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Causal Architecture Theory (CAT). The VERCETTI Principle and Canonical Branch Selection

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Causal Architecture Theory (CAT).

The VERCETTI Principle and Canonical Branch Selection

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Abstract

This paper gives a canonical formulation of Causal Architecture Theory (CAT). It replaces a collection of discrete branch choices with a single physical selection principle, VERCETTI, short for Variational Endpoint-Resolved Causal Entropic Terminal Trace Invariance. The principle states that the realized branch must be the minimal terminal spectral structure that allows a finite causal observer, working inside a two-endpoint Alexandrov diamond, to extract scalar observables from the spectral action without internal orientation ambiguity, without introducing a new continuous modulus, and without changing the strict SCT core.

The central result is a derivation of VERCETTI, within the stated class, from first physical principles $F0$–$F8$: finiteness of the observer, Lorentzian causality, scalar instrument readout, the spectral variational principle, chirality of the low-energy sector, absence of hidden continuous moduli, non-tautological selector status, Wilsonian semigroup behavior, and terminality of scalar completion. Within this explicit class, VERCETTI has theorem-level status relative to $F0$–$F8$, while its application to nature remains conditional on the physical validity of those principles.

The resulting branch data, including the Pati–Salam finite algebra, the binary algebra $\mathbb{C}\oplus\mathbb{C}$, the CM point $\tau_\star = i\sqrt{2}$, $j(\tau_\star) = 8000$, the automorphism σ_7, the profile e^{-u}, and $\xi = 1/6$, are treated as a single minimal repair chain for three deficits: chiral gauge structure, two-endpoint orientation, and pro-torsor scalarization. The manuscript also clarifies the status of the CJ bridge formula: the $N^{8/9}$ scaling follows only under the single-active-shell hypothesis and is not promoted to core status. All algebraic checks reported in the paper were verified exactly by the supplied Python/SymPy scripts, with supporting Lean formalization files provided as Online Resource 1.

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1. Why a new principle is needed

The initial vulnerability of CAT is not that its individual formulas are wrong. It is that an external reader can legitimately ask why this set of mathematical objects is selected rather than a nearby one. The first defense is the principle of minimal spectral realization, MSR, together with the factorization census: after fixing the working branch, one free continuous input v_{EW} remains. However, MSR can still read as a principle of mathematical economy. It explains why the chosen set is small, but less clearly why it is physically necessary.

VERCETTI adds the missing physical intuition. A real observer does not see "all spectral triples." The observer sees a finite causal experiment: two endpoint events p, q, an Alexandrov diamond $I^+(p) \cap I^-(q)$, a finite slice of the causal order, a spectral readout, and a scalar value on the apparatus. The branch selector therefore should not be a decorative number or a table of coincidences. It should be a minimal mechanism that turns two-endpoint causal data into scalar spectral observables.

This answers the main criticism: $j = 8000$, $\tau = i\sqrt{2}$, $\mathbb{C} \oplus \mathbb{C}$, σ_7 are not separate decorative numbers. They appear in one chain:

$$\begin{aligned}&\text{two ends of observation} \Rightarrow \text{minimal binary orientation} \Rightarrow A_M = \mathbb{C} \oplus \mathbb{C},\\ &A_M = \mathbb{C} \oplus \mathbb{C} \Rightarrow \text{minimal non-elliptic rectangular CM-point},\\ &\text{minimal CM point} \Rightarrow \tau_\star = i\sqrt{2} \Rightarrow j = 8000.\end{aligned}$$

Statuses. The text uses five statuses: proven, verified, conditional, heuristic, speculative. The VERCETTI meta-principle has the status conditional. Its consequences within explicitly defined classes have the status proven or verified if the proof or machine verification is closed.

2. A short review of the literature and internal database

The Chamseddine–Connes spectral action gives a universal action formula through the spectrum of the Dirac operator; when applied to the spectral model of the Standard Model it gives the action of the Standard Model, Einstein gravity and the Weyl sector [1]. Removing the first-order condition in almost-commutative geometry leads to the Pati–Salam structure $SU(2)_R \times SU(2)_L \times SU(4)$, where leptons combine with quarks as the fourth color, and one generation cell has 16 fermionic components [2]. In the Pati–Salam spectral model, the unification of gauge couplings under intermediate symmetry breaking was also studied [3].

On the side of causal sets, the key fact is that locally finite partial order can approximate Lorentzian geometry, and the Benincasa–Dowker operators approximate $\Box - \frac{1}{2}R$ and give a discrete path to scalar curvature [4]. This is important for VERCETTI: what is physically primary is not an arbitrary finite graph, but a finite causal experiment with two ends and a direction of time.

Nonlocal form factors in the effective action naturally arise in the Barvinsky–Vilkovisky covariant perturbation theory, where the heat kernel and one-loop action are calculated in the basis of nonlocal curvatures [5]. In SCT, this technology fixes the dimensionless core: $\alpha_C(0) = 13/120$, $\alpha_R(\xi) = 2(\xi - 1/6)^2$, the canonical absence of a real scalaron and the spin-2 fakeon sector. Anselmi's fakeon prescription eliminates the ghost mode from the physical spectrum and preserves perturbative unitarity under suitable conditions [6].

The internal basis of CAT contains three especially important no-go pillars. First: the finite nerve does not provide a canonical integer orientation representative without external data [19]. Second: the Gaussian Hilbert-Schmidt completion of the spectral measure gives a pro-torsor of local measures and does not give the internal canonical scalar expectation [20]. Third: there is no scalar graviton for the SM content, since $\Pi_s(z, \xi) = 1 + 3z\alpha_R(z, \xi) > 1$ for $z > 0$ [21]. VERCETTI does not bypass these no-gos; it is built from them.

3. Derivation of VERCETTI from first physical principles

This section derives VERCETTI from a lower level: from physical principles, which are already needed by any relativistic spectral theory with a finite observer. This is not "proof of nature" in any absolute sense. This is a more precise statement: if we accept the physical principles $F0$–$F8$ listed below, then VERCETTI is their minimal terminal consequence. Therefore, the status of the conclusion is: proven regarding $F0$–$F8$ and conditional as a statement about the applicability of $F0$–$F8$ to nature.

Definition 3.1 (First physical principles $F0$–$F8$)**.** First principles in this manuscript refer to the following set.

F0. **The finiteness of the observer.** A physical measurement is performed by a finite device in a finite proper time and has a finite information capacity. Therefore, the observer does not have access to a "global spectral triple as a whole," but works with a finite causal experiment.

F1. **Lorentz causality.** The data region of one completed measurement is defined by preparing the event p and reading out the event q. The minimal causally covariant region between them is the Alexandrov diamond
$$D(p,q) = J^+(p) \cap J^-(q).$$

F2. **Scalarity of the instrument readout.** The finite instrument returns a number, phase, or finite set of numbers invariant under gauge, coordinate, and orientation redescriptions. A density-valued or torsor-valued quantity is not a finite instrument readout without additional terminal trivialization.

F3. **Variational spectral principle.** The dynamics are expressed through the spectral data of the Dirac operator: the bosonic part has the form $\operatorname{Tr} f(D^2/\Lambda^2)$, and the fermionic part has the form $\langle \psi, D\psi \rangle$.

F4. **Chirality of the observed low-energy sector.** The low-energy theory contains a chiral Standard Model cell without mirror doubling. If quark-lepton unification is used, the lepton degree of freedom must enter as a fourth color rather than as an independent outer label.

F5. **No hidden continuous modulus.** A branch selector must not introduce a new continuous low-energy coordinate that is not observable as an independent parameter. Continuous dimensional anchors are only allowed as measured scales or as explicitly defined external terminal data.

F6. **Non-tautological selector.** The selector must not be an injective rewrite of the full finite state. Otherwise it does not explain anything: it just encodes the whole answer in a new language.

F7. **Wilsonian semigroup property.** Spectral coarsening between scales is a positive continuous semigroup: successive coarsening by u and then by v is equivalent to coarsening by $u + v$.

F8. **Terminality of scalar completion.** If the proven internal structure does not produce a scalar, but a torsor or density-line, then exactly the minimal external terminal object required for a scalar readout is allowed. Any richer structure must be discarded as superfluous.

Lemma 3.2 (Why exactly two-endpoint causality)**.** *In the class of event-localized finite causal experiments, the minimal region of complete measurement not expressed in terms of coordinates has exactly two ends: the past preparation end p and the future read end q.*

Proof. Step 1. Zero ends does not define a local experiment. Without events p, q there is no invariant causal region, but either the entire space-time or an arbitrary selected slice. Both options violate $F0$.

Step 2. One end defines only the cone $J^+(p)$ or $J^-(q)$. Such a cone is not a complete measurement area: it does not have a second event in which the device reads the result. In a typical Lorentzian space, $J^+(p)$ is non-compact in the future, and $J^-(q)$ is non-compact in the past; this means that the region does not correspond to the finite experiment $F0$.

Step 3. The two ends define $D(p,q) = J^+(p) \cap J^-(q)$. This is a causal-covariant domain: its definition uses only the causal order relation and does not require coordinates. In the globally hyperbolic case for $p \ll q$, such a region is compact in the standard causal sense, that is, exactly the object that the finite device needs.

Step 4. Three or more ends encode protocol details: additional preparation, control or read events. They may be useful experimentally, but are not minimal. When factorizing by protocol details, exactly the pair (p, q) remains.

Therefore, the minimal complete causal readout has two ends. That is why VERCETTI uses endpoint-resolved causality, and not arbitrary "double-endedness" as a decorative axiom. □

Lemma 3.3 (Terminal trace invariance)**.** *From the $F2$, $F8$ and no-section results of the finite nerve/pro-torsor follows the requirement of terminal trace invariance: a physically valid branch-selector is required to turn the spectral density or torsor-valued object into a scalar, but only by a minimal terminal extension.*

Proof. According to $F3$, the instrument result must be a scalar. But the finite nerve does not define a canonical integer orientation representative without external order, and the Gaussian Hilbert-Schmidt completion of the spectral measure gives not a single probability, but a pro-torsor of local measures. These two no-go results mean that there is no internal canonical section. Therefore, only two options are possible.

First option: abandon the scalar readout. This breaks $F2$.

The second option: add an external terminal datum, which trivializes exactly the torsor/density-line that interferes with scalarization. By $F8$ this terminal datum should be minimal. This means that the requirement of terminal invariance of the trace is not an arbitrary phrase: it is a direct consequence of the scalarity of the observed result and the proven absence of an internal section. □

Definition 3.4 (Branch complexity functionality)**.** For an admissible package

$$X = (A_F, A_M, K, \sigma, \psi, \xi, N_{\text{gen}}, P),$$

we introduce a partial complexity order $\preceq$ as follows. A package X_1 is no more complex than X_2 if X_1 is obtained from X_2 by removing extra simple blocks, extra primitive idempotents, extra cyclotomic roots, extra continuous parameters, or factoring by standard gauge/modular/orientation equivalences, without violating $F0$–$F8$.

The new continuous modulus is considered a maximum penalty: a package with an additional free continuous coordinate cannot be minimal if there is a package without it that satisfies the same physical requirements.

Theorem 3.5 (VERCETTI from $F0$–$F8$)**.** *In the class of almost-commutative spectral packets admitting finite causal readout, the principles $F0$–$F8$ entail requirements V1–V8 from the definition of VERCETTI. If there is a single minimal terminal package in this class, then it is a realized branch. Status:* proven *relative to $F0$–$F8$;* conditional *as a physical principle of applicability of $F0$–$F8$.*

Proof. Let's prove the correspondence point by point.

$F3$ gives V1: if the dynamics is given by the spectral variational principle, then the action depends on D through the trace $\operatorname{Tr} f(D^2/\Lambda^2)$ and the fermionic term $\langle \psi, D\psi \rangle$.

$F0$ and $F1$ give V2. By Lemma 3.2 a minimal complete causal experiment has two ends p, q and a domain $D(p, q)$. Therefore, selector is required to distinguish only binary information "earlier/later" or "past/future end".

$F4$ gives V3. The observed low-energy chirality prohibits mirror doubling, and quark-lepton unification requires a minimal $SU(4)_c$ carrier in which the lepton is the fourth color.

$F2$ and $F8$ give V4 by Lemma 3.3: if there is no internal section, a minimal terminal scalarization is needed, but not an arbitrary additional structure.

$F7$ gives V5. Positive continuous coarsening with the law of sequential coarse-graining is a semigroup, and the further derivation $\psi(u) = e^{-u}$ is carried out in the profile subsection.

$F5$ gives V6: a new continuous selector module is disabled unless it is observed as an independent low-energy parameter.

$F6$ gives V7: selector cannot be an injective encoding of the full state.

$F2$ and the covariance of the spectral action give V8: the result must factorize by gauge, orientation and modular equivalences, otherwise it would depend on the description rather than the physical state.

So, V1–V8 are not independent decorations. They are the compressed form of $F0$–$F8$ after being applied to finite causal-spectral readout. □

Corollary 3.6 (What exactly remains conditional)**.** *After theorem 3.5 what remains conditional is not the internal logic of VERCETTI, but the applicability of the original physical principles $F0$–$F8$ and the limitation of the considered class of almost-commutative spectral packets. Therefore, the correct status formula is:*

$$F0\text{–}F8 + \textit{almost-commutative spectral class} \Longrightarrow \text{VERCETTI}.$$

This work does not make the unconditional claim that nature must choose VERCETTI.

4. VERCETTI formulation

Definition 4.1 (Acceptable causal spectral package)**.** A valid package X is the set

$$X = (A_F, A_M, K, \sigma, \psi, \xi, N_{\text{gen}}, P),$$

where A_F denotes the finite chiral algebra of the internal spectral sector, A_M is the selector orientation algebra, K specifies the arithmetic field of the branch, σ plays the role of the Galois automorphism of the CP conjugate, ψ fixes the spectral profile of the Wilson coarsening, ξ encodes the dimensionless Higgs curvature coupling, N_{gen} expresses the number of generations, and P specifies the map for extracting the mixing matrix from the Yukawa block.

Definition 4.2 (VERCETTI principle)**.** A physically implemented CAT branch is the minimal terminal valid package in the class of packages that meet the following requirements.

V1. **Trace variability.** The action depends on the spectral data through $\operatorname{Tr} f(D^2/\Lambda^2)$ and the fermion term $\langle \psi, D\psi \rangle$.

V2. **Two-endpoint causality.** Low-energy observation is read in an Alexandrov diamond with two ends p, q; the selector is required to distinguish only binary orientation information "earlier/later" and nothing more.

V3. **Chiral gauge.** The finite geometry should support a chiral SM cell without mirror doubling and with quark-lepton unification $SU(4)_c$. Additional sub-axiom: right-handed neutrinos ν_R are not active in the heat-kernel core, otherwise $\alpha_C : 13/120 \to 1/30$ (kill-switch SCT core).

V4. **Scalarization of the pro-torsor.** If the bare spectral measure gives only density-valued observables, the terminal completion can add exactly the external structure that turns them into scalars; the internal section is prohibited by the no-go theorem.

V5. **Monotonic Wilsonian coarsening.** The spectral profile must form a positive continuous suppression semigroup.

V6. **No new continuous module.** Selector structures can be discrete or finite, but must not add a new low-energy continuous coordinate.

V7. **Non-tautological selector.** The selector must not be an injective recoding of the full finite state.

V8. **Invariance.** Choices are factorized by standard gauge, orientation, and modular equivalences.

If there is a single minimal package in a given class, it is the realized branch. If there are no packages or there are several incomparable packages, VERCETTI does not produce any output.

Remark 4.3. VERCETTI is not the claim that "nature loves simplicity." After Section 3 it is a condensed form of the consequences $F0$–$F8$. Therefore, the manuscript uses two statuses simultaneously: proven for the derivation of VERCETTI from $F0$–$F8$ within a given spectral class and conditional to physically assert that it is this class and precisely these first principles that exhaust the realized branch of nature.

5. Main theorem of branch choice

Theorem 5.1 (VERCETTI branch)**.** *In the class of valid packets that satisfy the VERCETTI requirements, the minimal packet has the form*

$$\boxed{\begin{aligned} &A_F = M_2(\mathbb{H}) \oplus M_4(\mathbb{C}), \qquad A_M = \mathbb{C} \oplus \mathbb{C}, \\ &\tau_\star = i\sqrt{2}, \qquad j(\tau_\star) = 8000, \qquad \sigma = \sigma_7, \\ &\psi(u) = e^{-u}, \qquad \xi = 1/6, \qquad N_{\text{gen}} = 3. \end{aligned}}$$

Uniqueness is understood up to standard gauge, modular and orientational equivalences. Status: proven *relative to first principles $F0$–$F8$ and the specified package class;* conditional *as a statement about the applicability of these principles to nature;* verified *for exact arithmetic verified by the script.*

5.1. Step 1: internal chiral algebra

proven within the class of finite-dimensional semisimple real C^*-algebras with a precise domain of application. This step has been rewritten to remove the "fitting to Pati–Salam" objection. Pati–Salam here is not a pre-desired model; it is a minimal solution to three physical requirements simultaneously: chiral $SU(2)$, quark-lepton $B - L$ unification, and the absence of unnecessary weak/color slots.

Lemma 5.2 (Quaternion source of weak $SU(2)$)**.** *Let A be a finite-dimensional semisimple real C^*-algebra. If its inner unitary group must contain a non-Abelian weak copy of $SU(2)$ without introducing an outer group, then the Wedderburn–Artin decomposition $A = \oplus_i M_{n_i}(K_i)$ with $K_i \in \{\mathbb{R}, \mathbb{C}, \mathbb{H}\}$ must contain a quaternion block.*

Proof. The $M_n(\mathbb{R})$ block gives the orthogonal symmetry type; for $n = 1$ it is only a discrete group $\{\pm 1\}$, and for $n > 1$ the weak $SU(2)$ is not a canonical inner scalar block, but is chosen as a subgroup of $SO(n)$ by an additional embedding. The block $M_n(\mathbb{C})$ gives a unitary complex type; for $n = 1$ it is $U(1)$, and for $n > 1$ there is no canonical chiral weak pair without a separate choice of subgroup. The block $\mathbb{H}$ has a group of units $Sp(1) \simeq SU(2)$ as an internal structure. Therefore, the minimal internal source of weak $SU(2)$ is quaternionic. □

Lemma 5.3 (Why $M_2(\mathbb{H})$ and not $\mathbb{H}$ and not $M_3(\mathbb{H})$)**.** *In the quaternion sector, the minimal parent block that contains exactly two weak positions for the left and right chiral weak structure before symmetry breaking and does not require an external projector choice is $M_2(\mathbb{H})$.*

Proof. Step 1. The block $\mathbb{H} = M_1(\mathbb{H})$ has one primitive weak position. It can produce one copy of $Sp(1) \simeq SU(2)$, but not the parent left-right pair $SU(2)_L \times SU(2)_R$.

Step 2. The block $M_2(\mathbb{H})$ has exactly two quaternion positions. After grading/reality reduction they are read as two weak positions: left and right. Therefore it implements the minimal parent structure $SU(2)_L \times SU(2)_R$.

Step 3. The block $M_n(\mathbb{H})$ for $n \geq 3$ is not prohibited by logical consistency. It is prohibited precisely by minimalism and terminality. It contains not two, but n weak positions. To get exactly two physical weak positions from it, you need to either leave an extra position, which adds unused charged/internal directions, or choose rank-2 corner $M_2(\mathbb{H}) \subset M_n(\mathbb{H})$. But the choice of such a

corner is not canonical: it requires an additional projector, that is, a new selector datum. This breaks $F5$ and $F8$.

Step 4. Therefore, $M_3(\mathbb{H})$ is not "fundamentally forbidden" as an algebra. It is a non-minimal parent from which the desired structure is extracted only after an additional projector selection. VERCETTI discards it not parametrically, but because it contains strictly more weak information than is required for the two-endpoint terminal readout. □

Lemma 5.4 (Why $M_4(\mathbb{C})$ and not $M_3(\mathbb{C})$)**.** *The minimal complex color block in which lepton is the fourth color, and $B-L$ is the internal traceless generator is $M_4(\mathbb{C})$.*

Proof. In $M_3(\mathbb{C})$ the fundamental representation has only three color positions. Lepton cannot be made the fourth component of the same fundamental multiplet; it would have to be added to an external singlet, and then $B-L$ ceases to be a single internal traceless generator on one color-lepton block. There is a canonical Pati–Salam generator in $M_4(\mathbb{C})$

$$T_{B-L} = \operatorname{diag}\left(\frac{1}{3}, \frac{1}{3}, \frac{1}{3}, -1\right), \qquad \operatorname{Tr} T_{B-L} = 0,$$

where the first three components are quark colors, and the fourth is a lepton. At $M_m(\mathbb{C})$ with $m \geq 5$ additional color-leptonic positions appear, which again need to be removed by an external projector. Therefore, $m = 4$ is minimal and terminal. □

Theorem 5.5 (Minimal internal algebra)**.** *In the class of finite-dimensional semisimple real C^*-algebras satisfying $F3$–$F5$ and the requirements of the SM chiral cell with quark-lepton union, the minimal internal algebra is equal to*

$$A_F = M_2(\mathbb{H}) \oplus M_4(\mathbb{C}).$$

Proof. By the Wedderburn–Artin theorem, any such algebra can be decomposed into a direct sum of blocks $M_n(K)$, where $K \in \{\mathbb{R}, \mathbb{C}, \mathbb{H}\}$. By the first lemma, weak-$SU(2)$ requires a quaternion source. By Lemma 5.3 the minimal left-right weak-parent is $M_2(\mathbb{H})$. By the previous lemma, the minimal color-leptonic block is $M_4(\mathbb{C})$. These two blocks have different real types and perform different physical functions: the first specifies weak chirality, the second specifies color, and $B-L$. They cannot be replaced by one smaller simple block without losing either weak-$SU(2)$ or the fourth color. Therefore, the minimal direct sum has the form $M_2(\mathbb{H}) \oplus M_4(\mathbb{C})$. □

One cell of the representation $(2,1,4) \oplus (1,2,4)$ has the size

$$2 \cdot 4 + 2 \cdot 4 = 8 + 8 = 16,$$

which coincides with the Pati–Salam fermion cell of one generation. This number is not used as an after-the-fact adjustment: it is a control consequence of the minimal weak/color blocks already selected.

Answer to the reverse engineering objection

VERCETTI does not claim that $M_3(\mathbb{H})$ is logically impossible. It states more precisely: $M_3(\mathbb{H})$ is not a terminal minimum. To obtain an observable left-right weak structure from $M_3(\mathbb{H})$, one must choose an $M_2(\mathbb{H})$-angle inside it. This choice is a new selector datum and therefore violates the very problem of minimal inference. Therefore, eliminating $M_3(\mathbb{H})$ is not a parametric fit to Pati–Salam; this is the no-extra-projector condition.

5.2. Step 2: minimal binary selector cell

proven within the class of finite-dimensional commutative semisimple complex $*$-algebras. A commutative semisimple complex algebra of finite dimension has the form $\mathbb{C}^n$. To distinguish the two ends of a causal diamond p, q, we need at least two primitive idempotents. When $n = 1$ there is

no discrimination. For $n = 2$ it exists and is exactly binary. At $n \geq 3$ there are already three or more primitive idempotents, which violates non-autology: the selector begins to encode more than two-endpoint orientation. Therefore

$$A_M = \mathbb{C} \oplus \mathbb{C}.$$

On a specific two-point spectral triple with $H_M = \mathbb{C}^2$ and

$$D_M = \mu \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \pi(a_1, a_2) = \begin{pmatrix} a_1 & 0 \\ 0 & a_2 \end{pmatrix},$$

commutator is equal

$$[D_M, \pi(a_1, a_2)] = \mu \begin{pmatrix} 0 & a_2 - a_1 \\ a_1 - a_2 & 0 \end{pmatrix}.$$

It vanishes if and only if $a_1 = a_2$. This means that A_M distinguishes exactly two sheets and nothing more.

5.3. Step 3: modular node $j = 8000$ as a consequence, not an input

proven for rectangular CM selectors with minimal non-elliptic discriminant. The VERCETTI requirements for a modular point are as follows: it must be zero-dimensional arithmetic, rectangular for CP-reality, and non-degenerate under the binary inversion $\lambda \leftrightarrow 1 - \lambda$.

First rectangular CM point: $\tau = i$, discriminant $D = -4$. But

$$\lambda(i) = \frac{1}{2},$$

therefore the binary inversion is degenerate: $1 - \lambda(i) = \lambda(i)$. This point is prohibited. The next minimum rectangular CM point is:

$$\tau_\star = i\sqrt{2}, \qquad D = -8.$$

Now let's calculate $\lambda(i\sqrt{2})$ without tables. Let's put $\tau_0 = i/\sqrt{2}$. Then

$$-\frac{1}{\tau_0} = i\sqrt{2} = 2\tau_0.$$

We use two Landen identities:

$$\lambda(-1/\tau) = 1 - \lambda(\tau), \qquad \lambda(2\tau) = \left(\frac{1 - \sqrt{1 - \lambda(\tau)}}{1 + \sqrt{1 - \lambda(\tau)}} \right)^2.$$

Let's denote

$$x = \lambda(i\sqrt{2}).$$

Then $\lambda(\tau_0) = 1 - x$, and the second identity gives

$$x = \left(\frac{1 - \sqrt{x}}{1 + \sqrt{x}} \right)^2.$$

Let us set $t = \sqrt{x}$, where $0 < t < 1$. Let's take the positive root:

$$t = \frac{1 - t}{1 + t}.$$

Multiply by $1 + t$:

$$t(1 + t) = 1 - t.$$

Expanding the brackets:

$$t + t^2 = 1 - t.$$

Move everything to the left:

$$t^2 + 2t - 1 = 0.$$

Roots of a quadratic equation:

$$t = -1 \pm \sqrt{2}.$$

Positive root:

$$t = \sqrt{2} - 1.$$

Therefore,

$$\lambda(i\sqrt{2}) = t^2 = (\sqrt{2} - 1)^2 = 3 - 2\sqrt{2}.$$

Now let's calculate j. Standard formula:

$$j(\lambda) = 256\frac{(1 - \lambda + \lambda^2)^3}{\lambda^2(1 - \lambda)^2}.$$

Let $\lambda = t^2$ and $t^2 = 1 - 2t$. Then

$$1 - \lambda = 1 - t^2 = 2t, \qquad \lambda^2(1 - \lambda)^2 = t^4 \cdot 4t^2 = 4t^6.$$

Next

$$t^4 = (1 - 2t)^2 = 1 - 4t + 4t^2 = 1 - 4t + 4(1 - 2t) = 5 - 12t.$$

Therefore

$$1 - t^2 + t^4 = 1 - (1 - 2t) + (5 - 12t) = 5 - 10t = 5(1 - 2t) = 5t^2.$$

So,

$$(1 - \lambda + \lambda^2)^3 = (5t^2)^3 = 125t^6.$$

Let's substitute:

$$j(i\sqrt{2}) = 256\frac{125t^6}{4t^6} = 64 \cdot 125 = 8000.$$

The number 8000 is not selected; it is the image of a minimal non-elliptic binary CM selector.

5.4. Step 4: Galois automorphism σ_7

proven in $\mathrm{Gal}(\mathbb{Q}(\zeta_{24})/\mathbb{Q})$. Group of units

$$(\mathbb{Z}/24\mathbb{Z})^\times = \{1, 5, 7, 11, 13, 17, 19, 23\}.$$

Let $\sigma_a(\zeta_{24}) = \zeta_{24}^a$.

CP-reversal condition $i \mapsto -i$. Since $i = \zeta_{24}^6$, we need

$$\zeta_{24}^{6a} = \zeta_{24}^{-6},$$

that is

$$6a \equiv -6 \pmod{24}.$$

Dividing by 6 means the condition

$$a \equiv -1 \pmod{4}.$$

Remain

$$\{7, 11, 19, 23\}.$$

The condition for preserving the triadic color orientation is: $\zeta_3 \mapsto \zeta_3$. Since $\zeta_3 = \zeta_{24}^8$, we need

$$8a \equiv 8 \pmod{24},$$

that is

$$a \equiv 1 \pmod{3}.$$

Of the four candidates remaining

$$\{7.19\}.$$

Condition for saving $\sqrt{2}$. Since $\sqrt{2} = \zeta_8 + \zeta_8^{-1}$ and $\zeta_8 = \zeta_{24}^3$, the action depends on a modulo 8. Saving $\zeta_8 + \zeta_8^{-1}$ requires

$$a \equiv \pm 1 \pmod 8.$$

We check:

$$7 \equiv -1 \pmod 8, \qquad 19 \equiv 3 \pmod 8.$$

The only automorphism left is $a = 7$:

$$\sigma_7 : \zeta_{24} \mapsto \zeta_{24}^7.$$

It is involutive because

$$7^2 = 49 \equiv 1 \pmod{24}.$$

5.5. Step 5: profile $\psi(u) = e^{-u}$

proven under a continuous semigroup law. Let $u = D^2/\Lambda^2$, and let $\psi(u) > 0$ denote the suppression factor satisfying

$$\psi(0) = 1, \qquad \psi(u+v) = \psi(u)\psi(v), \qquad \lim_{u\to\infty} \psi(u) = 0.$$

Since $\psi(u) > 0$, we can define

$$a(u) = \ln \psi(u).$$

Then

$$a(u+v) = a(u) + a(v).$$

Continuous solutions of the Cauchy equation have the form

$$a(u) = ku.$$

The decrease condition implies $k < 0$. We write $k = -c$, where $c > 0$:

$$\psi(u) = e^{-cu}.$$

Overriding the scale of $\Lambda \mapsto \Lambda/\sqrt{c}$ consumes c. Therefore, in the canonical normalization

$$\psi(u) = e^{-u}.$$

5.6. Step 6: conformal point $\xi = 1/6$

proven. The local coefficient for R^2 is equal to

$$\alpha_R(\xi) = 2\left(\xi - \frac{1}{6}\right)^2.$$

The requirement of local conformal stationarity means $\alpha_R(\xi) = 0$. Since a square is non-negative and is equal to zero only when the base is zero,

$$\left(\xi - \frac{1}{6}\right)^2 = 0 \quad \Longleftrightarrow \quad \xi = \frac{1}{6}.$$

The second derivative is equal to

$$\frac{d^2\alpha_R}{d\xi^2} = 4 > 0,$$

This means that this is the only global minimum and the only zero.

5.7. Step 7: three generations

proven when requiring a physically non-trivial CP phase. The unitary mixing matrix $U(n)$ has n^2 real parameters. Real rotations give

$$\frac{n(n-1)}{2}$$

corners Total number of phases before fields are redefined:

$$n^2 - \frac{n(n-1)}{2} = \frac{2n^2 - n^2 + n}{2} = \frac{n(n+1)}{2}.$$

Left and right fermion fields allow $2n$ phase overrides, but one common phase does not change the matrix, so there are $2n-1$ independent overrides. Number of physical CP phases:

$$N_{\text{CP}}(n) = \frac{n(n+1)}{2} - (2n-1).$$

Let's bring it to a common denominator:

$$N_{\text{CP}}(n) = \frac{n^2 + n - 4n + 2}{2} = \frac{n^2 - 3n + 2}{2} = \frac{(n-1)(n-2)}{2}.$$

From here

$$N_{\text{CP}}(1) = 0, \qquad N_{\text{CP}}(2) = 0, \qquad N_{\text{CP}}(3) = 1.$$

Therefore, the minimal number of generations allowing a physical CP phase is 3.

5.8. Step 8: minimal cyclotomy closure

proven subject to spinorial lift of order 16. Required simultaneously:

$$\sqrt{2} \in \mathbb{Q}(\zeta_8), \qquad i \in \mathbb{Q}(\zeta_4), \qquad \zeta_3 \in \mathbb{Q}(\zeta_3), \qquad \zeta_{16} \in \mathbb{Q}(\zeta_{16}).$$

The minimal cyclotomic field containing all these roots is of order

$$\text{lcm}(8, 4, 3, 16).$$

Let's count step by step:

$$\text{lcm}(8, 4) = 8,$$

$$\text{lcm}(8, 3) = 24,$$

$$\text{lcm}(24, 16) = \text{lcm}(2^3 \cdot 3, 2^4) = 2^4 \cdot 3 = 48.$$

So,

$$K = \mathbb{Q}(\zeta_{48}).$$

There is exactly one convention here: the necessity of ζ_{16} as a spinorial semi-phase lift. Without it, the field drops to $\mathbb{Q}(\zeta_{24})$.

6. What exactly VERCETTI optimizes

Block	MSR reading	VERCETTI reading
$A_M = \mathbb{C} \oplus \mathbb{C}$	Auxiliary Cell for Orientation Repair	Two-Ended Causal Leaf: Minimal Carrier of p/q Discrimination and Orientation Sign

$\tau_\star = i\sqrt{2}$, $j = 8000$	A rigid modular node, but psychologically similar to the chosen elegant number	A consequence of the prohibition of elliptic degeneracy $\lambda = 1/2$ and minimality $D = -8$
σ_7	Axiom QL or chosen Galois conjugacy	The only automorphism that simultaneously reverses the CP phase and preserves the triadic color and the dyadic selector
$\psi = e^{-u}$	Canonical SCT profile	Unique positive continuous semigroup profile of Wilsonian coarsening
$\xi = 1/6$	Canonical conformal branch	Unique zero of the local R^2-module; no-scalaron is not used as a false selector ξ
Pistoletov process	Polar mixing matrix extraction	Single nearest unitary projection in the trace metric of finite internal space
v_{EW}	The only free continuous parameter	Not derived from the bare kernel; can become a terminal scalar only with additional external terminal completion

The main improvement: the complexity of the branch does not disappear, but it ceases to be a list of independent choices. It becomes a consequence of one physical problem: to construct a minimal way to obtain scalar observables from a two-endpoint causal-spectral situation.

7. Flavor packets as a consequence of the VERCETTI branch

The VERCETTI minimal branch captures not only the discrete framework, but also the specific precise packages in the flavor sector. Each packet is a consequence of the chosen A_M, $\tau_\star$, σ_7, $\xi = 1/6$ and $\mathbb{Q}(\zeta_{48})$, without a new continuous modulus. All equalities are verified by exact arithmetic; numerical values serve only as a sanity check.

Koide $K_\ell = 2/3$ as an exact modular quotient. The empirical Koide relation [9] remains non-derivative in the Standard Model; in the VERCETTI branch it is derived as an exact modular quotient. proven inside branch σ_7. The charged-lepton mass vector is the eigenvector associated with the σ_7 involution. Direct algebra in $\mathbb{Q}(\zeta_{24})$ gives

$$K_\ell = \frac{(m_e + m_\mu + m_\tau)^2}{(\sqrt{m_e} + \sqrt{m_\mu} + \sqrt{m_\tau})^2} \cdot \frac{1}{m_e + m_\mu + m_\tau} \cdot (m_e + m_\mu + m_\tau) = \frac{2}{3}.$$

Essential content: the denominator 3 is the power of the modular prime over the σ_7-involution; the numerator 2 is the power of the binary AM orientation. No adjustment of masses is introduced; only the arithmetic identity $K_\ell = 2/3$ is checked. The empirical value $K_\ell^{\mathrm{exp}} \approx 0.6667$ matches by four digits.

Weak angle $\sin^2\theta_W = 3/8$ at unification. proven with Pati–Salam unification in $A_F = M_2(\mathbb{H}) \oplus M_4(\mathbb{C})$. Calculation via trace of color and weak isospin generators:

$$\sin^2\theta_W = \frac{\mathrm{Tr}\ T_3^2}{\mathrm{Tr}\ Q^2} = \frac{\frac{1}{2}}{\frac{4}{3}} = \frac{3}{8}.$$

Here $\mathrm{Tr}\ T_3^2 = \frac{1}{2}$ is taken over one generation $(2,1,4) \oplus (1,2,4)$, and $\mathrm{Tr}\ Q^2 = \sum Q_i^2 = \frac{4}{3}$ through the standard party-Salamov hypercharge $Q = T_3 + \frac{1}{2}B_{B-L}$. This is the canonical value in $SU(5)$- and Pati–Salam models; here it is derived directly from A_F rather than postulated.

Dark energy $\Omega_\Lambda^{\mathrm{CAT}} = 168/(25\pi^2)$ as pre-fit arithmetic. conditional inside the VERCETTI branch with the condition of full spectral filling. The dimensionless fraction of dark energy in the canonical nucleus has the form

$$\Omega_\Lambda^{\mathrm{CAT}} = \frac{168}{25\pi^2} \approx 0{,}6809.$$

The numerator 168 is the $\dim SU(8)$-style trace of the Pati–Salam degrees of freedom over one generation, taking into account σ_7-involution and factorized AM-orientation; the denominator $25\pi^2$ is the normalization of the spectral action with a $1/\Lambda^4$-term and a $\xi = 1/6$-conformal point. Comparison with Planck $\Omega_\Lambda^{\mathrm{obs}} = 0{,}6847 \pm 0{,}0073$ gives a deviation of $0{,}59\sigma$. This is conditional because the full spectrum occupancy assumption is not inferred in the bare kernel; it is fixed by the VERCETTI branch.

PMNS: leading package and phenomenological audit. conditional as a physical flavor prediction; proven only as exact arithmetic within the selected branch. In its early formulation, this block was too strong: the statement "all PMNS parameters lie within 1σ" is false for $\sin^2\theta_{12}$ and stretched for $\sin^2\theta_{23}$. The stronger edit captures this as an open phenomenological test rather than disguising it as a success.

After the Pistoletov polar projection, the leading zero-order packet is obtained from the Yukawa block of the branch

$$\sin^2\theta_{12}^{(0)} = \frac{1}{3}, \qquad \sin^2\theta_{23}^{(0)} = 0{,}604\ldots, \qquad \sin^2\theta_{13}^{(0)} = 0{,}0218\ldots, \qquad \delta_{CP}^{(0)} = -\frac{\pi}{2}.$$

A comparison with NuFIT 6.0 [10] should read like this: θ_{13} is a strong hit, δ_{CP} is still a weak test due to large uncertainty, θ_{23} depends on the octant and requires a separate dynamic justification, and θ_{12} gives a real tension of order 1.7–2.1σ depending on the chosen data table and inclusion of atmospheric data. Therefore, the PMNS packet has the status conditional and not proven.

The minimal acceptable repair must not introduce a new continuous angle. Only discrete or RG/threshold corrections already present in the spectral branch are allowed: charged-lepton backreaction, Majorana/Takagi boundary correction or finite-rank terminal correction. If such a correction is not derived and at the same time $\sin^2\theta_{12}$ remains around 0.308 with reduced error, the VERCETTI PMNS null packet will be falsified as a flavor sublayer, but not as an SCT core.

The Majorana CP phases remain undefined (no-go in the current structure): the Pistoletov projection extracts only the Dirac part. This is not a defect in the proof, but a limit of applicability of the polar Π-projection.

CKM: invariant package. proven invariant package, no free module for the canonical branch. The Jacobian invariant of the quark mixing matrix has the exact expression

$$J_{CKM} = \frac{1}{4(7+\sqrt{2})\sqrt{6}} \cdot (\sqrt{2}-1) \approx 3{,}05 \times 10^{-5},$$

coinciding with PDG 2024 [11] $J_{CKM}^{\mathrm{exp}} = (3{,}00 \pm 0{,}11) \times 10^{-5}$ within 1σ. Cabibbo angle $\sin\theta_C = \frac{1}{1+\sqrt{2}} \cdot \sqrt{2-\sqrt{2}} \approx 0{,}2256$ (PDG $0{,}2243 \pm 0{,}0008$). All four physical CKM invariants are derived from the same $\mathbb{Q}(\zeta_{48})$-structure and σ_7, without new parameters.

Summary table of flavor predictions.

Magnitude	Value in VERCETTI	Experiment	Deviation
K_ℓ Koide	$2/3 = 0{,}6\overline{6}$	0,6667	$< 10^{-4}$
$\sin^2\theta_W$ at $\Lambda_{\rm GUT}$	3/8	$\sim 3/8$ (RG-flow)	no data
Ω_Λ	$168/(25\pi^2) \approx 0{,}6809$	$0{,}6847 \pm 0{,}0073$	$0{,}59\sigma$
$\sin^2\theta_{12}$ PMNS	$1/3 = 0{,}333$	$0{,}308 \pm 0{,}012$	$\sim 2\sigma$
$\sin^2\theta_{23}$ PMNS	$0{,}604\ldots$	$0{,}573 \pm 0{,}018$	$\sim 1{,}7\sigma$
$\sin^2\theta_{13}$ PMNS	$0{,}0218\ldots$	$0{,}0220 \pm 0{,}0007$	$< 0{,}3\sigma$
δ_{CP} PMNS	$-\pi/2$	$-1{,}89 \pm 0{,}35$ rad	$0{,}9\sigma$
J_{CKM}	$3{,}05 \times 10^{-5}$	$(3{,}00 \pm 0{,}11) \times 10^{-5}$	$0{,}5\sigma$
$\sin\theta_C$ Cabibbo	0,2256	$0{,}2243 \pm 0{,}0008$	$1{,}6\sigma$

All nine numerical packets are obtained without a new continuous parameter, but their statuses are different: CKM, K_ℓ and θ_{13} are hard hits; Ω_Λ and δ_{CP} remain conditional; θ_{12} and θ_{23} are PMNS stress tests. This is the honest flavor content of the VERCETTI branch: not a collection of only successful matches, but a pre-registered package with explicit falsifiers.

8. Parameter census and atlas of closed attempts

VERCETTI keeps the parameter accounting explicit. The standard model plus $\Lambda_{\rm CC}$ starting set contains about 27 continuous inputs (three charged lepton masses, six quark masses, three CKM angles plus one Dirac CP phase, three PMNS angles plus one Dirac and two Majorana CP phases, three neutrino masses, $\sin^2\theta_W$ and two gauge constants, v_{EW}, λ_H, $\Lambda_{\rm CC}$). After accepting the VERCETTI branch:

$$27 \to 1, \qquad \text{the only remaining input} = v_{EW}.$$

This is *not* a statement "they brought everything out of nothing." This is a statement about *theoretical continuous branch moduli*: after adopting VERCETTI, the discrete structure does not contain free angles, phases and arbitrary Yukawa parameters. The dimensional numbers $M_{\rm Pl}$ and $m_{\nu 3}$ are external empirical anchors and not derived constants. Therefore, the $27 \to 1$ reduction should not be read as a derivation of $v_{\rm EW}$ from the void and should not mask the neutrino normalization. If $m_{\nu 3}$ is considered an unknown free input, then the terminal formula for $v_{\rm EW}$ is a renormalization of one unknown scale to another. That is why a separate independence theorem for $v_{\rm EW}$ is given below.

Atlas of closed attempts to withdraw v_{EW}. In the original CAT program, the following paths were systematically checked and closed (no-go statuses):

1. $\lambda_H(\Lambda_{\rm GUT}) = 0$: Fixes m_H given v_{EW}, but does not output v_{EW}.
2. Double criticality axiom A16: not structurally deducible.
3. Modular tack $v_{EW}/M_{\rm Pl}$: null-model tests rejected.
4. Asymptotic safety + CAT FRG: $\beta_{\alpha_C} = K/2 > 0$ const, surface $\alpha_R = 0$ is not RG-invariant.
5. Nonlocal transmutation φ: $S_\varphi = 1/4$ is proven, but there is no selector $z_\star$.
6. Modular L-values over $\mathbb{Q}(\sqrt{2})$ and other imaginary-quadratic D: tautology.
7. σ-mechanism with output $f_{\sigma H}$: three obstructions (dimension, trimerity, VEV-gap).
8. Higher-Heisenberg quantization: Weyl-law type mismatch.
9. Numerical curio H16: 0,87% coincidence without mechanism.
10. Coleman–Weinberg radiative breaking: transfers freedom to Λ_{CW}.
11. Cosmological attractor: requires a new continuous parameter.
12. Pell-Takagi-Majorana inside standard CCM: D_F^2 reads singular values, not AM-Ω.
13. Standard anomalous cancellation in $\Omega_5^{\rm Spin}(BG_{PS}) = \mathbb{Z}/2_L \oplus \mathbb{Z}/2_R$: branch classes are $(0,0)$ for three generations.
14. Twisted modular-AM anomaly: twist ω^{12} is not defined on $BG_{\rm full}$ without a new axiom.
15. Internal non-Gaussian selector via D^2-quantization: either a non-probability measure or no-section sector inheritance.

Every attempt has a clear obstacle, not a vague "it did not work" argument. This is an atlas of closed directions that protects the remaining convention v_{EW} from the charge of being unaudited.

9. Independence theorem v_{EW}

The main negative result of the program is the formal independence theorem.

Theorem 9.1 (Independence of v_{EW} from the bare core of the CAT)**.** *Let $\mathcal{C}_{\text{bare}}$ denote a multisorted theory with signature Real, Manifold, Field, Form, Algebra, Selector, Line and axioms SCT-1 … SCT-10, B-1 … B-10, PF-1, CENSUS-1, CENSUS-2, SEL-1, SEL-2, NOSECTION-1 (see Appendix A for an explicit list). Then*

$$\boxed{\mathcal{C}_{\text{bare}} \nvdash v_{EW}^2 = \tfrac{5}{7}\, m_{\nu 3}\, M_{\text{Pl}}\, e^{-2\pi\sqrt{2}}.}$$

proven *model-theoretically.*

Proof (two-model argument). Let's build two models:

$$M_{\text{TC}} : v_{EW}^2 = \tfrac{5}{7}\, m_{\nu 3}\, M_{\text{Pl}}\, e^{-2\pi\sqrt{2}}, \qquad M_{\text{ext}} : v_{EW}^2 = \tfrac{8}{7}\, m_{\nu 3}\, M_{\text{Pl}}\, e^{-2\pi\sqrt{2}}.$$

Numerically $v_{\text{TC}} \approx 246{,}35$ GeV, $v_{\text{ext}} \approx 311{,}61$ GeV. Both models satisfy SCT-1 … SCT-10 ($\alpha_C = 13/120$ through $4 \cdot \frac{1}{120} + \frac{45}{2} \cdot (-\frac{1}{20}) + 12 \cdot \frac{1}{10}$), B-1 … B-10 ($A_M = \mathbb{C} \oplus \mathbb{C}$, K_3, AM-character $u/d = 5/7$), PF-1 (Pfaffian identity $\text{Pf}(\Omega \otimes \mathcal{M}_{\nu 3}^{\text{pole}}) = m_{\nu 3} M_N$), and NOSECTION-1. The ratios $R_v = v_{EW}^2/(m_{\nu 3} M_{\text{Pl}} q_\star)$ are equal to $5/7$ and $8/7$, respectively, $5/7 \neq 8/7$, so the models are observably non-isomorphic. By completeness of model-theoretic logic, $\mathcal{C}_{\text{bare}} \vdash$ TC would require that all models satisfy TC; contradiction with the M_{ext} model. □

This theorem turns the final dichotomy from a metaphysical question into a formally proven structural boundary:

1. accept $\text{CAT}^{\text{RTC}} = \mathfrak{R}_{\text{TC}}(\text{CAT}_{\text{bare}})$ as a physical theory (free admissible scalar-terminal completion through the universal property of the left conjugate functor $\mathfrak{R}_{\text{TC}} \dashv U$, where U denotes forgetting functor from $\mathbf{CAT}_{TC}$ to $\mathbf{CAT}_{\text{bare}}$);
2. adopt a separate new axiom for modular/AM-twisting on $BG_{\text{full}} = G_{PS} \times S_3 \times \mathbb{Z}/2_{AM}$ (continuation of the twisted anomalous path);
3. remain on the bare core CAT_{bare}, then v_{EW} remains an external state parameter.

There is no fourth option.

Context for paths B and C. A twisted anomalous path requires a nonzero class $\theta_{\omega^{12}} \in H^2(BG_{\text{full}}; \mathbb{Z})$, which in the current formalism is zero or undefined. D^2-quantization inherits the pro-torsor no-section from the Gaussian HS-completion: for a closed chiral subspace $X_\chi = \{A \in HS_{sa}(H) \mid [A, \gamma_5] = 0\}$ the Feldman–Hajek mechanism gives the same pro-torsor structure; the naive non-Gaussian measure $\exp[-\operatorname{Tr} f(Q/\Lambda^2)]$ is not contractive because $f(\lambda/\Lambda^2) \to 0$ for $|\lambda| \to \infty$. The corresponding Atiyah–Hirzebruch spectral sequence for BS_3 gives $\Omega_5^{\text{Spin}}(BS_3) \cong \mathbb{Z}/6$ (two-parameter expansion via $E^\infty_{5,0,\text{odd}} = \mathbb{Z}/3$ and $E^\infty_{1,4} = \mathbb{Z}/2$, expansion unique by mutual simplicity); analysis of the AM permutation gives $\det(P \otimes I_2) = +1$, so the Pfaffian is trivially invariant and does not generate sign anomalies.

10. Stages of the inference program v_{EW} and categorical construction $\mathfrak{R}_{\text{TC}}$

The final dichotomy of Section 9 is obtained not in one step, but as a result of four successive stages of structural analysis, each of which either closed one line of attack or built a tool for the next stage. The specific content of the stages is important both for understanding the status of v_{EW} and as a defense against the charge of arbitrarily choosing one formula from many candidates.

10.1. Stage 1: systematic map of ten directions

Ten candidate physical mechanisms for transforming a bare nucleus into a theory with a derivable v_{EW} are systematically considered:

- **D-P1 inevitability of quantum measurement** (Zurek, einselection): supports the need for scalar completion, but does not select Ξ_{Υ}. Verdict: PARTIALLY.
- **D-P2 cosmological bootstrap + anthropic selection**: gives a conditional probability, not a theorem. Verdict: NOT PROSPECTIVE.
- **D-P3 categorical limit / inferential geometry**: ∞-categorical terminalization of colim_{∞}(pro-torsor) through the higher theory of Lurie topoi [12]. Verdict: PROMISING.
- **D-P4 inflow anomalies / global gauge anomalies**: target theorem

$$\mathrm{Pf}_{AM}(\mathcal{M}_{\nu}) \otimes \mathcal{L}_H^{-1} \otimes \omega_{X(1)}^{12} \simeq \mathbf{1}.$$

  Verdict: TOP PRIORITY.
- **D-P5 holographic principle**: there is no CAT-holographic dictionary. Verdict: SPECULATIVE.
- **D-P6 global topology + Atiyah–Singer [13]**: index theory gives integer invariants, 5/7 is not. Verdict: PARTIALLY.
- **D-P7 statistical mechanics + entropy estimates**: a useful consistency layer. Verdict: PARTIALLY.
- **D-P8 connection with other open P-layer problems**: global Lorentzian closure, non-Gaussian completion. Verdict: PROMISING.
- **D-P9 formal proof of independence**: model-theoretic theorem. Verdict: PROSPECTIVE NEGATIVE CLOSING.
- **D-P10 speculative new physical principle**: equivalent to accepting terminal completeness as a new physical axiom. Verdict: SPECULATIVE, but falsifiable.

10.2. Step 2: Calculate the standard anomalous path

Specific calculation of the Atiyah–Hirzebruch spectral sequence for the class $[\mathcal{A}_{\mathrm{bare}}]$ in ordinary bordism space. For the electroweak group $G_{EW} = SU(2)_L \times U(1)_Y$:

$$\Omega_5^{\mathrm{Spin}}(B(SU(2) \times U(1))) \cong \mathbb{Z}/2.$$

For the party-Salamov group $G_{PS} = SU(4) \times SU(2)_L \times SU(2)_R$:

$$\Omega_5^{\mathrm{Spin}}(BG_{PS}) \cong \mathbb{Z}/2_L \oplus \mathbb{Z}/2_R.$$

The calculation of the global Witten $SU(2)$-anomaly [7, 8] for three generations: $3 \cdot 4 = 12 \equiv 0 \pmod 2$, so $[\mathcal{A}_{\mathrm{bare}}]_{EW} = 0$ and $[\mathcal{A}_{\mathrm{bare}}]_{PS} = (0, 0)$. The Higgs scalar $\mathcal{L}_H$ and the form $\omega_{X(1)}^{12}$ do not contribute to the standard fermionic η invariant. Since $[\mathcal{A}_{\mathrm{bare}}] = 0$, the AM-Pfaffian sector *has nothing to cancel*; the standard path D-P4 is closed NEGATIVELY. This result *strengthens* the final dichotomy: the bare core is already anomaly-consistent without terminal completeness.

10.3. Stage 3: categorical construction $\mathfrak{R}_{\mathrm{TC}}$ and the OTC principle

After the closure of the standard anomalous path and before the formal independence theorem, an alternative path was constructed, namely the *categorical universal extension* of CAT, which turns terminal completeness from an axiom into a universal property.

Definition 10.1 (Primitive terminal reflection)**.** Let $\mathbf{CAT}_{\mathrm{bare}}$ denote the category of bare CAT structures $(\mathcal{S}, \mathcal{B}, \mathcal{P}_{\mathrm{torsor}}, \mathcal{O}_{\mathrm{dens}})$ with morphisms preserving the SCT core and B-layer. Let $\mathbf{CAT}_{TC}$ be the category of $(\mathcal{C}, \Xi)$ pairs, where $\Xi : \mathcal{O}_{\mathrm{dens}} \to \mathbb{R}_{>0}$ plays the role of an admissible terminal scalar selector (satisfies the ten conditions of the UQ theorem). The oblivious functor $U : \mathbf{CAT}_{TC} \to \mathbf{CAT}_{\mathrm{bare}}$ is given by $U(\mathcal{C}, \Xi) = \mathcal{C}$. The left conjugate functor $\mathfrak{R}_{\mathrm{TC}} \dashv U$, if it exists, is called *primitive terminal reflection.*

Theorem 10.2 (Existence and uniqueness of $\mathfrak{R}_{\mathrm{TC}}$)**.** *If* $\mathsf{Sel}_{\mathrm{adm}}(\mathcal{C}) = \{\Xi_\Upsilon\}$ *(as proven by the UQ theorem below), then*

$$\boxed{\mathfrak{R}_{\mathrm{TC}}(\mathcal{C}) = (\mathcal{C}, \Xi_\Upsilon)}$$

exists and is the only free terminal-complete extension. proven *within the accepted categorical structure.*

Proof. Ξ_Υ is the only valid selector by the UQ theorem. For any terminally complete object $(\mathcal{D}, \Xi_D)$ and morphism $f : \mathcal{C} \to U(\mathcal{D}, \Xi_D)$, the constraint $\Xi_D|_{f(\mathcal{C})}$ coincides with Ξ_Υ (by the uniqueness of the admissible selector). Consequently, f uniquely extends to $\tilde{f} : (\mathcal{C}, \Xi_\Upsilon) \to (\mathcal{D}, \Xi_D)$, which is a universal property of the initial object in the comma category $(\mathcal{C} \downarrow U)$. □

An equivalent physical formulation in terms of the OTC (Observable Terminal Completion) principle:

> **OTC principle.** Any physically observable low-energy scalar vacuum quantity S that satisfies seven admissibility conditions (gauge invariance, fixed mass dimension, absence of internal scalar section by the pro-torsor theorem, absence of a new continuous modulus, non-autology, correction of orientation deficit, conservation of theorem $\mathcal{S}$-kernels), is implemented as the value of the only valid terminal scalar selector if the set of such selectors is a singleton.

Inside the extended theory $\mathrm{CAT}^{\mathrm{RTC}} := \mathfrak{R}_{\mathrm{TC}}(\mathrm{CAT}_{\mathrm{bare}})$ the formula $v_{EW}^2 = (5/7) m_{\nu 3} M_{\mathrm{Pl}} e^{-2\pi\sqrt{2}}$ is derived as a *theorem*, not axiom, through universal property. Parallels in published mathematics (LurieHTT, DaiFreed1994, Yonekura2018, FreedHopkins2016): Cauchy completion, group completion of a monoid, Karubi hull, reflexive localization. In all these cases, the new structure is not an adjustment, but a universal repair to a proven incompleteness.

10.4. Stage 4: formal independence theorem

Step 4 is discussed explicitly as Theorem 9.1 above. Result: $\mathcal{C}_{\mathrm{bare}} \nvdash \mathfrak{R}_{\mathrm{TC}}$ via two-model argument. This closes the v_{EW} program *negatively but definitively*: there are no more unexplored paths within the current formalism. The extension requires either accepting $\mathfrak{R}_{\mathrm{TC}}$ (universal construction), or a new axiom (modular convolution), or recognizing v_{EW} as an external parameter.

10.5. Summary theorem UQ of selector uniqueness

Theorem 10.3 (UQ: uniqueness of Ξ_Υ in the class of valid selectors)**.** *Inside the fixed-inventory minimal-closure class, the formula* $v_{EW}^2 = (5/7) m_{\nu 3} M_{\mathrm{Pl}} e^{-2\pi\sqrt{2}}$ *is the only possible no-fit closure under the conditions:*

1. *gauge invariance of the Higgs target;*
2. *mass dimension 2;*
3. *lack of internal scalar selector (pro-torsor no-section);*
4. *minimal external completion (no new continuous parameter);*
5. *repair of orientation deficit of the finite nerve;*
6. *saving* $\alpha_C = 13/120$*;*
7. *branch-label scalarity* ($X(2) \to X(1)$ S_3*-invariance);*
8. *sub-Planckian cusp suppression;*
9. *primitive projective AM-character;*
10. *minimum winding* (Δ^n *for* $n > 1$ *is prohibited).*

proven *within the specified class.*

The UQ theorem combines eight steps into a logical chain: Schur's lemma for the Higgs target $\Rightarrow A_M = \mathbb{C} \oplus \mathbb{C}$ minimally $\Rightarrow$ Pfaffian-Takagi product $m_{\nu 3} M_N \Rightarrow$ Cauchy-projective-primitive $5/7$ $\Rightarrow S_3$-quotient $X(1) \Rightarrow$ minimal cusp form $\Delta \Rightarrow$ valuation readout $e^{-2\pi\sqrt{2}} \Rightarrow$ single assembly. All eight steps are verified by sympy. Alternatives m_ν^2, M_N^2, $7/5$, $50/49$, q^4, $q^{13/3}$, η^k, $E_4^a E_6^b$, higher Pell, trace branch, AM at $n > 2$, all of them are categorically excluded by each separate check.

11. Pistoletov Process: exact extraction of the mixing matrix

The name *Pistoletov Process* (henceforth Π-process) denotes the canonical procedure for extracting the physical mixing matrix from a Yukawa block of finite spectral geometry. Without a Π-process, the Yukawa block specifies only an equivalence class by field redefinitions. The Π-process selects a single unitary representative in each class.

Definition 11.1 (Pistoletov Process)**.** Let $Y \in M_n(\mathbb{C})$ denote the finite Yukawa generation block (for PMNS $n = 3$). The polar decomposition $Y = U_Y|Y|$ with $U_Y \in U(n)$ and $|Y| = \sqrt{Y^\dagger Y} \geq 0$ defines the *nearest unitary projection* in the trace norm $\|U - Y\|_{\mathrm{tr}}^2 = \mathrm{Tr}((U-Y)^\dagger(U-Y))$. The Π process returns U_Y.

Theorem 11.2 (Minimality and uniqueness of the Π-projection)**.** *The* Π*-process is well defined, minimal in the trace metric, and unique up to the right diagonal phase shift, absorbed by fermionic redefinitions.*

Proof. The polar decomposition exists and is unique for matrices with $\det Y \neq 0$. Minimality of the trace metric: for any $U \in U(n)$

$$\|U - Y\|_{\mathrm{tr}}^2 = 2\,\mathrm{Tr}\,|Y| - 2\,\mathrm{Re}\,\mathrm{Tr}(U^\dagger Y)$$

is minimized for $U = U_Y$ through the von Neumann inequality: $\mathrm{Re}\,\mathrm{Tr}(U^\dagger Y) \leq \mathrm{Tr}\,|Y|$ with equality only for $U = U_Y$. Right-hand phase freedom is absorbed by the redefinition of fermion fields and does not change the physical CP invariants. □

In the VERCETTI branch, the Π process is applied to a Yukawa block fixed by the σ_7 branch and the $\mathbb{Q}(\zeta_{48})$ structure. Result:

PMNS via Π-process. The Yukawa block of the neutrino sector in the σ_7 branch has the form

$$Y_\nu^{(\sigma_7)} = \frac{m_\nu^{(3)}}{\sqrt{2}} \begin{pmatrix} 1 & \alpha & \beta \\ \alpha & 1 & \gamma \\ \beta & \gamma & 1 \end{pmatrix}$$

where $\alpha, \beta, \gamma \in \mathbb{Q}(\zeta_{48})$ are σ_7-covariant coefficients. After Π-projection and left multiplication by the unitary representative of the charged lepton sector U_ℓ, the physical PMNS matrix is derived via

$$U_{\mathrm{PMNS}} = U_\ell^\dagger U_{Y_\nu}.$$

Direct algebra in $\mathbb{Q}(\zeta_{48})$ gives the mixing angles and Dirac CP phase from Section 7; Majorana CP phases remain undefined (Π-projection does not distinguish Dirac vs Majorana).

CKM via Π-process. Similarly, quark Yukawa blocks $Y_u, Y_d \in M_3(\mathbb{C})$ in the σ_7 branch give through $U_u^\dagger U_d =: U_{\mathrm{CKM}}$ an invariant CKM package with $J_{CKM} \approx 3{,}05 \times 10^{-5}$ and Cabibbo angle $\sin\theta_C \approx 0{,}2256$. All four physical CKM parameters are derived from the same σ_7 branch without new modules.

Limitations of the Π-process. The Π-process *does not* define:

- Majorana CP phases (they require a separate ν_R structure, which is disabled in the VERCETTI branch via the V3 sub-axioms);
- absolute normalizations $|Y|$ (unitary part only);
- spectral mass values (these are individual eigenvalues of $|Y|$).

Under these constraints, the Π-process is minimal: it extracts exactly what is determined by the σ_7-branch without additional axioms.

12. Complete atlas of no-go theorems VERCETTI

VERCETTI stands on seven no-go theorems, each of which rules out an entire direction of potential derivation. The atlas was collected explicitly for audit purposes.

1. **No-section theorem for Gaussian HS-completion.** In Gaussian Hilbert-Schmidt completion of the spectral measure there is no internal canonical scalar-probability trivialization. The global object is a pro-torsor of local measures, the limited density of the gauge group acts freely on the normalized projective representatives. proven [20].
2. **No-coherence finite nerve theorem.** Unordered simplex support does not specify canonical chain-level orientation without auxiliary ordering/orientation choice. Compatible orientation data exists, but the construction is noncanonical. proven in Lean 4 [19].
3. **No-scalaron theorem ($\Pi_s > 1$).** For the canonical SM content $\Pi_s(z,\xi) = 1 + 3z\alpha_R(z,\xi) > 1$ for $z > 0$, so there is no real scalar graviton pole. proven, tested for $\xi \in [-3,3]$ via spectral integral form [21].
4. **No-active-ν_R theorem.** Active right-handed neutrinos in the heat-kernel core give $N_D = 45/2 + 3/2 = 24$ and $\alpha_C \to 1/30$. Since $\alpha_C = 13/120$ is fixed by the theorem-grade core, ν_R must be sterile in the sense of the heat-kernel core. proven through direct counting.
5. **No-trace-branch theorem.** Trace metric $\|x\|^2_{\mathrm{tr}} = \mathrm{Tr}_{K/\mathbb{Q}}(x\bar{x})$ orientation-blind: $e_+ \leftrightarrow e_-$ leaves the norm invariant. Since AM was introduced specifically to fix the orientation, the trace branch $Z_\nu^{\mathrm{trace}} = 50/49$ is *categorically* forbidden, and not just numerically incorrect. proven.
6. **No-modular-ρ_ω theorem (no new axiom).** There is no canonical morphism $\rho_\omega : BG_{\mathrm{full}} \to \overline{X(1)}$ in the current formalism. The group $S_3 = X(2)/X(1)$ acts on the λ-labeling, but Δ is already at the $X(1)$ level and is S_3-invariant, so the finite S_3-monodromy of ω^{12} is trivial. proven if there is no new background symmetry.
7. **No-internal-Ξ_{int} theorem for D^2-quantization.** The Gaussian D^2-completion inherits the pro-torsor no-section through the closed chiral subspace $X_\chi \subset HS_{sa}(H)$. the naive non-Gaussian measure $\exp[-\mathrm{Tr}\, f(Q/\Lambda^2)]$ is not contractive because $f(\lambda/\Lambda^2) \to 0$ for $|\lambda| \to \infty$. proven via Phase 4 Stage 4 analysis.

Each of these seven no-go theorems has an obvious obstacle. Together they form the *protective environment* of the VERCETTI branch: any attempt to bypass Ξ_Υ as the only valid selector is required to violate one of them or introduce a new axiom outside the current formalism.

13. Correspondence of MSR theorems A–J and principles V1–V8

The meta-principle of MSR (minimal spectral realization) is expressed through theorems A–J. VERCETTI reproduces the same content through the eight principles V1–V8 more compactly. The correspondence is clear:

MSR-theorem	Contents	VERCETTI-equivalent
A	Minimum A_F: $M_2(\mathbb{H}) \oplus M_4(\mathbb{C})$ unique	V3 + Step 1
B	Minimality A_M: $\mathbb{C} \oplus \mathbb{C}$ unique binary	V2 + Step 2
C	Minimal non-elliptic rectangular CM selector	V2 + Step 3 ($\tau_\star = i\sqrt{2}$)
D	Uniqueness of σ_7 from CP, color, and $\sqrt{2}$ filter	Step 4
E	Uniqueness of polar extraction from the trace metric	Π-process (Section 11)
G	$N_{\mathrm{gen}} = 3$ at least for physical CP phase	Step 7
H	$\psi(u) = e^{-u}$ via the semigroup law	V5 + Step 5
I	Conformal point $\xi = 1/6$ from $\alpha_R(\xi) = 0$	V1 + Step 6
J	Minimum cyclotomic closure $\mathbb{Q}(\zeta_{48})$	Step 8

All ten MSR theorems (A–J minus F, which was transferred to Protocol F separately) have exact equivalents within the VERCETTI formulation. The main advantage of VERCETTI: instead of ten theorems motivated by mathematical parsimony, the eight principles V1–V8 are motivated

by a single physical problem, namely two-endpoint causal-spectral observability. The evidentiary content is preserved; the interpretive load is reduced.

14. Verification pipeline

VERCETTI inherits from the CAT an eight-layer verification pipeline, expanded by three infrastructure layers. Each structural result must pass through a minimum of four layers; each numeric value goes through six layers.

Eight layers of evidence-based verification.

1. **Analytical**: dimensions, limits, symmetries, reduction of poles.
2. **Numerical**: `mpmath` with ≥ 100-digit precision on ≥ 7 test points.
3. **Property-fuzzing**: `hypothesis` with ≥ 1000 cases on 12 properties.
4. **Literary**: comparison with Chamseddine–Connes, Vassilevich, Avramidi, Parker–Toms.
5. **Dual inference**: an independent method that differs in approach.
6. **Triple CAS**: SymPy $\times$ GiNaC $\times$ mpmath with ≥ 12 significant figures.
7. **Lean 4 formal**: PhysLean/Mathlib4 for rational identities.
8. **Multi-backend Lean**: LOCAL + Aristotle simultaneously PASS.

Three infrastructure layers.

9. **Quick sanity (simulation-in-the-loop)**: 18 quick checks $< 0{,}01$ s after each edit.
10. **Consistency DAG**: 19-node dependency graph; When a downstream node changes, all dependents are marked STALE.
11. **Provenance chain**: tracing any result to its postulate roots; `check_acyclic()` guarantees that there are no cycles.

Machine check for VERCETTI. Within the framework of this work, *Layer 2* (mpmath/SymPy with exact rational/algebraic arithmetic), partially *Layer 4* (literature references) and *Layer 9* (quick sanity via `cat_overview_verify.py`) were completed. The full Lean 4 formalization belongs to recommended future work and is outlined in Section 15 below.

15. Lean 4 formalization roadmap

Ten components for the complete formalization of the VERCETTI branch ($\sim$ 5500–7000 of Lean 4 lines on Mathlib 4.28):

1. Pell algebra: $\varepsilon = 1 + \sqrt{2}$, $\varepsilon^3 = 7 + 5\sqrt{2}$, K_3 as an anti-symplectic generator.
2. Galois decomposition in $\mathbb{Q}(\zeta_{24})$ and filter σ_7.
3. AM-Ω-readout: $u/d = 5/7$ via $K_3 e_+$.
4. Valuation-readout uniqueness lemma.
5. η/Δ-orders and Hodge-Mumford descent.
6. Pfaffian identity $\mathrm{Pf}(\Omega \otimes \mathcal{M}) = m_D^2$ for a 4×4 antisymmetric matrix.
7. $\varepsilon^2 \lambda(i\sqrt{2}) = 1$.
8. Anti-symplectic geodesic minimality.
9. $\alpha_C = 13/120$ arithmetic.
10. Closing the trail branch 50/49 as orientation-blind no-go.

Additionally for the independence theorem (Section 9): formalization of the two-model argument with explicit models M_{TC} and M_{ext}, verification of all 27 axioms of Appendix A, proof of $\mathcal{C}_{\mathrm{bare}} \nvdash \mathrm{TC}$ through soundness. Estimate: $\sim$ 1500–2000 additional lines of Lean 4. This formalization will change the status of Theorem 9.1 from proven (model-theoretic) to proven (formally verified in Mathlib).

16. Gravity and electromagnetism: precise formulation without metaphor

The phrase "gravity is electromagnetism" should not be included in a strict CAT as a literal statement. The correct wording is this.

Proposition 16.1 (Different types of the same spectral covariance)**.** *In almost-commutative spectral geometry, gravity and gauge fields are different types of fluctuations in spectral data. Gravity changes the commutative geometry, the main symbol of the Dirac operator and the causal cone. Electromagnetism is the internal $U(1)$ part of the finite sector gauge coupling. They are united by spectral action, but are not identical as physical fields.*

Proof. In the commutative part of the spectral triple, the Dirac operator has the main symbol $\gamma^\mu \xi_\mu$. The metric is recovered from the Clifford structure and spectral distance; changing the metric changes the light cone after Lorentzian continuation. Internal gauge fields arise from internal fluctuations $D \mapsto D + A + JAJ^{-1}$, where A denotes a single form of the algebra. For the $U(1)$ part this gives electromagnetic coupling. These operations live in the same spectral action, but act on different components of the data: the geometric main part and the internal connectivity. Therefore, the equality of the fields is false, and the single spectral origin is true. □

This reading is physically stronger: it does not promise "gravity as electromagnetism," but explains why both sectors emerge from the same operator without erasing their differences.

17. Where did the problem of quantization of gravity go

It did not disappear. VERCETTI bans the rhetorical "quantization is decided" and replaces it with three levels.

1. proven **one-loop spectral kernel**: coefficient $\alpha_C(0) = 13/120$, no-scalaron, conformal branch, Lorentzian TT-block at the level of flat space and main symbol.
2. conditional **fakeon interpretation of the spin-2 sector**: the spin-2 pole is not included in the asymptotic physical spectrum, but full rigor for the entire nonlocal Lorentzian operator requires a separate analysis.
3. no-go **intrinsic non-perturbative scalar expectation**: Gaussian Hilbert–Schmidt completion gives a pro-torsor, not a single background-independent scalar measure. Therefore v_{EW} cannot be derived from the bare kernel as an ordinary number without external terminal completion.

In other words, the quantization problem does not "disappear", but changes form: in CAT, the evidentiary advances lie in the one-loop spectral core and in the absence of a scalar, and the main non-perturbative remainder lies in the choice of the terminal scalar completion.

18. Conditional output of the CJ indicator and mandatory status correction

18.1. Active two-end shell theorem

Definition 18.1 (Single active shell hypothesis)**.** For a causal diamond in dimension d, assume that the asymptotic contribution of the Hasse estimator CJ is concentrated in one Poisson hull of the ordered-configuration space of dimension $2d + 1$. The thickness of this shell in terms of a dimensionless variable is equal to

$$\eta_N = N^{-1/(2d+1)}.$$

Theorem 18.2 (Conditional indicator CJ)**.** *Under the hypothesis of a single active shell, the mathematical expectation of the two-endpoint covariance Hasse estimator has an N-exponent*

$$\gamma_d = 1 - \frac{1}{2d+1} = \frac{2d}{2d+1}.$$

In four dimensions $\gamma_4 = 8/9$. Status: conditional.

Proof. There are N scattered elements in a causal diamond. If the entire volume were active, the linear total contribution would have a scale of N. But according to the hypothesis, only a thin shell of relative thickness η_N is active. Therefore, the effective number of active elements is

$$N_{\text{eff}} = N\eta_N.$$

We substitute the thickness:

$$N_{\text{eff}} = N \cdot N^{-1/(2d+1)}.$$

We use the power rule $N^a N^b = N^{a+b}$:

$$N_{\text{eff}} = N^{1-1/(2d+1)}.$$

We bring the indicator to a common denominator:

$$1 - \frac{1}{2d+1} = \frac{2d+1}{2d+1} - \frac{1}{2d+1} = \frac{2d}{2d+1}.$$

For $d = 4$:

$$\gamma_4 = \frac{2 \cdot 4}{2 \cdot 4 + 1} = \frac{8}{9}.$$

□

18.2. Exact coefficient subject to $N^{8/9}$

If we accept the original CJ-hypothesis from the manuscript [23], the coefficient has the form

$$C_0 = 4 \cdot \frac{8}{3} \cdot \frac{1}{9!} \cdot \frac{8\pi}{15} \cdot \frac{\pi}{24}.$$

We multiply the angular and volume factors:

$$\frac{8\pi}{15} \cdot \frac{\pi}{24} = \frac{8\pi^2}{360} = \frac{\pi^2}{45}.$$

Then

$$C_0 = 4 \cdot \frac{8}{3} \cdot \frac{1}{9!} \cdot \frac{\pi^2}{45}.$$

First

$$4 \cdot \frac{8}{3} = \frac{32}{3}.$$

So,

$$C_0 = \frac{32\pi^2}{3 \cdot 9! \cdot 45} = \frac{\pi^2}{1530900}.$$

Machine verification gives numerically

$$C_0 \approx 6.44692951929542 \times 10^{-6}.$$

18.3. Status after high N

It is important not to ignore the contradiction here. High precision numerical checks at $N = 50000$ weaken the naive $N^{8/9}$ expectation to an effective $\alpha_{\text{eff}} \approx 0{,}38$. Therefore, VERCETTI does not increase the CJ scaling to proven. The correct conclusion is:

> **If** one two-endpoint ordered-configuration shell of dimension $2d+1$ is active, then the indicator must be $2d/(2d+1)$ and in $d = 4$ equal to $8/9$. **If** large N stably yield $\alpha_{\text{eff}} \approx 0.38$, then it is the hypothesis of a single active shell that is violated, and not the SCT core and not the CAT branch.

This turns CJ from a "number to defend" into a diagnostic test of Hasse-estimator geometry. A specific candidate mechanism for $\alpha_{\text{eff}} < 8/9$: for large N, not one, but a family of shells of different dimensions $2d_k+1$ with weights $w_k(N)$ becomes active. The effective exponent is then the weighted average $\alpha_{\text{eff}} = \sum w_k(N)\gamma_{d_k}$, and the observed $\alpha_{\text{eff}} \approx 0{,}38$ corresponds to the dominance of the lower effective dimensions $d_{\text{eff}} \approx 0{,}3$. This is consistent with infrared dimensionality renormalization in spectral estimators and does not require violation of the VERCETTI branch.

19. Terminal candidate for v_{EW}

The bare core of the CAT does not output v_{EW}. This is not an opinion, but a consequence of the independence theorem and the absence of an internal scalar selector in the pro-torsor measure. Therefore, the formula below should not be called the central proven prediction of the kernel. In this manuscript, it receives a narrower and more honest status: conditional terminal seesaw bridge, which becomes physical only after a separate theorem on the uniqueness of the terminal scalar completion.

Definition 19.1 (Terminal seesaw bridge)**.** The terminal bridge TC consists of five conditions.

TC1. AM-orientation fixes the rational terminal weight $r_{AM} = 5/7$.

TC2. Modular node $\tau_\star = i\sqrt{2}$ fixes cusp-attenuation

$$q_\star = |e^{2\pi i \tau_\star}| = e^{-2\pi\sqrt{2}}.$$

TC3. Terminal heavy Majorana/Takagi scale is equal to

$$M_N = r_{AM}\, M_{\text{Pl}}\, q_\star.$$

TC4. For the heaviest light neutrino, a peer-to-peer type-I seesaw block is used

$$m_{\nu 3} = \frac{m_D^2}{M_N},$$

where m_D is the corresponding Dirac mass.

TC5. The final Higgs–Dirac spectral normalization of the block identifies m_D^2 with v_{EW}^2. If the normalization is written as $m_D = y_\nu v_{\text{EW}}/\sqrt{2}$, then this condition is equivalent to fixing $y_\nu^2/2 = 1$ in the terminal branch. Without this condition, the formula receives an additional factor $2/y_\nu^2$ and ceases to be a prediction.

Theorem 19.2 (Conditional output terminal-v_{EW})**.** *If the terminal seesaw bridge* TC *is satisfied, then*

$$v_{\text{EW}}^2 = \frac{5}{7}\, m_{\nu 3}\, M_{\text{Pl}}\, e^{-2\pi\sqrt{2}}.$$

Status: conditional *under TC1–TC5 conditions;* proven *as an algebraic consequence of these conditions.*

Proof. From TC2 we have $q_\star = e^{-2\pi\sqrt{2}}$. From TC1–TC3:

$$M_N = r_{AM} M_{\rm Pl} q_\star = \frac{5}{7} M_{\rm Pl} e^{-2\pi\sqrt{2}}.$$

From TC4:

$$m_{\nu 3} = \frac{m_D^2}{M_N}.$$

Multiply both sides by M_N:

$$m_{\nu 3} M_N = m_D^2.$$

According to TC5 $m_D^2 = v_{\rm EW}^2$. Therefore,

$$v_{\rm EW}^2 = m_{\nu 3} M_N = m_{\nu 3} \cdot \frac{5}{7} M_{\rm Pl} e^{-2\pi\sqrt{2}}.$$

□

Status of the cusp factor

The factor $e^{-2\pi\sqrt{2}}$ is not called "instantonic" without specifying the field and space. In this manuscript it has the exact status of modular cusp-attenuation: it is the absolute value of the q-parameter at the CM point $\tau_\star = i\sqrt{2}$. If a true instanton interpretation is later proposed, it must separately name the field, the Euclidean space, the action of the instanton configuration, and the measure of fluctuations. Until this point, the word "instanton" is not used as a physical explanation.

This way the formula gets a physical structure, and not just a numerical form. The product $m_{\nu 3} M_{\rm Pl}$ does not arise arbitrarily: in the seesaw mechanism, the product of a light neutrino mass by a heavy Majorana scale has the size and meaning of the square of the Dirac mass. The Planck mass appears only because TC3 selects the terminal Majorana scale as a Planck-anchored boundary scale, weakened by a modular cusp factor. If TC3 or TC5 is incorrect, the formula decays without damaging the SCT core.

Dimension check:

$$[m_{\nu 3}] = 1, \qquad [M_{\rm Pl}] = 1, \qquad [e^{-2\pi\sqrt{2}}] = 0, \qquad \left[\frac{5}{7}\right] = 0.$$

Therefore,

$$[v_{\rm EW}^2] = 1 + 1 = 2,$$

as required.

Numerical sanity-check, not used as evidence. Standardized neutrino anchor: $m_{\nu 3}$ from NuFIT 6.0 [10] normal hierarchy central value, $m_{\nu 3} \approx 50{,}3$ meV. With the total Planck mass $M_{\rm Pl} = 1{,}22091 \times 10^{19}$ GeV:

$$v_{\rm EW} = \sqrt{\tfrac{5}{7}\, m_{\nu 3}\, M_{\rm Pl}\, e^{-2\pi\sqrt{2}}} \approx 246{,}344\,\text{GeV},$$

deviation $+0{,}05\%$ from the observed $v_{\rm EW}^{\rm obs} = 246{,}22$ GeV. An alternative anchor through the atmospheric $\Delta m^2_{\rm atm}$: $m_{\nu 3} = \sqrt{2{,}50 \times 10^{-3}}$ eV ≈ 50 meV gives $v_{\rm EW} \approx 245{,}606$ GeV ($-0{,}25\%$). Neutrino anchor sensitivity at $\pm 0{,}3\%$ covers both values. Proximity to observation does not increase status to proven; to improve the status, an independent physical theorem TC1–TC5 is needed, and not a better numerical fit of the anchor.

Corollary 19.3 (Falsifying terminal-$v_{\rm EW}$)**.** *If it is proven that the terminal Majorana scale does not have the form $M_N = (5/7) M_{\rm Pl} e^{-2\pi\sqrt{2}}$, or that the final Higgs–Dirac normalization gives a free y_ν, then the terminal-$v_{\rm EW}$ formula loses its predictive power. In this case, the independence theorem of $v_{\rm EW}$ remains, and not a contradiction to the SCT-kernel.*

20. Machine verification

The verification reported in this manuscript was performed in Python/SymPy with exact arithmetic. The `cat_overview_verify.py` script does not use fitting; it tests exact rational and algebraic identities.

Check	Status	Result
$\alpha_C = 4 \cdot \frac{1}{120} + \frac{45}{2} \cdot (-\frac{1}{20}) + 12 \cdot \frac{1}{10}$	PASS	13/120
$\alpha_R(1/6) = 0$, $d^2\alpha_R/d\xi^2 = 4$	PASS	single conformal zero
$\lambda(i\sqrt{2}) = (\sqrt{2}-1)^2 = 3 - 2\sqrt{2}$	PASS	exact identity
$j(i\sqrt{2}) = 8000$	PASS	exact identity
Filter σ_a in $(\mathbb{Z}/24\mathbb{Z})^\times$	PASS	single $a = 7$
$7^2 \equiv 1 \pmod{24}$	PASS	involution
$\mathrm{lcm}(8,4,3,16) = 48$	PASS	$\mathbb{Q}(\zeta_{48})$
$N_{\mathrm{CP}}(1,2,3) = (0,0,1)$	PASS	minimum at three generations
CJ-coefficient	PASS	$C_0 = \pi^2/1530900$
Beta overlap $(d!)^2\binom{2d}{d}(2d+1) = (2d+1)!$	PASS	$d = 1, \ldots, 12$
Active shell indicator at $d = 4$	PASS	8/9
Binary selector accounting $\mathbb{C}^n$	PASS	$n = 2$ minimum
Terminal v_{EW} sanity-check	PASS	245.606 GeV at the specified anchors

Script result: 27/27 checks passed.

21. Predictions and falsifiers of VERCETTI

1. proven/conditional **There is no new selector algebra higher than $\mathbb{C}\oplus\mathbb{C}$ in the canonical branch.** If $\mathbb{C}^3$ or a continuous selector is required to save the flavor layer, the VERCETTI branch will be refuted as minimal.

2. proven $j = 8000$ **should remain a modular orbit and not a numerical fit parameter.** Any output that uses a neighboring j to better fit the data will violate the principle.

3. conditional **CJ-diagnostics.** If the Hasse estimator is indeed controlled by a single two-endpoint shell, the exponent in four dimensions is equal to 8/9. If large N consistently gives a different indicator, this falsifies not the CAT kernel, but the specific reading of the CJ estimator.

4. proven **No-scalaron is preserved.** In the canonical SCT core there is no real scalar graviton pole. The canonical verified lower bounds on the spectral slice should only be specified as follows: Eot-Wash PPN-1: $\Lambda > 2{,}38$ meV; torsion-balance+lab LT-3d: $\Lambda > 2{,}565$ meV; GWTC-3 no-scalaron: $\Lambda > 8{,}50$ meV, strongest.

5. conditional **Terminal v_{EW}.** The formula $v_{\mathrm{EW}}^2 = (5/7)m_{\nu 3}M_{\mathrm{Pl}}e^{-2\pi\sqrt{2}}$ is only valid as a terminal outer layer. If it is proven that physics does not implement such a terminal scalar selector, v_{EW} remains the only free continuous input of the canonical CAT.

6. proven **Koide $K_\ell = 2/3$ is stable.** Any deviation more than 10^{-4} from the exact value of 2/3 with future refinements of lepton masses falsifies the σ_7-branch.

7. conditional $\Omega_\Lambda^{\mathrm{CAT}} = 168/(25\pi^2) \approx 0{,}6809$**.** DESI Year-3, Planck NPIPE and ACT DR6 continue to narrow the observational range. A deviation of more than 3σ from 0,6809 will close the VERCETTI branch with the condition of full spectral filling.

8. conditional **PMNS octant θ_{23}.** The exact value $\sin^2\theta_{23} = 0{,}604\ldots$ uniquely fixes the upper octant. If JUNO/DUNE data show a stable low-octant value of $< 0{,}5$, the VERCETTI branch is refuted.

9. conditional $\delta_{CP}^{\mathrm{PMNS}} = -\pi/2$**.** T2K+NOvA converge in this area, but $\sigma \approx 0{,}9$. JUNO+DUNE will give $< 0{,}3\sigma$ by 2030. Deviation of $> 2\sigma$ from $-\pi/2$ will close the branch.

10. proven $J_{CKM} = (\sqrt{2}-1)/[4(7+\sqrt{2})\sqrt{6}] \approx 3{,}05 \times 10^{-5}$**.** Current PDG is $0{,}5\sigma$. Belle II and LHCb refinements will narrow the interval; deviation $> 2\sigma$ by 2028 will close the CKM branch.

11. conditional **Majorana nature of ν.** V3 sub-axiom prohibits active ν_R in the heat-kernel core, but the neutrino can be Majorana (sterile pole). If $0\nu\beta\beta$ consistently observes an effective mass $\langle m_{\beta\beta}\rangle \geq 25$ meV at $m_{\nu 3} \approx 50$ meV, this is consistent with the branch; the complete absence of the $0\nu\beta\beta$ signal below 5 meV with the current normalization will close the branch.

12. proven **Cyclotomic field $\mathbb{Q}(\zeta_{48})$.** Any feature of the flavor sector requiring roots ζ_n with $n \nmid 48$ falsifies the chosen minimal closure.

13. conditional **Spectral slice Λ.** GWTC-3 no-scalaron gives $\Lambda > 8{,}50$ meV. LISA, Einstein Telescope and Cosmic Explorer will tighten the limit to ~ 100 meV by 2035; the lower bound < 5 meV from any direct boundary experiment will close the no-scalaron theorem.

22. What this analysis does not show

1. It does not prove unconditionally that nature must satisfy VERCETTI. The analysis deduces VERCETTI from the first physical principles of $F0$–$F8$, but $F0$–$F8$ themselves remain physical axioms of the chosen class, and not mathematical theorems of nature. Therefore, the absolute status remains conditional.

2. It does not prove the complete derivation of v_{EW} from the bare kernel. On the contrary, it preserves no-go: without external terminal completion, v_{EW} remains a free continuous parameter of the canonical branch.

3. It does not prove the complete global Lorentzian closure of CAT/NCG. The one-loop TT block and the main symbol are strong results, but not a complete theory of all Lorentzian spectral triples.

4. It does not raise the CJ-scaling $N^{8/9}$ to the proven level. High N requires re-examining the hypothesis of a single active shell.

5. It does not prove the entire PMNS/CKM-flavor stack from first principles. It removes the arbitrariness of the key selectors A_M, $\tau_\star$, σ_7 and P, but does not complete the derivation of all F3 flavor coefficients. The tension $\sin^2\theta_{12}^{(0)} = 1/3$ against the central NuFIT value of about 0.308 is the real phenomenological risk of the branch, not a hidden detail.

6. It does not claim that "gravity is electromagnetism." It replaces it with a strict statement about different types of spectral covariance.

7. It does not prove the instanton mechanism for $e^{-2\pi\sqrt{2}}$. In this manuscript, this multiplier has the status of modular cusp-attenuation; instanton reading is disabled until a specific field, action, and configuration space are specified.

23. Conclusion

VERCETTI compresses the main discrete framework of CAT into one physically readable principle: the observed branch is a minimal terminal structure that allows a two-endpoint causal observer to obtain scalar spectral quantities without internal orientational and measure-theoretic arbitrariness. In this form, $\mathbb{C}\oplus\mathbb{C}$ becomes neither a name nor an ornament, but a minimal binary leaf of a causal experiment; $\tau_\star = i\sqrt{2}$ becomes the first non-degenerate rectangular CM-point after the forbidden elliptic $\lambda = 1/2$; $j = 8000$ becomes the calculated consequence; σ_7 becomes the only automorphism compatible with CP, color, and $\sqrt{2}$; $\psi = e^{-u}$ becomes the only semigroup profile.

The main optimization is not in adding new mathematics, but in removing psychological freedom of choice. What might otherwise look like a scattering of precise but strange structures becomes one general physical mechanism under VERCETTI. Where the output is not closed, especially for v_{EW}, the principle forbids pretending: either there is an outer terminal scalar layer, or v_{EW} remains the only free continuous input. This honest dichotomy is stronger than an artificial zero-parameter claim.

A. Appendix A: formal signature $\mathcal{C}_{\mathrm{bare}}$

Multisorted $\mathcal{C}_{\mathrm{bare}}$ theory for the Section independence theorem 9.

Sorts: Real (ordered real exponential field), Int, Manifold (smooth spinor manifold), Field (material and gauge fields), Form (differential/modular/Pfaffian forms), Algebra (finite real/complex $*$-algebras), Hilbert, DiracOp, Modular (modular points/functions/lines), Matrix2, Selector, Line.

Function symbols: Add, Mul, Inv, Exp, Sqrt : Real $\to$ Real; α_R : Real $\to$ Real; Pf : Form $\to$ Real; Tensor : Form $\times$ Form $\to$ Form; Valuation : Line $\to$ Real; OrdInf : Line $\to$ Int; OmegaPair : Vector $\times$ Vector $\to$ Real; ApplyMat : Matrix2 $\times$ Vector $\to$ Vector; HiggsScalar : Field $\to$ Real; DimR : Algebra $\to$ Int; IsTerminalSelector, IsAdmissible : Selector $\to$ Bool.

Constants: $\alpha_C, \xi_{\mathrm{conf}}, N_s, N_D, N_v$: Real; A_F, A_M : Algebra; KO_{total} : Int; $\tau_\star$: Modular; $\lambda_\star, \varepsilon, q_\star$: Real; Δ, ω^{12} : Line; Ω_{AM} : Form; e_+, e_- : Vector; K_3 : Matrix2; $M_{\mathrm{Pl}}, m_{\nu 3}, v_{EW}^2$: Real; Ξ_Υ : Selector; $\Xi_\Upsilon^{\mathrm{value}}, M_N$: Real; M_{pole} : Form.

Axioms (full list):

- **REAL-1**: Real is an ordered real exponential field, $\mathrm{Exp}(x) > 0$, $\mathrm{Sqrt}(2)^2 = 2$.
- **SCT-1**: $\alpha_C = 13/120$.
- **SCT-2**: $\alpha_R(x) = 2(x - 1/6)^2$ for all x :Real.
- **SCT-3**: $\xi_{\mathrm{conf}} = 1/6$, $\alpha_R(\xi_{\mathrm{conf}}) = 0$.
- **SCT-4**: $N_s = 4$, $N_D = 45/2$, $N_v = 12$.
- **SCT-5**: $\alpha_C = N_s \cdot (1/120) + N_D \cdot (-1/20) + N_v \cdot (1/10)$.
- **SCT-6**: $A_F = M_2(\mathbb{H}) \oplus M_4(\mathbb{C})$.
- **SCT-7**: $\dim_{\mathbb{R}} A_F = 16 + 32 = 48$.
- **SCT-8**: $KO_{\mathrm{total}} \equiv 2 \pmod 8$.
- **SCT-9**: right-handed neutrinos do not contribute to the heat-kernel core (no active ν_R): $N_D = 45/2$.
- **SCT-10**: there is no real scalar graviton: for all $z > 0$ and ξ, $\Pi_s(z, \xi) > 1$.
- **B-1**: $A_M = \mathbb{C} \oplus \mathbb{C}$.
- **B-2**: $\Omega(e_+, e_-) = 1$, $\Omega(e_-, e_+) = -1$, $\Omega(e_\pm, e_\pm) = 0$.
- **B-3**: $K_3 = [[7, 10], [5, 7]]$.
- **B-4**: $\mathrm{ApplyMat}(K_3, e_+) = 7e_+ + 5e_-$.
- **B-5**: $\varepsilon = 1 + \sqrt{2}$.
- **B-6**: $\lambda_\star = 3 - 2\sqrt{2} = \varepsilon^{-2}$.
- **B-7**: $\tau_\star = i\sqrt{2}$.
- **B-8**: $q_\star = \mathrm{Exp}(-2\pi\sqrt{2})$.
- **B-9**: Δ is the minimal scalar cusp-section of ω^{12} over $X(1)$.
- **B-10**: $\mathrm{OrdInf}(\Delta) = 1$, $\mathrm{Valuation}(\Delta) = q_\star$.

- **PF-1**: $\mathrm{Pf}(\mathrm{Tensor}(\Omega_{AM}, M_{\mathrm{pole}})) = m_{\nu 3} \cdot M_N$ whenever M_{pole} is a Takagi block with $m_D^2 = m_{\nu 3} M_N$.
- **CENSUS-1**: $v_{EW}^2 > 0$.
- **CENSUS-2**: $\mathcal{C}_{\mathrm{bare}}$ does not contain an axiom identifying v_{EW}^2 with $\Xi_\Upsilon^{\mathrm{value}}$.
- **SEL-1**: $\Xi_\Upsilon^{\mathrm{value}} = (5/7) \cdot m_{\nu 3} \cdot M_{\mathrm{Pl}} \cdot q_\star$.
- **SEL-2**: if Ξ_{phys} is admissible, then $\Xi_{\mathrm{phys}} = \Xi_\Upsilon$. Equivalent to: $\mathsf{Sel}_{\mathrm{adm}} = \{\Xi_\Upsilon\}$.
- **NOSECTION-1**: there is no internal canonical scalar-probabilistic trivialization in the Gaussian Hilbert–Schmidt completion; external terminal scalar completion requires terminal density data.

This signature is sufficient for the two-model independence argument of Theorem 9.1. The extension $T_{\mathrm{TC}} = \mathcal{C}_{\mathrm{bare}} + (v_{EW}^2 = \Xi_\Upsilon^{\mathrm{value}})$ is compatible with M_{TC}. The extension $T_{\mathrm{ext}} = \mathcal{C}_{\mathrm{bare}} + (\exists \xi_0 : v_{EW}^2 = \xi_0 \neq \Xi_\Upsilon^{\mathrm{value}})$ is compatible with M_{ext}. Both models satisfy all the listed axioms, which is proven by $\mathcal{C}_{\mathrm{bare}} \nvdash \mathrm{TC}$.

B. Appendix B: status taxonomy

Five statuses plus no-go are used:

- proven: the proof is closed in the specified mathematical class or formally verified.
- verified: exact-arithmetic verification passed in `cat_overview_verify.py`; result: 27/27 PASS.
- conditional: the statement depends on an explicitly stated additional condition (example: VERCETTI as a meta-principle; terminal candidate v_{EW}).
- heuristic: a heuristic argument that is physically strong, but does not reach the level of a theorem.
- speculative: A speculative hypothesis explicitly marked as such.
- no-go: proven ban; cannot be done in the current formalism without extending the axioms.

In the article, each statement has one of these markers, or is clearly marked in the text.

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All mathematical definitions, theorem statements, proofs, physical arguments, and final scientific conclusions were formulated and checked by the authors. AI-assisted code was independently validated against analytical results and precise arithmetic (see `cat_overview_verify.py`, 27/27 PASS).

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Conflict of interest

The authors declare that they have no competing financial, professional, or personal interests that could influence the content or conclusions of this work.

Data and code availability

No private or restricted datasets were used in this study. The manuscript uses public experimental inputs cited in the text and in the acknowledgements. The only generated outputs are the exact-arithmetic verification log for the 27 algebraic checks and the reviewer-facing figures in the supporting package. These outputs are reproducible from the formulas and parameter values stated in the manuscript. The Python, Lean, Wolfram, derivation, and verification files are supplied as Online Resource 1. Additional reproduction material is available from the corresponding author upon reasonable request.

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