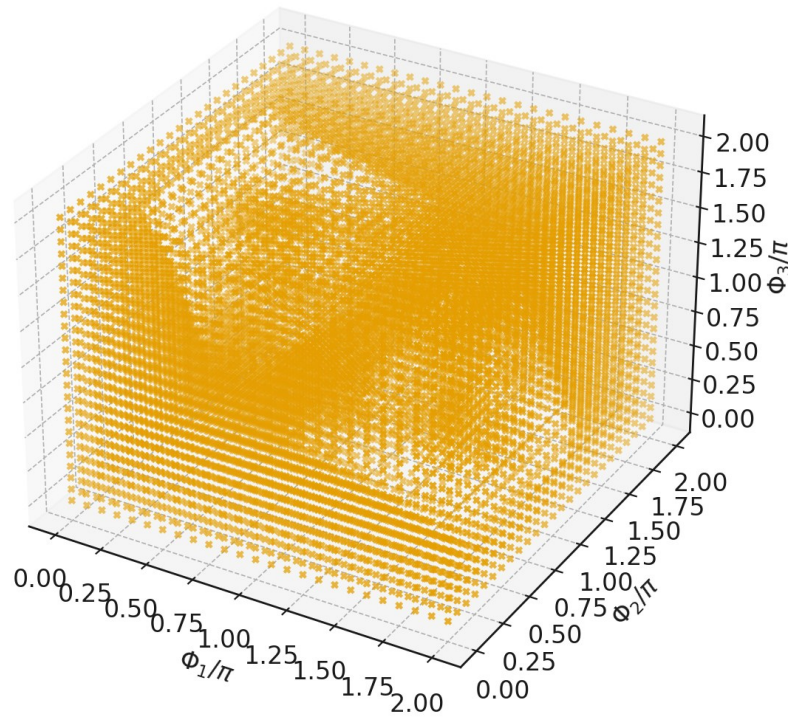


Three-ring system ( $G=0.9, K=0.3$ )  
Oscillatory phase triplets (phases normalized by  $\pi$ )

(A)



Three-ring system ( $G=0.8, K=0.3$ )  
Oscillatory phase triplets (phases normalized by  $\pi$ )

(B)

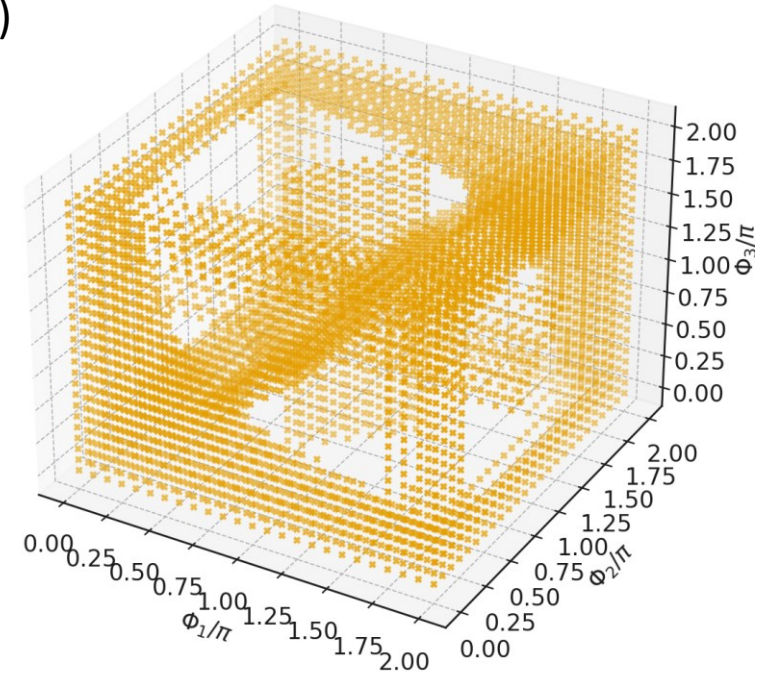
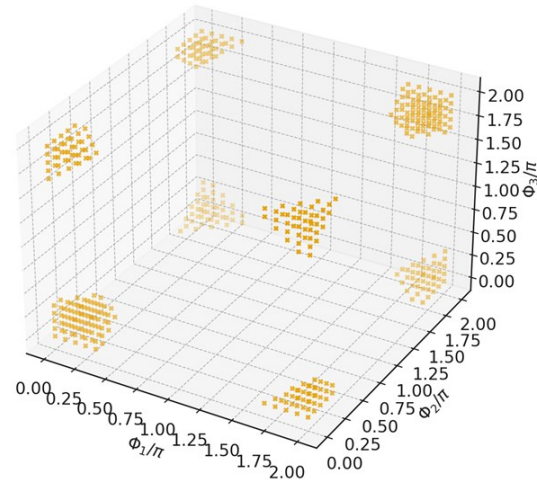


Figure [s1]: Results of numerical modeling showing the phase maps for the system of three coupled ARCs. The coupling coefficients between the three circuit is  $0.3 J_{12} = J_{13} = J_{23} = 0.3$ . (A) The amplification for all three circuits is taken to be  $0.9$ .  $G_1 = G_2 = G_3 = 0.9$ . (B) The amplification for all three circuits is taken to be  $0.8$ .  $G_1 = G_2 = G_3 = 0.8$ . The white region are the regions with no auto-oscillations.

Three-ring system ( $G=0.5$ ,  $K=0.3$ )  
Oscillatory phase triplets (phases normalized by  $\pi$ )

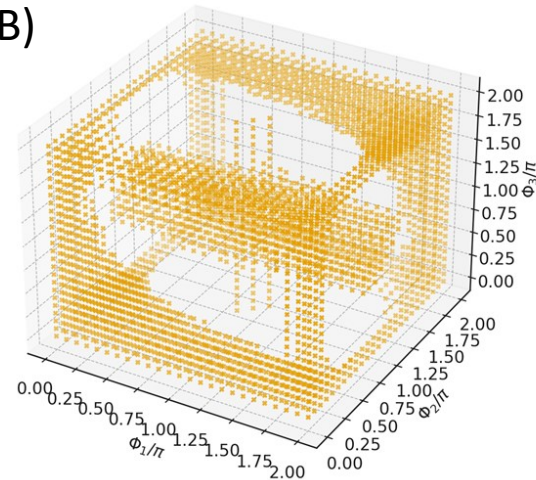
(A)



The same coupling coefficients  $K$ .

Three-ring system ( $G=0.7$ ,  $K_{12}=0.3$ ,  $K_{13}=0.2$ ,  $K_{23}=0.4$ )  
Oscillatory phase triplets (phases normalized by  $\pi$ )

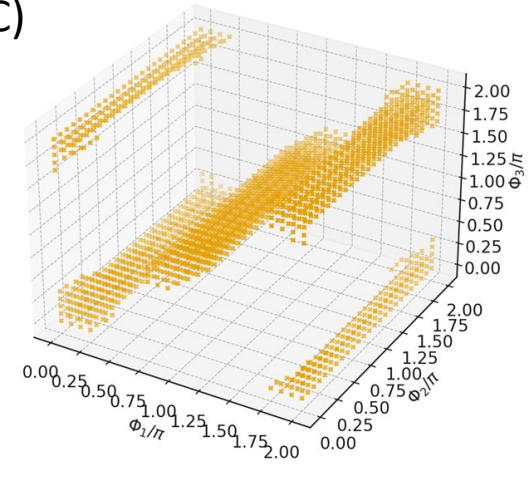
(B)



All coupling coefficients  $K$  are different.

Three-ring system ( $G=0.7$ )  
Strongly non-symmetric non-reciprocal coupling  
Oscillatory phase triplets (phases normalized by  $\pi$ )

(C)



The coefficients  $K$  are non-reciprocal.  
 $K_{12} \neq K_{21}$

Figure [s2]: Results of numerical modeling showing the phase maps for the system of three coupled ARCs. (A) The coupling coefficients between the three circuit is  $0.3 J_{12} = J_{13} = J_{23} = 0.3$ . The amplification for all three circuits is taken to be  $0.5$ .  $G_1 = G_2 = G_3 = 0.5$ . (B) The amplification for all three circuits is taken to be  $0.8$ .  $G_1 = G_2 = G_3 = 0.7$ .  $J_{12} = 0.3$ ;  $J_{13} = 0.2$ ;  $J_{23} = 0.4$ . (C) The amplification for all three circuits is taken to be  $0.5$ .  $G_1 = G_2 = G_3 = 0.7$ . The coupling coefficients are non-reciprocal  $J_{12} \neq J_{21}$ . The white region are the regions with no auto-oscillations.

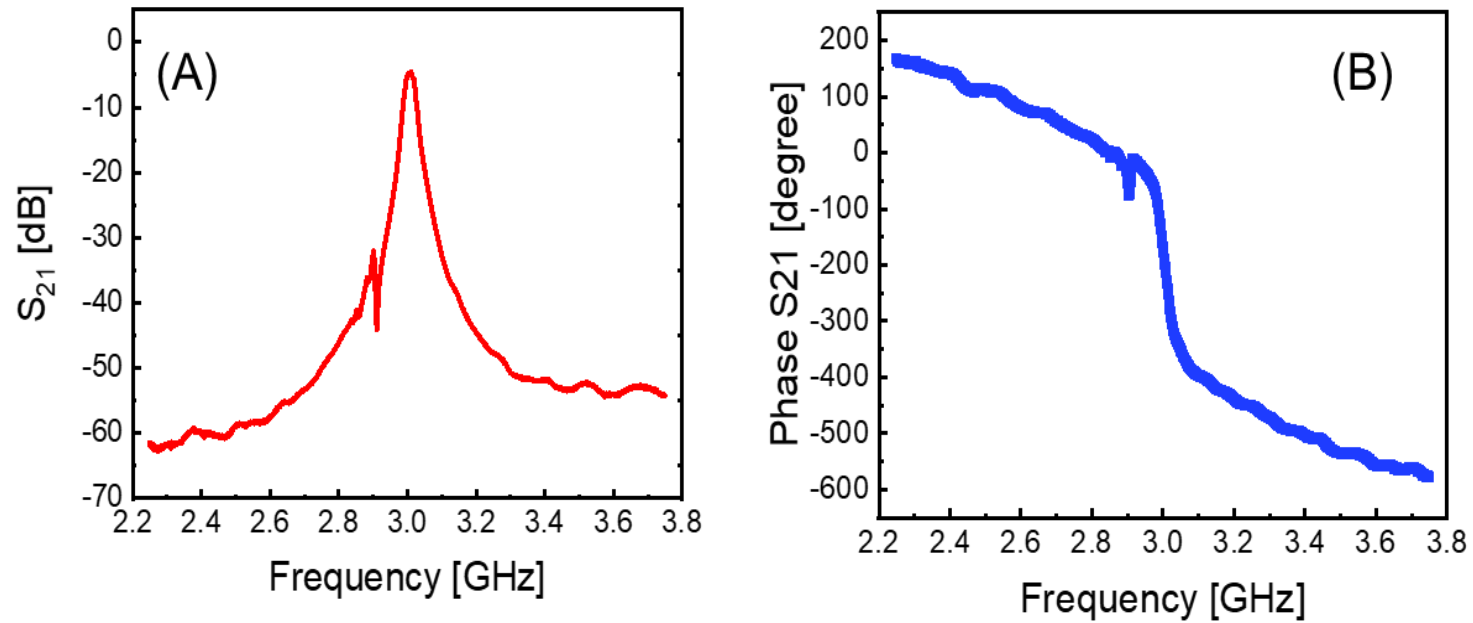


Figure [s3]: (A) Experimental data for the commercial filter Micro Lambda Wireless, Inc, model MLFD-40540. (A) S<sub>21</sub> parameter (amplitude); (B) S<sub>21</sub> parameter (phase shift). The bandpass frequency is set to 3GHz. The data are taken by VNA at UCR.