Supporting Information S1

S1.1 Expectations of $\widehat{A_{se}}$ and $\widehat{A_{sp}}$

Suppose $(TP, FN, FP, TN) \sim Multinomial(n, \boldsymbol{p})$ where $\boldsymbol{p} = (p_1, p_2, p_3, p_4)$. Then given that $TP + FN = m$, we know that the random variable TP follows a Binomial distribution:

$$TP|TP + FN = m \sim Binom(m, \frac{p1}{p1 + p2}).$$

The conditional expectation of TP is then given by:

$$\mathbb{E}(TP|TP + FN = m) = m\frac{p1}{p1 + p2}.$$

Therefore the conditional expectation of the ratio $\frac{TP}{TP+FN}$ is given by:

$$\mathbb{E}(\frac{TP}{TP + FN}|TP + FN = m) = \frac{1}{m}\mathbb{E}(TP|TP + FN = m) = \frac{1}{m}m\frac{p1}{p1 + p2} = \frac{p1}{p1 + p2}.$$

Since the conditional expectation of the proportion does not depend on the specific value of m (as long as $m > 0$), by the Law of Total Expectation, we have that the expected value of $\widehat{A_{se}} = \frac{TP}{TP+FN}$ is $\frac{p1}{p1+p2}$. Similarly, we can show that $\mathbb{E}(\frac{TN}{TN+FP}) = \mathbb{E}(\widehat{A_{sp}}) = \frac{p_4}{p_4+p_3}$.

S1.2 Crude estimates of A_{se} and A_{sp} and corresponding bias under a 'check the positives' design

Let us denote the crude sensitivity of Test A by $\widehat{A_{se}^+} = \frac{TP}{TP+FN}$ and the crude specificity by $\widehat{A_{sp}^+} = \frac{TN}{TN+FP}$ under a 'check the positives' design. The underlying probabilities for observing a TP, FN, FP and TN outcome under this design are given by:

$$\begin{aligned}
p_1^+ &= p_{111} = \pi(A_{se}B_{se} + c_1)\\
p_2^+ &= p_{011} = \pi((1 - A_{se})B_{se} - c_1)\\
p_3^+ &= p_{100} + p_{110} + p_{101} = (1 - \pi)(1 - A_{sp}) + \pi(A_{se}(1 - B_{se}) - c_1)\\
p_4^+ &= p_{000} + p_{010} + p_{001} = (1 - \pi)A_{sp} + \pi((1 - A_{se})(1 - B_{se}) + c_1),
\end{aligned}$$

respectively. Then the expectations of $\widehat{A_{se}^+}$ and $\widehat{A_{sp}^+}$, are given by:

$$\mathbb{E}(\widehat{A^+_{se}}) = \frac{p_1^+}{p_1^+ + p_2^+} = \frac{\pi(A_{se}B_{se} + c_1)}{\pi(A_{se}B_{se} + c_1) + \pi((1 - A_{se})B_{se} - c_1)} = A_{se} + \frac{c_1}{B_{se}},$$

$$\begin{aligned}\mathbb{E}(\widehat{A^+_{sp}}) &= \frac{p_4^+}{p_4^+ + p_3^+} = \frac{(1-\pi)(1-A_{sp}) + \pi(A_{se}(1-B_{se}) - c_1)}{(1-\pi)(1-A_{sp}) + \pi(A_{se}(1-B_{se}) - c_1) + (1-\pi)A_{sp} + \pi((1-A_{se})(1-B_{se}) + c_1)} \\ &= \frac{(1-\pi)A_{sp} + \pi((1-A_{se})(1-B_{se}) + c_1)}{1 - \pi B_{se}}.\end{aligned}$$

The conditional independence case then follows from setting $c_1 = 0$. The corresponding bias in then given by:

$$\text{Bias}(\widehat{A^+_{se}}) = \mathbb{E}(\widehat{A^+_{se}}) - A_{se} = \frac{c_1}{B_{se}} \leq \frac{\min(A_{se}, B_{se}) - A_{se}B_{se}}{B_{se}},$$

$$\begin{aligned}\text{Bias}(\widehat{A^+_{sp}}) = \mathbb{E}(\widehat{A^+_{sp}}) - A_{sp} &= \frac{(1-\pi)A_{sp} + \pi((1-A_{se})(1-B_{se}) + c_1)}{1 - \pi B_{se}} - A_{sp} \\ &= \frac{\pi((1-B_{se})(1-A_{se}-A_{sp}) + c_1)}{1 - \pi B_{se}}.\end{aligned}$$