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A Dual Hesitant Fuzzy Optimization Approach for Solving the Multi-Objective Multi-Item Just-in-Time Transportation Problem

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Abstract

This paper presents a study on the multi-objective multi-item Just-in-Time transportation problem (MJTP) under a dual hesitant fuzzy environment. In real-world transportation systems, uncertainty and ambiguity often arise in cost, profit, and delivery parameters; therefore, dual hesitant fuzzy numbers are employed to represent such imprecise information effectively. The Just-in-Time concept is incorporated to ensure that goods are delivered to destinations within the scheduled time, thereby reducing inventory holding costs and improving system efficiency. A mathematical model of the MJTP is formulated considering multiple conflicting objectives, including minimization of transportation cost, maximization of profit, and adherence to delivery schedules. To obtain a compromise solution, a goal programming approach is utilized to transform the multi-objective problem into a solvable form under the fuzzy environment. A numerical example is provided to demonstrate the applicability and effectiveness of the proposed model and solution procedure. The results confirm that the developed approach can handle uncertainty efficiently while satisfying time-dependent delivery requirements. Finally, conclusions and potential directions for future research are discussed.

Keywords: Just-in-time transportation problem; Multi-objective Just-in-time transportation problem; Mathematical model; $B(\lambda)$-problem; Multi-objective Just-in-time solution.

1 Introduction

Transportation is a fundamental component of modern logistics and supply chain management, and it plays a crucial role in decision-making processes involving the distribution of goods from multiple sources to multiple destinations. The classical Transportation Problem (TP) is a special case of linear programming that determines the optimal quantity of commodities to be transported from each origin to each destination in such a way that supply and demand constraints are satisfied while minimizing the total transportation cost. This model has been widely used in various real-life applications, including industrial production planning, logistics management, and resource allocation.

However, in real-world decision-making situations, transportation planning often involves multiple conflicting objectives, such as minimizing transportation cost, minimizing delivery time, maximizing service reliability, and improving customer satisfaction. The traditional single-objective transportation model is inadequate to address such complex situations. To overcome this limitation, multi-objective transportation problems have been introduced, in which several objective functions are optimized simultaneously. Charnes and Cooper [5] were among the first to develop solution procedures for decision-making problems involving multiple conflicting objectives, which significantly contributed to the development of multi-objective optimization theory.

With the increasing complexity of modern transportation systems, decision-makers face numerous challenges, including capacity limitations, environmental concerns, safety requirements, uncertainty in travel time, and energy efficiency considerations. Transportation systems typically involve multiple stakeholders with different and often conflicting objectives. Therefore, developing efficient mathematical models and solution techniques for transportation problems has become an important research area in operations research and management science.

The transportation problem was first formally investigated by Kantorovich [15], who developed an initial mathematical framework for optimal resource allocation. Hitchcock [13] later formulated the transportation problem for minimizing the cost of distributing goods from factories to customers. Since then, several researchers have extended the classical transportation model to incorporate additional objectives and constraints. Aneja and Nair [1] studied a bi-criteria transportation problem, while Hammer [12], Sharma and Swarup [39], Szwarc [41], and Bhatia et al. [2] investigated transportation problems with time minimization objectives. Liu [20] proposed a method for solving transportation problems with varying supply and demand conditions, and Sharma et al. [40] developed an iterative algorithm for hierarchical time minimization transportation problems.

In recent years, various extensions of the transportation problem have been proposed

to address real-world complexities. Damci-Kurt et al. [8] studied transportation problems with market choice, while Meiyi et al. [26] developed a fuzzy random transportation model for hazardous materials. Ebrahimnejad [9] proposed an improved method for solving fuzzy transportation problems using triangular fuzzy numbers. Prakash [29] studied transportation problems with multiple objectives involving cost and duration minimization, and Khan et al. [18] investigated bilevel transportation problems with time minimization objectives. Maity and Roy [24] developed a utility function approach for solving multi-objective transportation problems, and Yu et al. [46] proposed an interactive approach for multi-objective transportation problems with interval parameters. In practical transportation systems, decision-makers often encounter situations where transportation parameters such as cost, supply, and demand may have multiple possible values due to uncertainty and varying operational conditions. To address this issue, the concept of multi-choice transportation problems has been introduced. Chang [5, 6] developed multi-choice and revised multi-choice goal programming approaches for solving such problems. Roy et al. [36], Mahapatra et al. [21], and Roy [32] studied stochastic transportation problems under multi-choice environments involving different probability distributions. Mahapatra and Roy [37] investigated solid transportation problems under multi-choice conditions, and Maity and Roy [22] extended the multi-choice concept to multi-objective transportation problems using utility function approaches. Midya and Roy [27] developed a stochastic multi-objective transportation model with fixed charges, and Roy et al. [38] proposed a conic scalarization approach for solving multi-choice multi-objective transportation problems.

Despite these significant contributions, solving multi-objective multi-item Just-in-Time transportation problems under uncertain and multi-choice environments remains a challenging task. Therefore, this paper develops an efficient mathematical model and solution procedure to address this problem.

The remainder of this paper is organized as follows. Section 2 describes the problem environment and necessary preliminaries. Section 3 presents the mathematical formulation of the proposed transportation model. Section 4 provides the solution methodology. Section 5 illustrates the applicability of the proposed model through a numerical example. Finally, Section 6 concludes the paper and discusses future research directions.

2 Problem Environment

In modern manufacturing systems, industries require multiple types of raw materials to produce finished goods. These materials are usually procured from different suppli-

ers and transported to production facilities. Since these raw materials are often non-substitutable, the timely availability of each item is essential to ensure uninterrupted production. Any delay in delivery may disrupt the production schedule and cause significant operational losses. On the other hand, early delivery of materials may also create practical difficulties, such as increased inventory holding costs, storage limitations, and potential deterioration of perishable goods. Therefore, it is necessary to schedule transportation activities in such a way that the required materials are delivered precisely at the desired time.

In transportation planning, the delivery time plays a critical role in maintaining system efficiency and minimizing operational risks. If goods are not delivered within the specified time window, it may result in damage, spoilage, increased storage costs, or rejection by customers. Under such circumstances, transportation parameters such as cost and profit cannot always be treated as precise or deterministic values due to uncertainties in travel time, operational conditions, and environmental factors. Consequently, the selection of appropriate objective functions and the determination of optimal transportation policies become more complex. To address these challenges, it is necessary to incorporate reliability considerations into the transportation cost structure, which allows decision-makers to account for uncertainties and ensure more realistic and robust transportation planning.

In real-life logistics systems, transportation decisions must consider multiple performance criteria, such as minimizing transportation cost, minimizing delivery time, maintaining product quality, reducing fuel consumption, and maximizing service reliability. Moreover, in many practical situations, retailers may receive goods either from intermediate warehouses or directly from production facilities. Delivering goods directly from production units to retailers at the required time without unnecessary storage is consistent with the Just-in-Time (JIT) philosophy. The JIT approach aims to reduce inventory holding costs, improve operational efficiency, and ensure timely delivery of goods.

A Just-in-Time transportation problem (JTP) can be viewed as a special form of the classical transportation problem in which goods are transported from production centers to various destinations in such a way that deliveries are completed exactly at the required time. A typical JIT transportation network is illustrated in Figure 1.

In practical transportation scenarios, decision-makers often need to consider multiple and conflicting objectives simultaneously, such as minimizing transportation cost and maximizing profit while ensuring timely delivery. These objectives cannot always be combined into a single utility function without loss of important decision information. Therefore, it becomes necessary to formulate the problem as a MJTP, where different objective functions represent different performance criteria.

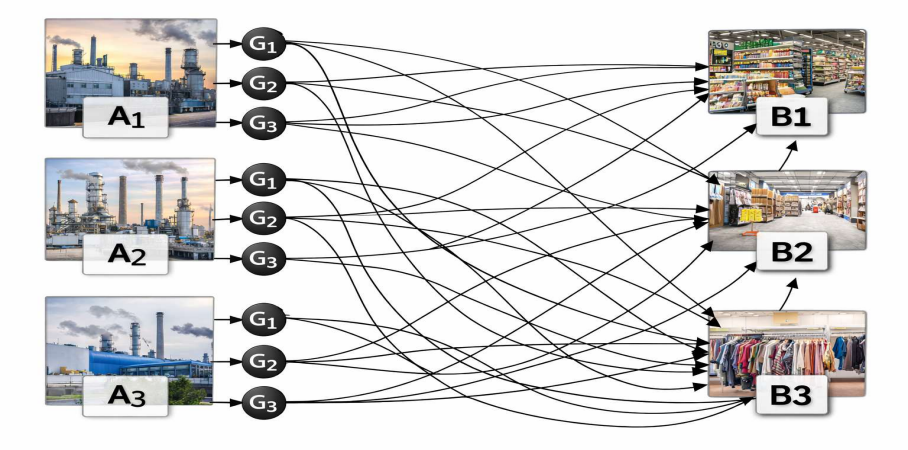


Figure 1: Network representation of a Just-in-Time transportation problem.

In the proposed framework, the decision-maker aims to determine an optimal transportation plan that minimizes transportation cost, maximizes overall system benefit, and ensures Just-in-Time delivery of multiple items. The developed model provides a realistic and efficient approach for handling complex transportation planning problems in modern logistics and supply chain systems.

3 Preliminaries

Since in this paper we tackle the uncertainty of MMJTP based on the dual hesitant fuzzy numbers, we recall the basic definitions and properties of hesitant fuzzy sets and dual hesitant fuzzy sets.

3.1 Hesitant Fuzzy Sets

The concept of a hesitant fuzzy set (HFS) was first introduced by Torra and Narukawa [42] to represent situations where the membership degree of an element cannot be expressed by a single value but rather by several possible values. Subsequently, Torra [43] provided the formal mathematical definition.

Definition 3.1 (Torra [43]). Let X be a reference set. A hesitant fuzzy set (HFS) H on X is defined by a mapping

$$h : X \to \wp([0,1]),$$

where $\wp([0,1])$ denotes the power set of the interval $[0,1]$. For each element $x \in X$, the function $h(x)$ returns a finite subset of $[0,1]$, representing all possible membership degrees of x in H.

Xu and Xia (2011) further expressed the HFS in a compact mathematical form as follows.
Definition 3.2 (Xu and Xia, 2011). A hesitant fuzzy set H can be represented as

$$H = \{(x_i, h(x_i)) : \ x_i \in X\}, \tag{3.1}$$

where $h(x_i)$ is a finite subset of $[0, 1]$ representing the possible membership degrees of the element $x_i \in X$. Each element of $h(x_i)$ is called a *hesitant fuzzy element (HFE)* and is denoted by h_i.
Definition 3.3. Let h_i be a hesitant fuzzy element corresponding to $x_i \in X$. The lower and upper bounds of h_i are defined as

$$h_i^- = \inf h(x_i), \quad h_i^+ = \sup h(x_i),$$

respectively.
Assumption. In this study, since transportation routes and decision alternatives are finite, the reference set X is assumed to be finite and non-empty. Accordingly, all HFSs and HFEs considered in this work are finite.
It is important to distinguish between an HFS and an HFE. An HFS assigns a set of possible membership values to each element of the reference set, whereas an HFE refers to the specific set of membership values corresponding to a single element.

3.2 Score Function of a Hesitant Fuzzy Element

Let $H = \{(x_i, h(x_i)) : \ x_i \in X\}$ be an HFS defined on the finite set $X = \{x_1, x_2, \ldots, x_n\}$. For a hesitant fuzzy element h_i containing k membership values, the score function $s(h_i)$ is defined as

$$s(h_i) = \frac{1}{k} \sum_{j=1}^{k} h_{ij}, \tag{3.2}$$

where h_{ij} represents the j-th membership value in h_i, and k is the cardinality of h_i. The score function provides a representative numerical value for comparison purposes.
For any two hesitant fuzzy elements h_1 and h_2:

- If $s(h_1) > s(h_2)$, then h_1 is said to be *superior* to h_2, denoted by $h_1 > h_2$.
- If $s(h_1) < s(h_2)$, then h_1 is said to be *inferior* to h_2, denoted by $h_1 < h_2$.
- If $s(h_1) = s(h_2)$, then h_1 and h_2 are said to be *equivalent*, denoted by $h_1 \sim h_2$.

The score function is adopted in this study to evaluate and rank hesitant fuzzy elements in the multi-choice transportation decision framework.

3.3 Dual Hesitant Fuzzy Sets

Dual hesitant fuzzy sets (DHFS) extend hesitant fuzzy sets by incorporating both membership and non-membership degrees.

Definition 3.4. Let X be a reference set. A dual hesitant fuzzy set (DHFS) D on X is defined as

$$D = \{(x, h(x), g(x)) : \ x \in X\}, \tag{3.3}$$

where

- $h(x)$ denotes the set of possible membership degrees of x, and
- $g(x)$ denotes the set of possible non-membership degrees of x,

such that for all $\gamma \in h(x)$ and $\eta \in g(x)$,

$$0 \leq \gamma \leq 1, \quad 0 \leq \eta \leq 1, \quad 0 \leq \gamma + \eta \leq 1. \tag{3.4}$$

The pair $d(x) = (h(x), g(x))$ is called a *dual hesitant fuzzy element (DHFE).*
Special cases of DHFEs include:

- Complete certainty: $(\{1\}, \{0\})$
- Complete uncertainty: $(\{0\}, \{1\})$
- Maximum hesitation: (h, g) where $h, g \subseteq [0, 1]$
- Empty information: $(\varnothing, \varnothing)$

In this study, the reference set X and all DHFS elements are assumed to be finite.

3.4 Arithmetic Operations on Dual Hesitant Fuzzy Elements

Let $d_1 = (h_{d_1}, g_{d_1})$ and $d_2 = (h_{d_2}, g_{d_2})$ be two DHFEs. The arithmetic operations are defined as follows:

Addition

$$d_1 \oplus d_2 = \{\{\gamma_1 + \gamma_2 - \gamma_1\gamma_2\}, \{\eta_1\eta_2\}\},$$

Subtraction

$$d_1 \ominus d_2 = \{\{\gamma_1\gamma_2\}, \{\eta_1 + \eta_2 - \eta_1\eta_2\}\},$$

where $\gamma_1 \in h_{d_1}$, $\gamma_2 \in h_{d_2}$, $\eta_1 \in g_{d_1}$, and $\eta_2 \in g_{d_2}$.

Scalar multiplication
For a positive scalar n,

$$nd = \{\{1-(1-\gamma)^n\}, \{\eta^n\}\}.$$

3.5 Ranking of Dual Hesitant Fuzzy Elements

Let $d = (h_d, g_d)$ be a DHFE with k membership values and k non-membership values. The score function is defined as

$$s_d = \frac{1}{k}\sum_{i=1}^{k} h_d(x_i) - \frac{1}{k}\sum_{i=1}^{k} g_d(x_i). \tag{3.5}$$

For any two DHFEs d_1 and d_2:

- If $s_{d_1} > s_{d_2}$, then d_1 is superior to d_2, denoted by $d_1 > d_2$.
- If $s_{d_1} < s_{d_2}$, then d_1 is inferior to d_2, denoted by $d_1 < d_2$.
- If $s_{d_1} = s_{d_2}$, then d_1 and d_2 are equivalent, denoted by $d_1 \sim d_2$.

The score function provides an effective ranking mechanism for comparing dual hesitant fuzzy elements in multi-objective decision-making problems such as transportation optimization under uncertainty.

4 Model Formulation

This section presents the mathematical formulation of the proposed multi-objective multi-item Just-in-Time transportation problem under a dual hesitant fuzzy environment. We begin with the classical multi-objective transportation problem and then extend it to the proposed model.

4.1 Classical Multi-objective Transportation Problem

Consider a transportation network consisting of s sources and t destinations. Let x_{pq} denote the quantity transported from source p to destination q. The multi-objective transportation problem (MOTP) can be formulated as follows:

Model 1

$$\begin{aligned} \min/\max \quad & Z^m = \sum_{p=1}^{s}\sum_{q=1}^{t} c_{pq}^m x_{pq}, \quad m = 1, 2, \ldots, M \\ \text{subject to} \quad & \sum_{q=1}^{t} x_{pq} \leq a_p, \quad p = 1, 2, \ldots, s, \\ & \sum_{p=1}^{s} x_{pq} \geq b_q, \quad q = 1, 2, \ldots, t, \\ & x_{pq} \geq 0, \quad \forall p, q. \end{aligned}$$

where c_{pq}^m denotes the transportation parameter associated with the m-th objective function, a_p and b_q represent the supply and demand, respectively.
The feasibility condition is given by

$$\sum_{p=1}^{s} a_p \geq \sum_{q=1}^{t} b_q.$$

—

4.2 Multi-objective Multi-item Transportation Problem

In practical situations, multiple commodities are transported simultaneously. Let

- $p = 1, 2, \ldots, s$ denote sources,
- $q = 1, 2, \ldots, t$ denote destinations,
- $r = 1, 2, \ldots, u$ denote commodities.

Define the following notation:

x_{pqr}	quantity of commodity r transported from source p to destination q
c_{pqr}^m	transportation parameter for objective m
a_{pr}	supply of commodity r at source p
b_{qr}	demand of commodity r at destination q

The mathematical model becomes:

Model 2

$$\begin{aligned}
\min/\max \quad & Z^m = \sum_{p=1}^{s}\sum_{q=1}^{t}\sum_{r=1}^{u} c_{pqr}^m x_{pqr}, \quad m = 1, \ldots, M \\
\text{subject to} \quad & \sum_{q=1}^{t} x_{pqr} \leq a_{pr}, \quad \forall p, r \qquad (4.6) \\
& \sum_{p=1}^{s} x_{pqr} \geq b_{qr}, \quad \forall q, r \qquad (4.7) \\
& x_{pqr} \geq 0. \qquad (4.8)
\end{aligned}$$

This model considers multiple conflicting objectives such as cost minimization and profit maximization.

4.3 Incorporation of Just-in-Time Constraint

In real supply chains, delivery within scheduled time is a critical requirement. Let

τ_{pqr} transportation time from source p to destination q for commodity r

σ_{qr} due time at destination q for commodity r

The Just-in-Time condition requires

$$\min_r \{\tau_{pqr} - \sigma_{qr}\} x_{pqr} = 0 \qquad (4.9)$$

This ensures that allocation is made only when delivery satisfies the scheduled time constraint.

Define the time deviation function

$$\tau'_{pqr} = \begin{cases} \tau_{pqr} - \sigma_{qr}, & \text{if } \tau_{pqr} > \sigma_{qr} \\ 0, & \text{otherwise} \end{cases} \qquad (4.10)$$

—

4.4 Multi-objective Just-in-Time Transportation Model

The problem is formulated as:

Model 3

$$\min \quad F(x) = \max\left\{\tau'_{pqr} \ : \ x_{pqr} \neq 0\right\} \tag{4.11}$$

$$\min \quad f_m(x) = \sum_p \sum_q \sum_r c^m_{pqr}\, x_{pqr}, \quad m = 1, 2, \ldots, M \tag{4.12}$$

$$\max \quad g_n(x) = \sum_p \sum_q \sum_r d^n_{pqr}\, x_{pqr}, \quad n = 1, 2, \ldots, N \tag{4.13}$$

$$\text{subject to} \quad \text{constraints (4.6)–(4.8).}$$

Here,

- $F(x)$ minimizes delivery time deviation,
- $f_m(x)$ minimizes transportation cost,
- $g_n(x)$ maximizes profit.

4.5 Dual Hesitant Fuzzy Representation

In practice, transportation costs and profits are uncertain. To model this uncertainty, dual hesitant fuzzy numbers are used.
Define dual hesitant fuzzy cost as

$$\tilde{c}^m_{pqr} = (h^m_{pqr}, g^m_{pqr}) \tag{4.14}$$

where

- h^m_{pqr} represents the degree of acceptance,
- g^m_{pqr} represents the degree of rejection.

The decision maker aims to

- maximize acceptance degree,
- minimize rejection degree.

4.6 Final Dual Hesitant Fuzzy Model

The final model is given as:
Model 4

$$\begin{aligned}
\min \quad & F(x) = \max\{\tau'_{pqr} : x_{pqr} \neq 0\}, && (4.15)\\
\max \quad & \sum_{p=1}^{s}\sum_{q=1}^{t}\sum_{r=1}^{u} h^m_{pqr}x_{pqr} \ (m = 1, 2, \ldots, M), && (4.16)\\
\min \quad & \sum_{p=1}^{s}\sum_{q=1}^{t}\sum_{r=1}^{u} g^m_{pqr}x_{pqr} \ (m = 1, 2, \ldots, M), && (4.17)\\
\min \quad & \sum_{p=1}^{s}\sum_{q=1}^{t}\sum_{r=1}^{u} h^n_{pqr}x_{pqr} \ (n = 1, 2, \ldots, N), && (4.18)\\
\max \quad & \sum_{p=1}^{s}\sum_{q=1}^{t}\sum_{r=1}^{u} g^n_{pqr}x_{pqr} \ (n = 1, 2, \ldots, N), && (4.19)\\
\text{subject to} \quad & (h^m_{pqr} + g^m_{pqr} - 1)x_{pqr} \leq 0, && (4.20)\\
& (h^n_{pqr} + g^n_{pqr} - 1)x_{pqr} \leq 0, && (4.21)\\
\text{and} \quad & \text{the constraints } (4.6) - (4.8).
\end{aligned}$$

4.7 Feasible and Pareto Optimal Solution

A feasible solution satisfies all supply, demand, and non-negativity constraints.
A Pareto optimal solution is defined as a feasible solution for which no objective function can be improved without worsening at least one other objective.
The objective is to determine a Pareto optimal Just-in-Time transportation plan that minimizes deviation and cost while maximizing system preference and profit.

Lemma 4.1 *Let D denote the set of all feasible solutions and let $x \in D$ be given. Then, for any $\lambda \geq 0$, the inequality $F(x) \leq \lambda$ holds if and only if*

$$g^m_{11}(\lambda, x) = f_{1m}(x), \quad g^m_{12}(\lambda, x) = f_{2m}(x), \quad \forall m = 1, 2, \ldots, M,$$

and

$$g^n_{21}(\lambda, x) = g_{1n}(x), \quad g^n_{22}(\lambda, x) = g_{2n}(x), \quad \forall n = 1, 2, \ldots, N,$$

where the functions F, f_{1m}, f_{2m}, g_{1n}, and g_{2n} are defined in Model 5, and the auxiliary functions g^m_{11}, g^m_{12}, g^n_{21}, and g^n_{22} are defined in equations (4.22)(4.25).

Proof.

From the definition of $F(x)$, we have

$$F(x) = \max\left\{\tau'_{pqr} : x_{pqr} \neq 0,\ p = 1, \ldots, s;\ q = 1, \ldots, t;\ r = 1, \ldots, u\right\}. \tag{4.22}$$

(Necessity) Suppose that $F(x) \leq \lambda$. Then, by definition,

$$\tau'_{pqr} \leq \lambda \quad \text{whenever } x_{pqr} \neq 0.$$

Thus, $(p, q, r) \in B(\lambda)$ whenever $x_{pqr} \neq 0$, and consequently

$$x_{pqr} = 0 \quad \text{for all } (p, q, r) \notin B(\lambda).$$

Now, consider

$$g^m_{11}(\lambda, x) = \sum_{(p,q,r)\in B(\lambda)} h^m_{pqr} x_{pqr} - l_1 \sum_{(p,q,r)\notin B(\lambda)} x_{pqr} \tag{4.23}$$

$$= \sum_{p=1}^{s}\sum_{q=1}^{t}\sum_{r=1}^{u} h^m_{pqr} x_{pqr} \tag{4.24}$$

$$= f_{1m}(x), \quad \forall m = 1, \ldots, M. \tag{4.25}$$

Similarly,

$$g^m_{12}(\lambda, x) = \sum_{(p,q,r)\in B(\lambda)} g^m_{pqr} x_{pqr} + L_1 \sum_{(p,q,r)\notin B(\lambda)} x_{pqr} \tag{4.26}$$

$$= \sum_{p=1}^{s}\sum_{q=1}^{t}\sum_{r=1}^{u} g^m_{pqr} x_{pqr} \tag{4.27}$$

$$= f_{2m}(x), \quad \forall m = 1, \ldots, M, \tag{4.28}$$

and

$$g^n_{21}(\lambda, x) = \sum_{(p,q,r)\in B(\lambda)} h^n_{pqr} x_{pqr} + L_2 \sum_{(p,q,r)\notin B(\lambda)} x_{pqr} \tag{4.29}$$

$$= \sum_{p=1}^{s}\sum_{q=1}^{t}\sum_{r=1}^{u} h^n_{pqr} x_{pqr} \tag{4.30}$$

$$= g_{1n}(x), \quad \forall n = 1, \ldots, N, \tag{4.31}$$

$$g^n_{22}(\lambda, x) = \sum_{(p,q,r)\in B(\lambda)} g^n_{pqr} x_{pqr} - l_2 \sum_{(p,q,r)\notin B(\lambda)} x_{pqr} \tag{4.32}$$

$$= \sum_{p=1}^{s}\sum_{q=1}^{t}\sum_{r=1}^{u} g^n_{pqr} x_{pqr} \tag{4.33}$$

$$= g_{2n}(x), \quad \forall n = 1, \ldots, N. \tag{4.34}$$

Thus, the necessity is established.

(Sufficiency) Conversely, assume that

$$g_{11}^m(\lambda, x) = f_{1m}(x), \quad g_{12}^m(\lambda, x) = f_{2m}(x), \quad \forall m,$$

and

$$g_{21}^n(\lambda, x) = g_{1n}(x), \quad g_{22}^n(\lambda, x) = g_{2n}(x), \quad \forall n.$$

From $g_{11}^m(\lambda, x) = f_{1m}(x)$, we obtain

$$\sum_{(p,q,r)\notin B(\lambda)} (l_1 + h_{pqr}^m) x_{pqr} = 0. \tag{4.35}$$

From $g_{12}^m(\lambda, x) = f_{2m}(x)$,

$$\sum_{(p,q,r)\notin B(\lambda)} (L_1 - g_{pqr}^m) x_{pqr} = 0. \tag{4.36}$$

Similarly,

$$\sum_{(p,q,r)\notin B(\lambda)} (L_2 - h_{pqr}^n) x_{pqr} = 0, \tag{4.37}$$

$$\sum_{(p,q,r)\notin B(\lambda)} (l_2 + g_{pqr}^n) x_{pqr} = 0. \tag{4.38}$$

Since l_1, L_1, l_2, and L_2 are sufficiently large positive constants, it follows that

$$x_{pqr} = 0 \quad \text{for all } (p, q, r) \notin B(\lambda).$$

Therefore,

$$\tau'_{pqr} \leq \lambda \quad \text{whenever } x_{pqr} \neq 0,$$

which implies

$$F(x) \leq \lambda. \tag{4.39}$$

Hence, the sufficiency is proved.

This completes the proof. □

Lemma 4.1 establishes an important characterization of feasible solutions of Model 5. Specifically, if a feasible solution $x \in D$ satisfies the conditions

$$g_{11}^m(\lambda, x) = f_{1m}(x), \quad g_{12}^m(\lambda, x) = f_{2m}(x), \quad \forall m = 1, 2, \ldots, M,$$

and

$$g_{21}^n(\lambda, x) = g_{1n}(x), \quad g_{22}^n(\lambda, x) = g_{2n}(x), \quad \forall n = 1, 2, \ldots, N,$$

then x corresponds either to a multi-objective just-in-time (MJT) solution of Model 5 or to a deviation solution whose deviation level does not exceed λ.

Motivated by this observation, we define the subset D_λ of feasible solutions as follows:

$$\begin{aligned} D_\lambda = \{x \in D : g_{11}^m(\lambda, x) = f_{1m}(x),\ g_{12}^m(\lambda, x) = f_{2m}(x), \\ g_{21}^n(\lambda, x) = g_{1n}(x),\ g_{22}^n(\lambda, x) = g_{2n}(x), \\ \forall m = 1, 2, \ldots, M,\ \forall n = 1, 2, \ldots, N\}, \quad \lambda \geq 0. \end{aligned} \tag{4.40}$$

It can be readily verified that this family of sets is monotonic with respect to λ, that is,

$$D_{\lambda_1} \subseteq D_{\lambda_2} \quad \text{whenever } \lambda_1 \leq \lambda_2.$$

Furthermore, Lemma 4.1 implies that the set D_λ is nonempty when

$$\lambda = \max\left\{\tau'_{pqr} : p = 1, 2, \ldots, s;\ q = 1, 2, \ldots, t;\ r = 1, 2, \ldots, u\right\}.$$

Theorem 4.1 *A feasible solution $\bar{x}$ is an MJT solution of Model 5 if and only if*

$$g_{11}^m(0, \bar{x}) = f_{1m}(\bar{x}), \quad g_{12}^m(0, \bar{x}) = f_{2m}(\bar{x}), \quad \forall m = 1, 2, \ldots, M,$$

and

$$g_{21}^n(0, \bar{x}) = g_{1n}(\bar{x}), \quad g_{22}^n(0, \bar{x}) = g_{2n}(\bar{x}), \quad \forall n = 1, 2, \ldots, N.$$

Proof.
First, assume that

$$g_{11}^m(0, \bar{x}) = f_{1m}(\bar{x}), \quad g_{12}^m(0, \bar{x}) = f_{2m}(\bar{x}), \quad g_{21}^n(0, \bar{x}) = g_{1n}(\bar{x}), \quad g_{22}^n(0, \bar{x}) = g_{2n}(\bar{x}).$$

Then, by Lemma 4.1, it follows that

$$F(\bar{x}) \leq 0.$$

Since, by definition, $F(x) \geq 0$ for all feasible solutions, we conclude that

$$F(\bar{x}) = 0.$$

Therefore, $\bar{x}$ is an MJT solution of Model 5.
Conversely, suppose that $\bar{x}$ is an MJT solution of Model 5. Then, by definition,

$$F(\bar{x}) = 0.$$

Applying Lemma 4.1 again, we immediately obtain

$$g_{11}^m(0, \bar{x}) = f_{1m}(\bar{x}), \quad g_{12}^m(0, \bar{x}) = f_{2m}(\bar{x}),$$

and

$$g_{21}^n(0, \bar{x}) = g_{1n}(\bar{x}), \quad g_{22}^n(0, \bar{x}) = g_{2n}(\bar{x}),$$

for all $m = 1, \ldots, M$ and $n = 1, \ldots, N$.
This completes the proof. □

Corollary 4.1 *Let $\bar{x}$ be a Pareto optimal solution of the $B(0)$-problem. Then $\bar{x}$ is a Pareto optimal MJT solution of Model 5 if and only if*

$$g_{11}^m(0,\bar{x}) = f_{1m}(\bar{x}), \quad g_{12}^m(0,\bar{x}) = f_{2m}(\bar{x}),$$

and

$$g_{21}^n(0,\bar{x}) = g_{1n}(\bar{x}), \quad g_{22}^n(0,\bar{x}) = g_{2n}(\bar{x}),$$

for all $m = 1,\ldots,M$ and $n = 1,\ldots,N$.

Proof.
The proof follows directly from Theorem 4.1 and Lemma 4.1.
If $\bar{x}$ is a Pareto optimal MJT solution, then by Theorem 4.1 the stated equalities hold. Conversely, if these equalities hold, then by Lemma 4.1 we obtain $F(\bar{x}) = 0$, implying that $\bar{x}$ is an MJT solution. Since $\bar{x}$ is already Pareto optimal, it is therefore a Pareto optimal MJT solution.
This completes the proof. □

Table 1: Tabular standard JTP for minimizing the transportation cost.

Dest. / ML	B_1	$\cdots$	B_n	Production
A_1	c_{111}		c_{1n1}	a_{11}
	$\vdots$	$\cdots$	$\vdots$	
	c_{11k}		c_{1nk}	a_{1k}
	t_{11}	$\cdots$	t_{1n}	
$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\vdots$
A_m	c_{m11}		c_{mn1}	a_{m1}
	$\vdots$	$\cdots$	$\vdots$	
	c_{m1k}		c_{mnk}	a_{mk}
	t_{m1}	$\cdots$	t_{mn}	
Sales Vol.	$b_{11},\ldots,b_{1k}$		$b_{n1},\ldots,b_{nk}$	
Due Time	t_1	$\cdots$	t_n	

Remark 3
We introduced lately in Theorem 2 that the sequence $\{D_\lambda\}$ is monotone with respect to λ. This monotone property of $\{D_\lambda\}$ and Theorem 4 not only theoretically show us that we may solve the corresponding $B(\lambda)$-problem instead of solving the MJTP transportation but also practically designate a method to find the optimal solutions: beginning

Table 2: Tabular standard JTP for maximizing the profit.

ML \ Dest.	B_1	$\cdots$	B_n	Production
A_1	d_{111}		d_{1n1}	a_{11}
	$\vdots$	$\cdots$	$\vdots$	
	d_{11k}		d_{1nk}	a_{1k}
	t_{11}	$\cdots$	t_{1n}	
$\vdots$	$\cdots$	$\cdots$	$\cdots$	$\vdots$
A_m	d_{m11}		d_{mn1}	a_{m1}
	$\vdots$	$\cdots$	$\vdots$	
	d_{m1k}		d_{mnk}	a_{mk}
	t_{m1}	$\cdots$	t_{mn}	
Sales Vol.	$b_{11}, \ldots, b_{1k}$		$b_{n1}, \ldots, b_{nk}$	
Due Time	t_1	$\cdots$	t_n	

with $\lambda = 0$, if $D_0 \neq \phi$, we have solved the problem by getting the optimal solution of the $B(0)$-problem; unless choose $\lambda = \min\{d_{pq} : \tau'_{pqr} \neq 0, 1 \leq p \leq s, 1 \leq q \leq t, 1 \leq r \leq u.\}$. Since $\{\tau'_{pqr}\}$ is a finite set and $\{D_\lambda\}$ is monotone, we may forever let λ be the subsequent minimal value of the remaining non-zero τ'_{pqr}s till the λ for which $D_\lambda \neq \phi$. This is the algorithm for the MJTP.

In light of this algorithm and conventional transportation algorithm, we give an unthinkable strategy to take care of MJTP. This technique continues as follows:

5 Solution Procedure

In this section, a systematic solution methodology is presented to solve the multi-objective multi-item Just-in-Time transportation problem (MMJTP). Since the proposed model involves dual hesitant fuzzy transportation parameters and multiple conflicting objectives, the problem is first transformed into an equivalent single-objective transportation problem. The resulting problem is then solved using a transportation algorithm based on productionmarketing coordination and Vogels Approximation Method (VAM).

5.1 Algorithm 1: Conversion of MMJTP into Single Objective Transportation Problem

Step 1: Convert all dual hesitant fuzzy transportation cost parameters into equivalent crisp values using the conversion rule $(c_d - s_d)$, where c_d and s_d denote the score and accuracy values of the dual hesitant fuzzy number, respectively, as defined in Eq. (3.5).

Step 2: Consider the MMJTP having M objective functions of maximization type, denoted by Z^m, $m = 1, 2, \ldots, M$, and N objective functions of minimization type, denoted by Z^n, $n = 1, 2, \ldots, N$.

Step 3: Normalize the objective functions as follows:

(a) For each maximization objective function Z^m, determine the maximum cost coefficient $c^m_{p'q'r'}$. Each cost coefficient is normalized as:

$$r^m_{pqr} = \frac{c^m_{pqr}}{c^m_{p'q'r'}}, \quad 0 \leq r^m_{pqr} \leq 1.$$

This normalization ensures comparability among different objective functions and preserves the maximization structure.

(b) For each minimization objective function Z^n, determine the maximum cost coefficient $d^n_{p'q'r'}$. Each cost coefficient is normalized as:

$$s^n_{pqr} = -\frac{d^n_{pqr}}{d^n_{p'q'r'}}, \quad -1 \leq s^n_{pqr} \leq 0.$$

This transformation converts the minimization objective into an equivalent maximization form.

Step 4: Assign normalized weights w_m and g_n to the maximization and minimization objective functions, respectively, such that

$$\sum_{m=1}^{M} w_m + \sum_{n=1}^{N} g_n = 1.$$

Step 5: Construct the equivalent single-objective transportation problem as:

$$\text{Maximize } z = \sum_p \sum_q \sum_r \left(\sum_{m=1}^{M} w_m r^m_{pqr} + \sum_{n=1}^{N} g_n s^n_{pqr} \right) x_{pqr}$$

subject to the supply, demand, and non-negativity constraints of the MMJTP.

Step 6: The resulting model is now a standard single-objective transportation problem (STP).

5.2 Algorithm 2: Optimal Solution Procedure for MMJTP

Step 1: Formulate the MMJTP based on the real-life transportation system, including supply, demand, transportation cost, profit, and delivery time parameters.

Step 2: Convert the MMJTP into the equivalent single-objective transportation problem using Algorithm 1.

Step 3: Define the modified transportation time parameter as:

$$\tau'_{pqr} = \begin{cases} \tau_{pqr} - \sigma_{qr}, & \text{if } \tau_{pqr} > \sigma_{qr}, \\ 0, & \text{if } \tau_{pqr} \leq \sigma_{qr}, \end{cases}$$

where τ_{pqr} represents the transportation time and σ_{qr} represents the required delivery time.

Step 4: Construct the modified transportation tableau by replacing τ_{pqr} with τ'_{pqr}.

Step 5: Define the modified transportation cost matrix as:

$$c_{pqr} = \begin{cases} c_{pqr}, & \text{if } \tau'_{pqr} = 0, \\ L, & \text{if } \tau'_{pqr} \neq 0, \end{cases}$$

where L is a sufficiently large positive constant representing infeasible allocations.

Step 6: Arrange the distinct values of τ'_{pqr} in ascending order as:

$$\lambda_1, \lambda_2, \ldots, \lambda_u.$$

Step 7: Apply Vogels Approximation Method (VAM) to obtain an initial feasible solution for the transportation problem.

Step 8: Check feasibility conditions:

- If all allocated cells satisfy $c_{pqr} < L$, the current solution is optimal.
- Otherwise, update the transportation tableau using the next value of λ_e and repeat the procedure.

Step 9: Repeat the above procedure until an optimal feasible solution is obtained.

Step 10: Compute the optimal values of the objective functions:

For maximization objectives:

$$Z^m = \sum_p \sum_q \sum_r c^m_{pqr} x_{pqr}, \quad m = 1, 2, \ldots, M.$$

For minimization objectives:

$$Z^n = \sum_p \sum_q \sum_r d^n_{pqr} x_{pqr}, \quad n = 1, 2, \ldots, N.$$

Step 11: Stop.

The obtained solution represents the optimal compromise solution of the MMJTP.

Importance of our methodology

- We propose a new way in which time and transportation cost both are minimized and profit is maximized.
- Various researcher proposed different transportation model but till now nobody discuss such of model.
- Time is minimizes by simple classical method.
- Generally researchers construct problems non-linear to minimize time but here we consider a linear problem to minimize the time.
- We are solve it by giving first priority on deliver goods in scheduled time one can give priority on other transportation cost.

6 Numerical example

To illustrate the applicability and effectiveness of the proposed multi-objective multi-item Just-in-Time transportation model, a real-life inspired logistics scenario is considered. Suppose a manufacturing company operates three production facilities, denoted by A_1, A_2, and A_3, located in different industrial regions. These facilities produce three different types of products, namely G_1, G_2, and G_3, which are required to be delivered to multiple market locations within specified time limits.

The production capacities of the manufacturing units vary depending on the available resources, machinery, and workforce. Specifically, production facility A_1 can supply 20,

25, and 15 units of products G_1, G_2, and G_3, respectively. Similarly, production facility A_2 can supply 32, 45, and 22 units, while production facility A_3 can supply 42, 55, and 35 units of products G_1, G_2, and G_3, respectively. The unit production costs associated with each product at different manufacturing facilities are provided in Table 3.

The manufactured products are required to be transported to four geographically dispersed market locations, denoted by B_1, B_2, B_3, and B_4. These market locations represent regional distribution centers that serve local retailers and customers. Each market location has specific demand requirements for the three types of products. In particular, market location B_1 requires 22, 30, and 12 units of products G_1, G_2, and G_3, respectively. Similarly, market locations B_2, B_3, and B_4 require (28, 16, 25), (27, 19, 30), and (32, 20, 15) units of products G_1, G_2, and G_3, respectively.

Since the company follows a Just-in-Time (JIT) logistics strategy to minimize inventory holding costs and improve operational efficiency, each market location specifies a maximum allowable delivery time. The required delivery time limits for market locations B_1, B_2, B_3, and B_4 are 10, 12, 9, and 8 hours, respectively. These time constraints ensure that the products arrive at the destinations without unnecessary delay or early delivery, thereby maintaining supply chain efficiency and reducing storage-related costs. The detailed demand quantities, product types, and delivery deadlines are summarized in Table 4.

The transportation process involves different routes connecting the production facilities and the market locations. Due to variations in distance, road conditions, fuel costs, and operational factors, the transportation cost, transportation time, and expected profit differ across routes and product types. The transportation cost matrix, profit matrix, and transportation time matrix are presented in Table 5, Table 6, and Table 7, respectively.

Under this transportation environment, the decision-maker aims to determine the optimal transportation plan that satisfies all supply and demand constraints while achieving multiple conflicting objectives. Specifically, the objectives of the decision-maker are:

- to ensure delivery of goods within the specified Just-in-Time schedule,
- to minimize the total transportation cost, and
- to maximize the overall transportation profit.

The proposed dual hesitant fuzzy optimization approach is applied to this transportation system to obtain the optimal allocation of products from production facilities to market locations. The results obtained from the proposed methodology demonstrate its effectiveness in handling MJTP under realistic operational conditions. Now first we

Table 3: Supplies.

Comm. / ML	G_1	G_2	G_3
A_1	40	25	15
A_2	32	15	22
A_3	42	45	35

Note: Comm.=commodities,ML=manufacturing

Table 4: Demands.

Comm. / Dest	G_1	G_2	G_3
B_1	22	30	12
B_2	28	16	25
B_3	27	19	30
B_4	32	20	15

Note: Dest=destination

convert the MJTP which are given by in Table 8 and Table 9 into single objective JTP.

The above results disclose to us that numerical illustration which we have considered do not have a JT solution, that is, this transportation problem does not have a solution for conveying every requested commodity to all destinations in time. The optimal deviation solution x of the problem comprises of $x_{111} = 8$, $x_{131} = 27$, $x_{221} = 28$, $x_{241} = 4$, $x_{311} = 14$, $x_{341} = 28$, $x_{112} = 5$, $x_{122} = 1$, $x_{132} = 19$, $x_{222} = 15$, $x_{312} = 25$, $x_{342} = 20$, $x_{123} = 3$, $x_{133} = 12$, $x_{223} = 22$, $x_{313} = 12$, $x_{333} = 18$, $x_{343} = 5$.
The deviation of this solution is $F(x) = 4$ and the total transportation cost $f(x) = 2623$ and also profit is $g(x) = 4556$

Table 5: Due times

Comm. / Dest	DT(G_1)	DT(G_2)	DT(G_3)
B_1	10	12	11
B_2	5	8	8
B_3	9	10	8
B_4	8	7	13

Note: DT=due time

Table 6: Shipping time.

Dest. / ML	B_1	B_2	B_3	B_4
A_1 (G_1)	8	6	10	12
A_1 (G_2)	6	7	12	11
A_1 (G_3)	10	9	9	14
A_2 (G_1)	10	7	8	9
A_2 (G_2)	12	9	7	12
A_2 (G_3)	15	9	8	11
A_3 (G_1)	13	9	6	8
A_3 (G_2)	14	12	7	10
A_3 (G_3)	11	8	9	12

Table 7: Unit transportation cost.

Dest. / ML	B_1	B_2	B_3	B_4
A_1	$(10, 12, 14)$	$(11, 14, 17)$	$(4, 9, 14)$	$(13, 19, 25)$
A_2	$(9, 11, 13)$	$(4, 7, 10)$	$(12, 17, 22)$	$(6, 12, 18)$
A_3	$(8, 10, 12)$	$(12, 15, 18)$	$(4, 8, 12)$	$(8, 14, 20)$

Table 8: Profit.

Dest. / ML	B_1	B_2	B_3	B_4
A_1	$(12, 14, 16)$	$(11, 12, 13)$	$(15, 18, 21)$	$(4, 6, 8)$
A_2	$(12, 15, 18)$	$(17, 22, 27)$	$(5, 8, 11)$	$(13, 14, 15)$
A_3	$(9, 16, 23)$	$(11, 15, 19)$	$(16, 20, 24)$	$(11, 12, 13)$

Table 9: JTP for minimizing the transportation cost.

Dest. / ML	B_1	B_2	B_3	B_4	Production
A_1	$(10, 12, 14)$	$(11, 14, 17)$	$(4, 9, 14)$	$(13, 19, 25)$	40
	$(10, 12, 14)$	$(11, 14, 17)$	$(4, 9, 14)$	$(13, 19, 25)$	25
	$(10, 12, 14)$	$(11, 14, 17)$	$(4, 9, 14)$	$(13, 19, 25)$	15
(G_1)	8	6	10	12	
(G_2)	6	7	12	11	
(G_3)	10	9	9	14	
A_2	$(9, 11, 13)$	$(4, 7, 10)$	$(12, 17, 22)$	$(6, 12, 18)$	32
	$(9, 11, 13)$	$(4, 7, 10)$	$(12, 17, 22)$	$(6, 12, 18)$	15
	$(9, 11, 13)$	$(4, 7, 10)$	$(12, 17, 22)$	$(6, 12, 18)$	22
(G_1)	10	7	8	9	
(G_2)	12	9	7	12	
(G_3)	15	9	8	11	
A_3	$(8, 10, 12)$	$(12, 15, 18)$	$(4, 8, 12)$	$(8, 14, 20)$	42
	$(8, 10, 12)$	$(12, 15, 18)$	$(4, 8, 12)$	$(8, 14, 20)$	45
	$(8, 10, 12)$	$(12, 15, 18)$	$(4, 8, 12)$	$(8, 14, 20)$	35
(G_1)	13	9	6	8	
(G_2)	14	12	7	10	
(G_3)	11	8	9	12	
Sales Vol.	$22, 30, 12$	$28, 16, 25$	$27, 19, 30$	$32, 20, 15$	
Due Time	10	5	9	8	
	12	8	10	7	
	11	8	8	13	

Table 10: JTP for maximizing the profit.

ML \ Dest.	B_1	B_2	B_3	B_4	Production
A_1	$(12, 14, 16)$	$(11, 12, 13)$	$(15, 18, 21)$	$(4, 6, 8)$	40
	$(12, 14, 16)$	$(11, 12, 13)$	$(15, 18, 21)$	$(4, 6, 8)$	25
	$(12, 14, 16)$	$(11, 12, 13)$	$(15, 18, 21)$	$(4, 6, 8)$	15
(G_1)	8	6	10	12	
(G_2)	6	7	12	11	
(G_3)	10	9	9	14	
Sales Vol.	$(20, 22, 24)$	$(23, 28, 33)$	$(25, 27, 29)$	$(27, 32, 37)$	
A_2	$(12, 15, 18)$	$(17, 22, 27)$	$(5, 8, 11)$	$(13, 14, 15)$	32
	$(12, 15, 18)$	$(17, 22, 27)$	$(5, 8, 11)$	$(13, 14, 15)$	15
	$(12, 15, 18)$	$(17, 22, 27)$	$(5, 8, 11)$	$(13, 14, 15)$	22
(G_1)	10	7	8	9	
(G_2)	12	9	7	12	
(G_3)	15	9	8	11	
A_3	$(9, 16, 23)$	$(11, 15, 19)$	$(16, 20, 24)$	$(11, 12, 13)$	42
	$(9, 16, 23)$	$(11, 15, 19)$	$(16, 20, 24)$	$(11, 12, 13)$	45
	$(9, 16, 23)$	$(11, 15, 19)$	$(16, 20, 24)$	$(11, 12, 13)$	35
(G_1)	13	9	6	8	
(G_2)	14	12	7	10	
(G_3)	11	8	9	12	
Sales Vol.	$22, 30, 12$	$28, 16, 25$	$27, 19, 30$	$32, 20, 15$	
Due Time	10	5	9	8	
	12	8	10	7	
	11	8	8	13	

Table 11: Crisp value for Table: 9

Dest. / ML	B_1	B_2	B_3	B_4	Production
A_1	12	14	9	19	40
	12	14	9	19	25
	12	14	9	19	15
(G_1)	8	6	10	12	
(G_2)	6	7	12	11	
(G_3)	10	9	9	14	
A_2	11	7	17	12	32
	11	7	17	12	15
	11	7	17	12	22
(G_1)	10	7	8	9	
(G_2)	12	9	7	12	
(G_3)	15	9	8	11	
A_3	10	15	8	14	42
	10	15	8	14	45
	10	15	4	14	35
(G_1)	13	9	6	8	
(G_2)	14	12	7	10	
(G_3)	11	8	9	12	
Sales Vol.	$22, 30, 12$	$28, 16, 25$	$27, 19, 30$	$32, 20, 15$	
Due Time	10	5	9	8	
	12	8	10	7	
	11	8	8	13	

Table 12: Crisp value for Table: 10

ML \ Dest.	B_1	B_2	B_3	B_4	Production
A_1	14	12	18	6	40
	14	12	18	6	25
	14	12	18	6	15
(G_1)	8	6	10	12	
(G_2)	6	7	12	11	
(G_3)	10	9	9	14	
A_2	15	22	8	14	32
	15	22	8	14	15
	15	22	8	14	22
(G_1)	10	7	8	9	
(G_2)	12	9	7	12	
(G_3)	15	9	8	11	
A_3	16	15	20	12	42
	16	15	20	12	45
	16	15	20	12	35
(G_1)	13	9	6	8	
(G_2)	14	12	7	10	
(G_3)	11	8	9	12	
Sales Vol.	$22, 30, 12$	$28, 16, 25$	$27, 19, 30$	$32, 20, 15$	
Due Time	10	5	9	8	
	12	8	10	7	
	11	8	8	13	

Table 13: Dual hesitant fuzzy number corresponding to each transportation cost in each node

Dest. / ML	B_1	B_2	B_3	B_4	Production
A_1	$\{\{0.5,0.3,0.1\},$ $\{0.4,0.6,0.9\}\}$	$\{\{0.6,0.5,0.2\},$ $\{0.4,0.3,0.7\}\}$	$\{\{0.8,0.6,0.3\},$ $\{0.1,0.2,0.6\}\}$	$\{\{0.5,0.3,0.1\},$ $\{0.5,0.6,0.9\}\}$	40
	$\{\{0.5,0.3,0.1\},$ $\{0.4,0.6,0.9\}\}$	$\{\{0.6,0.5,0.2\},$ $\{0.4,0.3,0.7\}\}$	$\{\{0.8,0.6,0.3\},$ $\{0.1,0.2,0.6\}\}$	$\{\{0.5,0.3,0.1\},$ $\{0.5,0.6,0.9\}\}$	25
	$\{\{0.5,0.3,0.1\},$ $\{0.4,0.6,0.9\}\}$	$\{\{0.6,0.5,0.2\},$ $\{0.4,0.3,0.7\}\}$	$\{\{0.8,0.6,0.3\},$ $\{0.1,0.2,0.6\}\}$	$\{\{0.5,0.3,0.1\},$ $\{0.5,0.6,0.9\}\}$	15
	0	0	1	4	
A_2	$\{\{0.7,0.6,0.2\},$ $\{0.1,0.4,0.7\}\}$	$\{\{0.8,0.5,0.1\},$ $\{0.1,0.4,0.8\}\}$	$\{\{0.6,0.4,0.1\},$ $\{0.2,0.6,0.8\}\}$	$\{\{0.9,0.5,0.3\},$ $\{0.1,0.4,0.7\}\}$	32
	$\{\{0.7,0.6,0.2\},$ $\{0.1,0.4,0.7\}\}$	$\{\{0.8,0.5,0.1\},$ $\{0.1,0.4,0.8\}\}$	$\{\{0.6,0.4,0.1\},$ $\{0.2,0.6,0.8\}\}$	$\{\{0.9,0.5,0.3\},$ $\{0.1,0.4,0.7\}\}$	15
	$\{\{0.7,0.6,0.2\},$ $\{0.1,0.4,0.7\}\}$	$\{\{0.8,0.5,0.1\},$ $\{0.1,0.4,0.8\}\}$	$\{\{0.6,0.4,0.1\},$ $\{0.2,0.6,0.8\}\}$	$\{\{0.9,0.5,0.3\},$ $\{0.1,0.4,0.7\}\}$	22
(G_1)	10	7	8	9	
(G_2)	12	9	7	12	
(G_3)	15	9	8	11	
A_3	$\{\{0.8,0.5,0.3\},$ $\{0.1,0.4,0.7\}\}$	$\{\{0.6,0.2,0.1\},$ $\{0.4,0.8,0.9\}\}$	$\{\{0.5,0.3,0.1\},$ $\{0.3,0.6,0.8\}\}$	$\{\{0.8,0.4,0.1\},$ $\{0.1,0.6,0.9\}\}$	42
	$\{\{0.8,0.5,0.3\},$ $\{0.1,0.4,0.7\}\}$	$\{\{0.6,0.2,0.1\},$ $\{0.4,0.8,0.9\}\}$	$\{\{0.5,0.3,0.1\},$ $\{0.3,0.6,0.8\}\}$	$\{\{0.8,0.4,0.1\},$ $\{0.1,0.6,0.9\}\}$	45
	$\{\{0.8,0.5,0.3\},$ $\{0.1,0.4,0.7\}\}$	$\{\{0.6,0.2,0.1\},$ $\{0.4,0.8,0.9\}\}$	$\{\{0.5,0.3,0.1\},$ $\{0.3,0.6,0.8\}\}$	$\{\{0.8,0.4,0.1\},$ $\{0.1,0.6,0.9\}\}$	35
(G_1)	13	9	6	8	
(G_2)	14	12	7	10	
(G_3)	11	8	9	12	
Sales Vol.	$22, 30, 12$	$28, 16, 25$	$27, 19, 30$	$32, 20, 15$	
Due Time	10	5	9	8	
	12	8	10	7	
	11	8	8	13	

Table 14: Dual hesitant fuzzy number corresponding to profit in each node

Dest. / ML	B_1	B_2	B_3	B_4	Production
A_1	$\{\{0.6, 0.2, 0.1\}, \{0.3, 0.8, 0.9\}\}$	$\{\{0.7, 0.4, 0.3\}, \{0.2, 0.4, 0.6\}\}$	$\{\{0.6, 0.5, 0.2\}, \{0.3, 0.4, 0.8\}\}$	$\{\{0.7, 0.4, 0.3\}, \{0.3, 0.6, 0.5\}\}$	40
	$\{\{0.6, 0.2, 0.1\}, \{0.3, 0.8, 0.9\}\}$	$\{\{0.7, 0.4, 0.3\}, \{0.2, 0.4, 0.6\}\}$	$\{\{0.6, 0.5, 0.2\}, \{0.3, 0.4, 0.8\}\}$	$\{\{0.7, 0.4, 0.3\}, \{0.3, 0.6, 0.5\}\}$	25
	$\{\{0.6, 0.2, 0.1\}, \{0.3, 0.8, 0.9\}\}$	$\{\{0.7, 0.4, 0.3\}, \{0.2, 0.4, 0.6\}\}$	$\{\{0.6, 0.5, 0.2\}, \{0.3, 0.4, 0.8\}\}$	$\{\{0.7, 0.4, 0.3\}, \{0.3, 0.6, 0.5\}\}$	15
(G_1)	8	6	10	12	
(G_2)	6	7	12	11	
(G_3)	10	9	9	14	
A_2	$\{\{0.5, 0.4, 0.3\}, \{0.3, 0.5, 0.6\}\}$	$\{\{0.6, 0.4, 0.3\}, \{0.3, 0.5, 0.6\}\}$	$\{\{0.7, 0.5, 0.3\}, \{0.3, 0.4, 0.6\}\}$	$\{\{0.8, 0.4, 0.2\}, \{0.2, 0.5, 0.7\}\}$	32
	$\{\{0.5, 0.4, 0.3\}, \{0.3, 0.5, 0.6\}\}$	$\{\{0.6, 0.4, 0.3\}, \{0.3, 0.5, 0.6\}\}$	$\{\{0.7, 0.5, 0.3\}, \{0.3, 0.4, 0.6\}\}$	$\{\{0.8, 0.4, 0.2\}, \{0.2, 0.5, 0.7\}\}$	15
	$\{\{0.5, 0.4, 0.3\}, \{0.3, 0.5, 0.6\}\}$	$\{\{0.6, 0.4, 0.3\}, \{0.3, 0.5, 0.6\}\}$	$\{\{0.7, 0.5, 0.3\}, \{0.3, 0.4, 0.6\}\}$	$\{\{0.8, 0.4, 0.2\}, \{0.2, 0.5, 0.7\}\}$	22
(G_1)	10	7	8	9	
(G_2)	12	9	7	12	
(G_3)	15	9	8	11	
A_3	$\{\{0.7, 0.3, 0.0.5\}, \{0.3, 0.6, 0.4\}\}$	$\{\{0.8, 0.3, 0.4\}, \{0.1, 0.6, 0.5\}\}$	$\{\{0.6, 0.4, 0.1\}, \{0.3, 0.6, 0.8\}\}$	$\{\{0.8, 0.4, 0.1\}, \{0.2, 0.6, 0.9\}\}$	42
	$\{\{0.7, 0.3, 0.0.5\}, \{0.3, 0.6, 0.4\}\}$	$\{\{0.8, 0.3, 0.4\}, \{0.1, 0.6, 0.5\}\}$	$\{\{0.6, 0.4, 0.1\}, \{0.3, 0.6, 0.8\}\}$	$\{\{0.8, 0.4, 0.1\}, \{0.2, 0.6, 0.9\}\}$	45
	$\{\{0.7, 0.3, 0.0.5\}, \{0.3, 0.6, 0.4\}\}$	$\{\{0.8, 0.3, 0.4\}, \{0.1, 0.6, 0.5\}\}$	$\{\{0.6, 0.4, 0.1\}, \{0.3, 0.6, 0.8\}\}$	$\{\{0.8, 0.4, 0.1\}, \{0.2, 0.6, 0.9\}\}$	35
(G_1)	13	9	6	8	
(G_2)	14	12	7	10	
(G_3)	11	8	9	12	
Sales Vol.	$22, 30, 12$	$28, 16, 25$	$27, 19, 30$	$32, 20, 15$	
Due Time	10	5	9	8	
	12	8	10	7	
	11	8	8	13	

Table 15: JTP for minimizing the transportation cost at minimum point.

Dest. / ML	B_1	B_2	B_3	B_4	Production
A_1	10	11	4	13	40
	10	11	4	13	25
	10	11	4	13	15
	0	0	1	4	
A_2	9	4	12	6	32
	9	4	12	6	15
	9	4	12	6	22
	0	1	0	1	
A_3	8	12	4	8	42
	8	12	4	8	45
	8	12	4	8	35
	0	0	0	0	
Sales Vol.	$22, 30, 12$	$28, 16, 25$	$27, 19, 30$	$32, 20, 15$	
Due Time	10	5	9	8	
	12	8	10	7	
	11	8	8	13	

Table 16: JTP for maximizing the profit at minimum point.

Dest. / ML	B_1	B_2	B_3	B_4	Production
A_1	12	11	15	4	40
	12	11	15	4	25
	12	11	15	4	15
	0	0	1	4	
A_2	12	17	5	13	32
	12	17	5	13	15
	12	17	5	13	22
	0	1	0	1	
A_3	9	11	16	11	42
	9	11	16	11	45
	9	11	16	11	35
(G_3)	0	0	0	0	
Sales Vol.	$22, 30, 12$	$28, 16, 25$	$27, 19, 30$	$32, 20, 15$	
Due Time	10	5	9	8	
	12	8	10	7	
	11	8	8	13	

Table 17: Single objective JTP.

ML \ Dest.	B_1	B_2	B_3	B_4	B_5	Production
A_1	0.015	−0.177	0.396	−0.726	0	40
	0.015	−0.177	0.396	−0.726	0	25
	0.015	−0.177	0.396	−0.726	0	15
	0	0	1	4	0	
A_2	0.121	0.640	−0.527	0.024	0	32
	0.121	0.640	−0.527	0.024	0	15
	0.121	0.640	−0.527	0.024	0	22
	0	1	0	1	0	
A_3	0.213	−0.119	0.490	−0.174	0	42
	0.213	−0.119	0.490	−0.174	0	45
	0.213	−0.119	0.490	−0.174	0	35
	0	0	0	0	0	
A_4	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	10
	0	0	0	0	0	
Sales Vol.	$22, 30, 12$	$28, 16, 25$	$27, 19, 30$	$32, 20, 15$	$5, 0, 0$	

Table 18: The Production marketing co-ordination table of B(1).

Dest. / ML	B_1	B_2	B_3	B_4	B_5	Production
A_1	0.064	0.199	−0.574	0.765	0	40
	0.064	0.199	−0.574	0.765	0	25
	0.064	0.199	−0.574	0.765	0	15
	0	0	1	4	0	
A_2	−0.013	−0.692	0.629	−0.304	0	32
	−0.013	−0.692	0.629	−0.304	0	15
	−0.013	−0.692	0.629	−0.304	0	22
	0	1	0	1	0	
A_3	0.086	0.276	−0.633	−0.032	0	42
	0.086	0.276	−0.633	−0.032	0	45
	0.086	0.276	−0.633	−0.032	0	35
	0	0	0	0	0	
Sales Vol.	22, 30, 12	28, 16, 25	27, 19, 30	32, 20, 15	5, 0, 0	
A_4	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0	0	10
	0	0	0	0	0	
Sales Vol.	0	0	0	0	0	

Table 19: The Production marketing co-ordination solution table of B(4).

Dest. / ML	B_1	B_2	B_3	B_4	B_5	Production
A_1	0.015[8]	−0.177	0.396[27]	−0.726	0[5]	40
	0.015[5]	−0.177[1]	0.396[19]	−0.726	0	25
	0.015	−0.177[3]	0.396[12]	−0.726	0	15
	0	0	1	4	0	
A_2	0.121	0.640[28]	−0.527	0.024[4]	0	32
	0.121	0.640[15]	−0.527	0.024	0	15
	0.121	0.640[22]	−0.527	0.024	0	22
	0	1	0	1	0	
A_3	0.213[14]	−0.119	0.490	−0.174[28]	0	42
	0.213[25]	−0.119	0.490	−0.174[20]	0	45
	0.213[12]	−0.119	0.490[18]	−0.174[5]	0	35
	0	0	0	0	0	
A_4	0	0	0	0	0	0
	0	0	0	0	0	0
	0	0	0	0[10]	0	10
	0	0	0	0	0	
Sales Vol.	$22, 30, 12$	$28, 16, 25$	$27, 19, 30$	$32, 20, 15$	$5, 0, 0$	

Table 20: Solution of MMJTP by taking weight $\mu_p = 0.5$ and $\lambda_t = 0.5$.

G_1	G_2	G_3
$A_1 \underset{1}{\rightarrow} B_1\ x_{111} = 8$	$A_1 \underset{2}{\rightarrow} B_1\ x_{112} = 5$	$A_1 \underset{3}{\rightarrow} B_1\ x_{113} = 0$
$A_2 \underset{1}{\rightarrow} B_1\ x_{211} = 0$	$A_2 \underset{2}{\rightarrow} B_1\ x_{212} = 0$	$A_2 \underset{3}{\rightarrow} B_1\ x_{213} = 0$
$A_3 \underset{1}{\rightarrow} B_1\ x_{311} = 14$	$A_3 \underset{2}{\rightarrow} B_1\ x_{312} = 25$	$A_3 \underset{3}{\rightarrow} B_1\ x_{313} = 12$
$A_1 \underset{1}{\rightarrow} B_2\ x_{121} = 0$	$A_1 \underset{2}{\rightarrow} B_2\ x_{122} = 1$	$A_1 \underset{3}{\rightarrow} B_2\ x_{123} = 3$
$A_2 \underset{1}{\rightarrow} B_2\ x_{221} = 28$	$A_2 \underset{2}{\rightarrow} B_2\ x_{222} = 15$	$A_2 \underset{3}{\rightarrow} B_2\ x_{223} = 22$
$A_3 \underset{1}{\rightarrow} B_2\ x_{321} = 0$	$A_3 \underset{2}{\rightarrow} B_2\ x_{322} = 0$	$A_3 \underset{3}{\rightarrow} B_2\ x_{323} = 0$
$A_1 \underset{1}{\rightarrow} B_3\ x_{131} = 27$	$A_1 \underset{2}{\rightarrow} B_3\ x_{132} = 19$	$A_1 \underset{3}{\rightarrow} B_3\ x_{133} = 12$
$A_2 \underset{1}{\rightarrow} B_3\ x_{231} = 0$	$A_2 \underset{2}{\rightarrow} B_3\ x_{232} = 0$	$A_2 \underset{3}{\rightarrow} B_3\ x_{233} = 0$
$A_3 \underset{1}{\rightarrow} B_3\ x_{331} = 0$	$A_3 \underset{2}{\rightarrow} B_3\ x_{332} = 0$	$A_3 \underset{3}{\rightarrow} B_3\ x_{333} = 18$
$A_1 \underset{1}{\rightarrow} B_4\ x_{141} = 0$	$A_1 \underset{2}{\rightarrow} B_4\ x_{142} = 0$	$A_1 \underset{3}{\rightarrow} B_4\ x_{143} = 0$
$A_2 \underset{1}{\rightarrow} B_4\ x_{241} = 4$	$A_2 \underset{2}{\rightarrow} B_4\ x_{242} = 0$	$A_2 \underset{3}{\rightarrow} B_4\ x_{243} = 0$
$A_3 \underset{1}{\rightarrow} B_4\ x_{341} = 28$	$A_3 \underset{2}{\rightarrow} B_4\ x_{342} = 20$	$A_3 \underset{3}{\rightarrow} B_4\ x_{343} = 5$

7 Results and Discussion

The transportation problem is one of the most significant decision-making models in operations research, with wide applicability in logistics, supply chain management, and industrial distribution systems. The classical transportation model primarily focuses on minimizing the total transportation cost while satisfying supply and demand constraints. In such models, transportation cost minimization is generally advantageous from the buyers' perspective, whereas profit maximization is beneficial for sellers. However, the classical formulation does not simultaneously address the conflicting interests of both buyers and sellers, nor does it consider uncertainties and practical limitations such as vehicle availability, delivery reliability, and Just-in-Time requirements.

Several researchers have extended the classical transportation problem by incorporating fuzzy parameters to handle uncertainty (cf. Chanas and Kuchta [4], Kaur and Kumar [16], Ebrahimnejad [9]). Although these approaches provide improved modeling flexibility, they generally emphasize cost minimization and do not fully capture the conflicting objectives of buyers and sellers under real-world operational constraints. In practical transportation systems, it is often difficult to simultaneously satisfy the cost minimization preference of buyers and the profit maximization objective of sellers due to limitations such as vehicle capacity, route availability, and delivery schedules. Under such conditions, customers may either pay a higher transportation cost to receive goods within the desired time or accept delayed delivery at a lower cost. These conflicting preferences make it difficult to obtain a balanced and unbiased solution using conventional approaches.

To address these challenges, the proposed methodology incorporates dual hesitant fuzzy sets to represent transportation cost and profit parameters more realistically. This framework allows the simultaneous consideration of multiple possible cost and profit values, reflecting the uncertainty and hesitation inherent in real-world transportation systems. The numerical example presented in this study includes hesitant fuzzy parameters representing minimum and maximum transportation costs and profits for different items and routes. In general, minimum transportation costs are preferred by buyers, while maximum profits are preferred by sellers. Therefore, dual hesitant membership and non-membership functions are introduced to capture these conflicting preferences effectively. Initially, the transportation model is solved using the minimum transportation cost and minimum profit values. The corresponding optimal transportation plan is obtained as follows:

$$x_{111} = 22,\ x_{131} = 13,\ x_{221} = 28,\ x_{241} = 4,\ x_{331} = 14,\ x_{341} = 28,$$

$$x_{112} = 24,\ x_{122} = 1,\ x_{222} = 15,\ x_{312} = 6,\ x_{332} = 19,\ x_{342} = 20,$$

$$x_{113} = 5,\ x_{133} = 10,\ x_{223} = 22,\ x_{333} = 20,\ x_{343} = 15,\ x_{413} = 7,\ x_{423} = 3.$$

For this solution, the deviation value is obtained as $F(x) = 4$, the total transportation cost is $f(x) = 1661$, and the total profit is $g(x) = 3720$. This solution is clearly favorable from the buyers' perspective, as it minimizes transportation cost across all routes.

Next, the transportation model is solved using the maximum transportation cost and maximum profit values. The resulting optimal transportation plan is obtained as follows:

$$x_{121} = 28,\ x_{131} = 7,\ x_{241} = 32,\ x_{311} = 22,\ x_{331} = 20,$$

$$x_{122} = 16,\ x_{132} = 9,\ x_{242} = 15,\ x_{312} = 30,\ x_{332} = 10,\ x_{342} = 5,$$

$$x_{123} = 8,\ x_{133} = 7,\ x_{223} = 17,\ x_{313} = 12,\ x_{333} = 23,\ x_{443} = 10.$$

In this case, the deviation value is $F(x) = 4$, the total transportation cost is $f(x) = 3816$, and the total profit is $g(x) = 6453$. This solution is more favorable from the sellers' perspective, as it maximizes profit and ensures easier allocation of transportation resources.

A comparative analysis of the obtained results shows that the transportation cost and profit values satisfy the following relationships:

$$1661 < 2623 < 3816,$$

$$3720 < 4556 < 6453.$$

These results indicate that the solution obtained using the proposed dual hesitant fuzzy approach lies between the extreme solutions corresponding to minimum and maximum hesitant fuzzy values. Therefore, the proposed approach provides a balanced compromise solution that considers both cost minimization and profit maximization simultaneously.

The results demonstrate that the proposed dual hesitant fuzzy transportation model successfully resolves the conflict between buyers' and sellers' objectives and provides an unbiased and realistic transportation plan. Furthermore, the proposed methodology can effectively handle bi-objective and multi-objective decision-making problems involving conflicting objectives. By incorporating both membership and non-membership functions, the model provides improved decision support under uncertain environments.

The proposed solution procedure is computationally efficient and can be applied to a wide range of real-world transportation and logistics problems. In particular, the methodology can be extended to solve other multi-objective optimization problems, including route selection problems and supply chain optimization models. Therefore, the proposed dual hesitant fuzzy transportation framework provides a valuable and practical tool for solving complex decision-making problems in uncertain transportation environments.

8 Conclusion and Outlook

In this paper, a mathematical framework has been developed to address the multi-objective multi-item Just-in-Time transportation problem under a realistic decision-making environment. The proposed model incorporates multiple conflicting objectives, including the minimization of transportation cost and the maximization of system benefit, while ensuring timely delivery of goods according to the Just-in-Time philosophy. The integration of reliability considerations into the transportation cost structure enhances the practical applicability of the model and allows decision-makers to account for uncertainties associated with real-world transportation systems.

A suitable solution methodology has been presented to transform the proposed model into an equivalent solvable form, and an algorithmic procedure has been provided to obtain the optimal transportation plan. The numerical example demonstrates the effectiveness, feasibility, and applicability of the proposed approach in solving complex transportation problems involving multiple items and multiple objectives. The results indicate that the developed model provides a reliable and efficient decision-support tool for optimizing transportation planning in modern logistics and supply chain systems.

The proposed study can be extended in several directions for future research. The model may be further generalized by incorporating different types of uncertainty, such as fuzzy, intuitionistic fuzzy, or stochastic parameters. Additional real-world constraints, including vehicle capacity limitations, time windows, and environmental considerations, can also be incorporated to improve the realism of the model. Furthermore, advanced optimization techniques and metaheuristic algorithms may be employed to solve large-scale transportation problems efficiently. The proposed framework can also be applied to other logistics and supply chain optimization problems, such as inventory management, distribution planning, and productiontransportation coordination.

Overall, the proposed approach provides a flexible and effective foundation for modeling and solving complex MJTPs and offers significant potential for practical applications and future research developments.

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