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An Interactive Pedagogical Framework for the Axiomatic Construction of Natural Numbers in High Achieving Groups

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Research Article

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Abstract

Traditional pedagogical approaches in university-level mathematics often struggle to maintain student engagement during highly abstract topics. This paper presents an interactive methodology implemented within the "SABAH" excellence groups—a strategic initiative by the Ministry of Science and Education of the Republic of Azerbaijan. The curriculum for "Numerical Systems" was restructured into student-led research modules focusing on Peano axioms. By leveraging the competitive and high-potential nature of the SABAH initiative, we transformed abstract number theory into a collaborative research project. This study details the implementation of the "Four-Group Research Model," the multi-dimensional assessment matrix, and the qualitative outcomes of the project. Findings suggest that shifting the epistemological responsibility from the instructor to the students significantly enhances engagement and retention in abstract mathematical logic.

Keywords: Peano axioms, Axiomatic construction, Active learning, SABAH groups, Mathematics education, Inquiry-based learning.

1. Introduction

In contemporary higher education, the transition from passive lecture-based learning to active participation is essential for fostering deep conceptual understanding. For complex subjects like "Numerical Systems," a student-centered approach is particularly effective. Over-reliance on traditional seminars often leads to monotony, especially when dealing with the foundational logic of mathematics, such as the axiomatic construction of the set of natural numbers (N).

Building upon preliminary pedagogical observations (Nabiyev, 2025), this study provides a comprehensive analytical framework for implementing student-led research modules within elite academic cohorts. The core objective is to bridge the gap between abstract algebraic theory and pedagogical practice by employing an inquiry-based model.

2. The Context: The SABAH Excellence Initiative

The methodology was implemented within the SABAH groups, a strategic excellence project in Azerbaijan's higher education system. The acronym 'SABAH' represents three core pillars: *Savadlı* (Educated), *Bacarıqlı* (Capable), and *Hazırlıqlı* (Prepared). Established to align workforce training with global academic standards, SABAH groups consist of high-potential students selected through a rigorous multi-stage competitive process. This intellectually rigorous environment provided the ideal setting for exploring the foundational logic of numerical systems through a decentralized instructional model.

3. Theoretical Framework: Peano Axioms

The sessions focused on the axiomatic construction of (N) based on Giuseppe Peano's foundational logic. The curriculum was structured around the following five axioms:

- **A1:** $1 \in N$.
- **A2:** Every $a \in N$ has a unique successor a'.
- **A3:** 1 is not the successor of any element ($a' \neq 1$).
- **A4:** If $a' = b'$, then $a = b$.
- **A5 (Induction):** If a set $M \subset N$ contains 1 and the successor of every element in M, then $M = N$ (Peano, 1881).



Figure 1. The instructor facilitating the session and managing the peer-assessment table on the interactive whiteboard.

4. Methodology: The Four-Group Research Model

The students were divided into four research units, each tasked with presenting a specific pillar of the natural number system. This forced students to transition from passive listeners to "subject matter experts" (Nabiyev, 2026).

Group I: The Genesis of N

Focused on the first principles and the successor function. They demonstrated how the set is built from unity and proved the uniqueness of the natural number model up to isomorphism.

Group II: Algebraic Operations

This group defined addition $(+)$ and multiplication $(\cdot)$ through inductive recursion:

1. $a + 1 = a'$,
2. $a + b' = (a + b)'$,
3. $a \cdot 1 = a$,
4. $a \cdot b' = a \cdot b + a$.

Technical Highlight: The group provided a formal proof for the commutativity of addition $(a + b = b + a)$, sparking a significant classroom debate on the limits of intuition versus formal proof.

Group III: Order and Trichotomy

Explored the "Law of Trichotomy," proving that for any natural numbers a and b, exactly one of the following holds: $a = b$, $a > b$, or $a < b$.

Group IV: Inverse Operations and Limitations

Discussed subtraction and division as inverse operations. By proving that $a - b$ is only possible if $a > b$, they demonstrated the "algebraic incompleteness" of N, leading to the logical necessity of Integers and Rational numbers.



Figure 2. Group II presenting the axiomatic foundations of addition and multiplication using digital slides.

5. Results and Discussion: Peer-Assessment and Engagement

A critical component of this framework was the "Multi-Dimensional Assessment Matrix." By shifting the evaluative power to the peers, a "cognitive conflict" was created, forcing students to defend their mathematical logic.

Table 1: Multi-Dimensional Assessment Matrix

Criteria	Weight	Description
Scientific Rigor	40%	Accuracy of Peano axiom applications and formal proofs.
Pedagogical Clarity	20%	Quality of visual aids and clarity of explanation.
Peer Q&A Defense	20%	Critical thinking during interactive defense sessions.

Criteria	Weight	Description
Collaborative Effort	20%	Evidence of teamwork and equal participation.

Comparative observations indicated that students in the SABAH groups exhibited significantly higher engagement rates when tasked with defending Peano's axioms to their peers compared to traditional lecture formats. The classroom transformed from a static environment into a dynamic research floor, as evidenced by the visual documentation (Figures 1–4).



Figure 3. Students explaining the theoretical framework of the Axiomatic Theory to their peers.

6. Conclusion

The SABAH group experiment demonstrates that student-led research modules are highly effective for teaching abstract branches of mathematics. By shifting the burden of proof to the students, we foster a generation of mathematicians capable of independent inquiry. Shifting the epistemological responsibility not only improves retention but also humanizes the abstract nature of algebraic constructions.

inquiry.



Figure 4. Collaborative learning environment: Group members and the instructor during the final discussion phase.

Declarations

Ethical approval. This study was conducted in accordance with the ethical standards of Azerbaijan State Pedagogical University. The research protocol and the use of visual documentation involving human participants were formally reviewed and approved by the University Ethics Committee (Protocol No: 2026/01-ASPU) in accordance with the 1964 Helsinki declaration and its later amendments.

Consent to participate. Informed consent was obtained from all individual participants included in the study. Since participants were students, informed consent was also obtained from their parents and/or legal guardians. Participation was voluntary.

Consent to publish. Informed consent (Consent to Participate and Consent to Publish) was obtained from all participants and their legal guardians for the publication of identifiable images within this manuscript. All student data has been anonymized to ensure privacy.

Data Availability Statement. The datasets generated during and/or analysed during the current study are available from the corresponding author on reasonable request.

Disclosure Statement (Conflict of Interest). The author reports there are no competing financial or personal interests that could have appeared to influence the work reported in this paper.

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