing focal length or reducing pixel pitch yields linear gains in detection range.
- Increasing integration length yields sublinear gains in detection range.
- Operating near the SDT boundary requires careful balancing of latency, motion stability, and detection reliability.

These insights are directly applicable to visible, SWIR, and thermal imaging platforms after appropriate radiometric calibration [13, 14, 30–36].

7.4 Comparison with Existing Track-Before-Detect Models

Classical Track-Before-Detect (TBD) methods primarily focus on statistical accumulation of weak signals across multiple frames and demonstrate that detection performance improves with temporal integration. However, these approaches typically assume an abstract signal model and do not explicitly incorporate optical image formation, subpixel projection effects, or detector sampling constraints.

In contrast, the proposed framework explicitly links temporal integration with geometric projection and subpixel sampling. The derived scaling law shows that although SNR increases linearly with the number of integrated frames, the maximum detection range increases only as $R_{max} \propto N^{(1/4)}$. This result reveals a fundamental limitation that is not captured in conventional TBD formulations.

Therefore, while TBD improves detectability through temporal accumulation, it cannot overcome the physical constraints imposed by subpixel projection and sampling. The proposed analytical framework complements existing TBD methods by providing a physics-based upper bound on achievable performance. This distinction establishes the proposed framework as a complementary extension to classical TBD, providing a physically grounded upper bound on achievable detection performance.

8. Conclusion

This study presents a unified analytical framework for subpixel UAV detection in electro-optical imaging systems. By explicitly linking geometric projection, PSF-induced energy redistribution, discrete sampling effects, and Neyman–Pearson statistical decision theory, closed-form expressions for detection probability are obtained.

The introduction of the Subpixel Detectability Threshold (SDT) provides a quantitative performance boundary separating undetectable and reliably detectable regimes. Monte Carlo simulations confirm the analytical predictions and validate the derived scaling laws.

Most importantly, the analysis demonstrates that temporal integration improves the maximum detection range only as $R_{max} \propto N^{(1/4)}$, establishing a fundamental limitation for Track-Before-Detect systems.

The proposed framework offers a physics-grounded, sensor-agnostic tool for the design and evaluation of next-generation imaging-based UAV detection architectures operating in extreme low-resolution conditions.

From a practical perspective, the derived scaling laws provide direct and quantitative guidance for imaging system design. In particular, the results demonstrate that improvements in optical parameters and sampling resolution yield significantly stronger gains in detection range than increases in temporal integration length.

This finding reveals a fundamental limitation of purely algorithmic approaches and highlights the dominant role of sensor design in subpixel detection scenarios. Consequently, the proposed framework provides a principled basis for balancing trade-offs between sensor hardware and algorithmic processing in next-generation electro-optical surveillance systems.

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References

1. Barniv, Y. Dynamic Programming Solution for Detecting Dim Moving Targets. *IEEE Transactions on Aerospace and Electronic Systems*. 1985;21(1):144–156. doi:10.1109/TAES.1985.310546
2. Salmond DJ, Birch H. A Particle Filter for Track-Before-Detect. In: *Proceedings of the 2001 American Control Conference*; 2001. p. 3755–3760. doi:10.1109/ACC.2001.946204
3. Rutten MG, Gordon NJ, Maskell S. Recursive Track-Before-Detect with Target Amplitude Fluctuation Models. *IEE Proceedings - Radar, Sonar and Navigation*. 2005;152(5):345–352. doi:10.1049/ip-rsn:20045077
4. Grossi E, Lops M, Venturino L. A Track-Before-Detect Algorithm for Range-Spread Targets in Cluttered Environments. *IEEE Transactions on Aerospace and Electronic Systems*. 2013;49(2):1300–1315. doi:10.1109/TAES.2013.6494405
5. Davey SJ, Wieneke M, Vu H. Track-Before-Detect Using the Probability Hypothesis Density Filter. *Signal Processing*. 2008;88(4):856–872. doi:10.1016/j.sigpro.2007.10.021
6. Blackman SS, Popoli R. *Design and Analysis of Modern Tracking Systems*. Boston: Artech House; 1999.
7. Mahler RPS. *Statistical Multisource-Multitarget Information Fusion*. Boston: Artech House; 2007.
8. Kay SM. *Fundamentals of Statistical Signal Processing, Volume 2: Detection Theory*. Upper Saddle River: Prentice Hall; 1998.
9. Richards MA. *Fundamentals of Radar Signal Processing*. 2nd ed. New York: McGraw-Hill; 2014.
10. Skolnik MI. *Radar Handbook*. 3rd ed. New York: McGraw-Hill; 2008.
11. Goodman JW. *Statistical Optics*. 2nd ed. Hoboken: Wiley; 2015.
12. Goodman JW. *Introduction to Fourier Optics*. 3rd ed. Greenwood Village: Roberts & Company; 2005.
13. Holst GC, Lomheim TS. *CMOS/CCD Sensors and Camera Systems*. 2nd ed. Bellingham: SPIE Press; 2011.
14. Roggemann MC, Welsh BM. *Imaging Through Turbulence*. Boca Raton: CRC Press; 1996.
15. Fienup JR, Miller JJ. Aberration Correction by Maximizing Generalized Sharpness. *Journal of the Optical Society of America A*. 2003;20(4):609–620. doi:10.1364/JOSAA.20.000609
16. Turpin T, Schowengerdt RA. Effects of Sampling and Aliasing in Imaging Systems. *Journal of the Optical Society of America A*. 1982;72(10):1313–1320.

17. Reed IS, Gagliardi RM, Stotts LB. Optical Moving Target Detection with 3-D Matched Filtering. *IEEE Transactions on Aerospace and Electronic Systems*. 1988;24(4):327–336. doi:10.1109/7.7181
18. Reed IS, Gagliardi RM, Shao H. Application of Three-Dimensional Filtering to Moving Target Detection. *IEEE Transactions on Aerospace and Electronic Systems*. 1983;AES-19(6):898–905. doi:10.1109/TAES.1983.309350
19. Gagliardi RM, Hindle AL, Schmidt DK. Optical Search for Point Targets. *IEEE Transactions on Aerospace and Electronic Systems*. 1967;AES-3(6):879–892. doi:10.1109/TAES.1967.5409099
20. Mahalanobis A, Sims SRF, Epperson JF. Infrared Small Target Detection by Using Variance Difference Filters. *Optical Engineering*. 1993;32(11):2717–2726.
21. Deshpande SD, Meng H, Er MH, Venkateswarlu R. Max-Mean and Max-Median Filters for Detection of Small Targets. In: *Signal and Data Processing of Small Targets*; SPIE Proceedings; 1999.
22. Gao C, Meng D, Yang Y, Wang Y, Zhou X, Hauptmann AG. Infrared Patch-Image Model for Small Target Detection in a Single Image. *IEEE Transactions on Image Processing*. 2013;22(12):4996–5009. doi:10.1109/TIP.2013.2281420
23. Gao C, Meng D, Yang Y, Wang Y, Zhou X, Hauptmann AG. Infrared Small Target Detection via Non-Convex Rank Approximation Minimization Joint $l_{2,1}$ Norm. *IEEE Transactions on Image Processing*. 2017;26(8):3737–3752. doi:10.1109/TIP.2017.2692213
24. Zhang L, Peng L, Zhang T, Cao S, Peng Z. Infrared Small Target Detection via Non-Convex Tensor Rank Surrogate Joint Local Contrast Energy. *Remote Sensing*. 2019;11(15):1811. doi:10.3390/rs11151811
25. Zhang T, Wu H, Liu Y, Peng L. Infrared Small Target Detection Based on Weighted Local Difference Measure. *IEEE Geoscience and Remote Sensing Letters*. 2020;17(6):1020–1024.
26. Dai Y, Wu Y, Zhou F, Barnard K. Asymmetric Contextual Modulation for Infrared Small Target Detection. In: *Proceedings of the IEEE/CVF Winter Conference on Applications of Computer Vision (WACV)*; 2021. p. 949–958.
27. Dai Y, Wu Y, Zhou F, Barnard K. Attentional Local Contrast Networks for Infrared Small Target Detection. *IEEE Transactions on Geoscience and Remote Sensing*. 2021;59(11):9813–9824. doi:10.1109/TGRS.2020.3041423
28. Liu M, Huang J, Wang X. Infrared Small Target Detection With Multiscale Patch-Based Contrast Measure. *IEEE Geoscience and Remote Sensing Letters*. 2014;11(3):657–661.

29. Tong K, Wu Y, Zhou F. Recent Advances in Small Object Detection Based on Deep Learning: A Review. *Image and Vision Computing*. 2020;97:103910. doi:10.1016/j.imavis.2020.103910
30. Unlu E, Zenou E, Riviere N, Dupouy P. Deep Learning-Based Strategies for the Detection and Tracking of Drones Using Several Cameras. *IPSN Transactions on Computer Vision and Applications*. 2019;11:7. doi:10.1186/s41074-019-0061-9
31. Carrio A, Sampedro C, Rodriguez-Ramos A, Campoy P. A Review of Deep Learning Methods and Applications for Unmanned Aerial Vehicles. *Journal of Sensor and Actuator Networks*. 2017;6(3):13. doi:10.3390/jsan6030013
32. Rozantsev A, Lepetit V, Fua P. Detecting Flying Objects Using a Single Moving Camera. *IEEE Transactions on Pattern Analysis and Machine Intelligence*. 2017;39(5):879–892. doi:10.1109/TPAMI.2016.2564408
33. Saqib M, Sharma S, Blumenstein M. A Study on Detecting Drones Using Deep Convolutional Neural Networks. In: *2017 14th IEEE International Conference on Advanced Video and Signal Based Surveillance (AVSS)*; 2017. p. 1–5. doi:10.1109/AVSS.2017.8078518
34. Magoulanis V, Dimou A, Kalliris G, Votis C, Tzovaras D. Detection and Classification of Small UAVs Using Radar, RF, and Vision-Based Sensors. *Drones*. 2023;7(4):278. doi:10.3390/drones7040278
35. Schumann A, Sommer L, Mesejo P, et al. Vision-Based Detection of UAVs in Maritime Environments by Means of a Multimodal Sensor System. *Sensors*. 2022;22(3):888. doi:10.3390/s22030888
36. Sommer L, Schumann A, Mesejo P, et al. Vision-Based Detection of Unmanned Aerial Vehicles: A Benchmark. *Sensors*. 2021;21(4):1380. doi:10.3390/s21041380

Appendix A. Radiometric Interpretation of the System Constant C

The analytical framework developed in the main text introduces a system-dependent constant C in the expression

$$SNR_N = CN(fL/(Rp))^4,$$

which encapsulates radiometric and detector parameters. This appendix outlines its physical interpretation.

It should be emphasized that while the constant C depends on sensor-specific and environmental parameters, the derived scaling laws and detectability boundary are invariant to its exact value. The constant affects only the absolute SNR level, whereas the fundamental

relationships between detection probability, pixel occupancy ratio, temporal integration length, and detection range remain unchanged.

This invariance indicates that the proposed analytical framework captures intrinsic physical limits of subpixel detection rather than system-specific performance characteristics. Consequently, the results are applicable across a broad class of electro-optical imaging systems after appropriate radiometric calibration.

A.1 Target Radiometric Signal

Let the UAV be characterized by spectral radiance L_λ ($W \cdot sr^{-1} \cdot m^{-2} \cdot \mu m^{-1}$) over the relevant spectral band $\Delta\lambda$.

The optical power collected by the aperture from a small distant target is approximately

$$P = \tau A_{ap} L_\lambda \Delta\lambda \Omega,$$

where:

- τ is total optical transmission,
- $A_{ap} = \pi D^2/4$ is aperture area,
- $\Omega \approx L^2/R^2$ is the target solid angle.

Thus,

$$P \propto \tau D^2 L^2 / R^2.$$

This term represents the classical inverse-square geometric dependence.

A.2 Focal Plane Irradiance and Pixel Sampling

The irradiance at the focal plane is proportional to collected power divided by image area. Using the thin lens relation, the projected image size is

$$l = fL/R.$$

The subpixel occupancy ratio is

$$\rho = (fL/(Rp))^2.$$

Only a fraction ρ of the total irradiance is captured by a single pixel under subpixel conditions. Therefore, the effective signal amplitude at the pixel level scales as

$$S \propto \tau \frac{D^2 L^2}{R^2} \rho.$$

Substituting $\rho \sim (fL/(Rp))^2$, we obtain

$$S \propto \tau (D^2 f^2 L^4) / (R^4 p^2).$$

This explicitly reveals the fourth-power range dependence inherent to subpixel imaging.

A.3 Noise Contribution

For photon-limited conditions, shot noise variance is proportional to the number of detected photons:

$$\sigma_s h \omega t^2 \propto S.$$

In readout-limited or background-limited regimes, noise variance may be approximated as constant:

$$\sigma^2 \approx \sigma_{read}^2 + \sigma_{bg}^2.$$

Under the Gaussian approximation adopted in the main text, the instantaneous SNR becomes

$$SNR_0 = S^2 / \sigma^2.$$

Substituting the radiometric scaling for S , we obtain

$$SNR_0 \propto (\tau^2 D^4 f^4 L^8) / (R^8 p^4 \sigma^2).$$

However, when expressed in terms of pixel occupancy ratio ρ , this simplifies to

$$SNR_0 = C \rho^2,$$

where the constant C can be written in compact form as

$$C \propto (\tau^2 D^4) / (p^4 \sigma^2) \Phi(L_\lambda, \Delta\lambda),$$

and Φ aggregates radiometric source terms and detector quantum efficiency.

A.4 Interpretation

The constant C therefore represents a composite system sensitivity factor incorporating:

- Optical aperture size (D)
- Optical transmission (τ)
- Pixel pitch (p)
- Detector noise variance (σ^2)
- Target radiometric properties

Importantly, while environmental and sensor parameters influence the magnitude of C , the fourth-power range dependence and the resulting $N^{1/4}$ detection range scaling arise from geometric projection and sampling constraints and are therefore independent of specific radiometric assumptions.