

# **Supplementary materials of Quantitative evaluation of the percolation threshold and backbone fraction of rocks with transversely isotropic cracks**

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Tables S1–S4 summarize the estimates of  $\rho'_{3c}$  and  $\Delta_L$  of individual cases and their standard errors. Figure S1 illustrates the backbone fractions of the anisotropic cases in which  $\kappa=\pm 10$ ,  $a=2.7$ ,  $r_{min}/r_{max}=0.1$ , and a constant crack aspect ratio was assumed. In this figure, the curves of the backbone fractions with respect to  $\rho'_3$  are depicted, as well as the backbone fractions with respect to  $\Delta\rho'_3(=\rho'_3-\rho'_{3c})$ .

**Table S1.** Summary of  $\rho'_{3c}$  and  $\Delta_L$  for the isotropic cases (1/2). The values in parentheses are the standard errors of the estimates.

$a$	$r_{min}/r_{max}$	$L/r_{max}$	$\rho'_{3c}$	$\Delta_L$
–	1	6	2.452 (0.011)	0.571 (0.011)
		8	2.385 (0.006)	0.411 (0.006)
		10	2.324 (0.004)	0.322 (0.004)
		12	2.327 (0.003)	0.294 (0.003)
		16	2.302 (0.002)	0.206 (0.001)
1.8	0.3	4	2.572 (0.010)	0.612 (0.009)
		5	2.543 (0.006)	0.473 (0.006)
		6	2.434 (0.004)	0.423 (0.004)
		7	2.401 (0.003)	0.357 (0.003)
		8	2.398 (0.002)	0.314 (0.002)
1.8	0.1	4	2.649 (0.006)	0.633 (0.005)
		5	2.553 (0.003)	0.479 (0.003)
		6	2.517 (0.002)	0.422 (0.002)
		7	2.495 (0.002)	0.350 (0.002)
		8	2.461 (0.001)	0.319 (0.001)
1.8	0.03	4	2.689 (0.003)	0.605 (0.003)
		5	2.555 (0.002)	0.496 (0.002)
		6	2.518 (0.001)	0.446 (0.001)
		7	2.474 (0.001)	0.347 (0.001)
		8	2.462 (0.001)	0.319 (0.001)
1.8	0.01	4	2.619 (0.002)	0.667 (0.002)
		5	2.599 (0.001)	0.501 (0.001)
		6	2.479 (0.001)	0.408 (0.001)
		7	2.463 (0.001)	0.382 (0.001)
		8	2.474 (0.001)	0.309 (<0.001)

**Table S2.** Summary of  $\rho'_{3c}$  and  $\Delta_L$  for the isotropic cases (2/2). The values in parentheses are the standard errors of the estimates.

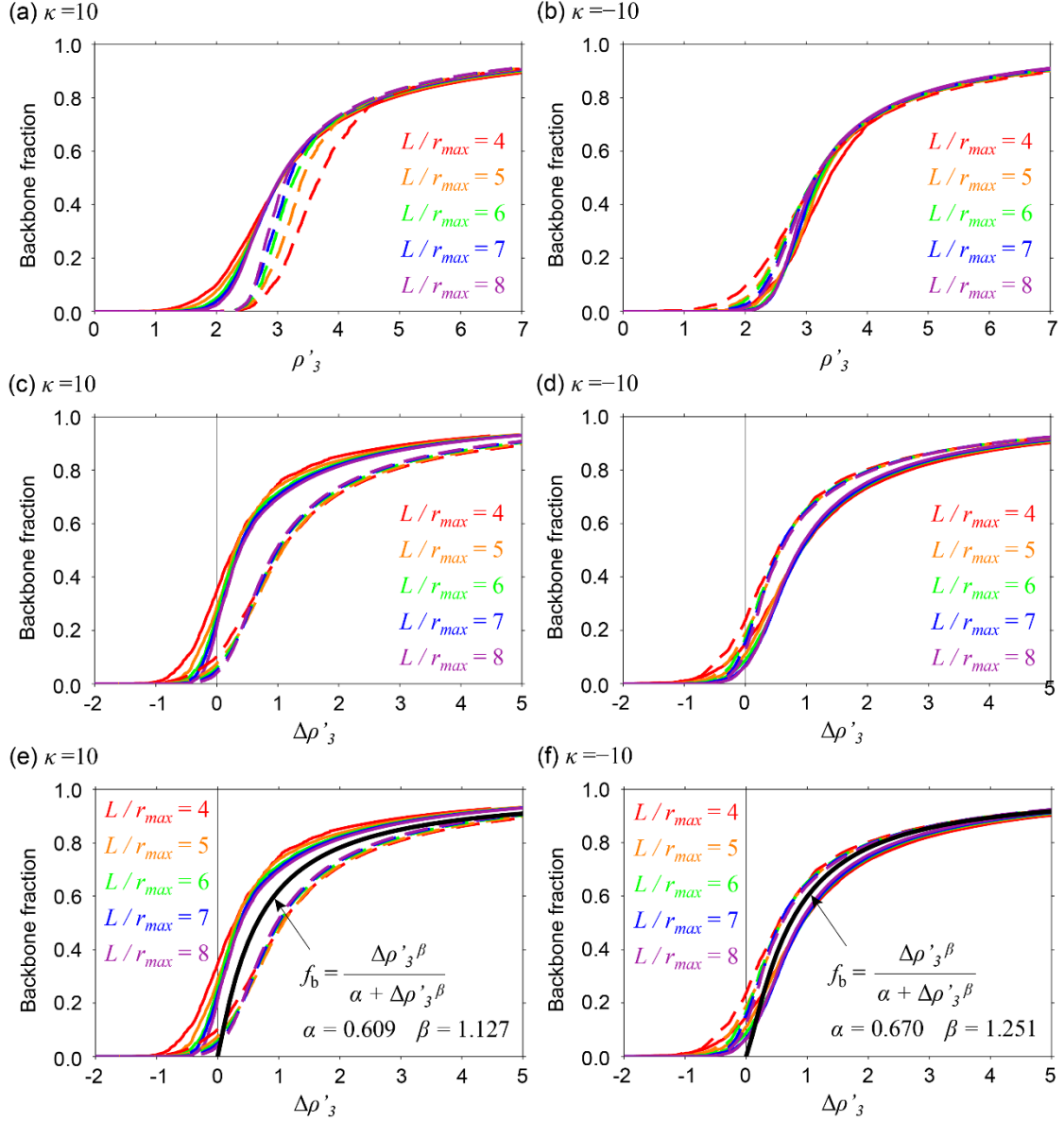
$a$	$r_{min}/r_{max}$	$L/r_{max}$	$\rho'_{3c}$	$\Delta_L$
2.4	0.1	4	2.658 (0.004)	0.591 (0.004)
		5	2.589 (0.003)	0.496 (0.003)
		6	2.531 (0.002)	0.402 (0.002)
		7	2.520 (0.001)	0.344 (0.001)
		8	2.491 (0.001)	0.299 (0.001)
2.7	0.1	4	2.706 (0.003)	0.576 (0.003)
		5	2.602 (0.002)	0.464 (0.002)
		6	2.563 (0.001)	0.378 (0.001)
		7	2.535 (0.001)	0.324 (0.001)
		8	2.497 (0.001)	0.278 (0.001)
3.0	0.1	4	2.692 (0.003)	0.526 (0.003)
		5	2.610 (0.002)	0.421 (0.002)
		6	2.590 (0.001)	0.358 (0.001)
		7	2.537 (0.001)	0.315 (0.001)
		8	2.530 (0.001)	0.271 (0.001)
3.6	0.1	4	2.666 (0.002)	0.433 (0.002)
		5	2.630 (0.001)	0.372 (0.001)
		6	2.606 (0.001)	0.286 (0.001)
		7	2.556 (0.001)	0.251 (0.001)
		8	2.580 (<0.001)	0.224 (<0.001)

**Table S3.** Summary of  $\rho'_{3c}$  and  $\Delta_L$  for the anisotropic cases (1/2). For all cases,  $r_{min}/r_{max} = 0.1$ . The values in parentheses are the standard errors of the estimates. The z-axis corresponds to the symmetric axis of the transverse isotropy of the crack orientation.

$a$	$\kappa$	$L/r_{max}$	$\rho'_{3c}$ (z-axis)	$\Delta_L$ (z-axis)	$\rho'_{3c}$ (x-axis)	$\Delta_L$ (x-axis)
1.8	-10	4	2.153 (0.005)	0.565 (0.005)	2.858 (0.005)	0.623 (0.005)
		5	2.116 (0.003)	0.459 (0.003)	2.730 (0.003)	0.497 (0.003)
		6	2.170 (0.002)	0.396 (0.002)	2.629 (0.002)	0.424 (0.002)
		7	2.173 (0.002)	0.366 (0.002)	2.573 (0.002)	0.354 (0.002)
		8	2.183 (0.001)	0.310 (0.001)	2.542 (0.001)	0.299 (0.001)
1.8	-6	4	2.262 (0.005)	0.578 (0.005)	2.832 (0.005)	0.616 (0.005)
		5	2.199 (0.003)	0.492 (0.003)	2.703 (0.003)	0.513 (0.003)
		6	2.200 (0.002)	0.386 (0.002)	2.615 (0.002)	0.428 (0.002)
		7	2.223 (0.002)	0.366 (0.002)	2.562 (0.002)	0.360 (0.002)
		8	2.209 (0.001)	0.305 (0.001)	2.523 (0.001)	0.289 (0.001)
1.8	-3	4	2.356 (0.005)	0.582 (0.005)	2.749 (0.005)	0.631 (0.005)
		5	2.283 (0.003)	0.475 (0.003)	2.668 (0.004)	0.527 (0.003)
		6	2.301 (0.002)	0.398 (0.002)	2.590 (0.002)	0.445 (0.002)
		7	2.302 (0.002)	0.379 (0.002)	2.545 (0.002)	0.359 (0.002)
		8	2.282 (0.001)	0.322 (0.001)	2.510 (0.001)	0.314 (0.001)
1.8	0	4	2.649 (0.006)	0.633 (0.005)	2.649 (0.006)	0.633 (0.005)
		5	2.553 (0.003)	0.479 (0.003)	2.553 (0.003)	0.479 (0.003)
		6	2.517 (0.002)	0.422 (0.002)	2.517 (0.002)	0.422 (0.002)
		7	2.495 (0.002)	0.350 (0.002)	2.495 (0.002)	0.350 (0.002)
		8	2.461 (0.001)	0.319 (0.001)	2.461 (0.001)	0.319 (0.001)
1.8	3	4	2.977 (0.005)	0.624 (0.005)	2.341 (0.005)	0.583 (0.005)
		5	2.833 (0.003)	0.545 (0.003)	2.288 (0.003)	0.477 (0.003)
		6	2.738 (0.002)	0.409 (0.002)	2.347 (0.002)	0.422 (0.002)
		7	2.643 (0.002)	0.351 (0.002)	2.349 (0.002)	0.360 (0.002)
		8	2.581 (0.001)	0.305 (0.001)	2.292 (0.001)	0.305 (0.001)
1.8	6	4	3.271 (0.005)	0.653 (0.004)	2.094 (0.004)	0.529 (0.004)
		5	3.044 (0.003)	0.471 (0.003)	2.078 (0.003)	0.471 (0.003)
		6	2.855 (0.002)	0.386 (0.002)	2.103 (0.002)	0.393 (0.002)
		7	2.732 (0.001)	0.334 (0.001)	2.097 (0.001)	0.318 (0.001)
		8	2.672 (0.001)	0.291 (0.001)	2.131 (0.001)	0.295 (0.001)
1.8	10	4	3.520 (0.004)	0.674 (0.004)	1.910 (0.003)	0.504 (0.003)
		5	3.185 (0.002)	0.496 (0.002)	1.942 (0.002)	0.434 (0.002)
		6	3.011 (0.002)	0.385 (0.002)	1.971 (0.002)	0.341 (0.002)
		7	2.861 (0.001)	0.320 (0.001)	1.993 (0.001)	0.313 (0.001)
		8	2.769 (0.001)	0.264 (0.001)	2.000 (0.001)	0.267 (0.001)

**Table S4.** Summary of  $\rho'_{3c}$  and  $\Delta_L$  for anisotropic cases (2/2). For all cases,  $r_{min}/r_{max} = 0.1$ . The values in parentheses are the standard errors of the estimates. The z-axis corresponds to the symmetric axis of the transverse isotropy of the crack orientation.

$a$	$\kappa$	$L/r_{max}$	$\rho'_{3c}$ (z-axis)	$\Delta_L$ (z-axis)	$\rho'_{3c}$ (x-axis)	$\Delta_L$ (x-axis)
2.7	-10	4	2.228 (0.003)	0.545 (0.003)	2.842 (0.003)	0.548 (0.003)
		5	2.256 (0.002)	0.419 (0.002)	2.702 (0.002)	0.432 (0.002)
		6	2.217 (0.001)	0.360 (0.001)	2.675 (0.001)	0.361 (0.001)
		7	2.227 (0.001)	0.312 (0.001)	2.638 (0.001)	0.297 (0.001)
		8	2.265 (0.001)	0.275 (0.001)	2.579 (0.001)	0.271 (0.001)
2.7	-6	4	2.266 (0.003)	0.527 (0.003)	2.823 (0.003)	0.549 (0.003)
		5	2.298 (0.002)	0.436 (0.002)	2.692 (0.002)	0.441 (0.002)
		6	2.262 (0.001)	0.366 (0.001)	2.661 (0.001)	0.362 (0.001)
		7	2.266 (0.001)	0.310 (0.001)	2.614 (0.001)	0.308 (0.001)
		8	2.291 (0.001)	0.296 (0.001)	2.560 (0.001)	0.278 (0.001)
2.7	-3	4	2.368 (0.003)	0.567 (0.003)	2.795 (0.003)	0.566 (0.003)
		5	2.374 (0.002)	0.419 (0.002)	2.656 (0.002)	0.423 (0.002)
		6	2.351 (0.001)	0.385 (0.001)	2.633 (0.001)	0.366 (0.001)
		7	2.364 (0.001)	0.307 (0.001)	2.589 (0.001)	0.325 (0.001)
		8	2.360 (0.001)	0.301 (0.001)	2.545 (0.001)	0.279 (0.001)
2.7	0	4	2.706 (0.003)	0.576 (0.003)	2.706 (0.003)	0.576 (0.003)
		5	2.602 (0.002)	0.464 (0.002)	2.602 (0.002)	0.464 (0.002)
		6	2.563 (0.001)	0.378 (0.001)	2.563 (0.001)	0.378 (0.001)
		7	2.535 (0.001)	0.324 (0.001)	2.535 (0.001)	0.324 (0.001)
		8	2.497 (0.001)	0.278 (0.001)	2.497 (0.001)	0.278 (0.001)
2.7	3	4	2.954 (0.003)	0.571 (0.003)	2.425 (0.003)	0.558 (0.003)
		5	2.806 (0.002)	0.418 (0.002)	2.401 (0.002)	0.426 (0.002)
		6	2.735 (0.001)	0.353 (0.001)	2.382 (0.001)	0.348 (0.001)
		7	2.696 (0.001)	0.321 (0.001)	2.353 (0.001)	0.306 (0.001)
		8	2.629 (0.001)	0.263 (0.001)	2.363 (0.001)	0.280 (0.001)
2.7	6	4	3.164 (0.002)	0.560 (0.002)	2.178 (0.002)	0.491 (0.002)
		5	2.974 (0.002)	0.411 (0.002)	2.150 (0.002)	0.394 (0.002)
		6	2.839 (0.001)	0.350 (0.001)	2.166 (0.001)	0.328 (0.001)
		7	2.765 (0.001)	0.293 (0.001)	2.178 (0.001)	0.292 (0.001)
		8	2.706 (0.001)	0.242 (0.001)	2.182 (0.001)	0.265 (0.001)
2.7	10	4	3.413 (0.002)	0.518 (0.002)	2.008 (0.002)	0.468 (0.002)
		5	3.115 (0.001)	0.393 (0.001)	1.992 (0.001)	0.376 (0.001)
		6	2.936 (0.001)	0.325 (0.001)	2.043 (0.001)	0.321 (0.001)
		7	2.837 (0.001)	0.281 (0.001)	2.057 (0.001)	0.291 (0.001)
		8	2.754 (0.001)	0.233 (0.001)	2.079 (0.001)	0.247 (0.001)



**Figure S1.** Backbone fractions of the anisotropic cases in which  $\kappa=\pm 10$ ,  $a=2.7$ , and  $r_{\min}/r_{\max}=0.1$  under the assumption of a constant crack aspect ratio. The solid-colored lines are the backbone fractions for the direction parallel to the symmetric axis of anisotropy, i.e., the z-axis, whereas the broken lines are those for the direction perpendicular to the symmetric axis. The horizontal axes of (a) and (b) are  $\rho'_3$ , whereas those of the other panels are  $\Delta\rho'_3(=\rho'_c-\rho'_{3c})$ . The solid black lines in (e) and (f) are the backbone fraction extrapolated for  $L/r_{\max}=\infty$ , which is based on Eq. (11) in the main text.