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Research Article

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Posted Date: August 13th, 2026

DOI: <https://doi.org/10.21203/rs.3.rs-9502773/v1>

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Additional Declarations: No competing interests reported.

Surface Extraction for Industrial CT Based On Adaptive Particle System

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Abstract

To achieve high-quality and high-precision surface extraction from industrial CT data, this paper proposes an adaptive surface sampling algorithm based on a particle system. The algorithm first constructs a multi-scale voxel pyramid and performs adaptive subdivision in a coarse-to-fine manner, incorporating curvature information at each scale. The subdivided voxel units serve as the fundamental units for particle behavior. Unlike traditional surface extraction methods based on particle systems, this paper decouples the force mechanism acting on particles in the time domain and adopts an adaptive weighted projection of grayscale and second-order gradient to replace the isosurface gravitational force, driving particles to sample at precise edges. Within the 26-neighborhood of the voxel unit containing the target particle, interacting particles are searched and repulsive forces are calculated to improve computational efficiency. Meanwhile, the tangential component of the repulsive force is extracted to enhance particle distribution while reducing the impact on projection accuracy. Furthermore, to increase sampling density under minimal system energy variation, this paper proposes an incremental particle insertion mechanism based on the Voronoi diagram.

Experimental results demonstrate that, compared to traditional surface extraction algorithms, the proposed method significantly improves the uniformity of particle distribution and robustness against noise and artifacts while maintaining sampling accuracy. It enables adaptive sampling with fewer sampling points, showing superior engineering practical value in industrial metrological CT applications.

Keywords: Surface extraction, Voxel pyramid, Multi-scale, Weighted projection, Particle system, Adaptive sampling

1 Introduction

1.1 Background

As product complexity and quality requirements continue to increase in engineering applications, traditional contact-based inspection methods often struggle to meet the demands of high-precision measurement when acquiring surface information of complex workpieces. This is due to the interference effects caused by physical contact and the inherent limitations of the detection range. Moreover, these methods are unable to capture the internal contour information of the workpiece. In addition, the inspection process of such methods is relatively cumbersome and time-consuming, which severely limits the overall efficiency of industrial production processes and makes it difficult to meet the core requirements of modern manufacturing for efficient quality control.

Against this background, Industrial Computed Tomography (ICT)[1] has emerged as an advanced non-contact and non-destructive testing technique. Its core advantage lies in its ability to intuitively and accurately reflect the geometric information of the inspected workpiece in the form of tomographic slice images, thereby providing quantitative basis for processes such as product quality assessment, performance verification, and reliability analysis.

In industrial applications, surface extraction is often performed on CT scan data to obtain a measurable 3D mesh model. Based on this model, internal details of complex structures can be quantitatively observed, enabling rapid localization of issues such as dimensional deviations and internal defects[2]. However, due to the influence of artifact interference and limitations in the measurement accuracy of scanning equipment, the effectiveness of the extraction algorithm can be significantly reduced, and reconstruction errors may even occur[3]. Therefore, developing high-precision surface extraction algorithms is of great significance for the application value of industrial CT in dimensional metrology.

1.2 Related Research

Surface extraction aims to recover a continuous geometric representation from discrete data. Currently, static surface extraction algorithms are widely used and can be mainly categorized into methods based on spatial geometric decomposition and methods based on surface topology tracking.

The Marching Cubes (MC) algorithm is the most classic method based on spatial geometric decomposition[4], proposed by Lorensen and Cline in 1987. This algorithm generates triangular facets within voxel units using lookup tables and linear interpolation. Its surface extraction results are highly robust, but it suffers from issues such as topological ambiguity, difficulty in preserving sharp features, and poor mesh quality. In 2002, Ju proposed the Dual Contouring (DC) algorithm[5], which employs adaptive octree subdivision[6] and quadratic error function minimization to generate optimal vertices within voxel units for isosurface reconstruction. This algorithm can effectively preserve sharp features, but it is relatively complex to implement and prone to topological ambiguities, mesh self-intersections, and vertex instability. In the same year,

Ohtake proposed a dual dual mesh optimization method[7], which projects mesh centroids onto an implicit surface and minimizes the quadratic distance error from vertices to tangent planes, achieving high-precision reconstruction of sharp features and significantly improving mesh quality. However, it cannot change the initial topology and incurs high computational costs. In 2004, Nielson proposed the Dual Marching Cubes algorithm[8], which connects dual vertices generated inside four voxel units sharing a common edge into quadrilateral faces to form a dual mesh, thereby reducing the low-quality triangles commonly found in MC isosurfaces.

Algorithms based on surface topology tracking generate triangles by continuously expanding or advancing a front. A classic example is the Marching Triangles (MT) algorithm proposed by Hilton and Illingworth in 1997[9]. This algorithm starts from an initial seed triangle and progressively estimates the positions of new vertices through boundary expansion, projecting them onto an implicit surface. At the same time, it uses 3D Delaunay surface constraints to ensure that newly added triangles do not self-intersect or overlap, ultimately completing the surface mesh extraction. However, this algorithm can only handle relatively simple topological structures. Furthermore, to avoid reconstruction errors, the advancing front must be capable of detecting and avoiding self-intersections. Although some heuristic methods are built in, such behavior often leads to a degradation in the quality of triangles in the corresponding regions, violating the original intent of the algorithm. Despite subsequent improvements made by many researchers[10–13], the aforementioned issues remain unresolved.

In recent years, dynamic surface extraction algorithms have gradually gained attention[14]. Unlike static methods, this class of approaches treats the surface as a dynamic system capable of autonomously deforming, diffusing, and converging to the target morphology under the drive of an energy field. Among these, surface reconstruction methods based on particle systems use a large collection of discrete particles to represent the surface. Driven jointly by data constraint forces and inter-particle interaction forces, the particles continuously move and eventually become uniformly distributed on the isosurface, reaching mechanical equilibrium. This method was originally proposed by Witkin and Heckbert[15]. Due to the large number of interactions among particles, the computational efficiency of the algorithm is low, and the final sampling result is merely a surface point cloud lacking topological information. In 1997, Heckbert introduced binning techniques[16], which limit neighbor searches in energy and velocity calculations to small regions through spatial partitioning, thereby improving computational efficiency. In 2007, Miriah Meyer et al. combined dynamic particle systems with Delaunay surface reconstruction theory[17], ensuring that particle distribution meets the geometric conditions for Delaunay sampling, thus guaranteeing topological correctness of the generated mesh. In 2012, Mark Kim proposed using GPUs to accelerate the iterative process of particle systems[18], significantly improving the sampling efficiency of particle systems. Although surface reconstruction methods based on particle systems have made progress in the aforementioned studies, they still face challenges in metrological CT applications, such as difficulty in ensuring surface accuracy, susceptibility to noise and artifacts, and insufficient adaptive sampling capability, making it difficult to achieve an effective balance between accuracy and storage space. In summary, although static surface extraction methods are widely used, they

still have shortcomings in the field of industrial CT metrology. First, static methods cannot ensure sampling quality while performing surface sampling, and are prone to introducing issues such as long, thin triangles and even topological errors. The mesh vertices in the surface extraction results are sampled non-uniformly and exhibit poor distribution quality, causing many inconveniences for subsequent processing operations. Second, the extraction effectiveness of traditional algorithms is often highly dependent on the choice of threshold values and is easily disturbed by inherent artifacts, severely affecting reconstruction accuracy[19, 20]. Moreover, these algorithms lack sufficient adaptive sampling capability, maintaining high sampling rates even in flat regions, which leads to a significant increase in data volume and severely impacts the processing efficiency of subsequent workflows. In addition, traditional surface extraction algorithms based on particle systems suffer from low computational efficiency due to the large number of inter-particle force interactions. The coupling of repulsive and attractive forces during iteration can also affect the reconstruction outcome. When increasing the sampling rate, the splitting mechanism of traditional particle systems tends to cause a sharp decline in system equilibrium, further affecting algorithmic efficiency and even leading to system imbalance and reconstruction errors. To address the above issues, this paper improves the force mechanism of particles based on traditional particle system surface extraction algorithms and inserts particles in a manner that minimizes changes in system energy. Additionally, by incorporating a multi-scale voxel pyramid and following a coarse-to-fine reconstruction order, the correct contour is obtained at coarse scales, while details are gradually enriched during the scale refinement process, ultimately achieving adaptive sampling. Under the premise of ensuring sampling accuracy, the proposed method is able to obtain adaptive sampling results with good distribution quality.

1.3 Contributions of this Work

Building upon traditional particle system-based surface extraction algorithms, this paper introduces multi-scale pyramid characteristics and proposes an adaptive surface extraction method. First, the algorithm constructs a pyramid structure and the corresponding curvature field from CT slice data and performs adaptive region refinement in a coarse-to-fine order based on curvature information. The refined voxel units serve as the basic units for particle behavior. Voxel units at each level control parameters such as the number of internal particles and their repulsion radius according to their refinement degree, thereby achieving curvature-adaptive sampling.

To address the low computational efficiency caused by the calculation of repulsive forces among a large number of particles in particle systems, this paper treats repulsion as a local short-range force, allowing only particles within the repulsion radius to interact, thereby improving computational efficiency. Furthermore, the force mechanism is decoupled and adjusted. An adaptive weighted projection of grayscale and second-order gradient is adopted to drive particles to precisely settle on the true isosurface position. Meanwhile, the tangential component of the repulsive force is extracted to enhance the effectiveness of particle distribution optimization. To address the issue that traditional particle systems tend to disrupt the system equilibrium state when increasing the sampling rate through a splitting mechanism, this paper proposes an

incremental particle insertion mechanism based on Voronoi diagrams, which effectively reduces the system energy fluctuation caused by particle insertion behavior.

In the experimental section, the effectiveness of the proposed method is validated through comparison with classical surface extraction algorithms. The conclusion section identifies the limitations of the method.

Compared with the previously proposed surface extraction methods based on surface tracking[21], the method proposed in this paper introduces multi-scale pyramid characteristics and particle system. It not only can effectively reduce the sampling rate while maintaining the sampling accuracy, but also significantly improves the uniformity of particle distribution and the stability of size accuracy.

2 Method

2.1 Multi-scale Adaptive Refinement Strategy

Directly extracting surface meshes from raw CT volume data using threshold-based methods is susceptible to artifact interference, which may cause the reconstruction process to fall into local optima or produce topological errors. Although traditional filtering methods can suppress artifacts to some extent, they often exacerbate the partial volume effect of the data, thereby introducing significant errors. To address these issues, this paper adopts a method that combines multi-scale characteristics with edge-preserving filtering.

This section provides a detailed introduction to our multi-scale curvature-adaptive nested refinement method. First, a voxel pyramid is constructed from the original CT data through downsampling, and based on this, the curvature field and gradient field at each scale are computed. Then, following a coarse-to-fine reconstruction order, scale refinement is performed on high-curvature regions identified at the coarse scale, serving as a necessary prerequisite for adaptive sampling.

2.1.1 Construction of Voxel Pyramid

The construction of the voxel pyramid is achieved through edge-preserving filtering and downsampling of the CT data, in which guided filtering[22] is extended to three dimensions as the filtering algorithm. After the voxel pyramid is constructed, the gradient field and curvature field are further built using the voxel information at each level.

The gradient field is used to describe the rate of change of voxel gray values in the CT image within three-dimensional space. Its direction points toward the direction of the fastest increase in gray value, and its magnitude reflects the strength of the edge. The gradient at any arbitrary point in space is a three-dimensional vector, defined as shown in Equation (1). The gradient of discrete data is typically computed using difference methods.

$$\nabla F = \left(\frac{\partial F}{\partial x}, \frac{\partial F}{\partial y}, \frac{\partial F}{\partial z} \right) \quad (1)$$

When calculating the curvature field to achieve adaptive sampling, this paper characterizes the level of detail of an implicit surface by its local degree of curvature in space.

The method proposed by Ron Goldman[23] is used to compute the mean curvature at each voxel position. The relevant formulas are as follows:

$$\kappa = \frac{\nabla F \cdot H(F) \cdot \nabla F^T - |\nabla F|^2 \text{tr}(H)}{2|\nabla F|^3} \quad (2)$$

where $H(F)$ is the Hessian matrix of the grayscale function F :

$$H(F) = \begin{bmatrix} \frac{\partial^2 F}{\partial x^2} & \frac{\partial^2 F}{\partial x \partial y} & \frac{\partial^2 F}{\partial x \partial z} \\ \frac{\partial^2 F}{\partial y \partial x} & \frac{\partial^2 F}{\partial y^2} & \frac{\partial^2 F}{\partial y \partial z} \\ \frac{\partial^2 F}{\partial z \partial x} & \frac{\partial^2 F}{\partial z \partial y} & \frac{\partial^2 F}{\partial z^2} \end{bmatrix} \quad (3)$$

However, traditional central difference methods suffer from directional bias on high-frequency or oblique structures. Due to their anisotropy, the response on oblique axes differs from that on axial axes in discrete grids, thereby affecting reconstruction accuracy. When the gradient direction is at 45° , the error reaches its maximum[24]. To mitigate this issue, the Scharr operator adopts a smoothing kernel of the form $[a, b, a]^T$ based on the Sobel operator and minimizes the error function of directional responses, ensuring that the response magnitude of the gradient operator remains as consistent as possible across all directions. Therefore, this paper adopts the 3D Scharr operator as a replacement for central differencing.

Furthermore, we apply the Scharr operator to the derivation of the Laplacian operator and extend it to three dimensions, convolving the CT data to obtain the divergence field of the second-order gradient. Its zero-crossings serve as precise edges, acting as a correction term in the subsequent particle system to guide accurate particle projection. The constructed gradient field and curvature field play crucial roles in subsequent steps such as particle projection iteration and curvature-adaptive sampling.

2.1.2 Nested Multi-Scale Refinement

In the discrete space partitioning of a 3D scalar field, the cube formed by four adjacent voxels on each of two adjacent slices is typically used as the basic processing unit, referred to as a voxel unit, as shown in Figure 1, where the integer coordinates (i, j, k) serve as the reference for the voxel unit. By comparing the grayscale values of the eight vertices of a voxel unit with a given threshold, if a sign change exists, it indicates that the isosurface passes through that voxel unit. For ease of explanation, such units are referred to as 'response units' in the remainder of this paper.

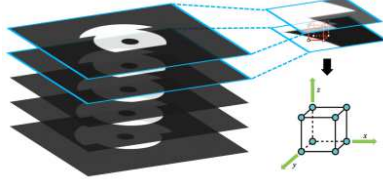


Fig. 1 Voxel unit

We assign a corresponding scale level l to each voxel unit to characterize its degree of detail. Starting from the coarsest scale, the set of all response units is defined as the 'response region', and the scale level l of these voxel units is marked as the maximum number of levels L . Subsequently, the curvature statistical threshold θ of all voxel units within the response region is calculated, from which the 'high-curvature region' is identified to represent the parts with significant geometric features.

When calculating the curvature statistical threshold, the mean curvature at the center of each voxel unit within the response region is sampled to construct a curvature sample set, and its mean μ and standard deviation are σ calculated, which reflect the overall level of curvature in the region and the degree of dispersion of its distribution, respectively. The statistical threshold is determined by formula $\theta = \mu + k \cdot \sigma$, where k is an adjustable parameter. All voxel units with curvature values higher than θ are marked as the 'high-curvature region', serving as a search guide mask for subsequent refinement reconstruction.

If the high-curvature region is directly mapped to the fine scale, important geometric features may be missed, leading to inconsistent reconstruction results across scales and making it difficult for particles to fully cover the surface, resulting in aliasing artifacts. To address this, this paper applies morphological dilation to the high-curvature region, with a dilation width of n voxels. The set of all response units in the processed result is defined as the 'candidate region' and mapped to the fine scale for refinement. In experiments, n is typically set to 1 or 2, which yields good sampling results. For ease of illustration, Figure 2 shows the scale refinement process in 2D form.

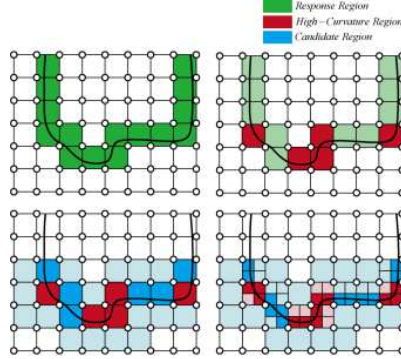


Fig. 2 Scale refinement process

In the subsequent process, the refined region serves as the starting point, where new response regions are screened and their scale level l is decreased by 1. The above steps of curvature statistical threshold calculation, morphological dilation, and scale refinement are repeated until the finest scale is reached, thereby completing the curvature-adaptive nested refinement process. The refinement effect at each level is shown in Figure 3.

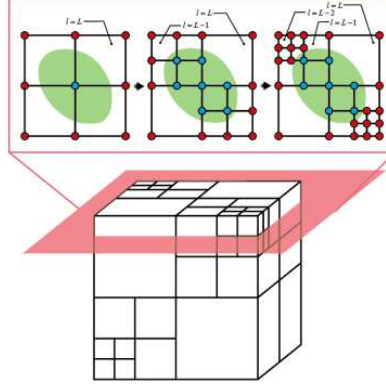


Fig. 3 Detailed effects at all levels

2.2 Extended Particle System

Based on traditional particle system-based surface extraction algorithms, this paper adjusts the force mechanism during particle iteration by decoupling the repulsive and attractive forces acting on particles in the time domain. In each iteration cycle, an adaptive weighted projection of grayscale information and second-order gradient is first used to drive particles to precisely settle on the true isosurface. Subsequently, repulsion is treated as a local short-range force, and its tangential component is extracted to perform repulsive interactions among particles for distribution optimization. Furthermore, the average particle displacement is used as an approximation of the 'energy gradient' to evaluate the equilibrium state of the system. When the system approaches equilibrium, incremental particle insertion is achieved by constructing a Voronoi diagram.

The overall workflow of this method mainly includes stages such as pyramid construction, particle system initialization, nested scale refinement, local decoupled iteration, system equilibrium checking, and incremental particle insertion. Based on the constructed pyramid information, the particle system is first initialized to form a basic coverage of the target surface. The initialization results are then iteratively optimized by updating particle positions, and the overall equilibrium state of the system is evaluated during the iteration process. If the equilibrium condition is not met, the iteration continues; when the system reaches equilibrium, new particles are incrementally inserted into high-curvature regions according to the pyramid adaptive refinement

results, and the above process is repeated until the finest level of refinement is completed.

2.2.1 Particle System Initialization

The particle system is initialized at the coarsest scale of the voxel pyramid. First, all 'response units' are traversed, and one particle is created as an initial particle at the approximate position corresponding to the mesh center generated by the cube in the lookup table of the MC algorithm.

After initialization, the Mersenne Twister algorithm is used as the random number generator to introduce small perturbations, thereby avoiding local extrema caused by the symmetry of the initial particle distribution, while ensuring the periodicity and uniformity of the random sequence, as shown in Figure 4. This mechanism provides a diversified initial state for subsequent iterations, avoiding local minima traps caused by distribution symmetry, thus enhancing the adaptability of the particle system to geometric shapes.

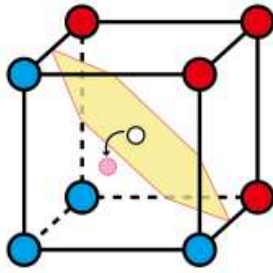


Fig. 4 Initialization with accompanying random perturbations

2.2.2 Gradient-Weighted Projection

The second-order gradient reflects the rate of change of the gradient magnitude. When its value equals zero, it corresponds to the position of the maximum magnitude of the first-order gradient. Using the zero-crossings of the second-order gradient for boundary localization helps to obtain sub-pixel boundaries more accurately[25]. This paper utilizes the divergence field of the second-order gradient constructed earlier and achieves a smooth transition between the grayscale term and the second-order gradient term through adaptive weight scheduling. First, $\phi(p)$ is defined to represent the degree of deviation of the target particle from the isosurface:

$$\phi(p) = f(p) - f_0 \quad (4)$$

where p represents the spatial position of the target particle. $f(p)$ represents the scalar field function value at that position, and f_0 is the scalar value of the target isosurface. The projection correction constructs a Newton iteration scheme based on first-order Taylor expansion. Its core is local linear approximation, expanding $\phi(p + \Delta p)$ to first order at p :

$$\phi(p + \Delta p) \approx \phi(p) + \nabla f(p) \cdot \Delta p \quad (5)$$

Set the above approximation to zero, and take the gradient vector $\nabla f(p)$ as its correction direction:

$$\Delta p = \lambda \nabla f(p) \quad (6)$$

where $\nabla f(p)$ can be obtained by interpolation from the first-order gradient field constructed earlier, and λ is the step size factor. Substituting equation (6) into equation (5) yields:

$$\phi(p) + \lambda \|\nabla f(p)\|^2 = 0 \quad (7)$$

$$\lambda = -\frac{\phi(p)}{\|\nabla f(p)\|^2} \quad (8)$$

Substituting equation (8) into equation (6) yields the Newton iteration step size for grayscale projection:

$$\Delta p_{\text{grayscale}} = -\frac{\phi(p)}{\|\nabla f(p)\|} \cdot \frac{\nabla f(p)}{\|\nabla f(p)\|} \quad (9)$$

This projection process exhibits quadratic convergence in the neighborhood of the solution, where $\left\| \frac{\phi(p)}{\|\nabla f(p)\|} \right\|$ serves as an adaptive step size that dynamically adjusts the projection distance based on the degree of deviation. Due to the truncation of higher-order terms, a single projection correction cannot guarantee that the particle lands precisely at the target position; therefore, multiple iterations are required for gradual approximation. The equilibrium of the particle system is achieved through such multiple iterations, which aligns well with the mechanism of the particle system. The purpose of second-order gradient projection is to position the particle at the Laplacian zero-crossing. i.e. $\nabla f(p) = 0$. Similar to grayscale projection, the first-order gradient is used to guide the iteration direction of this step. By introducing a step size factor based on the Laplacian value, the iteration step size for second-order gradient projection is obtained:

$$\Delta p_{\text{laplacian}} = -\frac{\nabla^2 f(p)}{\|\nabla f(p)\|} \cdot \frac{\nabla f(p)}{\|\nabla f(p)\|} \quad (10)$$

By combining grayscale projection and second-order gradient projection, and taking a weighted sum of the two, the gradient-weighted projection of the proposed algorithm is obtained:

$$\Delta p = \alpha \cdot \Delta p_{\text{grayscale}} + \beta \cdot \Delta p_{\text{laplacian}} \quad (11)$$

The weight coefficients α and β are dynamically adjusted during the iteration process. In the early stage of iteration, the grayscale term dominates, leveraging its quadratic convergence property to quickly pull particles to the vicinity of the surface. As the iteration proceeds, the weight of the second-order gradient gradually increases, allowing particles to finely adjust to the edge positions indicated by the zero-crossings of the second-order gradient after approaching the surface. This method exhibits gradient-adaptive characteristics: in regions with small gradients, the step size is appropriately

enlarged to accelerate convergence, while in regions with large gradients, the step size is reduced to ensure accuracy. Repeating the above process until the particle positions stabilize enables precise projection of particles onto the edge positions on the isosurface. The projection process is illustrated in Figure 5.

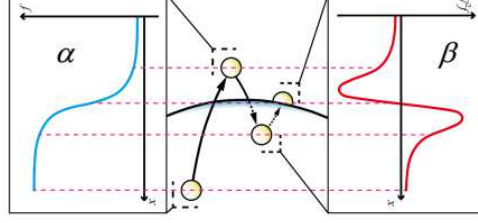


Fig. 5 “Gray-Two step Gradient” Weighted Projection

2.2.3 Local Short-Range Repulsion

During the iteration process, repulsive interactions are used to optimize the distribution of particles. Since the influence of repulsive force is weak when particles are far apart, it is reasonable to treat the repulsive force between particles as a local short-range force. We define the particle repulsion interaction to mainly consist of three steps: searching for neighboring particle sets, calculating the repulsive force, and iterating the tangential repulsive force.

Searching for the neighboring particle sets

This step is a prerequisite for calculating the repulsive force between particles, using the adaptively subdivided voxel units of the pyramid as the basic containers for particle behavior. This paper defines the repulsion radius h_{suff} of a target particle to be related to the scale level l of the voxel unit to which it belongs, and numerically equal to the voxel size σ_i at scale l .

$$h_{\text{cutoff}} = h_{\text{base}} \cdot \frac{\Delta^{(i)}}{\Delta^{(i-1)}} = h_{\text{base}} \cdot 2^{i-1-i} \quad (12)$$

where L is the maximum scale level of the pyramid, and h_{max} is the cell size at the finest scale. During the neighborhood search process, all voxels within the voxel unit containing the target particle and its 26-neighborhood are traversed, and other particles contained within these voxels are treated as the neighboring particle set. It is worth noting that when calculating the repulsive force in subsequent steps, the coordinates of the activated particles involved and their repulsion radii need to be mapped to the current scale to ensure consistency of scale.

Calculating the repulsive force

After determining the neighboring particle set, the repulsive force acting on the target particle is defined as the vector sum of the forces exerted by each neighboring particle. For any pair of interacting particles i and j , the relative position vector $\vec{d}_{ij} = \vec{p}_i - \vec{p}_j$ is first calculated.

We introduce a repulsion weight factor ϱ based on radial distance to achieve the short-range characteristic of the repulsive force. i.e. the closer the distance between particles, the stronger the repulsive force; when the distance approaches the cutoff radius h_{cutoff} , the repulsive force tends to zero. The weight factor ϱ is defined as follows:

$$\omega = \left(1 - \frac{r}{h_{cutoff}}\right)^2 \quad (13)$$

The relative position vector $\vec{d}_{ij}$ is normalized, and the unit vector $\hat{d}_{ij}$ is used as the direction of the repulsive force. The magnitude is modulated by the weight factor ω . Finally, the repulsive force $\vec{f}_{ij} = \hat{d}_{ij} \cdot \omega$ contributed by particle j to the target particle i is obtained.

Iterating the tangential repulsive force

The repulsive force $\vec{f}_g$ is projected onto the normal direction $\vec{n}_i$ of the target particle i to obtain the tangential component $\vec{f}_v^T = \vec{f}_v - \vec{n}_i (\vec{n}_i \cdot \vec{f}_v)$, and $\vec{f}_g^T$ is used as the actual acting repulsive force to reduce the impact on projection accuracy during the repulsive force iteration process, as shown in Figure 6.

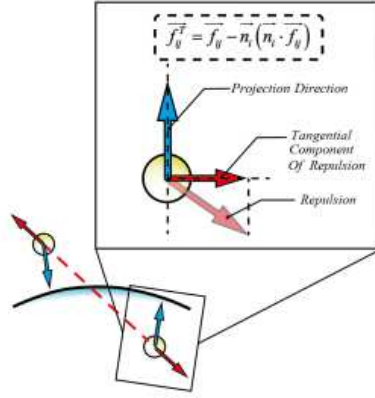


Fig. 6 The tangential component of the repulsive force

The total repulsive force on each particle is the vector sum of the contributions from all particles in its neighborhood:

$$\vec{F}_i = \sum_{j \in \mathbb{N}, j \neq i} \vec{f}_j \quad (14)$$

where N_i is the set of all neighboring particles of the target particle i within the cutoff radius h_{cutoff} . The total repulsive force is multiplied by the iteration step size to obtain the displacement $\Delta \vec{p}_i = \vec{F}_i \cdot step$ of the target particle for the current iteration. Through testing, a step size of 0.02 achieves a good balance between iteration accuracy and efficiency. The displacement is added to the current position of the particle, completing one iteration of repulsion-driven movement.

2.2.4 System Stability Check

The physical essence of system equilibrium is energy minimization. The equilibrium state corresponds to a local minimum of the total energy, and the driving force of its evolution originates from energy gradient descent. When the system reaches equilibrium, for each particle p_i , the energy gradient should be zero:

$$\frac{dp_i}{dt} - \nabla E_{\text{total}} = 0 \quad (15)$$

where E_{total} is the total energy function of the system. Considering the complexity of calculating the energy gradient, this method uses the average displacement of particles before and after iteration to characterize the energy gradient:

$$\Delta^{(t)} = \frac{1}{N} \sum_{i=1}^N \|p_i^{(t)} - p_i^{(t-1)}\| \quad (16)$$

However, during the iterative process of numerical implementation, when the system approaches equilibrium, the average displacement may change extremely slowly. Continuing the iteration at this stage has a very weak effect on particle distribution optimization but significantly impacts computational efficiency. Therefore, this paper defines a threshold θ , which is proportional to the particle repulsion radius h_{cutoff}

$$\varepsilon = \beta \cdot h_{\text{cutoff}} \quad (17)$$

where β represents the convergence threshold coefficient, which is set to 0.01 in the experiments of this paper. When the condition $\Delta^{(\ell)} < \varepsilon$ is satisfied for M consecutive iterations, the system is considered to have approached an equilibrium state near the energy minimum, as shown in Figure 7. M is a user-defined parameter; the larger its value, the more stable the system stability check becomes.

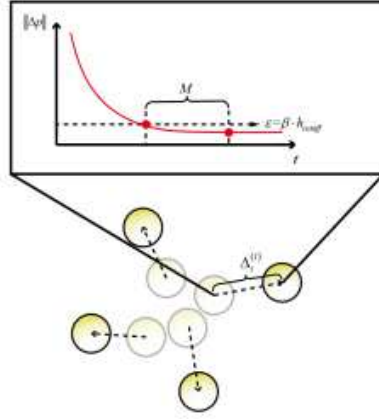


Fig. 7 Principle of System Stability Testing

2.2.5 Incremental Insertion

Particle insertion operations are performed within the high-curvature voxel units identified at each level of the voxel pyramid. This is the key for the proposed algorithm to achieve adaptive progressive refinement and enrich geometric details. Furthermore, to minimize the disturbance of the insertion operation to the system equilibrium state, we propose an incremental particle insertion mechanism guided by Voronoi diagrams[26, 27].

Considering the complexity of constructing a 3D Voronoi diagram, this paper constructs a 2D local Voronoi diagram to simplify the process. First, based on the currently stabilized particle distribution, a local orthogonal basis is constructed according to the gradient vector at the center of the target voxel unit, and the corresponding tangent plane is established. The activated particles within the local neighborhood are projected onto this tangent plane, and a 2D Voronoi diagram is constructed using the set of projected points. The area of each Voronoi cell is then calculated to quantify the local influence range of each particle. By comparing the sizes of these ranges, the largest uncovered region in the current particle distribution is identified. Finally, the centroid of the identified region is mapped back to 3D space and projected onto the isosurface, and the projected point is used as the insertion position for a new particle, as shown in Figure 8.

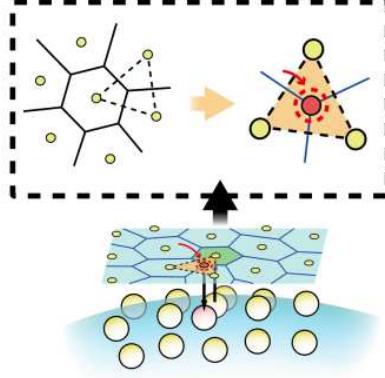


Fig. 8 Principle of Incremental Insertion

Direct projection in high-curvature regions may introduce deviations from the actual particle distribution due to normal-dimensionality reduction. To address this, in the nearest neighbor determination stage of Voronoi diagram construction, this paper introduces a correction term ϖ to the Euclidean distance of the 2D projected points, as shown in Equation (18). The projection distance is corrected $d_\omega(p) = \varpi \cdot d(p)$ based on local curvature and the angle between normals, so that the 2D Voronoi analysis results more accurately reflect the true void distribution on the 3D surface:

$$\omega = (1 + \alpha \cdot \kappa(p) |\cos \theta|) \quad (18)$$

where α is the correction weight coefficient, $\kappa(p)$ is the curvature at the position of the target particle, and θ is the angle between the normal at that point and the projection tangent plane. Since the insertion point is located at the center of the local void, its initial position is close to the equilibrium state, significantly reducing the number of adjustments required in subsequent relaxation iterations. Furthermore, to ensure that the algorithm can continue to execute under any circumstances, this method incorporates a fallback mechanism. When the number of local particles is insufficient or Voronoi construction fails, the original random offset scheme is used instead.

3 Experiment

In the experiment, the geometric data measured by a high-precision coordinate measuring machine (CMM) is used as the reference benchmark [28]. The absolute deviation between the surface extraction results of the proposed algorithm and traditional algorithms and the CMM measurements is taken as the evaluation metric for assessing the surface extraction effect. Considering the systematic errors in the measurement process, statistical indicators including maximum deviation, standard deviation, and residuals during geometric feature fitting are used for analysis to evaluate the surface extraction effect of the algorithms.

In this study, a high-precision metrological CT platform independently developed by the research group was used to acquire CT data. The system comprises a Varian Paxscan L08 detector, a WOX CMX350 X-ray source, along with motion stages and control systems, as illustrated in Fig.9.

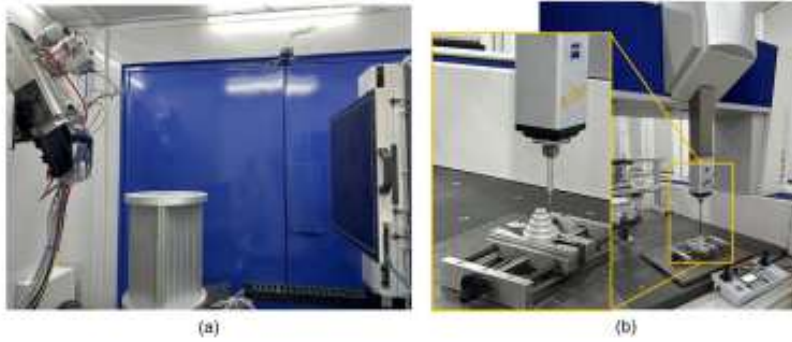


Fig. 8 Experimental equipment: a CT, b CMM

Fig. 9 Experimental equipment

In the experiment, two aluminum workpieces from the research group were used as CT scanning samples, and A ZEISS PRISMO navigator CMM (coordinate measuring machine) was used to obtain the actual dimensional data of the workpieces. As shown in Figure 10, both workpieces contain cylindrical and planar geometric structures, as well as features such as steps and holes, offering good structural representativeness and effectively evaluating the capability of industrial CT in the field of dimensional

metrology. For ease of description, these two workpieces are referred to as Sample 1 and Sample 2 hereafter.

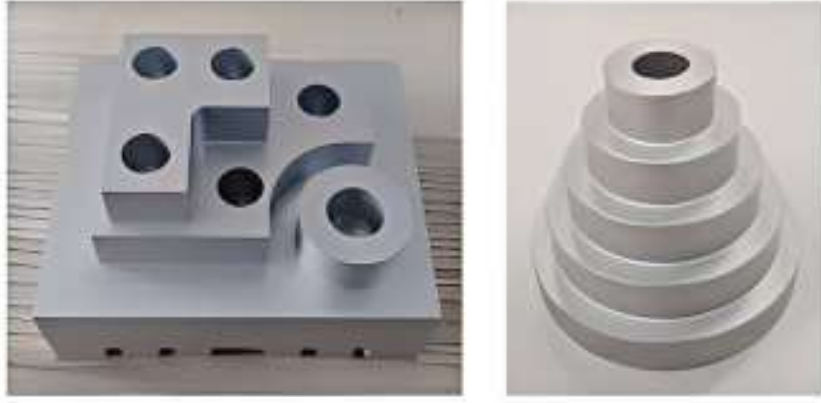


Fig. 10 Experimental samples

We set the tube voltage to 120 kV and the tube current to $600 \mu A$. The detector operates at a frequency of 4 frames per second, and 20 image averages are performed per angle to suppress noise[29]. During the scanning process, the sample was placed on the rotary table at an inclined angle to reduce cone-beam artifacts in CT reconstruction[30, 31].

3.1 Sampling Results and Algorithm Operational Efficiency

The surface extraction refinement process of the proposed algorithm for Sample 1 and Sample 2 is shown in Figure 11. It can be observed that as the scale gradually refines, the adaptively refined pyramid information drives particles to increase sampling density in high-curvature regions while maintaining a lower sampling rate in flat regions, thereby achieving curvature-adaptive sampling. In the figure, high-curvature regions are marked with red point clouds. Moreover, minimizing sampling density while ensuring sampling accuracy not only helps reduce storage overhead but also improves the efficiency of subsequent processing steps.

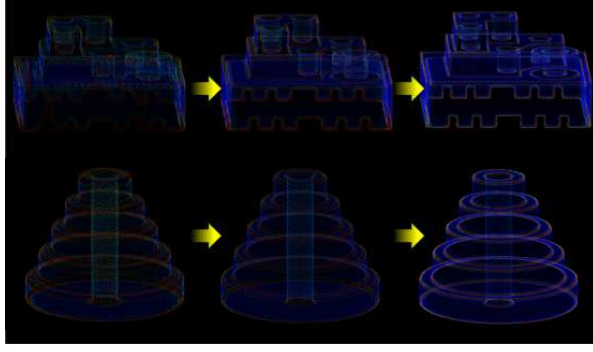


Fig. 11 Surface extraction refinement

Figure 12 shows the cross-sectional views of the surface extraction results for Sample 1 and Sample 2, providing a more intuitive illustration that the point cloud distribution is uniform in flat regions, while higher sampling density is present in detailed regions, with a smooth transition between the two.

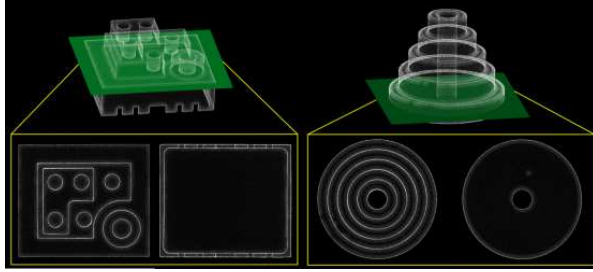


Fig. 12 Cross-sectional sampling effect

We compared the MC, DMC, and the proposed algorithm in terms of total sampling rate and computational efficiency. Since the proposed method involves a large number of particle interactions and iterations, and adopts an incremental particle insertion approach that requires new iteration processes after each insertion, it has a certain disadvantage in runtime compared to the MC and DMC methods. However, because we decouple the forces acting on particles and treat repulsion as a local short-range force for localized computations, while also employing adaptive sampling to significantly reduce the overall particle sampling rate, the proposed algorithm achieves a marked improvement in surface extraction efficiency compared to traditional particle system methods.

Table 1 compares the sampling scale and sampling efficiency of the four methods on different samples. For Sample 1 and Sample 2, the computation times of the MC and DMC methods are relatively close, as the computational effort of these algorithms is proportional to the number of voxel units. The runtime of the proposed method is longer than that of MC and DMC, but remains significantly lower than that of traditional particle system methods. In addition, a comparative analysis of the data size of

the sampling results was conducted. Compared with the sampling results of traditional surface extraction algorithms, the proposed method reduces the number of vertices by approximately 70% due to the use of curvature-adaptive sampling, effectively reducing the storage burden while ensuring reconstruction accuracy.

Table 1 Sampling results

Method		MC	DMC	Our
Sample 1	Number of Sampling Points	1049662	1060140	307843
	Sampling time	5.046	34.118	2281.178
Sample 2	Number of Sampling Points	915686	988464	285977
	Sampling time	4.698	31.198	1858.603

3.2 Sampling Distribution Quality

High-quality surface sampling can faithfully reconstruct the geometric features of a curved surface its quality essentially depends on whether the point set distribution can approximate an isotropic uniform state at the local scale. From a geometric perspective, minimizing the potential energy of a particle system induces a hexagonal structure, meaning that a high-quality sampling point distribution tends toward an ideal regular hexagonal tiling.

To this end, when evaluating surface sampling quality, this paper constructs a tangent plane at the target point. The projection points of the six nearest neighboring sampling points around the target point onto the tangent plane form a hexagon. The degree of similarity between this hexagon and a regular hexagon is used to evaluate the distribution quality at the target point. The normalized weighted result R of the angular deviations and side length differences of the hexagon’s adjacent edges is calculated to quantify whether the local arrangement approximates an ideal hexagonal tiling structure, as shown in Figure 13. A smaller weighted result R indicates that the distribution is closer to a regular hexagonal tiling, and thus the sampling distribution quality is higher.

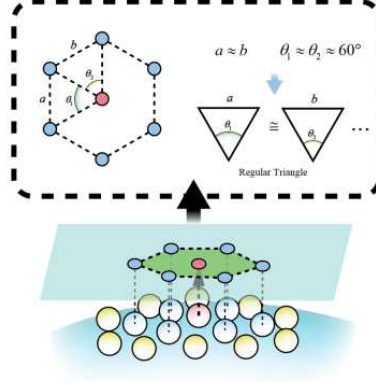


Fig. 13 Sampling distribution quality assessment principle

We perform weighted color rendering for each point in the sampling result based on the R value, intuitively presenting the overall sampling quality in the form of a colored point cloud. A color closer to blue indicates higher sampling distribution quality, while a color closer to red indicates lower quality. Figures 14 and 15 show the sampling quality distribution maps for Sample 1 and Sample 2, respectively, and the statistical results of the sampling distribution quality are presented in the form of histograms.

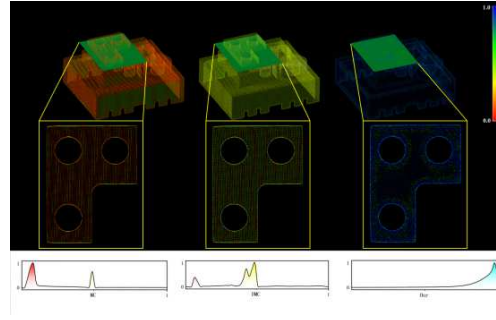


Fig. 14 The quality comparison chart of sampling distribution for Sample 1

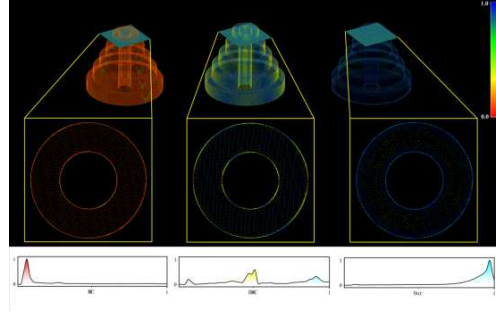


Fig. 15 The quality comparison chart of sampling distribution for Sample 2

As can be seen from the figure, the sampling points of the MC algorithm exhibit a noticeable clustering phenomenon in certain voxel units. The DMC algorithm generates sampling points inside voxel units with a uniformly arranged voxel grid, resulting in a certain improvement in sampling distribution quality compared to MC. In contrast, the sampling points generated by the proposed method benefit from iterative optimization of particle repulsion, achieving significantly better sampling distribution quality. Further statistical analysis using histograms shows that the overall distribution quality of MC sampling results is generally poor. DMC shows improvement, with sampling quality reaching a moderate level overall. The proposed method achieves the highest overall sampling quality.

3.3 Dimensional Accuracy and Geometric Accuracy

In this study, the commercial metrology software GOM Inspect was used as the measurement platform. Mesh models generated by different algorithms were imported, and the best Gaussian fitting function in the software was used to measure the cylindrical and planar features in the models. The measurement results were then compared with the reference values obtained from a coordinate measuring machine (CMM). Additionally, statistical indicators such as maximum deviation, standard deviation, and residuals output by GOM Inspect were analyzed.

We use cylinder diameter to evaluate dimensional accuracy and cylindricity and flatness to evaluate geometric accuracy. Sample 1 mainly involves the analysis of four cylindrical surfaces D1-D4 and four planar surfaces $x = 0$, $x = L_x$, $y = 0$, and $y = 0$, as shown in Figure 16. Sample 2 mainly involves the analysis of five cylindrical surfaces D1-D5. Subsequently, we compare different features of the test specimens in terms of dimensional accuracy and geometric accuracy. The measurement results from GOM Inspect are presented in tabular form, and the dimensional deviations of the measurement results relative to the CMM reference values are visualized as line charts.

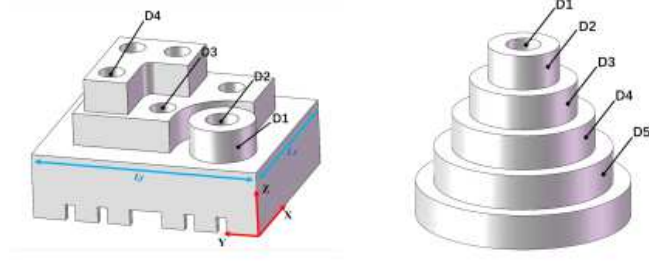


Fig. 16 Labeling of model geometric features

Dimensional Accuracy

The measurement results for the cylindrical diameters in Sample 1 and Sample 2 are recorded in Table 2. A line chart is plotted using the standard deviation of each feature as the dimensional deviation, as shown in Figure 17. In the figure, the horizontal axis represents each feature structure, the left vertical axis indicates the dimensional deviation of the measurement results relative to the CMM reference value, corresponding to the data points in the line chart, and the right vertical axis represents the range of deviation band width characterized by the standard deviation. As can be seen from the figure, because the proposed method uses second-order gradient correction for the sampling points, it achieves an overall improvement in dimensional accuracy compared to other methods, and the deviation range is relatively stable. For Sample 1, the measurement accuracy of the proposed algorithm is significantly improved compared to traditional algorithms. For Sample 2, there is no significant improvement in measurement accuracy for features D2 and D5 compared to traditional algorithms, but the proposed method performs better for the remaining features.

Table 2 Dimensional accuracy measurement results (mm)

Method		CMM	MC	DMC	MC(Otsu)	Our
Sample 1	D1	19.860	19.907	19.905	20.075	19.876
	D2	10.121	10.053	10.053	9.842	10.145
	D3	8.121	8.032	8.032	7.732	8.157
	D4	8.121	8.033	8.033	7.779	8.170
Sample 2	D1	10.087	10.016	10.007	10.009	10.116
	D2	20.039	20.065	20.052	20.051	20.028
	D3	30.006	30.085	30.065	30.064	30.019
	D4	40.014	40.106	40.078	40.077	40.016
	D5	50.041	50.102	50.068	50.066	50.019

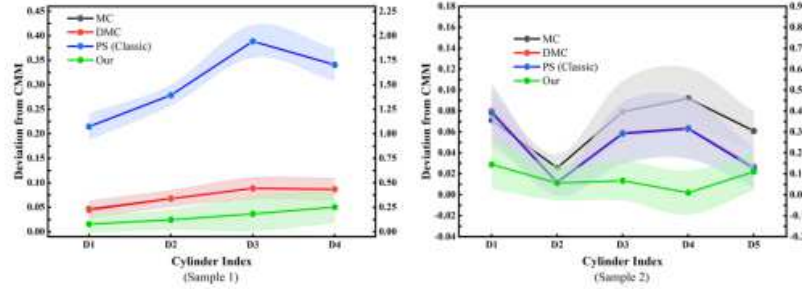


Fig. 17 Deviation in dimensional accuracy

Geometric Accuracy

We evaluate the geometric accuracy of the algorithm using cylindricity and flatness. The measurement results are recorded in Tables 3 and 4 and visualized in the form of line charts. Since the second-order gradient is sensitive to noise, improving dimensional accuracy through the second-order gradient may sacrifice surface flatness to some extent. However, benefiting from the multi-scale pyramid characteristics and the improved weighted projection, the proposed algorithm can mitigate this phenomenon to a certain degree. As shown in Figure 18, for Sample 1, the cylindricity and flatness measured by the proposed algorithm are relatively close to those of the MC and DMC algorithms, while the MC algorithm using the Otsu threshold performs poorly. For Sample 2, the geometric accuracy distributions of different algorithms are relatively similar. The measurement results of the proposed method are slightly inferior to those of traditional algorithms, but this trade-off for improved dimensional accuracy remains within an acceptable range.

Table 3 Geometric accuracy measurement results for sample 1 (mm)

	CMM	mc	Dmc	MC(Otsu)	Our
D1	0.0303	0.1124	0.1093	0.1797	0.1173
D2	0.0241	0.1157	0.1109	0.1501	0.1173
D3	0.0236	0.1332	0.1269	0.2016	0.1379
D4	0.0557	0.1163	0.1114	0.1996	0.1262
P1	0.0097	0.1533	0.1500	0.0180	0.1672
P2	0.0091	0.2172	0.2068	0.3522	0.1995
P3	0.0091	0.0947	0.1474	0.1974	0.1966
P4	0.0087	0.1610	0.1539	0.2472	0.1613

Table 4 Geometric accuracy measurement results for sample 2 (mm)

	D1	D2	D3	D4	D5
CMM	0.0009	0.0010	0.0015	0.0021	0.0022
Mc	0.1835	0.0875	0.1736	0.1601	0.1069
Dmc	0.1534	0.0830	0.1667	0.1513	0.0938
MC(Otsu)	0.1384	0.0874	0.1734	0.1599	0.1065
Our	0.1996	0.0822	0.1820	0.1790	0.1203

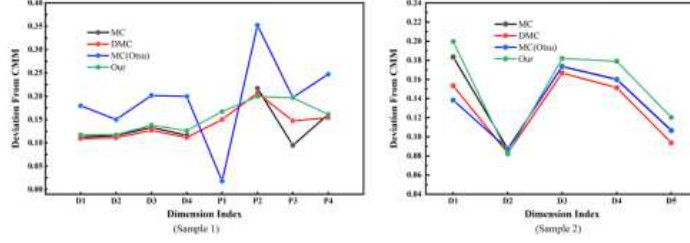


Fig. 18 Deviation of geometric accuracy

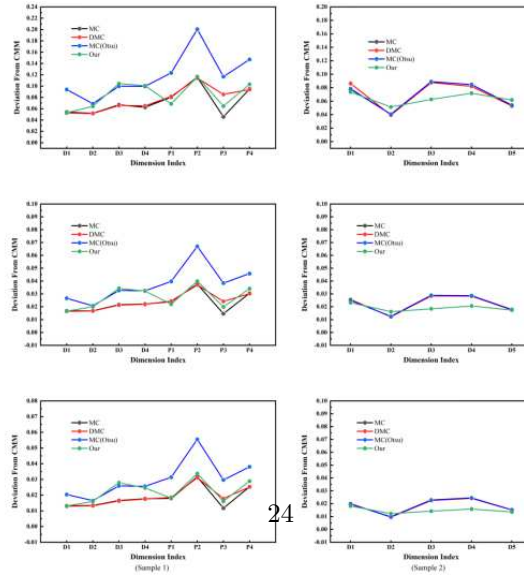
Tables 5 and 6 record the fitting statistical indicators for each geometric feature, including maximum absolute deviation (MaxDev), standard deviation (StdDev), and residual (Res), as shown in Figure 19. Overall, the proposed method maintains a comparable level of statistical indicators to traditional methods, and for certain features, its performance is even superior to some traditional algorithms. For the three indicators of Sample 1, except for the D3 feature, which is slightly inferior to the traditional algorithm, the performance of the other features is close to that of the traditional algorithm. For Sample 2, although the proposed algorithm is slightly inferior to the traditional algorithm in D2 and D5, it exhibits more stable statistical characteristics overall compared to traditional algorithms, demonstrating better performance.

Table 5 Fitted data for sample 1 (mm)

Statistics	Method	D1	D2	D3	D4	P1	P2	P3	P4
Max Dev	MC	0.0540	0.0519	0.0669	0.0625	0.0805	0.1158	0.0456	0.0949
	DMC	0.0525	0.0515	0.0658	0.0651	0.0816	0.1142	0.0853	0.0940
	PS(Classic)	0.0939	0.0688	0.0997	0.0995	0.1233	0.2005	0.1166	0.1470
	Our	0.0528	0.0641	0.1044	0.1006	0.0687	0.1166	0.0646	0.1031
Std Dev	MC	0.0168	0.0168	0.0216	0.0221	0.0239	0.0376	0.0146	0.0305
	DMC	0.0164	0.0167	0.0213	0.0219	0.0244	0.0370	0.0242	0.0302
	PS(Classic)	0.0266	0.0207	0.0327	0.0324	0.0397	0.0670	0.0384	0.0458
	Our	0.0166	0.0200	0.0343	0.0322	0.0220	0.0399	0.0199	0.0341
Res	MC	0.0132	0.0134	0.0166	0.0177	0.0179	0.0316	0.0117	0.0253
	DMC	0.0129	0.0132	0.0163	0.0175	0.0185	0.0311	0.0178	0.0252
	PS(Classic)	0.0204	0.0165	0.0259	0.0256	0.0314	0.0555	0.0297	0.0380
	Our	0.0131	0.0160	0.0279	0.0246	0.0181	0.0337	0.0161	0.0289

Table 6 Fitted data for sample 2 (mm)

Statistics	Method	D1	D2	D3	D4	D5
Max Dev	MC	0.0778	0.0407	0.0890	0.0846	0.0542
	DMC	0.0862	0.0394	0.0876	0.0820	0.0529
	PS(Classic)	0.0784	0.0406	0.0889	0.0846	0.0541
	Our	0.0735	0.0513	0.0626	0.0718	0.0618
Std Dev	MC	0.0250	0.0125	0.0289	0.0286	0.0178
	DMC	0.0256	0.0122	0.0284	0.0283	0.0175
	PS(Classic)	0.0250	0.0125	0.0289	0.0286	0.0178
	Our	0.0232	0.0162	0.0184	0.0206	0.0174
Res	MC	0.0197	0.0098	0.0229	0.0245	0.0152
	DMC	0.0200	0.0095	0.0225	0.0242	0.0151
	PS(Classic)	0.0197	0.0097	0.0228	0.0244	0.0152
	Our	0.0182	0.0122	0.0142	0.0158	0.0136

**Fig. 19** Comparison plot of fitted data

4 Conclusion

This paper proposes a surface extraction algorithm for industrial metrological CT data, capable of generating high-quality adaptive sampling point clouds from CT scan data and subsequently fitting mesh models that meet the requirements of high-precision metrological applications. Building upon traditional particle system-based surface extraction methods, this paper makes improvements in the following aspects: First, a multi-scale voxel pyramid is constructed through edge-preserving filtering and downsampling. The pyramid is adaptively refined based on mean curvature, and the refined voxel units serve as the basic containers for particle behavior. Second, a force mechanism with repulsive and attractive forces decoupled in the time domain is adopted. A grayscale-second-order gradient weighted projection replaces the traditional isosurface attraction to drive particles for precise sampling. Repulsion is treated as a local short-range force, with interacting particles searched within a 26-neighborhood for iterative computation to improve algorithm efficiency. Simultaneously, the tangential component of the repulsive force is extracted to optimize particle distribution uniformity while reducing the impact on projection accuracy. Furthermore, an incremental particle insertion method based on a local Voronoi diagram is proposed. After the iteration approaches equilibrium, regions with sparse particle distribution are estimated, and new particles are inserted in a manner that minimizes system energy changes. Experimental comparisons demonstrate that, compared to traditional surface extraction algorithms, the proposed algorithm exhibits superior adaptive sampling quality, significantly reduces the overall sampling rate, and achieves good reconstruction of most geometric features, showing clear comprehensive advantages.

However, the proposed algorithm still has certain limitations. First, the final output of the algorithm is a point cloud object containing discrete spatial coordinates and normal vectors, lacking topological connectivity between vertices. This makes it difficult to directly apply to subsequent geometric feature extraction and quantitative analysis tasks that require a continuous surface representation, necessitating post-processing steps such as surface fitting. Second, although the algorithm leverages multi-scale characteristics to enhance robustness against noise and artifacts to some extent, improper control of scale level settings can affect sampling correctness. When the number of pyramid levels is too high, the coarse scales become blurred due to downsampling operations, potentially losing some detailed information. The subsequent progressive refinement process is then constrained by the structure reconstructed at the coarse scales, making it difficult to effectively express details. To address these limitations, we hope to improve the algorithm in future work by constructing a voxel pyramid that preserves topological structure. In the initialization stage, referencing the topological construction approach of the DMC (Dual Marching Cubes) algorithm, a rough mesh model with complete topological information could be built, with details gradually enriched in subsequent processes. Additionally, since the algorithm requires storing voxel, gradient, and curvature information at each level of the voxel pyramid, it imposes high demands on runtime memory. Future work may consider storing only valid regions rather than the entire region to reduce memory overhead.

Author Contribution L X: Is responsible for researching the overall design concept and optimizing the theoretical framework. JT W: Conducts experimental-related work, obtains experimental data and results. Is responsible for writing and revising the paper, and also performs polishing and adjustment on the complete paper.

Data Availability No datasets were generated or analysed during the current study.

Declarations

Competing interests The authors declare no competing interests.

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