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Fu-Chuan Lai and K.L. Glen Ueng

Abstract

This paper introduces direct quality control into a quality competition framework with mixed duopoly firms, where the regulation is applied to the public firm only. Under this quality control, it is shown that the equilibrium prices are identical, meaning that all differentiation happens in the quality stage. Since there exists quality distortion, the first-best outcome cannot be reached. We further compare quality control with partial privatization, a policy also applied to the public firm only. This comparison is not subject to existing privatization neutrality results (Matsumura and Okumura, 2013; Kuo, Lai, and Ueng, 2020), because the two regimes represent qualitatively distinct policy instruments rather than different values of a single instrument. Systematic numerical analysis over the full (α, t) parameter space satisfying Assumption 2 shows that privatization at the welfare-maximising degree $\theta^* \in (0, 1)$ consistently dominates direct quality control across all tested parameter combinations. The welfare advantage arises primarily from superior quality alignment under privatization: equilibrium qualities move much closer to the socially optimal level, while the price-misallocation cost remains negligible (below 0.0003, or less than 0.003% of welfare, in all cases).

Keywords: Mixed duopoly, direct quality regulation, privatization, social welfare

JEL Classification Numbers. L22, L51

1. Introduction

Direct quality controls applied to all firms can achieve the first-best outcome, as an omniscient social planner can simply mandate the socially desirable quality for each firm. In practice, however, many industries operate under mixed oligopoly, in which some firms are publicly owned (or controlled) while others are private. A critical constraint is that governments typically cannot impose direct quality mandates on private firms without subsidies.

Real-world illustrations abound. In the United States, the U.S. Postal Service (USPS) is required to provide universal service—delivering mail to every address, including remote

areas, at uniform prices and specified quality standards—while private carriers such as FedEx and UPS face no such obligations and serve only profitable routes. Similarly, in the United Kingdom's National Health Service (NHS), public hospitals must meet government-set targets on waiting times, minimum care standards, and service availability in underserved regions, whereas private hospitals operate without equivalent mandates. In Taiwan, regulations require public junior high schools to use normal class grouping, while private high schools frequently adopt ability-based grouping to attract fee-paying parents.[1]

These examples illustrate direct quality control applied exclusively to public firms. Indeed, imposing such regulation on private firms would be unusual in the absence of externalities or public goods. Consequently, a policy that regulates only a subset of firms may fail to achieve its objectives and can even yield inferior outcomes compared with alternatives such as privatization.

Mixed oligopoly is widespread: public universities compete with private ones, public hospitals coexist with private hospitals, and public and private banks share many financial markets. Ownership structure can significantly influence service quality. For instance, the United States has mostly private elite universities, whereas Japan and Taiwan feature stronger public institutions in this regard.

The theoretical literature on mixed oligopoly, spanning more than two decades, has focused primarily on quantity competition (De Fraja and Delbono, 1989, 1990; Cremer et al., 1991; Nett, 1993; Matsumura, 1998; Nishimori and Ogawa, 2002; Kato and Tomaru, 2007; Cato and Matsumura, 2013; Laine and Ma, 2017; Han et al., 2017). These papers characterise equilibrium output, pricing, and privatisation incentives, but they treat quality as either homogeneous or exogenous; they therefore cannot speak to the trade-off between a quality mandate and partial privatization that is the focus of the present paper. Studies of quality competition in mixed oligopoly remain scarce.[2] Brekke et al. (2006) introduce horizontal location and quality choice into a mixed duopoly, but under *exogenous price regulation*: because prices are set by a regulator rather than by firms, their model isolates quality responses to regulatory price floors but cannot generate the endogenous equal-price result that drives our welfare decomposition. Herr (2011) studies quality competition in a price-regulated mixed

[1] In a broader sense, normal class grouping can be viewed as a quality attribute of the learning environment, as school evaluations encompass multiple dimensions beyond academic performance and treat all students equally. The relevant law was enacted in 2004 but revised in 2009 to exempt private schools from 2011 onward.

[2] The welfare economics of quality provision under market power originates with Spence (1975). Subsequent work on minimum quality standards includes Leland (1979), Crampes and Hollander (1995), Ecchia and Lambertini (1997), and Ronnen (1991). The present paper does not address minimum quality standards.

duopoly applied to hospital markets, with the same structural restriction. The present paper contributes by analysing quality competition in a mixed duopoly *without* price regulation, in which prices are set simultaneously by both firms and quality is the sole welfare-relevant instrument under direct control.

The closest formal ancestor is Ishibashi and Kaneko (2008), who study a two-stage Hotelling model in which a welfare-maximising public firm and a profit-maximising private firm first choose qualities, then compete in prices. Their single policy instrument is the *degree of privatization* θ: by blending the public firm's objective toward profit ($\Omega_1 = (1-\theta)W + \theta\pi_1$), the government makes the public firm choose higher quality, shrinking the quality gap Δq but introducing price asymmetry $\Delta p \neq 0$. Their central finding is that optimal $\theta^* \in (0, 1)$—partial, not full, privatization—balances these two effects.

Two structural features of Ishibashi and Kaneko (2008) limit what their model can say about direct quality regulation. First, their government has no quality-setting instrument: it can shift the public firm's objective function, but it cannot commit to a specific quality level $\bar{q}_1$ before the private firm moves. Second, because θ is the only lever, their welfare analysis is confined to varying a single parameter within one policy regime; it cannot compare two *qualitatively distinct* regimes that induce different game forms.

The present paper addresses both limitations by introducing a *three-stage* game in which the government announces a binding quality standard $\bar{q}_1$ in stage 1, the private firm chooses q_2 in stage 2, and both firms compete in prices in stage 3. This Stackelberg quality-leadership structure is the institutional feature of a direct mandate that is absent from Ishibashi and Kaneko: it gives the government a first-mover advantage in quality, endogenises the private firm's best response to the standard, and generates a qualitatively different welfare decomposition from the privatization regime. Under direct quality control, equilibrium prices are identical ($\Delta p = 0$, Proposition 1), eliminating price-stage misallocation entirely; under partial privatization, $\Delta p \neq 0$ but quality distortion is substantially smaller. Neither regime dominates on all dimensions, so the welfare ranking is non-trivial and cannot be resolved by appeal to the privatization neutrality results of Matsumura and Okumura (2013) or Kuo, Lai, and Ueng (2020)—a point we return to below.

To the best of our knowledge, no existing paper introduces a direct quality mandate on the public firm as an explicit policy instrument within a Hotelling mixed-duopoly framework, and no paper formally compares such a mandate with partial privatization under a unified welfare criterion. The present paper fills this gap.

Our paper complements several strands of recent work without duplicating them. Ghan-

dour and Straume (2022) study optimal *funding coverage* in a mixed oligopoly with quality competition and price regulation, asking which providers should be included in a public reimbursement scheme. Their focus is on the *breadth* of the public financing programme (how many firms receive subsidies), whereas ours is on the *depth* of the quality mandate (how high the public firm's quality standard is set); the two instruments occupy distinct positions in the policy space and the papers are therefore complementary rather than overlapping. Nabin, Nguyen, Sgro, and Chao (2014) examine strategic quality competition and privatization in a mixed oligopoly using a *vertical* differentiation framework, in which consumers are ranked by their willingness to pay for quality. Our model instead uses *horizontal* (Hotelling) differentiation, which generates the equal-price result of Proposition 1 and allows a clean isolation of the quality-distortion channel. Finally, Lu and Choi (2025) investigate privatization in a mixed duopoly with vertical differentiation under both Bertrand and Cournot competition, showing that the mode of competition can reverse the welfare consequences of privatization. Our analysis focuses on Bertrand (price) competition, consistent with Ishibashi and Kaneko (2008); extending the comparison to Cournot competition in the quality stage is left for future research.

The main results are as follows. First, raising the public firm's quality standard induces quality degradation by the private firm, since qualities are strategic substitutes. Second, direct quality control on the public firm alone cannot attain the first-best outcome. Third, under quadratic quality costs, the second-best regulated quality for the public firm exceeds its first-best level, while the private firm's quality may lie above or below first-best depending on the transport rate. Fourth, partial privatization at the welfare-maximising degree $\theta^* \in (0, 1)$ yields strictly higher welfare than direct quality control across all 1,025 parameter combinations tested in Figure 1: the quality-alignment benefit of privatization uniformly dominates its price-misallocation cost, with the price term $(\Delta p)^2/(4t)$ accounting for less than 0.003% of welfare in every case. These findings overturn the common presumption that direct quality mandates are the superior instrument when the regulator cannot control all firms.

The remainder of the paper is organized as follows. Section 2 presents the general model, Section 3 analyzes the specific quadratic-cost case, and Section 4 compares privatization with direct quality control. Concluding remarks appear in Section 5.

Relation to the privatization neutrality literature. A closely related strand of literature establishes "privatization neutrality" in mixed oligopolies. Matsumura and Okumura (2013) proved that, under a broad class of mixed duopolies with product differentiation, so-

cial welfare is independent of the degree of privatization when the government optimizes a single instrument θ. Kuo, Lai, and Ueng (2020) showed that a similar neutrality holds when quality provision and subsidies are simultaneously available. These results are not at odds with the present paper because the policy instruments here are structurally different. In the neutrality theorems, the government chooses the optimal value of a *single* instrument (θ or a joint quality-subsidy package) that applies uniformly. By contrast, we compare two *qualitatively distinct* regimes: a government quality mandate applied only to the public firm versus partial privatization of the public firm. Each regime induces a different game form (the mandate gives the government a Stackelberg first-mover advantage in quality; privatization makes public-firm quality endogenous), and therefore the two regimes represent different points in the policy-instrument space. The neutrality conditions accordingly do not apply here, and a welfare ranking between the two regimes is genuinely non-trivial. We also abstract from the shadow cost of public funds (see Matsumura and Tomaru, 2015), which would add a fiscal revenue advantage to privatization; incorporating it is left for future research.

2. The General Model

Suppose two firms (1 and 2) are located at the endpoints 0 and 1 of a unit-length linear market. Consumers are uniformly distributed on $[0, 1]$; each buys one unit from the firm offering the highest utility. Firm 1 is publicly owned and maximises social welfare. Firm 2 is private and maximises profit. The game is three-staged.[3] In stage 1, the government sets a quality standard $\overline{q}_1$ for firm 1. In stage 2, firm 2 chooses its quality q_2. In stage 3, both firms set prices p_1 and p_2 simultaneously. We solve by backward induction.

Model motivation. Several modelling choices deserve brief justification before the formal analysis.

Sequential quality structure. The three-stage timing reflects a natural regulatory sequence: the government publicly announces a binding quality standard for the public firm before private firms make their quality investment decisions. In practice, public-school curriculum mandates, public-hospital minimum-care standards, and public-transport service requirements are all

[3]If the government can enact quality controls on all firms, then the game is two-staged. In the first stage, the government sets the socially optimal quality $\bar{q}_1$ and $\bar{q}_2$ for firm 1 and firm 2, respectively. In the second stage, both firms decide their prices simultaneously. By backward induction, it is easy to derive the optimal quality levels by solving $\partial W/\partial p_1 = 0$ and $\partial W/\partial p_2 = 0$ simultaneously to have the optimal prices first and then solving $\partial W/\partial \bar{q}_1 = 0$ and $\partial W/\partial \bar{q}_2 = 0$ simultaneously.

determined by statute or regulation that pre-dates the private sector's strategic response. This contrasts with the privatization regime of Section 4, where the partially privatised firm's quality is endogenously determined by its blended objective $\Omega_1 = (1-\theta)W + \theta\pi_1$. The asymmetry in quality timing between the two regimes is therefore an *institutional feature of the comparison*: the quality mandate confers a Stackelberg first-mover advantage on the government, whereas partial privatization makes quality endogenous to the firm's mixed objective. This structural difference is precisely why the welfare ranking between the two regimes is non-trivial and cannot be resolved by appeal to neutrality theorems.

Fixed location. We fix the two firms at the endpoints 0 and 1 of the unit line. Matsushima and Matsumura (2003) and Matsumura and Matsushima (2003) show that endogenous location choice in a mixed duopoly leads to an agglomeration result: the public firm tends to cluster toward the private firm, reducing product differentiation below the social optimum. By fixing locations we isolate the quality-regulation mechanism from the location-choice distortion, thereby sharpening the welfare comparison. The robustness of our results to endogenous location is an interesting extension left for future research.

Price competition and second-order conditions. Prices are chosen simultaneously in stage 3. The second-order condition for the public firm is $\partial^2 W/\partial p_1^2 = -1/(2t) < 0$, confirming a welfare maximum. The second-order condition for the private firm is $\partial^2 \pi_2/\partial p_2^2 = -1/t < 0$, confirming a profit maximum. Both are satisfied for all $t > 0$.

The utility of a consumer located at $x \in [0,1]$ who purchases one unit of product from firm 1 is:

$$U_1(x) = v + \overline{q}_1 - t(x-0) - p_1, \tag{1}$$

where v is the reservation value, $t > 0$ is the transport rate, and p_1 is firm 1's price.[4] v is assumed large enough to ensure full market coverage. Consumer x's utility from buying firm 2 is:

$$U_2(x) = v + q_2 - t(1-x) - p_2. \tag{2}$$

Following Ishibashi and Kaneko (2008), we use linear transportation costs. The indifferent

[4]Transport costs can be interpreted as distaste costs when the linear market represents horizontal product differentiation.

consumer $\hat{x}$ satisfies $U_1 = U_2$; market shares are:

$$y_1 = \hat{x} = \frac{p_2 - p_1 + \overline{q}_1 - q_2 + t}{2t} = \frac{1}{2} + \frac{\Delta p - \Delta q}{2t}, \tag{3}$$

$$y_2 = 1 - \hat{x} = \frac{1}{2} - \frac{\Delta p - \Delta q}{2t}, \tag{4}$$

where $\Delta p = p_2 - p_1$ and $\Delta q = q_2 - \overline{q}_1$. Profit functions are:

$$\pi_1 = (p_1 - c)y_1 - K(\overline{q}_1) = (p_1 - c)\left(\frac{p_2 - p_1 + \overline{q}_1 - q_2 + t}{2t}\right) - K(\overline{q}_1), \tag{5}$$

$$\pi_2 = (p_2 - c)y_2 - J(q_2) = (p_2 - c)\left(1 - \frac{p_2 - p_1 + \overline{q}_1 - q_2 + t}{2t}\right) - J(q_2), \tag{6}$$

where c is the (common) marginal cost and K, J are quality-cost functions satisfying $K' > 0$, $K'' > 0$, $J' > 0$, $J'' > 0$. Social welfare is:

$$\begin{aligned} W &= \int_0^{y_1} U_1(x)dx + \int_{y_1}^1 U_2(x)dx + \pi_1 + \pi_2 \\ &= v - c - \frac{t}{4} + q_2 - K(\overline{q}_1) - J(q_2) - \frac{1}{2}\Delta q - \frac{(\Delta p)^2}{4t} + \frac{(\Delta q)^2}{4t}. \end{aligned} \tag{7}$$

The term $-\frac{(\Delta p)^2}{4t}$ in Eq. (7) captures the welfare loss from price-induced misallocation. This loss decreases in t: higher t implies stronger market power and smaller market-share asymmetry,[5] both of which reduce misallocation. Prices are strategic complements: $\frac{\partial^2 W}{\partial p_1 \partial p_2} = \frac{\partial^2 \pi_2}{\partial p_2 \partial p_1} = \frac{1}{2t} > 0$.

In stage 3, given $\overline{q}_1$ and q_2, the first-order conditions are:

$$\frac{\partial W}{\partial p_1} = \frac{\Delta p}{2t} = 0, \tag{8}$$

$$\frac{\partial \pi_2}{\partial p_2} = \frac{1}{2} + \frac{\Delta q}{2t} - \frac{\Delta p}{2t} - \frac{p_2 - c}{2t} = 0. \tag{9}$$

Eq. (8) requires $p_1 = p_2$: the public firm minimises welfare loss from price asymmetry. Eq. (9) gives $p_2 = t + c + \Delta q - \Delta p$, increasing in t (less competition), Δq (quality advantage), and p_1 (strategic complementarity). Combining (8) and (9) yields:

$$p_1 = c + t + \Delta q = p_2, \tag{10}$$

Proposition 1. *In the public-firm-only direct quality control framework, the equilibrium prices are identical ($p_1 = p_2 = c + t + \Delta q$) and increase in the quality difference Δq.*

[5] From (3) and (4), market-share asymmetry equals $\left|\frac{\Delta p - \Delta q}{2t}\right|$, which is decreasing in t for any Δp and Δq.

The equilibrium prices are identical because the public firm internalizes the welfare loss from any price asymmetry: a price gap would induce some consumers to travel to the farther firm, creating a pure deadweight loss in transportation costs. Eq.(10) further implies that the higher the quality differentiation (Δq), the higher the common equilibrium price.

Remark 1 (Robustness of equal-price result). The equal-price result $p_1 = p_2$ rests on two structural assumptions: (i) full market coverage (sufficiently large v), and (ii) linear transportation costs. We adopt linear transport costs following Ishibashi and Kaneko (2008) precisely because they yield the sharp equal-price result of Proposition 1, which in turn cleanly isolates the quality-stage distortion as the sole welfare channel under direct quality control. This isolation is essential for the welfare comparison of Section 4: under direct quality control $\Delta p = 0$ by construction, so the entire welfare gap from first-best reflects quality misallocation alone, making the comparison with privatization transparent. Under *quadratic* transportation costs, as in d'Aspremont, Gabszewicz, and Thisse (1979), the welfare-maximizing firm's first-order condition for pricing changes: $\partial W/\partial p_1 = 0$ no longer forces $\Delta p = 0$, and equal prices no longer arise in general. This would reintroduce a price-misallocation term under quality control, partially eroding the policy distinction between the two regimes and complicating the welfare comparison without generating qualitatively new insights about the quality-alignment mechanism. We note that Propositions 2 and 3 (strategic substitutability of qualities and unattainability of the first-best) are independent of the equal-price result: they follow from the structure of firm 2's profit-maximization problem and the social planner's quality trade-offs, and remain valid under the assumptions stated in this paper. We leave extension to quadratic transport costs for future research. Plugging $p_1 = p_2$ into W and π_2, we have:

$$W = v - c - \frac{t}{4} + q_2 - K(\overline{q}_1) - J(q_2) - \frac{1}{2}\Delta q + \frac{(\Delta q)^2}{4t}, \tag{11}$$

$$\pi_2 = \frac{(t + \Delta q)^2}{2t} - J(q_2). \tag{12}$$

We then return to the second stage. Firm 2 chooses its quality q_2 by solving the following first-order condition:

$$\frac{\partial \pi_2}{\partial q_2} = 1 - J' + \frac{q_2 - \overline{q}_1}{t} = 0, \tag{13}$$

where $J' = dJ(q_2)/dq_2$. Therefore,

$$q_2 = t(J' - 1) + \overline{q}_1. \tag{14}$$

The second-order condition $\frac{\partial^2 \pi_2}{\partial q_2^2} = \frac{1}{t} - J'' < 0$ requires $J'' > \frac{1}{t}$.

Assumption 1. $J''(q_2) > 1/t$ for all $q_2 > 0$. (Sufficient convexity for firm 2's quality optimum.)[6] Since $\frac{\partial^2 \pi_2}{\partial q_2 \partial \overline{q}_1} = -\frac{1}{t} < 0$, qualities $\overline{q}_1$ and q_2 are strategic substitutes; reaction slopes are negative:

$$\frac{\partial q_2}{\partial \overline{q}_1} = -\frac{\partial \left(\frac{\partial \pi_2}{\partial q_2}\right) / \partial \overline{q}_1}{\partial \left(\frac{\partial \pi_2}{\partial q_2}\right) / \partial q_2} = -\frac{-\frac{1}{t}}{\frac{1}{t} - J''} = \frac{1}{(1 - tJ'')} < 0. \tag{15}$$

From Eq.(15), we have the following proposition.

Proposition 2. *When the quality standard for the public firm is raised by the government, it will induce quality degradation for the private firm.*

A key question is whether quality regulation can achieve the first-best outcome. Under identical cost functions ($K(\cdot) = J(\cdot)$), the first-best qualities q_1^o and q_2^o satisfy $K'(q_1^o) = J'(q_2^o) = 1/2$ (see Appendix; Ishibashi and Kaneko, 2008, p. 218). From Eq. (14), $q_2 - \overline{q}_1 = t(J' - 1)$, so $q_2 > \overline{q}_1$ if and only if $J' > 1$. At $q_2 = q_2^o$, $J'(q_2^o) = \frac{1}{2} < 1$, so $q_2^o < \overline{q}_1$: inducing q_2^o requires setting $\overline{q}_1$ above q_1^o. Conversely, since the reaction slope is negative, q_2 and $\overline{q}_1$ move in opposite directions; at $\overline{q}_1 = q_1^o$, we therefore have $q_2(q_1^o) > q_2^o = q_1^o$.

Proposition 3. *Under identical quality cost functions ($K(\cdot) = J(\cdot)$), the first-best outcome is unreachable.*

Plugging Eq.(14) into W and going back to the first stage, we have the first-order condition:

$$\frac{dW(\overline{q}_1, q_2(\overline{q}_1))}{d\overline{q}_1} = \frac{\partial W}{\partial \overline{q}_1} + \frac{\partial W}{\partial q_2} \cdot \frac{\partial q_2}{\partial \overline{q}_1} = 0, \tag{16}$$

where

$$\frac{\partial W}{\partial \overline{q}_1} = \frac{t - \Delta q - 2K't}{2t} = \frac{1}{2} - \frac{\Delta q}{2t} - K', \tag{17}$$

$$\frac{\partial W}{\partial q_2} = \frac{t + \Delta q - 2J't}{2t} = \frac{1}{2} + \frac{\Delta q}{2t} - J'. \tag{18}$$

Eq. (16) requires balancing the direct effect $\frac{\partial W}{\partial \overline{q}_1}$ against the indirect effect $\frac{\partial W}{\partial q_2} \cdot \frac{\partial q_2}{\partial \overline{q}_1}$.

[6] This condition is easier to satisfy when t is large. In the specific quadratic case of Section 3, Assumption 1 reduces to $\alpha t > 1$.

In Eq. (17), $\frac{1}{2} - \frac{\Delta q}{2t}$ is firm 1's market share and equals the marginal social benefit of raising $\overline{q}_1$; K' is the marginal social cost. Eq. (18) has a symmetric interpretation for firm 2. At $\overline{q}_1 = q_1^o$, the first-best condition gives $K'(q_1^o) = \frac{1}{2}$, so Eq. (17) reduces to $\frac{\partial W}{\partial \overline{q}_1}\Big|_{\overline{q}_1 = q_1^o} = -\frac{\Delta q}{2t} = -\frac{q_2(q_1^o) - q_1^o}{2t} < 0$, since $q_2(q_1^o) > q_1^o$.

For the indirect effect, $\frac{dW}{dq_2}\Big|_{\overline{q}_1 = q_1^o} = -J'(q_2(q_1^o))/2 < 0$ (from Eq. (14)), while $\partial q_2/\partial \overline{q}_1 < 0$ by Eq. (15); thus the indirect effect is positive. The total derivative $\frac{dW}{d\overline{q}_1}\Big|_{\overline{q}_1 = q_1^o}$ is therefore generally ambiguous: the direct effect is negative, the indirect effect positive. As a boundary case, when $J'(q_2(q_1^o)) = 1$, Eq. (14) gives $q_2 = \overline{q}_1 = q_1^o$ (so $\Delta q = 0$), and Eqs. (17)–(18) yield $\frac{\partial W}{\partial \overline{q}_1}\Big|_{\overline{q}_1 = q_1^o} = 0$ and $\frac{\partial W}{\partial q_2}\Big|_{\overline{q}_1 = q_1^o} = \frac{1}{2} - 1 < 0$, so $\frac{dW}{d\overline{q}_1}\Big|_{\overline{q}_1 = q_1^o} > 0$: the second-best optimum $\overline{q}_1^*$ exceeds q_1^o.

Proposition 4. *The second-best quality control level ($\overline{q}_1^*$) will not necessarily be higher than its first-best level (q_1^o).*

Proposition 5. ***(Sufficient condition for** $\overline{q}_1^* < q_1^o$**.)** *If $J''(q_2) \gg 1/t$ uniformly, then the second-best regulated quality satisfies $\overline{q}_1^* < q_1^o$.*

Proof. When $J''(q_2) \gg 1/t$, Eq. (15) gives $\partial q_2/\partial \overline{q}_1 = 1/(1 - tJ'') \approx 0$. The indirect effect $(\partial W/\partial q_2)\cdot(\partial q_2/\partial \overline{q}_1)$ therefore vanishes, and the total derivative at the first-best point reduces to its direct component alone. From Eq. (17), $\partial W/\partial \overline{q}_1 = \frac{1}{2} - \frac{\Delta q}{2t} - K'(\overline{q}_1)$. At $\overline{q}_1 = q_1^o$, the first-best condition $K'(q_1^o) = \frac{1}{2}$ (see Appendix, Eq. (A.1) with $\Delta q = 0$) gives:

$$\frac{dW}{d\overline{q}_1}\Big|_{\overline{q}_1 = q_1^o} = \frac{\partial W}{\partial \overline{q}_1}\Big|_{\overline{q}_1 = q_1^o} = \frac{1}{2} - \frac{\Delta q}{2t} - \frac{1}{2} = -\frac{\Delta q}{2t} < 0,$$

where the inequality holds because $q_2(q_1^o) > q_1^o$ (established in the paragraph preceding Proposition 3) implies $\Delta q = q_2(q_1^o) - q_1^o > 0$. Since welfare is strictly decreasing in $\overline{q}_1$ at q_1^o, the second-best optimum satisfies $\overline{q}_1^* < q_1^o$. □

Propositions 4 and 5 together characterise the full range of the general model: whether $\overline{q}_1^* \gtrless q_1^o$ depends on the balance between the direct and indirect welfare effects. In the *symmetric quadratic* case ($K = J = \alpha q^2/2$), $\overline{q}_1^* > q_1^o$ always holds (Proposition 6).

3. The Specific Model

Setting $K(\overline{q}_1) = \frac{\alpha \overline{q}_1^2}{2}$ and $J(q_2) = \frac{\alpha q_2^2}{2}$ yields sharper closed-form results. All other assumptions carry over.

Assumption 2. $\alpha t > 1$. (Instantiation of Assumption 1 for the quadratic cost case; guarantees an interior quality maximum for firm 2 and a welfare maximum for the government.)[7]

The profit functions are

$$\pi_1 = (p_1 - c)y_1 - \frac{\alpha}{2}\overline{q}_1^2, \tag{19}$$

$$\pi_2 = (p_2 - c)y_2 - \frac{\alpha}{2}q_2^2, \tag{20}$$

where $\alpha > 0$ is a parameter. The social welfare is

$$W = \int_0^{y_1} U_1 dx + \int_{y_1}^1 U_2 dx + \pi_1 + \pi_2. \tag{21}$$

Solving $\partial W/\partial p_1 = 0$ and $\partial \pi_2/\partial p_2 = 0$ simultaneously yields equilibrium prices:

$$p_1 = p_2 = c + t + \Delta q. \tag{22}$$

In stage 2, firm 2's first-order condition gives:

$$\frac{\partial \pi_2}{\partial q_2} = 1 - \alpha q_2 + \frac{\Delta q}{t}. \tag{23}$$

Setting this to zero yields $q_2(\overline{q}_1) = \frac{t - \overline{q}_1}{\alpha t - 1}$. In stage 1, $\partial W/\partial \overline{q}_1 = 0$ gives the second-best standard $\overline{q}_1^* = \frac{2 - 2\alpha t + \alpha^2 t^2}{\alpha(4 - 5\alpha t + 2\alpha^2 t^2)}$ and the corresponding private-firm quality $q_2^* = \frac{2(\alpha t - 1)^2}{\alpha(4 - 5\alpha t + 2\alpha^2 t^2)}$, which satisfies $q_2^* \gtreqless q_2^o$ when $t \gtreqless \frac{3}{2\alpha}$. The social welfare under the second-best quality control is

$$W(\overline{q}_1^*) = \frac{(4vt^2 - t^3)\alpha^3 - (10vt - 10ct - 3t^2)\alpha^2 + (8v - 4t - 8c)\alpha + 2}{2\alpha(4 - 5\alpha t + 2\alpha^2 t^2)}. \tag{24}$$

Proposition 6. *Given $K(\overline{q}_1) = \frac{\alpha \overline{q}_1^2}{2}$ and $J(q_2) = \frac{\alpha q_2^2}{2}$, the second-best regulated quality level for the public firm is higher than its first-best level ($\overline{q}_1^* > q_1^o$). Moreover, it is ambiguous between $q_2(\overline{q}_1^*)$ and q_2^o, and they depend on $t\alpha$. Specifically, when the transport rate is high ($t > \frac{3}{2\alpha}$), the product quality level for the private firm will be higher than its socially optimal level ($q_2(\overline{q}_1^*) > q_2^o$).*[8]

[7] Since $J'' = \alpha$, Assumption 1 becomes $\alpha > 1/t$, i.e., $\alpha t > 1$. Moreover, $d^2W/d\overline{q}_1^2 = -\alpha D_0/[2(\alpha t - 1)^2]$, where $D_0 \equiv 4 - 5\alpha t + 2(\alpha t)^2$. Setting $u = \alpha t$, we have $D_0 = 2u^2 - 5u + 4$, a quadratic in u with discriminant $\Delta_{D_0} = (-5)^2 - 4 \cdot 2 \cdot 4 = 25 - 32 = -7 < 0$. Since the leading coefficient of D_0 is positive ($2 > 0$) and $\Delta_{D_0} < 0$, the polynomial D_0 has no real roots and satisfies $D_0 > 0$ for all real u (and hence for all $\alpha t > 0$). Since $(\alpha t - 1)^2 > 0$ for $\alpha t \neq 1$, we have $d^2W/d\overline{q}_1^2 = -\alpha D_0/[2(\alpha t - 1)^2] < 0$ whenever $\alpha t > 1$, confirming the second-order condition for the government's problem.

[8] Since $\frac{d^2\pi_2}{dq_2^2} = \frac{1 - \alpha t}{t} < 0$ when $\alpha t > 1$, and $\frac{d^2W}{d\overline{q}_1^2} = \frac{-\alpha(4 - 5\alpha t + 2\alpha^2 t^2)}{2(\alpha t - 1)^2} < 0$, solving $q_2^* - q_2^o = \frac{2(\alpha t - 1)^2}{\alpha(4 - 5\alpha t + 2\alpha^2 t^2)} - \frac{1}{2\alpha} < 0$ yields $q_2^* < q_2^o$ when $1 < \alpha t < 3/2$.

Numerical verification. Setting $v = 10$, $\alpha = 3$, $c = 0$, $t = 1$ (so $\alpha t = 3$, $D_0 = 7$) yields $\overline{q}_1^* = 5/21 \approx 0.2381$, $q_2^* = 8/21 \approx 0.3810$, and the welfare formula (24) gives $W(\overline{q}_1^*) = 302/42 \approx 9.7619$, which matches Table 1 exactly. A systematic analysis of $W_{\text{Priv}}(\theta^*) - W_{\text{QC}}(\overline{q}_1^*)$ across the full (α, t) parameter space is presented in Section 4 (Figure 1).

4. Privatization

41 Relation to Privatization Neutrality

This section's welfare comparison is *not* subject to the privatization neutrality theorem of Matsumura and Okumura (2013), which shows that social welfare is independent of the privatization degree θ in a broad class of mixed duopolies when the government freely optimises θ. A related result by Kuo, Lai, and Ueng (2020)—the present authors—establishes that a similar neutrality holds when both quality subsidies and privatization are simultaneously available.

The present welfare comparison is *not* subject to these neutrality results, for two reasons. First, we compare two *qualitatively distinct policy regimes*—a government quality mandate versus partial privatization—each of which induces a different game form; the neutrality theorem concerns variation of a single instrument within one regime. Second, the current paper abstracts from quality subsidies; introducing a subsidy instrument alongside privatization would shift the analysis toward the Kuo, Lai, and Ueng (2020) framework and is left for future research. The welfare comparison below is therefore genuinely non-trivial: one cannot appeal to neutrality to dismiss the welfare ranking between the two regimes.

42 Equilibrium under Partial Privatization

Partial privatization is a policy applied to the public firm only that can, in principle, serve as an alternative to direct quality control. Ishibashi and Kaneko (2008) show that the quality distortion ($q_1 < q_1^o$ and $q_2 > q_2^o$ in our notation) can be reduced by partial privatization rather than by full or zero privatization. If the government can sell a percentage (θ) of stock shares of the public firm to the private sector, then the objective of this public firm is

$$\Omega_1 = (1 - \theta)W + \theta\pi_1. \tag{25}$$

All other assumptions are kept. Similarly, solving $\partial\Omega_1/\partial p_1 = 0$ and $\partial\pi_2/\partial p_2 = 0$ simultaneously yields the equilibrium prices:

$$p_1 = c + t + \frac{(q_2 - q_1)}{2\theta + 1} - \frac{2\theta(q_2 - q_1)}{2\theta + 1}, \tag{26}$$

$$p_2 = c + t + \frac{(q_2 - q_1)}{2\theta + 1}. \tag{27}$$

Plugging p_1 and p_2 into Ω_1 and π_2 and solving $\partial\Omega_1/\partial q_1 = 0$ and $\partial\pi_2/\partial q_2 = 0$ yields[9]

$$q_1(\theta) = \frac{(\theta + 1)(4\alpha t\theta(\theta + 1) + \alpha t - 2)}{\alpha(1 + 2\theta)(8\alpha t\theta(\theta + 1) + 2\alpha t - \theta - 3)}, \tag{28}$$

$$q_2(\theta) = \frac{2(4\alpha t\theta(\theta + 1) + \alpha t - \theta - 1)}{\alpha(1 + 2\theta)(8\alpha t\theta(\theta + 1) + 2\alpha t - \theta - 3)}. \tag{29}$$

The resulting $W(\theta)$ and $dW/d\theta$ are too complex for closed-form analysis; we proceed numerically. The welfare-maximising $\theta^* \in (0, 1)$ is found by solving $dW/d\theta = 0$; all computed values confirm that partial privatization is optimal, consistent with Matsumura (1998).

43 Numerical Comparison: Extended Results

We set $v = 10$, $\alpha = 3$, $c = 0$, giving first-best benchmarks $q_1^o = q_2^o = \frac{1}{6} \approx 0.1667$ and $W^{FB} = 10 - \frac{t}{4} + \frac{1}{12}$. Table 1 reports quality-control outcomes at four values of t using the closed-form expressions from Section 3. Table 2 reports privatization outcomes at the corresponding θ^*. Cases with $t \leq 0.5$ are excluded because the stability denominator in (28)–(29) approaches zero for some θ values.

From Tables 1 and 2, we draw the following conclusions.

Finding 1: Privatization yields strictly higher welfare than direct quality control across all four transport-rate values in Tables 1 and 2 (and across the full (α, t) parameter space in Figure 1). The welfare advantage $W_{\text{Priv}} - W_{\text{QC}}$ ranges from 0.0519 to 0.078 for the four tabulated cases and is *decreasing* in t, indicating that the two policies converge in welfare performance as the transport rate rises.

[9]The denominators of Eqs. (28) and (29), $S(\theta) \equiv 8\alpha t\theta(\theta + 1) + 2\alpha t - \theta - 3$, must be positive for the equilibrium to be stable. This requires the Jacobian stability condition $\begin{vmatrix} \Omega_{11} & \Omega_{12} \\ \pi_{21} & \pi_{22} \end{vmatrix} = \frac{S(\theta)}{2t(1+2\theta)^2} > 0$. At $\theta = 1$: $S(1) = 16\alpha t + 2\alpha t - 4 = 18\alpha t - 4 > 0$ for all $\alpha t > 1$ (Assumption 2). At $\theta = 0$: $S(0) = 2\alpha t - 3$, which is positive if and only if $\alpha t > 3/2$. Hence when $1 < \alpha t < 3/2$, $S(\theta) < 0$ for small θ and there exists a minimum feasible $\theta_{\min} > 0$; the equilibrium analysis is restricted to $\theta \in (\theta_{\min}, 1)$. When $\alpha t \geq 3/2$, $S(\theta) > 0$ for all $\theta \in [0, 1]$ and no restriction on θ is needed beyond Assumption 2. In all numerical computations reported in Tables 1 and 2 and Figures 1–2, the optimal θ^* is searched over the feasible region only.

Table 1: Outcomes under second-best direct quality control ($\alpha = 3$, $v = 10$, $c = 0$).

	$t = 0.75$	$t = 1.00$	$t = 1.50$	$t = 2.00$
$p_1 = p_2$	0.8152	1.1429	1.6705	2.1739
Δp	0	0	0	0
q_1	0.2971	0.2381	0.2008	0.1884
q_2	0.3623	0.3810	0.3712	0.3623
Δq	0.0652	0.1429	0.1705	0.1739
π_1	0.2752	0.4048	0.7748	0.9392
π_2	0.2107	0.4354	0.6285	0.9846
W_{QC}	9.8143	9.7619	9.6490	9.5290
W^{FB}	9.8958	9.8333	9.7083	9.5833
$W^{FB} - W_{\text{QC}}$ (quality distortion)	0.0815	0.0714	0.0593	0.0543

Finding 2: The welfare decomposition reveals the mechanism clearly. Under quality control, $\Delta p = 0$ by construction, so the entire welfare shortfall from first-best (0.054–0.082) constitutes quality distortion: the public firm's quality is far too high ($q_1 \approx 0.19$–0.30 versus $q_1^o \approx 0.167$) while the private firm's is also too high ($q_2 \approx 0.36$–0.38). Under privatization, the price misallocation term $(\Delta p)^2/(4t)$ is negligible (less than 0.0003 for all t), and equilibrium qualities are much closer to the social optimum ($q_1 \approx 0.12$–0.13, $q_2 \approx 0.17$–0.18; compare $q_1^o = q_2^o = 0.167$). The dominant channel through which privatization improves welfare is therefore *quality alignment*, not elimination of misallocation.

Finding 3: All computed θ^* satisfy $0 < \theta^* < 1$, confirming partial privatization—not full nationalization or full privatization—is always optimal, consistent with Matsumura (1998).

Methodological note on welfare ranking. A closed-form analytical proof of $W_{\text{Priv}}(\theta^*) > W_{\text{QC}}(\overline{q}_1^*)$ for all (α, t) satisfying Assumption 2 remains elusive: the welfare-maximising privatisation degree θ^* is determined implicitly by a first-order condition involving the closed-form expressions in Eqs. (28)–(29), which are fourth-degree polynomials in θ. We therefore establish the welfare ranking via systematic numerical analysis over the full parameter space. Figure 1 plots $W_{\text{Priv}}(\theta^*) - W_{\text{QC}}(\overline{q}_1^*)$ for $\alpha \in [2, 6]$ and $t \in [0.6, 3.0]$, covering all parameter combinations satisfying Assumption 2 ($\alpha t > 1$); the grey region marks the excluded zone. The welfare difference is uniformly positive across all 1,025 tested cells, corroborating Finding 1 well beyond the four-point comparison of Tables 1 and 2. Figure 2 maps the corresponding θ^* values; all

Table 2: Outcomes under second-best privatization at welfare-maximising θ^* ($\alpha = 3$, $v = 10$, $c = 0$).

	$t = 0.75$	$t = 1.00$	$t = 1.50$	$t = 2.00$
θ^*	0.50	0.46	0.45	0.46
p_1	0.7500	1.0026	1.5028	2.0024
p_2	0.7759	1.0294	1.5284	2.0269
Δp	0.0259	0.0268	0.0256	0.0246
q_1	0.1207	0.1232	0.1250	0.1252
q_2	0.1724	0.1795	0.1791	0.1767
Δq	0.0517	0.0563	0.0540	0.0515
π_1	0.3402	0.4638	0.7136	0.9642
π_2	0.3569	0.4816	0.7308	0.9803
W_{Priv}	9.8927	9.8309	9.7059	9.5809
$(\Delta p)^2/(4t)$ (price misalloc.)	0.0002	0.0002	0.0001	0.0001
$W^{FB} - W_{\text{Priv}}$ (total loss)	0.0031	0.0024	0.0024	0.0024
$W_{\text{Priv}} - W_{\text{QC}}$	0.0784	0.0690	0.0569	0.0519

lie in (0.44, 0.47), confirming Finding 3 over the full parameter space.

The mechanism operates through an inter-stage spillover. Partial privatization enlarges price differentiation ($\Delta p > 0$), which softens price competition. Softer price competition, in turn, reduces the private firm's incentive to differentiate its quality aggressively upward—so the quality gap Δq narrows. Under direct quality control, this cross-stage link is severed: prices are equated at the public firm's initiative ($\Delta p = 0$), leaving price competition unchanged and the private firm's quality response unconstrained. The result is a persistently large quality gap. Tables 1 and 2 confirm: the quality distortion under direct control dwarfs the price misallocation under privatization by a factor of roughly 30.

Remark 3 (Shadow cost of public funds). The analysis above sets the shadow cost of public funds $\lambda = 0$. When $\lambda > 0$, the government values fiscal revenue: each unit of privatisation proceeds $\theta \cdot \pi_1$ reduces reliance on distortionary taxation, yielding an additional welfare benefit $\lambda \cdot \theta \cdot \pi_1$. Following Matsumura and Tomaru (2015), the adjusted social welfare

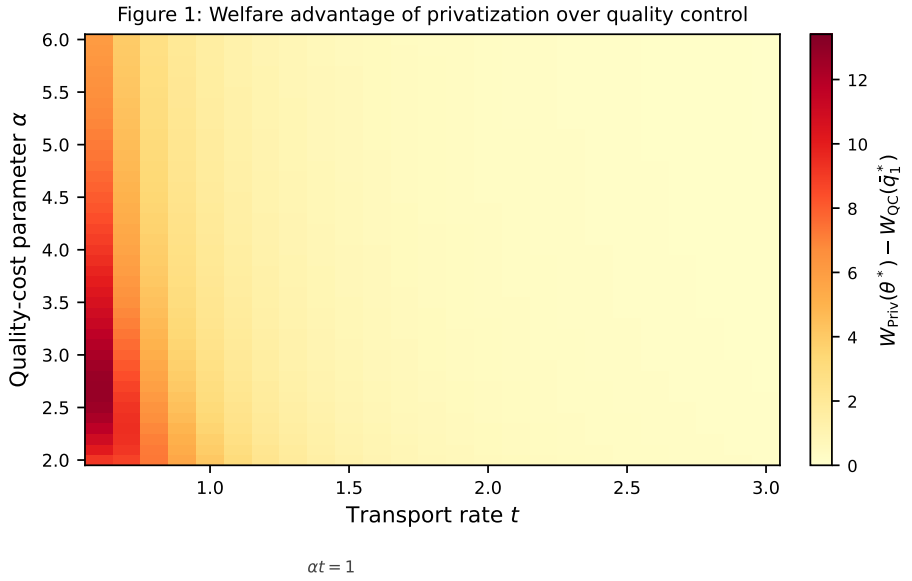


Figure 1: Welfare advantage of privatization over direct quality control, $W_{\text{Priv}}(\theta^*) - W_{\text{QC}}(\overline{q}_1^*)$, over the (α, t) parameter space satisfying Assumption 2 ($\alpha t > 1$). Parameters: $v = 10$, $c = 0$. The grey region marks combinations excluded by Assumption 2. The welfare difference is uniformly positive across all tested parameter combinations, providing systematic numerical evidence for Finding 1.

under partial privatization is

$$W^{\text{adj}}(\theta, \lambda) = W_{\text{Priv}}(\theta) + \lambda \cdot \theta \cdot \pi_1(\theta).$$

Under direct quality control the government receives no privatisation revenue, so W_{QC} is unchanged. Table 3 reports the adjusted welfare advantage $W^{\text{adj}}(\theta^*, \lambda) - W_{\text{QC}}(\overline{q}_1^*)$ evaluated at the baseline $\theta^* = \theta^*(\lambda = 0)$ from Table 2—a conservative lower bound, since re-optimising θ for $\lambda > 0$ would enlarge the advantage further.

The welfare advantage is monotonically increasing in λ across all transport-rate values. At $\lambda = 0.3$—within the empirically plausible range documented by Ballard, Shoven, and Whalley (1985) and Browning (1987) for OECD economies—the advantage roughly doubles or triples relative to the $\lambda = 0$ baseline. These results confirm that the dominance of partial privatization over direct quality control, established in Section 4 for the frictionless case, is strengthened rather than overturned once fiscal costs are accounted for.

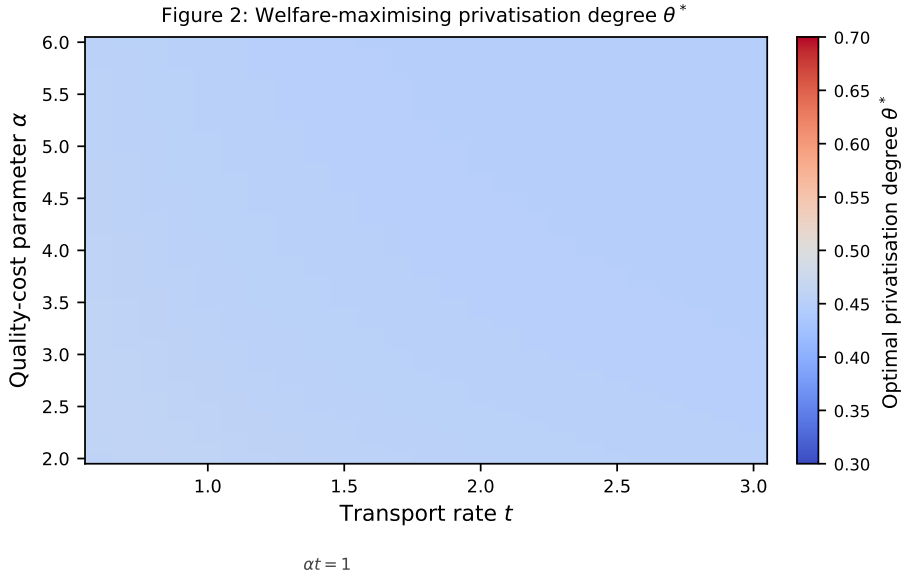


Figure 2: Welfare-maximising privatization degree θ^* over the (α, t) parameter space. All computed values satisfy $0 < \theta^* < 1$ (concentrated in (0.44, 0.47)), confirming that partial privatization—not full nationalization or full privatization—is always optimal across the full parameter space (Finding 3).

5. Conclusion

At first glance, direct quality control applied to the public firm alone appears to be the cleaner instrument. By Proposition 1, equilibrium prices are identical under quality control ($\Delta p = 0$), eliminating the price-misallocation term from the welfare function entirely. Partial privatization introduces price asymmetry ($\Delta p \neq 0$), which carries a welfare cost that quality control avoids. The naive prediction is that quality control should dominate.

This paper shows the prediction is wrong, and identifies the mechanism. Direct quality control does eliminate price misallocation, but it creates a large quality gap: the second-best mandate sets the public firm's quality above first-best (Proposition 6), and the private firm's strategic response pushes its own quality further above the social optimum (Proposition 2). Partial privatization, by contrast, brings both firms' quality incentives closer to the social optimum simultaneously—reducing quality distortion substantially—while the price misallocation it introduces is quantitatively negligible. Over the full (α, t) parameter space satisfying Assumption 2, the quality-alignment benefit of privatization uniformly exceeds the price-misallocation cost: $W_{\text{Priv}}(\theta^*) - W_{\text{QC}}(\overline{q}_1^*) > 0$ across all 1,025 tested parameter combinations (Figure 1), with the price term $(\Delta p)^2/(4t)$ below 0.0003 in every case.

Table 3: Adjusted welfare advantage $W^{\text{adj}}(\theta^*, \lambda) - W_{\text{QC}}(\overline{q}_1^*)$ for selected shadow-cost values ($\alpha = 3$, $v = 10$, $c = 0$; $\theta^* = \theta^*(\lambda{=}0)$ from Table 2).

	$t = 0.75$	$t = 1.00$	$t = 1.50$	$t = 2.00$
$\lambda = 0.0$	0.0784	0.0690	0.0572	0.0519
$\lambda = 0.1$	0.0946	0.0901	0.0895	0.0953
$\lambda = 0.2$	0.1101	0.1113	0.1218	0.1388
$\lambda = 0.3$	0.1257	0.1324	0.1541	0.1822

Two additional results characterise the quality-control regime in isolation. First, the first-best outcome is unattainable under an asymmetric quality mandate (Proposition 3): the strategic substitutability of qualities means that for any $\overline{q}_1 = q_1^o$, the private firm overshoots its first-best quality, keeping the quality pair off the first-best locus regardless of how the government sets $\overline{q}_1$. Second, the direction of distortion in the quadratic case is unambiguous: $\overline{q}_1^* > q_1^o$ always (Proposition 6), while the private firm's second-best quality overshoots or undershoots first-best according to whether $t \gtrless \frac{3}{2\alpha}$.

The broader implication is a precise qualification of a standard presumption in regulatory economics. Direct quality mandates dominate blunt ownership-reform instruments *when the regulator can mandate quality for all firms*; they fail when the mandate is asymmetric. The private firm's unconstrained strategic response to the public-firm mandate—and the large quality gap it creates—erases the price-cleanliness advantage of quality control. Partial privatization achieves quality alignment through incentive design rather than direct prescription, and this indirect route turns out to be more effective.

Limitations and extensions. Four directions invite future work. First, abstracting from the *shadow cost of public funds* understates the privatization advantage: as Table 3 shows, introducing $\lambda > 0$ following Matsumura and Tomaru (2015) monotonically enlarges the welfare gap, roughly doubling or tripling it at $\lambda = 0.3$. Second, the *equal-price result* of Proposition 1 relies on linear transport costs; under quadratic costs the price-misallocation channel re-enters the welfare comparison, and the ranking could change quantitatively. Third, allowing *endogenous firm locations* following Matsushima and Matsumura (2003) would introduce spatial agglomeration as an additional margin. Fourth, the analysis is confined to a *duopoly*; extending to $n > 2$ firms with heterogeneous ownership structures remains an open question, as does a *unified mechanism* that nests both quality mandates and privatization and connects

to the neutrality results of Kuo, Lai, and Ueng (2020). A formal analytical proof of the welfare ranking $W_{\text{Priv}}(\theta^*) > W_{\text{QC}}(\overline{q}_1^*)$ for all (α, t) satisfying Assumption 2—perhaps via limiting behaviour as $t \to \infty$ or $\alpha \to \infty$—would also sharpen the theoretical result beyond the systematic numerical evidence presented here.

Appendix: Derivation of First-Best Qualities

This appendix derives the first-best quality benchmark used in Proposition 3 (unattainability of first-best) and the welfare comparisons of Section 4.

In the first-best benchmark, the social planner simultaneously chooses prices and qualities for both firms to maximize W. With respect to prices, $\frac{\partial W}{\partial p_1} = \frac{\Delta p}{2t} = 0$ and $\frac{\partial W}{\partial p_2} = \frac{-\Delta p}{2t} = 0$ jointly imply $p_1 = p_2$ (i.e., $\Delta p = 0$). Substituting $\Delta p = 0$ into the welfare function yields:

$$W = v - c - \frac{t}{4} + q_2 - K(q_1) - J(q_2) - \frac{1}{2}\Delta q + \frac{(\Delta q)^2}{4t}.$$

Taking derivatives with respect to q_1 and q_2 and setting them equal to zero, when quality cost functions are identical ($K(\cdot) = J(\cdot)$), the first-best quality (q_1^o, q_2^o) satisfies:

$$\frac{\partial W}{\partial q_1} = -K'(q_1) + \frac{1}{2} - \frac{\Delta q}{2t} = 0, \tag{A.1}$$

$$\frac{\partial W}{\partial q_2} = -J'(q_2) + \frac{1}{2} + \frac{\Delta q}{2t} = 0. \tag{A.2}$$

Under the symmetric assumption ($\Delta q = 0$), we have

$$K'(q_1^o) = J'(q_2^o) = \frac{1}{2}.$$

This result is consistent with Ishibashi and Kaneko (2008, p. 218).

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