Supplementary Information: Supplementary Material 2

Methodological proofs and climatic restriction profiles

Each figure (Fig. S1 and Fig. S2) consists of two complementary panels designed to diagnose the climatic relationship of the species along a specific environmental gradient. The left panel (a), the Gaussian Suitability Profile, represents the empirical occupancy of the species in environmental space. The bell-shaped curve illustrates theoretically modeled probability density, where the peak (centered on the green dashed line) identifies the climatic optimum—the specific value where the species is most frequently observed. A narrow, steep curve indicates high niche specialization and conservatism, while a broader curve reflects greater tolerance or exposure to a wider range of conditions for that specific variable.

The right panel (b), the VERA Quadratic Restriction Intensity, translates these occupancy patterns into a standardized measure of environmental stress or "restriction." The red parabolic curve represents the squared *z*-score (Z^2), which quantifies the penalty as a locality deviate from the species' climatic optimum. The blue dots represent individual occurrence records mapped onto this penalty surface. Points positioned higher on the Y-axis indicate individuals persisting under significant climatic restriction for that variable. In VERA diagnostics, the variable that produces the highest Z^2 value at a given pixel is identified as the Most Limiting Variable (MLV), representing the primary climatic bottleneck preventing further range expansion.

Note on Reproducibility: Diagnostic profiles for all other environmental variables utilized in this study can be generated using the standardized visualization protocols provided in the SI_2_VERA_Plotting_R_Code.txt file.

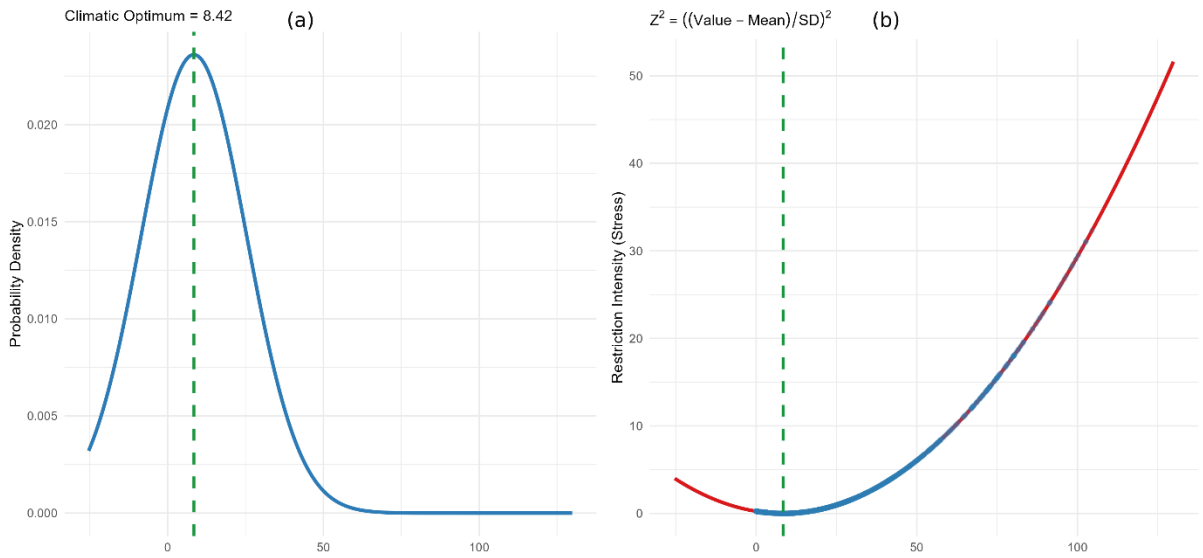

Fig. S1 VERA diagnostic profiles for PET of the Coldest Quarter in *L. canadensis*. (a) Gaussian Suitability Profile showing the probability density of occurrences; the dashed

green line indicates the climatic optimum (mean). (b) VERA Quadratic Restriction Intensity curve (Z^2); blue points represent individual occurrence records, illustrating the stress penalty relative to the niche-centroid distance

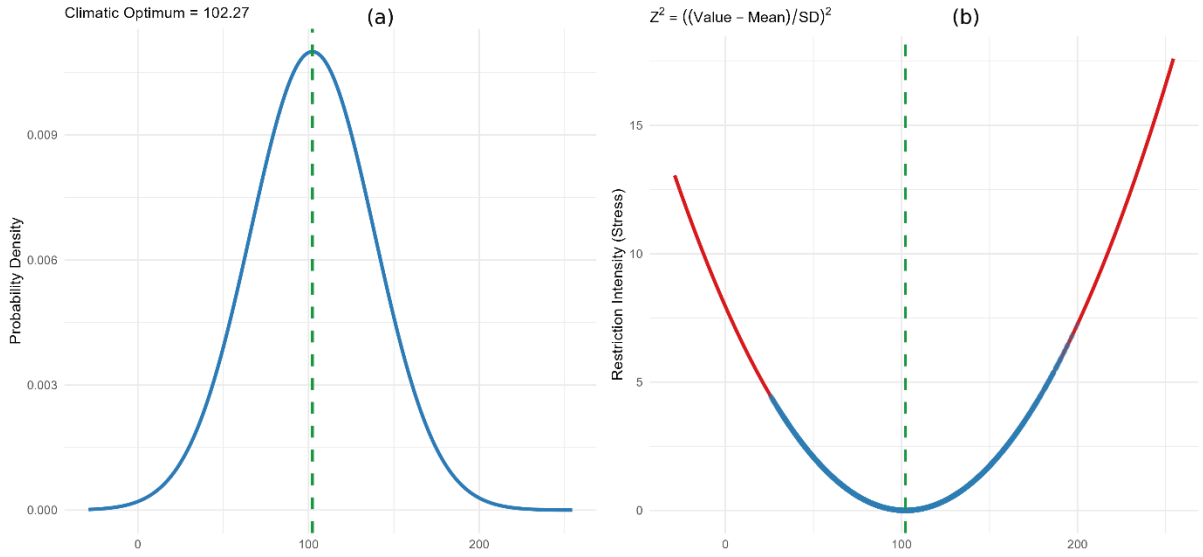

Fig. S2 VERA diagnostic profiles for PET of the Coldest Quarter in *L. rufus*. (a) Gaussian Suitability Profile showing the probability density of occurrences; the dashed green line indicates the climatic optimum (mean). (b) VERA Quadratic Restriction Intensity curve (Z^2); blue points represent individual occurrence records, illustrating the stress penalty relative to the niche-centroid distance