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Zhicheng Lei

lei.zc2019@outlook.com

East China University of Technology

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Scale-dependent truncated Lévy modeling of Gamma-ray log increments for characterizing stratigraphic heterogeneity in IODP well logs

Zhicheng Lei, School of Artificial Intelligence and Information Engineering, East China University of Technology

Corresponding E-mail: leizc2019@outlook.com

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Zhicheng Lei: Writing – review & editing, Writing – original draft, Visualization, Validation, Software, Resources, Methodology, Formal analysis, Conceptualization.

Abstract: Gamma-ray (GR) well logs record lithologic variability across multiple scales, but their increment statistics are often analyzed under stationary Lévy assumptions. In this study, we introduce a scale-dependent framework based on the gradually truncated Lévy flight model to characterize nonstationary increment behavior in 23 International Ocean Discovery Program (IODP) wells. This approach treats the stability index α and truncation scale l_c as functions of lag scale, combining Lévy-stable core distributions with exponential truncation of large deviations. A robust estimation pipeline is developed that integrates core-distribution fitting, tail-sensitive truncation detection, and profile-likelihood optimization. Mean squared displacement (MSD) analysis reveals a crossover from super-diffusive ($\beta > 1$) to sub-diffusive ($\beta < 1$) regimes at scales of approximately 0.8–1.2 m, reflecting changes in dominant stratigraphic controls. α and l_c exhibit scale-dependent evolution, from interface-dominated increments at small scales to bounded variability in larger

stratigraphic units. The proposed framework enhances stochastic modeling by extending traditional Gaussian assumptions and provides a versatile tool for quantifying anomalous diffusion and stratigraphic heterogeneity in well-log data. The Python implementation is open-source, ensuring reproducibility and flexibility in broader geological contexts.

1. Introduction

Subsurface geological records exhibit scale-dependent variability (Sabatier et al., 2022; Plantz et al., 2024), reflecting shifts in the dominant processes that organize lithologic and stratigraphic structure (Yoshida et al., 2007; Ganti et al., 2014; Catuneanu, 2019). Fine-resolution borehole signals often display intermittent, heavy-tailed distributions, while at coarser resolution, these signals tend toward smoother, near-Gaussian behavior (Guadagnini et al., 2018). This transition suggests that geological variability is governed by stratigraphic architecture rather than by a single scale-invariant stochastic process.

Gamma-ray (GR) logs, as continuous, high-resolution proxies for lithological variability (Pettigrew et al., 2024), provide a unique opportunity to test this hypothesis. Previous studies have described well-log increments using Lévy stable models to account for non-Gaussian, heavy-tailed fluctuations and scale-dependent heterogeneity (Painter, 1995, 1996; Painter and Paterson, 1994; Liu and Molz, 1997). However, classical Lévy processes assume stationarity and, in their pure form, infinite variance—assumptions that may be inconsistent with finite, stratified geological systems. Although truncated Lévy models address variance divergence (Gupta and Campanha, 1999), there is a lack of efficient, systematic fitting methods tailored to geological data.

Lévy models have been successfully applied in fields such as finance and physics (Mantegna and Stanley, 1995; Constantinides and Savelev, 2013), where parameter estimation assumes

stationarity, a condition that often holds in systems with consistent, repeatable behaviors. However, geological systems, especially those represented by GR well logs, exhibit considerable complexity and hierarchical structure, making the stationary assumption unrealistic. Consequently, the traditional methods used to estimate Lévy parameters, which are based on stationary distributions, are not suitable for the irregular and stratified nature of geological data. Although some studies have applied Lévy models to well-log data (Painter, 1995, 1996; Painter and Paterson, 1994), the methods for estimating the parameters remain underdeveloped and not optimized for geological contexts. This highlights the need for new, scale-dependent approaches to estimate the Lévy parameters in geological systems more accurately.

To address this gap, we propose a novel computational framework that models both the stability index α and truncation scale l_c as scale-dependent parameters, offering a more accurate representation of GR increment statistics. The approach integrates a robust algorithmic pipeline that combines core-mass matching, tail-sensitive truncation detection, and profile-likelihood optimization to efficiently estimate these parameters.

In summary, this study introduces a scale-dependent framework for analyzing GR increments, using the truncated Lévy model to capture the heavy-tailed fluctuations and hierarchical variability characteristic of geological systems. This work provides a more realistic method for well-log modeling, minimizing the impact of extreme fluctuations while offering an efficient computational workflow for estimating the truncated Lévy parameters and scaling exponent β . Ultimately, this framework enhances our ability to capture the complexity of geophysical signals, providing a more reliable basis for geological interpretation and advancing the field of stochastic modeling in Earth sciences.

2. Data and Methodology

2.1 Dataset

To validate the proposed scale-dependent Lévy framework, we analyze gamma-ray (GR) logs from 23 wells drilled during multiple International Ocean Discovery Program (IODP) expeditions. Most logs were acquired at a vertical sampling interval of 0.1524 m, with one well sampled at 0.0254 m (Table 1). These high-resolution measurements provide continuous records of lithological variability and are well suited for investigating the statistical structure of GR fluctuations across multiple spatial scales.

2.2 Increment statistics of GR logs

The GR increment is defined as

$$\Delta X_{\Delta t} = GR(t + \Delta t) - GR(t) \quad (1)$$

where $GR(t)$ denotes the GR value at depth index t and Δt is the index lag. The corresponding physical separation equals $\Delta t \cdot \Delta h$, where Δh is the depth sampling interval.

Increment statistics are widely used in stochastic transport and anomalous diffusion analyses because they characterize scale-dependent variability and correlations in the underlying process (Bouchaud and Georges, 1990; Metzler and Klafter, 2000).

2.3 Scale-dependent Lévy model

Increment distributions are modeled using the gradually truncated Lévy flight (GTLF) framework (Gupta and Campanha, 1999, 2000). In this model, a symmetric Lévy stable kernel governs the central part of the distribution, while large increments are gradually suppressed by an exponential truncation beyond a cutoff scale l_c . Both the stability index α ($0 < \alpha \leq 2$) and the

truncation scale l_c are treated as scale-dependent parameters.

The probability density function (PDF) of the increment size x at scale Δt is expressed as:

$$P(x) = \begin{cases} cL_\alpha(x, \Delta t), & |x| \leq l_c \\ cL_\alpha(x, \Delta t) \exp \left\{ -\left(\frac{|x| - l_c}{k} \right)^\tau \right\}, & |x| > l_c \end{cases} \quad (2)$$

where c is a normalizing constant, l_c is the physical cutoff scale, and k and τ control the truncation of large increments. The symmetrical Lévy kernel is given by

$$L_\alpha(x, \Delta t) = \frac{1}{\pi} \int_0^\infty \exp(-\gamma \Delta t q^\alpha) \cos(qx) dq \quad (3)$$

where γ is the Lévy scale factor. In practical computations, the Lévy kernel was evaluated numerically using the numerical implementation of the symmetric Lévy stable distribution, with stability parameter α , skewness parameter $\beta = 0$, location 0, and scale parameter $(\gamma \Delta t)^{1/\alpha}$.

2.4 Scale diagnostics of GR increments

To characterize scale dependence of GR increments, both the complementary cumulative distribution function (CCDF) and the mean squared displacement (MSD) are analyzed.

2.4.1 Complementary cumulative distribution function

The empirical CCDF is defined as

$$\text{CCDF}(x) = P(|\Delta X_{\Delta t}| > x), \quad (4)$$

CCDF representations are widely used in heavy-tailed statistics because they avoid binning bias associated with histogram-based PDFs and provide stable diagnostics of tail behavior on logarithmic scales (Clauset et al., 2009). This representation facilitates discrimination among Gaussian, Lévy, and truncated Lévy regimes.

2.4.2 Mean squared displacement

The MSD is computed as

$$\text{MSD}(\Delta t) = \langle (\Delta X_{\Delta t})^2 \rangle, \quad (5)$$

where $\langle \cdot \rangle$ denotes the ensemble average over increments at lag Δt . The MSD is assumed to scale with observation window according to a power-law relationship,

$$\text{MSD}(\Delta t) \sim (\Delta t)^\beta, \quad (6)$$

where the scaling exponent β characterizes the correlation structure of the increments. Values of $\beta > 1$ indicate persistent, super-diffusive behavior, $\beta \approx 1$ corresponds to uncorrelated or diffusive dynamics, and $\beta < 1$ indicates anti-persistent, sub-diffusive behavior. This scaling relation has been widely used in anomalous diffusion and Lévy flight/walk analyses (Hao et al., 2026). Estimation of β across scales thus provides a quantitative measure of the evolving temporal (or vertical) correlation and intermittency of GR increments.

The scaling exponent β was estimated from the relationship between MSD and the lag scale Δt in log-log space. To capture the scale-dependent behavior of the diffusion process, a local regression approach was adopted. For each intermediate lag value, $\beta(\Delta t)$ was calculated using a three-point moving regression, in which a linear fit was performed using the neighboring lag scales $(\Delta t_{i-1}, \Delta t_i, \Delta t_{i+1})$. The slope of the fitted line provides the local scaling exponent β at scale Δt , corresponding to a discrete approximation of the logarithmic derivative (Dieterich et al., 2008)

$$\beta(\Delta t) = \frac{d \log \text{MSD}}{d \log \Delta t} \quad (7)$$

The MSD was first computed using discrete lag steps Δt . These were subsequently converted to physical depth scales $\Delta h = \Delta t \cdot \Delta h$. This approach allows the identification of scale-dependent variations in the diffusion regime while maintaining sufficient numerical stability given the limited number of lag scales.

2.5 Parameter estimation

The overall parameter estimation workflow is summarized in Fig. 1. The parameter estimation algorithm consists of three main components: (1) core-mass matching to estimate Lévy stability index α , (2) quantile-based estimation of the scale parameter γ , and (3) CCDF-based initialization of the truncation scale l_c , followed by profile-likelihood optimization of the GTLF parameters. The details of each component are described in the following sections.

2.5.1 Estimation of Lévy parameters α and γ

According to Eq. (2), the Lévy model parameters α and γ are mainly determined by increments smaller than the truncation scale l_c , corresponding to the core region of the distribution. In contrast, the tail region is mainly influenced by truncation effects and extreme events, which can bias parameter estimation. Therefore, parameter inference is restricted to the central portion of the distribution.

The core region is defined using a quantile-based threshold

$$r_{\text{core}} = Q_{q_{\text{core}}}(|X|), \quad (8)$$

where $Q_q(|X|)$ denotes the empirical q -th quantile of $|X|$. In this study, $q_{\text{core}} = 0.2$ unless otherwise specified. This choice balances statistical stability and reduced contamination from tail fluctuations. Only increments satisfying $|X| \leq r_{\text{core}}$ are used to constrain estimate α .

(1) Estimation of α

An initial α_0 is iteratively refined by matching the integrated mass of the empirical and theoretical PDFs within the core region. The estimator is motivated by probability-mass matching within the central region of the distribution. Since the Lévy parameters α and γ primarily control the core shape, matching the integrated mass in this region provides a stable estimator that is less

sensitive to tail truncation.

Let $f_{emp}(x)$ denote the empirical density estimated from the normalized histogram with fixed bin width. The symmetric Lévy PDF is $f_{\alpha,\gamma}(x)$ with scale

$$\text{scale} = (\gamma\Delta t)^{1/\alpha} \quad (9)$$

Define

$$M_{emp} = \int_{|x| \leq r_{core}} f_{emp}(x) dx, \quad (10)$$

$$M_{levy}(\alpha) = \int_{|x| \leq r_{core}} f_{\alpha,\gamma}(x) dx. \quad (11)$$

The empirical mass M_{emp} within the core region was obtained by summing the histogram densities multiplied by their corresponding bin widths. To better represent the heavy-tailed nature of the data, logarithmic binning was used when constructing the histogram. The probability mass of the Lévy model $M_{levy}(\alpha)$ was estimated by numerically integrating the model probability density function $f_{\alpha,\gamma}(x)$ over the same interval using the trapezoidal rule.

The parameter α was iteratively updated as

$$\alpha \leftarrow \alpha * (1 + \text{step} * \text{sign}(M_{levy}(\alpha) - M_{emp})) \quad (12)$$

where step is a small positive constant controlling the update magnitude. The iteration stops when

$$|M_{levy}(\alpha) - M_{emp}| < 0.02 M_{emp} \quad (13)$$

All α values satisfying Eq. (13) are retained, defining an admissible interval:

$$\alpha_{CI} = [\alpha_{min}, \alpha_{max}], \quad (14)$$

and the reported estimate is

$$\alpha = \frac{\alpha_{min} + \alpha_{max}}{2}. \quad (15)$$

This tolerance-based interval acts as a core-mass consistency confidence band rather than a

likelihood-based interval.

(2) Estimation of γ

Following the quantile-based estimation strategy for stable distributions (McCulloch, 1986), given α , the Lévy scale parameter γ is estimated as:

$$\gamma_q = \frac{1}{\Delta t} \left(\frac{Q_q(|X|)}{c_q(\alpha)} \right)^\alpha, \quad (16)$$

where

$$c_q(\alpha) = F_\alpha^{-1} \left(\frac{1+q}{2} \right), \quad (17)$$

is the corresponding theoretical quantile of a unit-scale symmetric Lévy distribution, and F_α^{-1} denotes the inverse cumulative distribution function (CDF) of a unit-scale symmetric Lévy stable distribution with stability index α .

The final estimate of γ is obtained as a weighted geometric mean:

$$\log \gamma = \sum_i w_i \log \gamma_{q_i} \quad (18)$$

with normalized weights:

$$w_i \leftarrow \frac{w_i}{\sum_j w_j} \quad (19)$$

This procedure reduces sensitivity to extreme quantiles. The dispersion of $\log \gamma_q$ across quantiles provides an empirical uncertainty estimate for γ .

2.5.2 Initial detection of cutoff l_c

The cutoff scale is initially detected by comparing empirical and theoretical CCDFs. The empirical CCDF of the absolute increments is defined as

$$\text{CCDF}_{\text{emp}}(x) = \frac{1}{N} \sum_{i=1}^N \mathbf{1}(|\Delta X_i| > x) \quad (20)$$

where $\mathbf{1}(\cdot)$ denotes the indicator function.

The theoretical Lévy CCDF is computed from the symmetric Lévy stable density with stability

index α and scale parameter γ ,

$$\text{CCDF}_{\text{levy}}(x) = \int_x^\infty f_{\alpha,\gamma}(u) du \quad (21)$$

The integral was evaluated numerically using cumulative trapezoidal quadrature on the discretized PDF.

The logarithmic deviation between empirical and theoretical CCDFs is then evaluated as

$$\Delta(x) = \log \text{CCDF}_{\text{emp}}(x) - \log \text{CCDF}_{\text{levy}}(x) \quad (22)$$

The cutoff l_c is determined by scanning $\Delta(x)$ along the ordered empirical values x . A moving window containing k consecutive points is used to detect persistent deviations from the Lévy model. In this study $k = 10$ was used. The cutoff l_c is defined as the smallest x for which the following conditions are satisfied:

The mean deviation within the window satisfies

$$\bar{\Delta} < \log(0.75), \quad (23)$$

indicating that the empirical CCDF is at least 25% smaller than the Lévy prediction. At least 70% of successive differences in $\Delta(x)$ within the window are negative, indicating a persistent decreasing trend. The resulting cutoff provides an initial estimate of the truncation scale, which is subsequently used as the starting point for the likelihood optimization of the truncated Lévy model.

2.5.3 Maximum likelihood estimation

Given fixed α and γ , the GTLF model (Eq.2) can be rewritten as:

$$g(x; l_c, k) = c(l_c, k) f_{\alpha,\gamma}(x) \exp \left[- \left(\frac{(|x| - l_c)_+}{k} \right)^\tau \right], \quad (24)$$

where $(|x| - l_c)_+ = \max(|x| - l_c, 0)$, and $\tau = 2 - \alpha$ (Gupta and Campanha, 1999).

The normalization constant $c(l_c, k)$ satisfies:

$$\int_{-\infty}^{\infty} g(x; l_c, k) dx = 1. \quad (25)$$

Using symmetry,

$$c(l_c, k) = \left[\int_{-l_c}^{l_c} f_{\alpha, \gamma}(x) dx + 2 \int_{l_c}^{\infty} f_{\alpha, \gamma}(x) \exp \left[- \left(\frac{x-l_c}{k} \right)^\tau \right] dx \right]^{-1}. \quad (26)$$

Define:

$$\text{core}(l_c) = \int_{-l_c}^{l_c} f_{\alpha, \gamma}(x) dx \quad (27)$$

$$\text{tail}(l_c, k) = \int_{l_c}^{\infty} f_{\alpha, \gamma}(x) \exp \left[- \left(\frac{x-l_c}{k} \right)^\tau \right] dx \quad (28)$$

Then:

$$c(l_c, k) = \frac{1}{\text{core}(l_c) + 2 \text{tail}(l_c, k)}. \quad (29)$$

For independent increments x_i , the likelihood is:

$$L(l_c, k) = \prod_{i=1}^n c(l_c, k) f_{\alpha, \gamma}(x_i) \exp \left\{ - \left[\frac{(|x_i| - l_c)_+}{k} \right]^\tau \right\}, \quad (30)$$

The negative log-likelihood (NLL) is:

$$NLL(l_c, k) = -n \log c(l_c, k) - \sum_{i=1}^n \log f_{\alpha, \gamma}(x_i) + \sum_{i=1}^n \left[\frac{(|x_i| - l_c)_+}{k} \right]^\tau \quad (31)$$

Optimization is performed using profile likelihood to estimate l_c and k . For fixed l_c

$$\hat{k}(l_c) = \arg \min_k NLL(l_c, k). \quad (32)$$

No closed-form solution exists because k appears both in the exponential penalty and in the normalization constant $c(l_c, k)$. Numerical bounded optimization in log-space is used:

$$k = e^\theta. \quad (33)$$

where $\theta \in \mathbb{R}$ is an unconstrained variable. This transformation guarantees $k > 0$ and improves numerical stability.

The profile likelihood is:

$$NLL_p(l_c) = NLL(l_c, \hat{k}(l_c)). \quad (34)$$

The maximum likelihood estimate (MLE) of the cutoff parameter is obtained as:

$$\hat{l}_c = \arg \min_{l_c} NLL_p(l_c). \quad (35)$$

Confidence intervals are determined using the likelihood-ratio criterion:

$$NLL_p(l_c) - NLL_p(\hat{l}_c) \leq \frac{1}{2} \chi_{1,0.95}^2 \quad (36)$$

corresponding to the 95% confidence level for one parameter.

After obtaining $\hat{l}_c$, the optimal k is:

$$\hat{k} = \hat{k}(\hat{l}_c). \quad (37)$$

According to Eq. (29), the normalization constant becomes:

$$\hat{c} = \frac{1}{\text{core}(\hat{l}_c) + 2 \text{tail}(\hat{l}_c, \hat{k})} \quad (38)$$

All numerical integrations were performed using trapezoidal quadrature on discretized grids.

The proposed workflow integrates core-mass-based Lévy parameter estimation, CCDF-based cutoff detection, and profile-likelihood optimization, providing a robust computational framework for inferring scale-dependent truncated Lévy models from heavy-tailed geophysical signals.

The implementation was carried out in Python and applied independently to each well and lag scale. For every well and scale Δt , we report: (1) Lévy stability index α and its admissible interval; (2) Lévy scale parameter γ ; (3) cutoff length l_c with profile-likelihood confidence intervals; (4) the GTLF parameters k and c . All estimated values are listed in Table S1.

3. Results

3.1 Multi-scale intermittency of GR increments

The scale dependence of GR increments was evaluated using index lags $\Delta t =$

{1, 3, 8, 10, 20, 50, 100, 250, 500, 1000}, spanning three orders of magnitude.

Fig.1 shows well 349-U1431E as a representative example. The GR record exhibits intermittent fluctuations (Fig.2a), and the corresponding increment distributions display pronounced heavy tails at small scale $\Delta t = 1$ (Fig.2b, c). In the tail region, the GTLF model provides a consistently better than both the Gaussian and pure Lévy-stable models (Fig.2d).

The evolution of the Lévy index α with respect to Δt is characterized by initial oscillations at smaller scales, followed by a steady monotonic increase for intervals greater than 3 m (Fig.2e). Meanwhile, the cutoff parameter l_c displays a consistent positive correlation with the depth interval, increasing monotonically throughout the observed scale (Fig.2f).

3.2 Scale-dependent statistics

Across the 23 wells, most wells exhibit peak excess kurtosis at small to intermediate sampling intervals ($\Delta t < 50m$), indicating highly non-Gaussian distributions with heavy tails. As Δt increases, the excess kurtosis for the majority of wells declines significantly, eventually approaching zero at large scales ($\Delta t > 100m$) (Fig.3a).

The Lévy-stability index α displays considerable inter-well variability across all scales. Although fluctuations are present, many wells show a general upward trend in α as Δt increases, particularly at scales larger than 10 m (Fig.3b).

Both the cutoff length l_c (Fig.3c) and MSD (Fig.3d) exhibit strikingly similar scaling characteristics across the 23 wells. Both parameters show a strictly monotonic increase with respect to the depth interval Δt . At smaller scales ($\Delta t < 10m$), this growth is rapid and approximately follows a power-law trend. However, as Δt increases further, both l_c and MSD demonstrate a clear plateau effect, where the growth rate slows markedly and gradually approaches saturation at the

largest sampled intervals.

3.3 Crossover behavior in MSD scaling

Fig.4a shows that at small sampling intervals ($\Delta t < 1m$), the MSD exponent β is consistently greater than 1, indicating a super-diffusive regime. As the depth interval increases, β decreases sharply, crossing the Brownian threshold ($\beta = 1$) at a specific scale Δt^* . Beyond this point, the system enters a sub-diffusive regime ($\beta < 1$) at larger scales.

The crossover scales Δt^* exhibit distinct clustering depending on the expedition group (Fig. 4b). Across the full set of 23 wells, the mean crossover scale is $0.968 \pm 0.234m$ (Table 2).

Expedition 359 shows the largest and most consistent crossover values ($1.215 \pm 0.046m$), whereas Expedition 356 displays a moderate crossover scale ($0.999 \pm 0.282m$). Expedition 367/368 yields a comparable value of $0.985 \pm 0.119m$.

3.4 Relationships among GR-increment statistical parameters

Fig.5a–b shows the relationships between the MSD scaling exponent β and the Lévy parameters for different expeditions. Overall, neither the Lévy stability index α nor the truncation scale l_c exhibits a clear monotonic dependence on β . However, the expedition-level trends display distinct patterns. For Exp 359, both α and l_c remain nearly constant across the investigated β range. In contrast, Exp 356 and Exp 367/368 show non-monotonic variations in α with increasing β : α first increases and then decreases for Exp 356, whereas it first decreases and then increases for Exp 367/368. The truncation scale l_c also shows no systematic dependence on β , but the distributions of l_c differ among expeditions, indicating a clear separation between datasets. Meanwhile, the α values from different expeditions largely overlap and appear interwoven.

Fig.5c–d examine the relationships between the Lévy parameters and the variability of GR increments. The Lévy stability index α shows a fluctuating pattern with respect to the standard deviation of GR increments $\sigma_{\Delta GR}$, without a clear overall trend. In contrast, the truncation scale l_c exhibits a strong positive relationship with $\sigma_{\Delta GR}$, following an approximately linear or power-law scaling across wells (Fig. 5d). This relationship indicates that the cutoff scale of the truncated Lévy process increases systematically with the variability of GR increments.

4. Discussion

4.1 Scale-dependent Lévy statistics and geological heterogeneity

4.1.1 Geological interpretation of Lévy parameters

The Lévy parameters derived from GR increments quantify non-Gaussian variability and finite-scale truncation in well-log signals. Within the truncated Lévy framework, the stability index α controls the decay the increment distribution, while the cutoff scale l_c constrains the effective upper bound of fluctuations (Gupta and Campanha, 2000). For GR increments $\Delta GR = GR(t + \Delta t) - GR(t)$, the exceedance probability approximately follows a truncated power-law relationship

$$P(|\Delta GR| > x) \sim x^{-\alpha} \left[- \left(\frac{(|x| - l_c)_+}{k} \right)^\tau \right].$$

In this study, α is estimated from the Lévy core (Sect. 2.5.1) and therefore reflects the scaling behavior of small-to-moderate fluctuations, serving as a proxy for the overall non-Gaussianity. Smaller α values indicate more irregular fluctuation structures, whereas values approaching 2 suggest a tendency toward Gaussian behavior.

From a geological perspective, GR increments capture lithological variability across depth

intervals Δt , with large increments typically associated with strong facies contrasts. Accordingly, α reflects the intensity and organization of stratigraphic heterogeneity.

Across the 23 wells, α values generally range between ~ 0.6 and 1.2 over three orders of magnitude in Δt (Fig. 6; Fig. 3b), indicating persistent non-Gaussian behavior. Systematic differences among wells suggest variability in lithological organization across depositional settings.

The cutoff parameter l_c characterizes the effective upper bound of Lévy-type fluctuations. At small Δt , variability is typically limited, resulting in low l_c . As Δt increasing, the sampling window integrates broader stratigraphic structures, expanding the fluctuation range. Once Δt approaches characteristic stratigraphic scales, such as depositional cycles or facies stacking patterns, l_c tends to stabilize or fluctuate, reflecting the bounded nature of geological variability (Fig. 3c). Variations in stratigraphic architecture therefore led to distinct l_c behaviors among wells (Fig. 6).

4.1.2 Stratigraphic interpretation: a case study from Expedition 356

The wells from Expedition 356 provide a useful case study for linking scale-dependent Lévy statistics to stratigraphic architecture. Expedition 356 drilled a transect across the Northwest Shelf of Australia, sampling carbonate-dominated successions deposited in shelf to slope environments (Gallagher et al., 2017). Across Sites U1460–U1464, two dominant regimes are observed: (i) inter-lithology contrasts at small Δt , and (ii) intra-unit or large-scale structural variability at larger Δt .

The stability index α exhibits systematic scale dependence across Sites U1460–U1464 (Fig. 7), reflecting differences in lithological interaction structures. At Site U1463, α exhibits relatively low values at small to intermediate Δt . The lithological column indicates frequent alternations between mudstone and carbonate facies (Fig. 8), producing strong lithological contrasts over short depth intervals. Based on core descriptions, the effective thickness of lithological cycles—

approximated by the thinner component within mudstone–carbonate couplets—ranges from ~0.1 to 39.16 m. The observed transition of α from lower to higher values occurs at $\Delta t = 38.1\text{ m}$ (Fig.7), which closely matches the upper bound of this range. This correspondence suggests that, at small Δt , GR increments are strongly influenced by inter-lithology contrasts, whereas at Δt approaching the cycle scale, the increments increasingly integrate complete depositional units, leading to a more stable fluctuation structure and increased α . The corresponding CCDF patterns and GTLF fits show systematic changes in distribution shape with increasing Δt , consistent with this scale-dependent transition (Fig.9).

Site U1464 exhibits a contrasting behavior. The upper interval consists of interbedded carbonate and mudstone units with relatively strong vertical continuity (maximum thicknesses ~94 m and ~146.6 m, respectively), while the lower interval is dominated by a thick carbonate unit (~524.9 m) (Fig.8). The depth offset between the top of the lower carbonate unit and the overlying thick mudstone interval is approximately 169.8 m. At small to intermediate Δt , the sampling window is typically confined within individual lithological units, and GR increments are therefore dominated by intra-lithology variability, resulting in relatively stable fluctuations and higher α values. As Δt increases toward scales comparable to the separation between major lithological units, the increments begin to incorporate contrasts between thick mudstone and carbonate intervals, as well as variability within thick carbonate successions (e.g., differences spanning tens to hundreds of meters). This leads to an increased contribution from large-amplitude variations and is associated with a decrease in α . Consistently, α begins to decline at $\Delta t > 38.1\text{ m}$ and reaches lower values at $\Delta t = 152.4\text{ m}$ (Fig. 7), suggesting a transition from intra-unit to inter-unit variability at larger structural scales.

At Site U1462, shipboard physical property data reveal a dominant 20–30 m cyclicity in magnetic susceptibility and NGR (Gallagher et al., 2017). The increase of α from 0.91 at $\Delta t=15.24$ m to 1.20 at 38.1 m, followed by a plateau at larger scales (Fig. 7), is consistent with a transition at the lithological cycle scale. Below this scale, variability is influenced by frequent facies transitions, whereas above it, GR increments increasingly reflect variability within coherent depositional units.

Sites U1459 and U1461 are dominated by thick carbonate successions with relatively homogeneous lithology (Fig.8). In these settings, the contribution of inter-lithology contrasts appears limited across the investigated scales, and α remains relatively stable (Fig.7).

These patterns are also reflected in l_c . Relatively homogeneous sites (U1459, U1461, U1462) show early saturation of l_c , whereas heterogeneous sites (U1463, U1464) exhibit continued growth, indicating expansion of variability with scale (Fig.7).

Overall, the scale-dependent evolution of α is interpreted to reflect a transition in the dominant source of GR variability, from inter-lithology contrasts at small scales to variability within lithological cycles or larger stratigraphic units at increasing scales. The corresponding crossover scales are therefore more plausibly associated with characteristic lithological organization (e.g., cycle thickness or unit-scale architecture) rather than with individual bed thicknesses alone.

4.1.3 Comparison across depositional settings

A comparison across expeditions reveals that non-Gaussian, Lévy-type behavior is a consistent feature of GR increments, while its scale dependence varies systematically with depositional environment. Wells from Expedition 359, which primarily sample carbonate platform sediments in the Maldives region (Betzler et al., 2016), exhibit relatively stable α values across scales (Fig.7).

In addition, the cutoff parameter l_c remains low (generally below ~ 5) and reaches saturation at small scales ($\Delta t = 7.62$ m) (Figs.6, 7). This indicates that variability is effectively bounded within individual lithological units and does not expand with scale. Consistently, the MSD analysis yields the largest crossover scale of 1.215 ± 0.046 m (Fig.4b, Table 2), implying sustained correlation over relatively long depth intervals and a well-organized vertical structure.

In contrast, wells from Expeditions 367 and 368, drilled along the South China Sea margin, show markedly different behavior. These successions are characterized by frequent interbedding of sandstone, mudstone, carbonate, igneous, and volcanoclastic or metasediments (Sun et al., 2018; Jian et al., 2018), resulting in strong lithological heterogeneity. Correspondingly, α exhibits pronounced variability across scales, and l_c increases continuously with Δt without reaching a clear saturation plateau (Figs.6, 7). Their smaller crossover scales of 0.985 ± 0.119 m (Fig.4b, Table 2) indicate rapid loss of correlation and limited vertical coherence. Together, these features suggest that variability is not intrinsically bounded but instead expands progressively with scale.

Expedition 356 exhibits intermediate behavior between these two end members. The cutoff parameter l_c initially varies but gradually approaches a stable value with increasing scale, while the crossover scale indicates moderate persistence (Fig.4b, Table 2). This pattern reflects a partially organized system (Fig.8), where variability is only weakly constrained at small scales but becomes increasingly bounded as correlation develops.

A unified interpretation is provided by the relationship between l_c and the MSD exponent β (Fig.5b). For Expedition 359, l_c remains nearly constant across the full range of β , indicating that fluctuation amplitudes are largely decoupled from correlation structure and are controlled by intrinsic lithological organization. In Expedition 356, l_c decreases and approaches a stable value

with increasing β , suggesting a progressive coupling between variability and persistence. In contrast, Expeditions 367 and 368 show an approximately linear increase of l_c with β , indicating a strong positive coupling in which larger fluctuations emerge as correlation strengthens, consistent with highly heterogeneous successions lacking a dominant stratigraphic scale.

Overall, although the specific scaling behavior of α and l_c varies across depositional environments, the persistent presence of Lévy-type statistics suggests that such behavior reflects intrinsic properties of vertically heterogeneous stratigraphic systems, rather than site-specific anomalies.

4.2 Stratigraphic architecture encoded in GR increment statistics

The results presented in Section 4.1 demonstrate that the Lévy parameters α , l_c , and the MSD exponent β exhibit coherent, scale-dependent evolution across all studied wells. Such behavior is inconsistent with stationary increment processes and instead indicates that GR increment statistics reflect systematic shifts in the dominant data-generating mechanisms.

Rather than representing a single stochastic regime, GR increments arise from a hierarchy of processes associated with stratigraphic organization, which can be decomposed into three scale-dependent components: (i) interface-dominated increments associated with abrupt lithological boundaries, (ii) intra-lithology variability within individual units, and (iii) inter-lithology contrasts between larger-scale stratigraphic units. As Δt increases, the relative contributions of these components shift progressively, leading to systematic changes in fluctuation structure and the emergence of non-stationary statistical behavior (Figs.3, 4).

Within this framework, the Lévy parameters provide complementary constraints on different aspects of stratigraphic variability. The stability index α reflects the degree of intermittency and

the relative contribution of large fluctuations, and can therefore be interpreted as a proxy for the organization of lithological heterogeneity. The cutoff scale l_c characterizes the effective upper bound of fluctuations and tracks how the amplitude of variability expands or becomes constrained by stratigraphic architecture. The MSD exponent β quantifies the correlation structure of increments and provides a measure of vertical coherence. Their coupled evolution thus encodes both the intensity and spatial organization of geological variability across scales.

A key implication is that the apparent tendency toward Gaussian behavior at large scales is not universal but depends on the degree of stratigraphic organization. When the sampling window is dominated by a single lithology, increments are primarily controlled by intra-lithology variability, with additional contributions from sparse lithological interfaces or event beds. In this case, the central part of the distribution progressively approaches a Gaussian form due to aggregation, but the tails remain heavier than Gaussian, reflecting intermittent contributions from lithological boundaries or rare depositional events. This pattern is consistent with observations from 359-U1464, where the dominance of thick carbonate units reduces lithological heterogeneity within the sampling window, thereby promoting Gaussian-like central behavior while preserving heavy tails from sparse boundaries or event beds (Figs.10, 11).

In contrast, when multiple lithologies with comparable thicknesses coexist within the sampling window, GR increments arise from a mixture of intra-lithology differences and inter-lithology contrasts. Because these components have comparable statistical weight, the resulting increment distribution may deviate from unimodality and exhibit weak multimodal structure. This behavior is observed in wells such as 356-U1464 (Figs.12,13), where thick mudstone and carbonate units alternate and compete. In such cases, the unimodal GTLF model still provides a useful first-order

approximation, but the inferred parameters effectively average over heterogeneous sub-populations, contributing to increased dispersion in α at larger scales.

These observations can be summarized within a scale-dependent conceptual framework (Fig. 14), in which GR increment statistics reflect a hierarchy of geological processes rather than a scale-invariant random process. Apparent anomalous diffusion in well-log data therefore emerges from stratigraphic heterogeneity and scale-dependent aggregation, rather than from a stationary Lévy process. Models that assume scale-invariant increments risk conflating observational scale with intrinsic process behavior and may overlook the hierarchical nature of sedimentary systems.

4.3 Implications for stochastic modelling of well logs

These statistical characteristics have important implications for stochastic modelling of well logs. Traditionally, well logs have been modeled as self-affine stochastic signals exhibiting fractal scaling behavior, often represented using Gaussian fractal processes such as fractional Brownian motion or fractional Gaussian noise (Lu et al., 2003; Adoko and Madani, 2024). However, these models, which assume Gaussian increments, predict exponentially decaying tails and fail to capture the heavy-tailed fluctuations frequently observed in GR increments. As evidenced in Figs. 2d, 9, 10, 11, 12 and 13, large-amplitude fluctuations occur much more frequently than Gaussian models predict, suggesting that Lévy-type stochastic processes may provide a more accurate description of well-log variability.

Because l_c shows a strong relationship with the variability of GR increments (Fig.5d), it may serve as a key parameter for calibrating stochastic well-log simulations. By incorporating scale-dependent Lévy statistics, stochastic models can better represent the complexity of stratigraphic variability and improve uncertainty analysis in stratigraphic interpretation.

The coherent co-evolution of α , l_c , and β with increasing Δt (Figs.3b, c and 4a) indicates that GR increments do not follow a stationary Lévy process but instead reflect scale-dependent changes in governing mechanisms (Fig.14). The distribution of Δt at which $\beta = 1$ (Fig. 4d) shows that the transition from persistent to anti-persistent behavior occurs over systematically different scale ranges among expeditions. Because $\beta = 1$ defines the statistical boundary between these regimes, it provides a natural measure of transition scale. This crossover is not universally consistent but varies with the underlying depositional architecture, as discussed in Section 4.1.3. Thus, the crossover point represents the interaction between geological processes and scale-dependent statistical behavior, rather than a fixed threshold.

These findings suggest that traditional Gaussian models may not capture the full complexity of stratigraphic variability, particularly in systems with significant lithological heterogeneity. Incorporating crossover behavior and scale-dependent Lévy statistics into the modeling process will improve the realism of well-log simulations, especially when simulating complex, heterogeneous geological formations with abrupt facies transitions.

5. Conclusions

This study investigates the scale-dependent statistical behavior of GR increments from 23 well logs, applying a truncated Lévy framework to capture the heavy-tailed fluctuations and scale-dependent variability observed in the data. The results provide significant insights into the hierarchical and non-Gaussian nature of well-log data, demonstrating that the Lévy stability index α , cutoff scale l_c , and MSD exponent β are essential parameters for understanding the complex stratigraphic variability across different scales.

Through detailed parameter estimation, we have developed a robust computational workflow

based on the core-mass matching method, quantile-based γ estimation, and CCDF-based cutoff detection. These methodologies ensure that the Lévy parameters are estimated accurately, minimizing the impact of extreme tail fluctuations. The profile-likelihood optimization procedure further refines the truncation scale l_c and the scaling parameter k , providing a precise fit to the empirical data. This workflow, implemented in Python, allows for efficient and reproducible analysis of heavy-tailed geophysical signals, with all key parameters (α , γ , l_c , k , c) and their respective confidence intervals reported for each well and lag scale.

The results show that GR increment statistics are governed by scale-dependent transitions between different geological mechanisms, rather than by a stationary stochastic process. Apparent Gaussian behavior at large scales depends on lithological composition and does not represent a universal limit.

Comparisons across depositional settings reveal that while Lévy-type behavior is ubiquitous, its scale dependence reflects differences in stratigraphic organization. Systems with strong vertical coherence exhibit bounded variability, whereas highly heterogeneous systems show continued expansion of variability with scale.

These findings highlight the limitations of Gaussian-based models and support the use of truncated Lévy approaches for modeling well-log variability. Incorporating scale-dependent behavior and crossover dynamics provides a more realistic representation of stratigraphic heterogeneity in complex geological systems.

Data and Code Availability

Gamma-ray log data analyzed in this study are publicly available through the International Ocean Discovery Program (IODP) data repository (<https://iodp.tamu.edu/>). Processed datasets and the

Python scripts used for scale-dependent GTLF parameter estimation and MSD analysis are archived at [GitHub\(https://github.com/leizc2019/Scale-dependent-levy-GR.git\)](https://github.com/leizc2019/Scale-dependent-levy-GR.git) with DOI: <https://doi.org/10.5281/zenodo.18831588>.

Declaration of competing interest

The authors declare that they have no known competing financial interest or personal relationships that could have appeared to influence the work reported in this paper.

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Figures

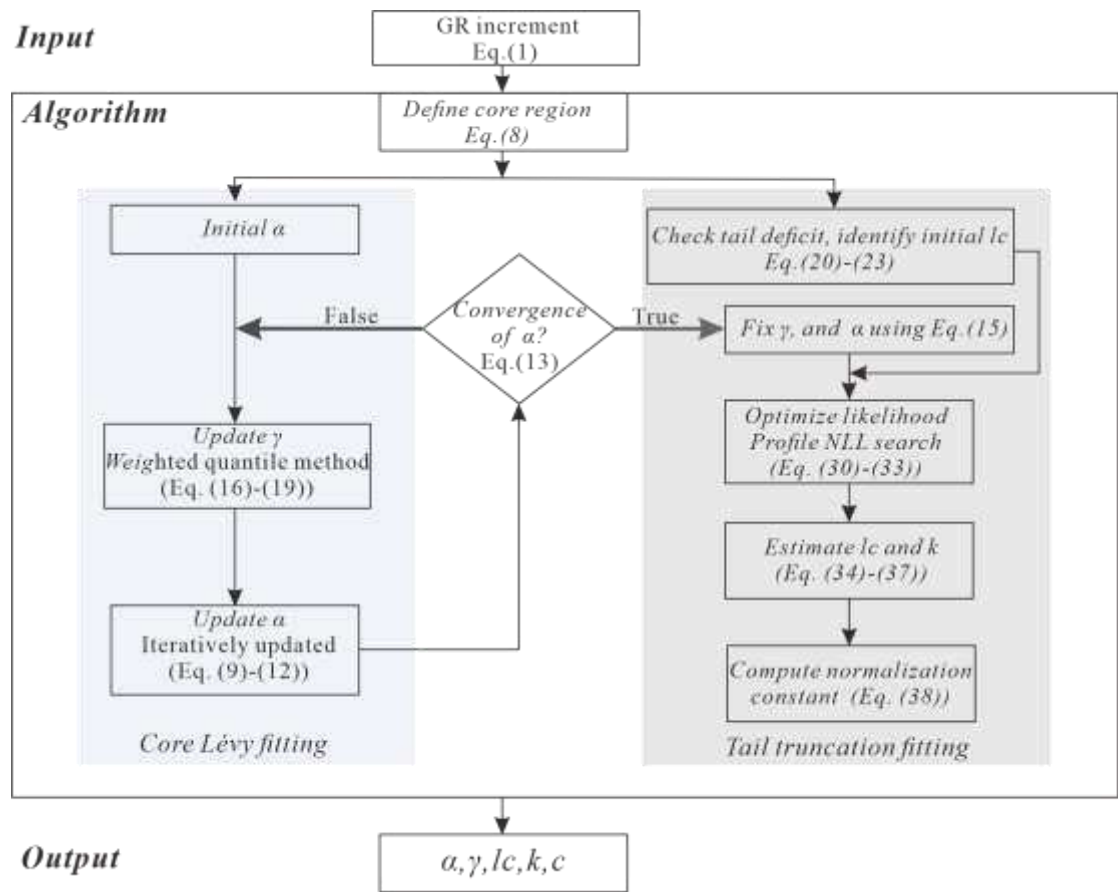


Figure 1. Flowchart of the parameter estimation algorithm for the gradually truncated Lévy flight (GTLF) model based on GR increments. The procedure consists of two stages: (1) estimation of the core Lévy parameters α and γ using core-mass matching and quantile-based methods, and (2) estimation of the truncation parameters l_c and k through CCDF-based initialization followed by profile-likelihood optimization.

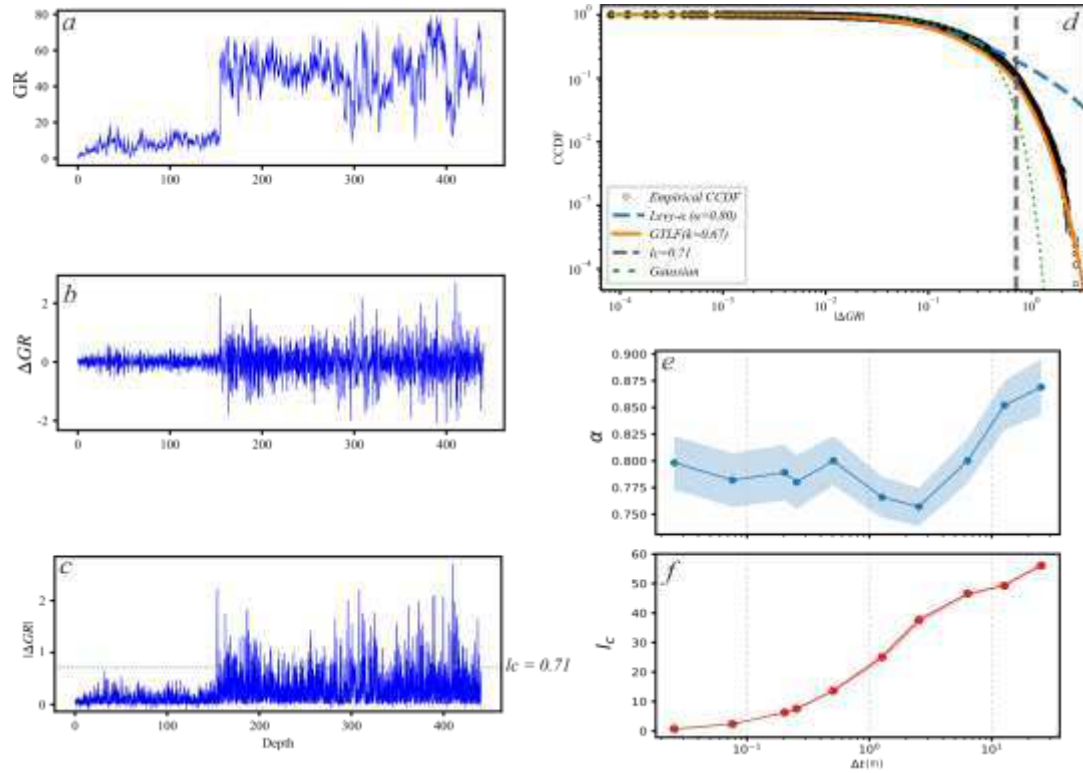


Figure 2. Representative GR record and increment distributions for well 349-U1431E. (a) Original GR log exhibiting intermittent fluctuations. (b) First-order increments ($\Delta t = 1$). (c) Absolute increments $|\Delta GR|$, the green dashed line indicates the estimated truncation scale l_c . (d) Complementary cumulative distribution function (CCDF) of the increments on a log-log scale. Open circles denote empirical data. The distributions are compared with a Lévy-stable model ($\alpha = 0.8$), a Gaussian model, and the GTLF model with estimated parameters $\alpha = 0.8$, $k = 0.67$, and $l_c = 0.71$. (e) Evolution of the stability index α as a function of time scale Δt , where the shaded area represents the admissible interval defined in Eq. (14). (f) Evolution of the truncation scale l_c

across different Δt , with the shaded region indicating the 95% confidence interval defined in Eq. (37).

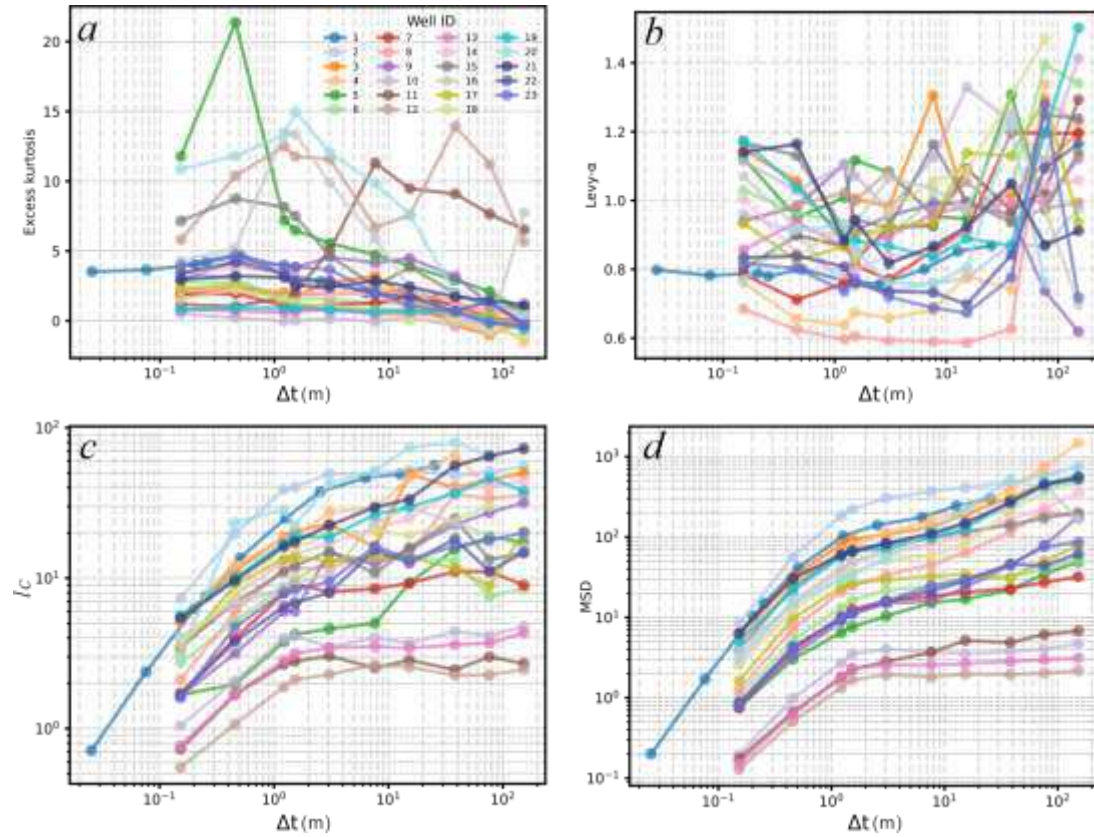


Figure 3. Scale-dependent evolution of stochastic parameters and scaling exponents of GR increments across the 23 study wells. Colors denote individual well IDs and are consistent across all subsequent figures.

(a) Excess kurtosis is plotted as a function of lag scale Δt . A general decrease toward zero is observed with increasing scale, although non-monotonic variations occur in several wells.

(b) Variation of the Lévy stability index (α) with lag scale Δt . The index spans a broad range ($0.6 < \alpha < 1.5$) and shows well- and scale-dependent non-monotonic behavior, reflecting varying intermittency across geological settings.

(c) Characteristic truncation scale l_c versus Δt , showing systematic growth with increasing lag scale.

(d) Mean squared displacement (MSD) of GR increments as a function of Δt (log-log scale). Linear trends in log-log space indicate approximate power-law scaling, from which the MSD exponent β is derived.

(d) Statistical distribution of the crossover scale Δt^* (where $\beta = 1$) grouped by expedition. The parameter Δt^* marks the critical lag scale at which the system shifts from super-diffusive ($\beta > 1$) to sub-diffusive ($\beta < 1$) behavior. Expeditions 367 and 368 are combined due to similar tectonic positions within the same geological setting and broadly comparable statistical behavior.

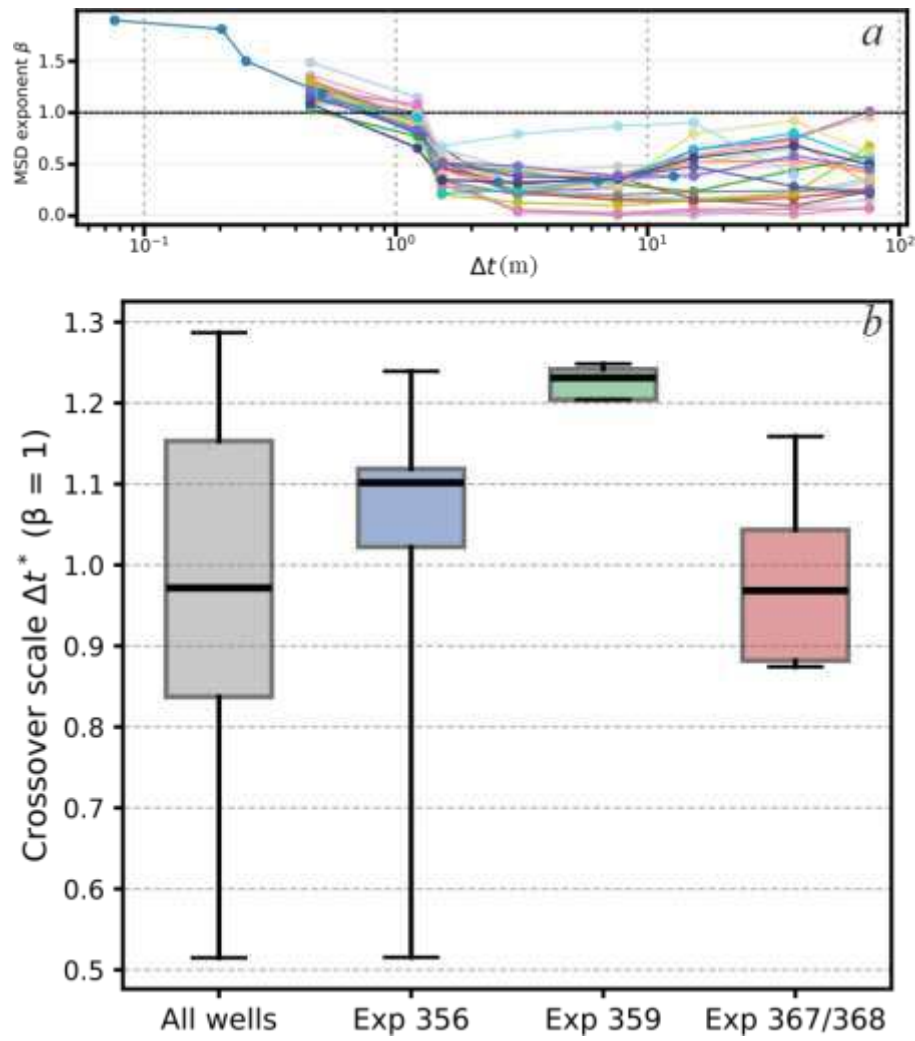


Figure 4. Scale-dependent MSD exponents and crossover scales. (a) The MSD exponent β as a function of the depth interval Δt for all studied wells. The horizontal dashed line at $\beta = 1$ denotes the threshold for normal diffusive (Brownian motion), where Δt^* represents the crossover scale at which β intersects this threshold. (b) Box plots showing the distribution of the crossover scale Δt^* for the entire dataset ("All wells") and categorized by expedition groups (Exp 356, Exp 359, and Exp 367/368). The thick central lines, box limits, and whiskers represent the median, interquartile range, and full data range, respectively.

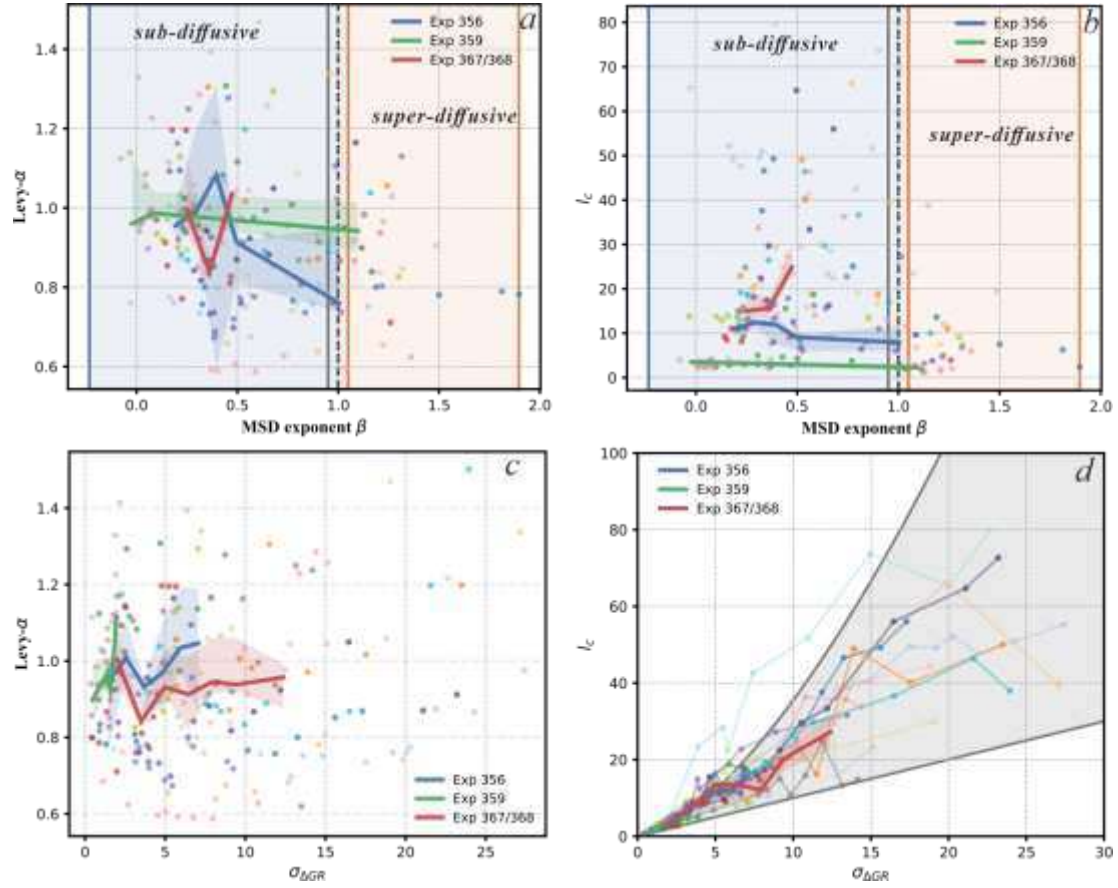
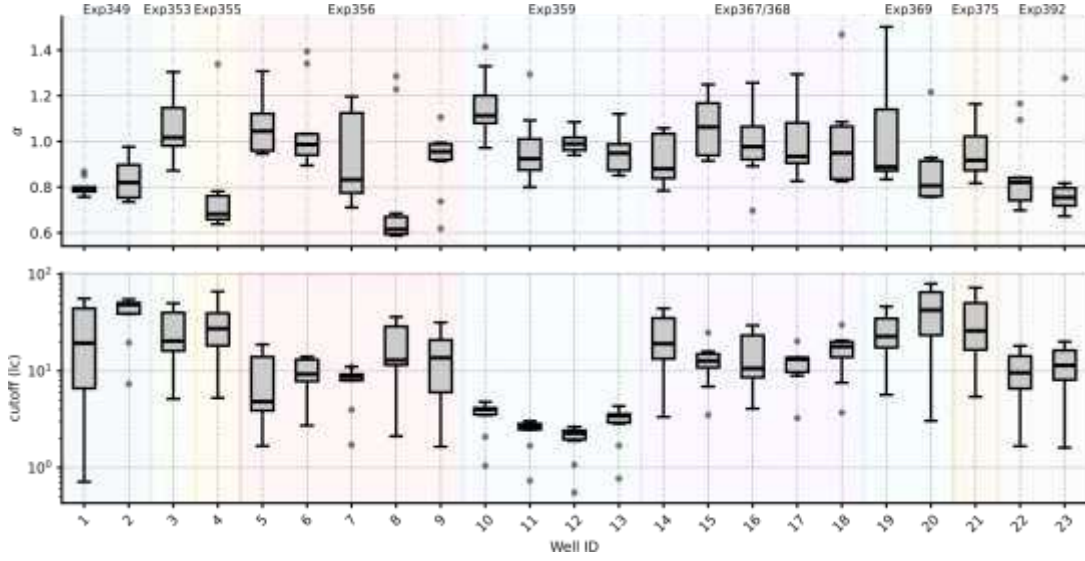


Figure 5. Relationships among statistical parameters of GR increments.

- (a) α versus β . Dispersion in α increase toward lower β values. The dashed line at $\beta = 1$ marks the transition between sub- and super-diffusive regimes. Shaded backgrounds denote sub-diffusive ($\beta < 0.95$), transitional ($0.95 \leq \beta \leq 1.05$) and super-diffusive ($\beta > 1$) regimes. Colored lines indicate the average trends for different expeditions: Exp 356 (blue), Exp 359 (green), and Exp 367/368 (red). Shaded envelopes represent the variability around the mean trend.
- (b) l_c versus β .
- (c) α versus $\sigma_{\Delta GR}$. Inter-well heterogeneity dominates and systematic correlation is weak.
- (d) l_c versus $\sigma_{\Delta GR}$. A strong, approximately linear to power correlation indicates that truncation scales closely track fluctuation intensity. The shaded band denotes the scaling envelope $l_c \sim \sigma_{\Delta GR}^n$ with $1 \leq n \leq 1.5$.

660



661

662 **Figure 6. Boxplots showing the distribution of fitted Lévy stability index α (top panel) and**

663 **cutoff scale l_c (bottom panel) across all analyzed lag scales Δt for the 23 IODP wells. Each**

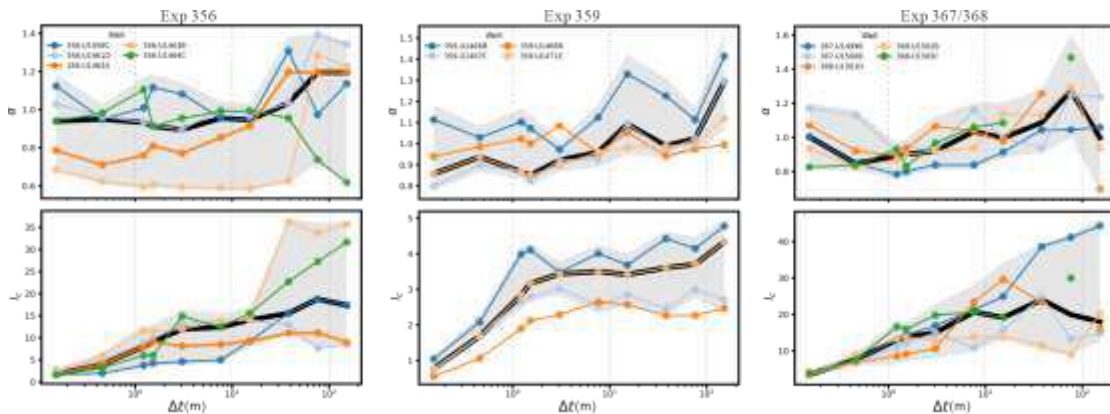
664 **boxplot summarizes the ensemble of parameter estimates obtained from independent GTLF fits at**

665 **every discrete lag Δt per well. The boxes represent the interquartile range (25th–75th percentiles),**

666 **with the median indicated by the central line; whiskers extend to the most extreme values within 1.5**

667 **$\times$ IQR, and individual points denote outliers.**

668



669

670 **Figure 7. Scale-dependent evolution of Lévy stability index α (top row) and truncation cutoff**

671 **scale l_c (bottom row) as a function of lag interval Δt (converted to physical depth in meters)**

for wells grouped by IODP expedition. Each panel shows results from one expedition: Expedition 356 (left), Expedition 359 (center), and Expeditions 367/368 (right). Individual well trajectories are color-coded and labeled. Thick black lines represent the median parameter value across all wells in the expedition at each Δt . Shaded regions indicate the admissible range (confidence band) for each well's parameter estimates, derived from the core-mass matching tolerance criterion for α and profile-likelihood intervals for l_c .

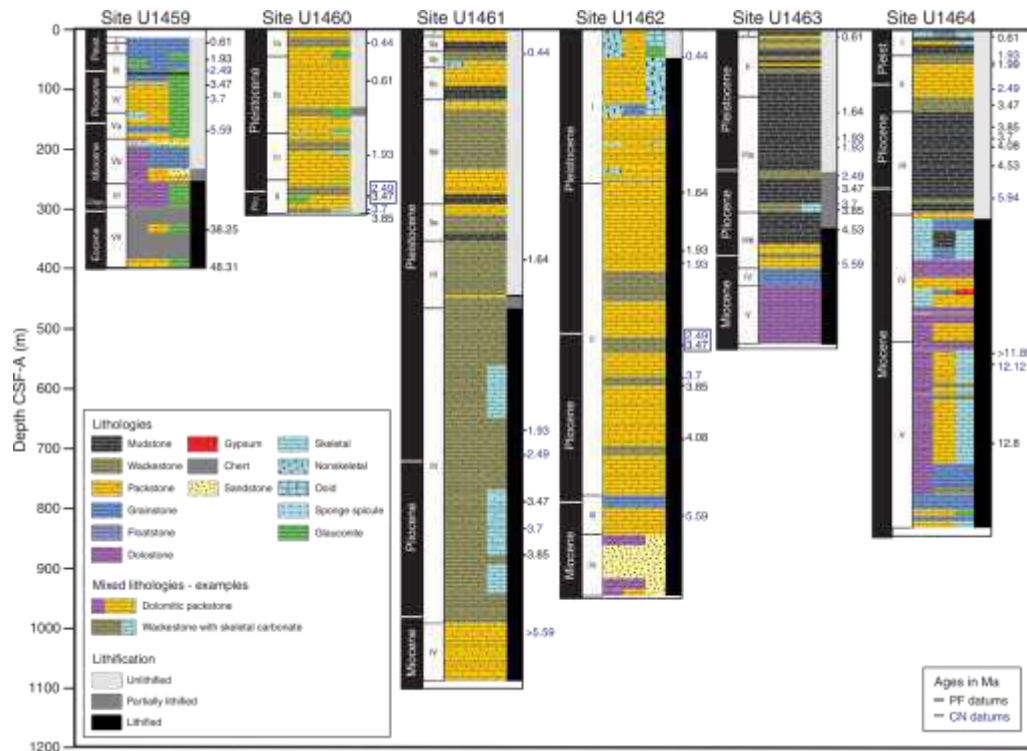


Fig.8 Summary of lithology and unit boundaries for each site, expedition 356, modified after Expedition 359 Scientists, Gallagher et al., (2017).

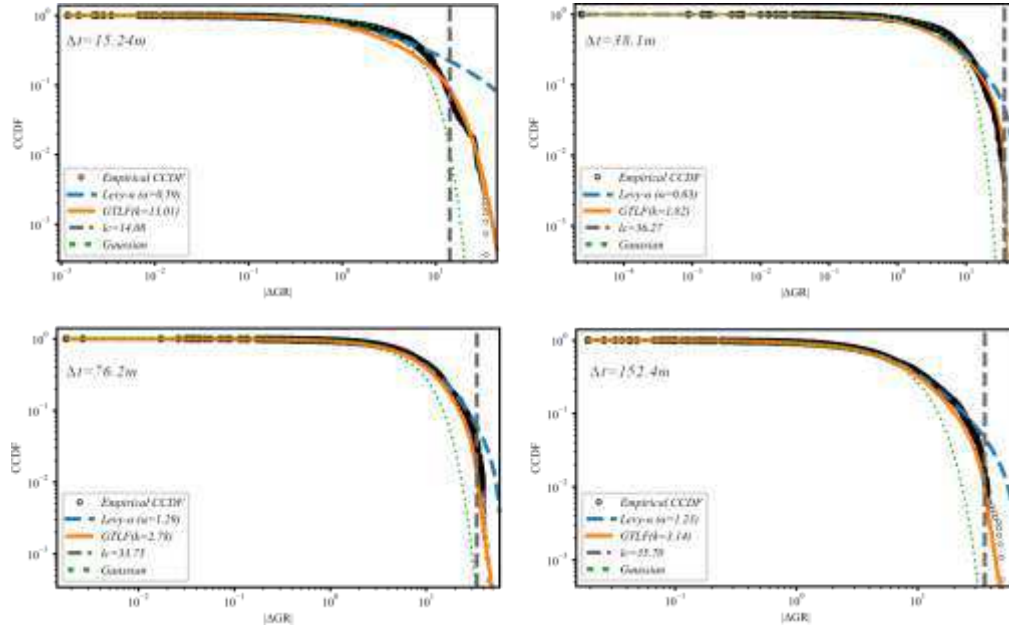


Figure 9. CCDFs of absolute GR increments at four representative lag scales Δt (15.24 m, 38.1 m, 76.2 m, 152.4 m) for well 356-U1463, comparing empirical CCDFs (black circles) with fits from the GTLF model (orange solid line), pure Lévy stable distribution (blue dashed line), and Gaussian distribution (green dotted line). Vertical dashed lines mark the fitted truncation scale l_c for each GTLF model. The distribution shape changes systematically with increasing Δt : at smaller to intermediate scales ($\Delta t = 15.24$ m to 38.1 m), the empirical CCDF exhibits pronounced heavy tails corresponding to a low stability index ($\alpha \approx 0.6$), while at larger scales ($\Delta t = 76.2$ m and 152.4 m), the tails become progressively less extreme and α increases and stabilizes around 1.2. All CCDFs are displayed on log–log axes to emphasize tail behavior. These patterns illustrate the overall scale-dependent evolution of increment statistics, as also shown in the full α vs. Δt trend in Fig. 7.

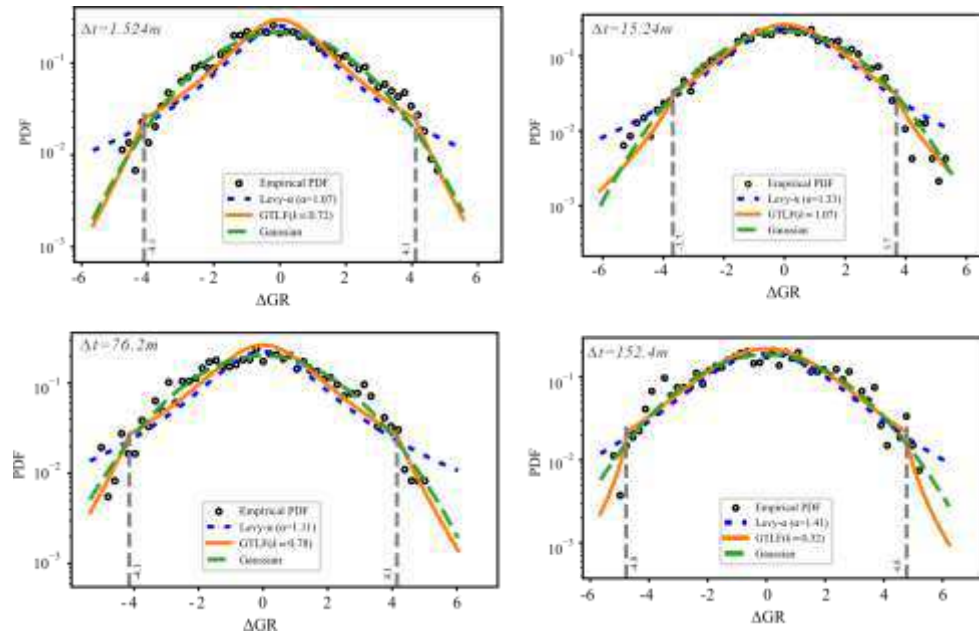


Figure10. PDFs of GR increments at four representative lag scales Δt (1.524 m, 15.24 m, 76.2 m, 152.4 m) for well 359-U1466 (Expedition 359), comparing empirical PDF (black circles) with fits from GTLF model (orange solid line), pure symmetric Lévy stable distribution (blue dashed line), and Gaussian distribution (green dashed line). At smaller scales ($\Delta t = 1.524$ m), the empirical distribution shows heavy tails and a peaked center, well-captured by the GTLF model. As Δt increases (to 15.24 m, 76.2 m, and 152.4 m), the central part of the empirical PDF approaches the Gaussian shape, with the GTLF model consistently providing the best fit across scales, maintaining heavy tails in the far extremes. This illustrates the transition from heavy-tailed behavior to Gaussian-like central behavior, reflecting the hierarchical and bounded variability characteristic of carbonate platform settings in Expedition 359.

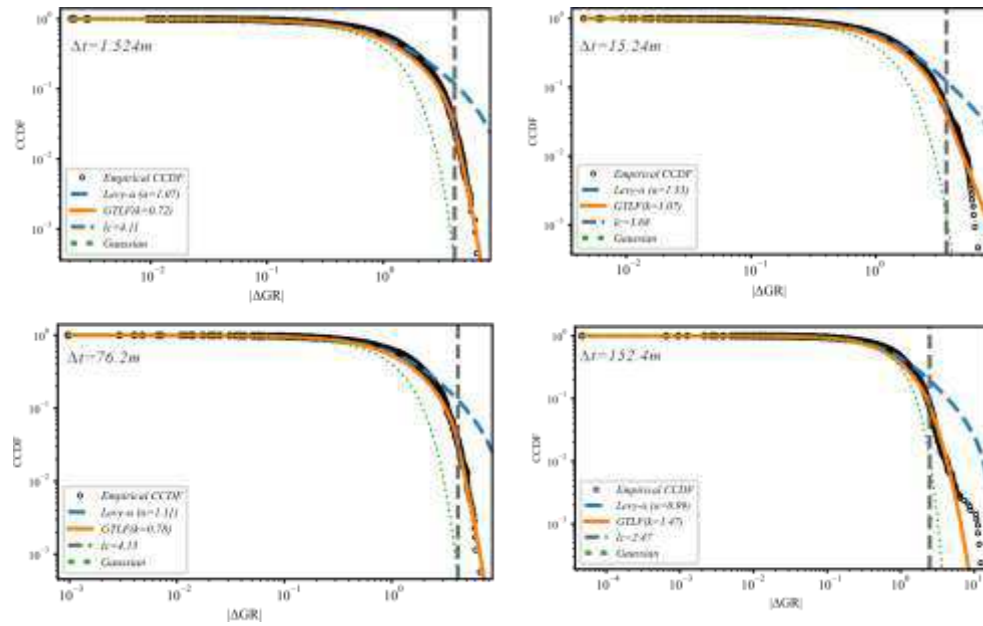


Figure 11. CCDFs of absolute GR increments at four representative lag scales Δt (1.524 m, 15.24 m, 76.2 m, 152.4 m) for well 359-U1466 (Expedition 359), comparing empirical CCDFs (black circles) with fits from the GTLF model (orange solid line), pure Lévy stable distribution (blue dashed line), and Gaussian distribution (green dotted line). The GTLF model provides the best fit at all scales, especially in the tail regions. At smaller Δt (1.524 m and 15.24 m), the empirical CCDF exhibits heavy tails, which deviate from both the Gaussian and pure Lévy models. As Δt increases (to 76.2 m and 152.4 m), the empirical tail gradually becomes closer to the Gaussian, though it remains heavier in the extremes, consistent with the stratigraphic bounding behavior observed in Expedition 359.

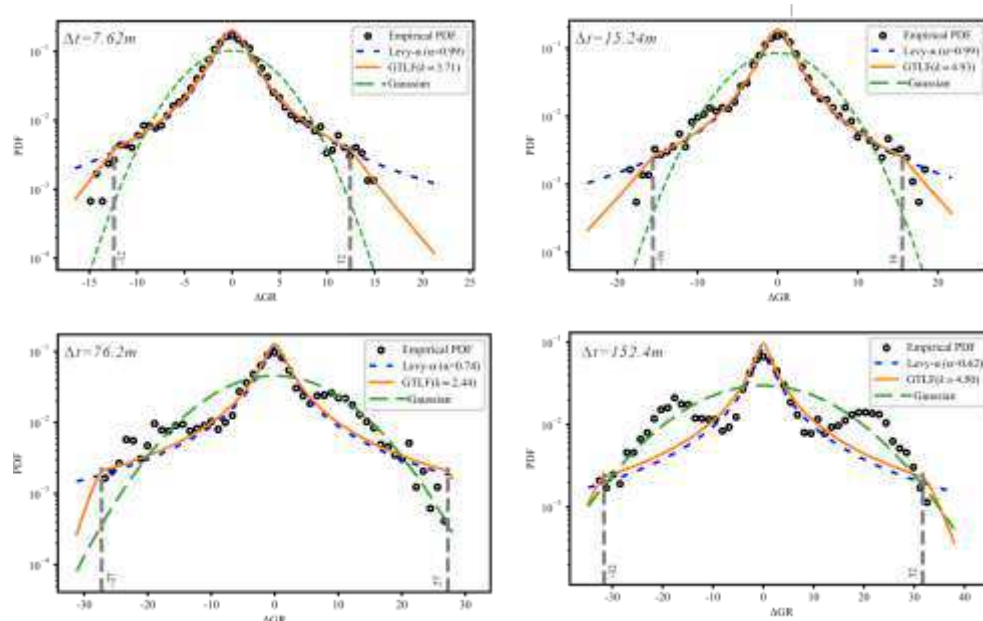


Figure 12. PDF of GR increments at different Δt (7.62 m, 15.24 m, 76.2 m, 152.4 m) for well 356-U1464, comparing empirical PDF (black circles) with fits from GTLF model (orange solid line), pure symmetric Lévy stable distribution (blue dashed line), and Gaussian distribution (green dashed line). At smaller Δt (top-left), the distribution closely follows a GTLF distribution ($\alpha = 0.99$), as Δt increases, the distribution transitions from a unimodal to a bimodal (multi-peak) structure (bottom panels), with a decrease in the stability index α , reflecting the competition between intra-lithology variability and inter-lithology contrasts and a shift toward a more complex stratigraphic variability.

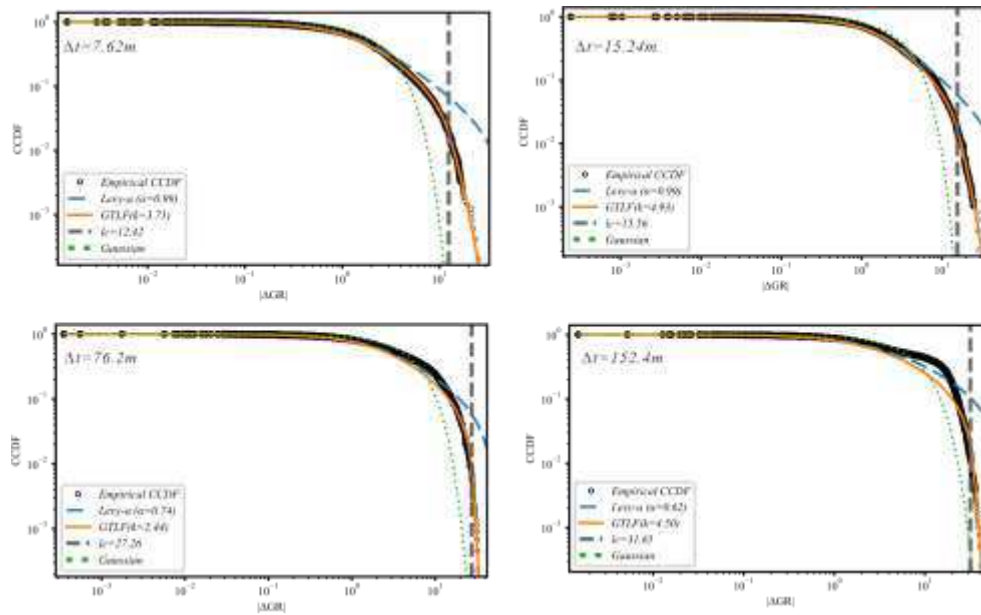


Figure 13. CCDFs of GR increments at different Δt (7.62 m, 15.24 m, 76.2 m, 152.4 m) for well 356-U1464, comparing empirical CCDFs (black circles) with fits from the GTLF model (orange solid line), pure Lévy stable distribution (blue dashed line), and Gaussian distribution (green dotted line). The GTLF model provides the best fit at smaller Δt , where the distribution is dominated by intra-lithology variability (top-panels). However, at larger Δt (bottom-panels), as the intra-lithology differences and inter-lithology contrasts begin to compete and the GR increments exhibit multi-peak behavior as shown in Fig.12, the GTLF model's fit worsens, with the model failing to adequately capture the increasing complexity of the distribution. This highlights the diminishing performance of the GTLF model when the distribution becomes more complex.

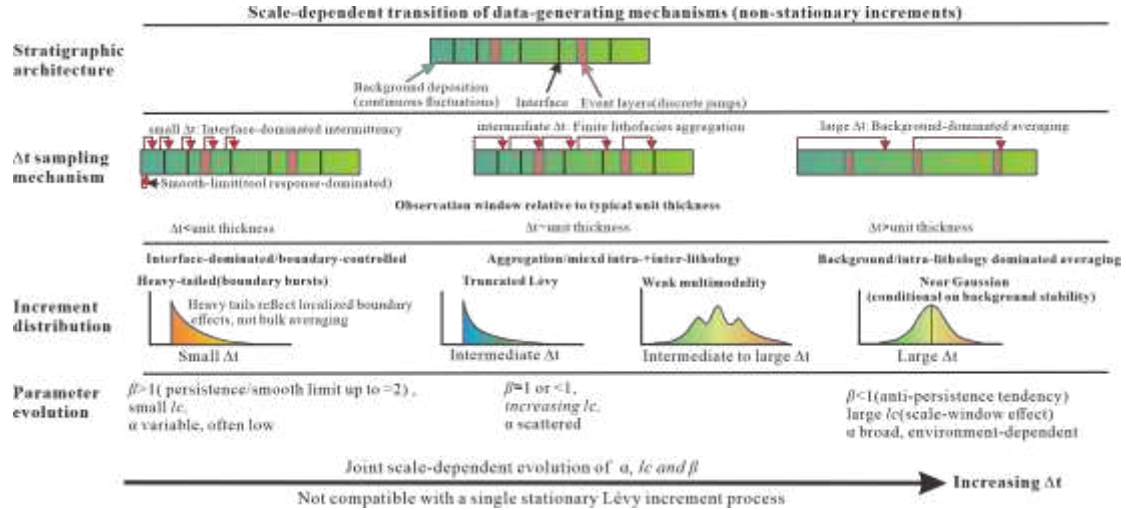


Figure 14. Scale-dependent transition of GR increment statistics and underlying mechanisms.

Schematic showing how stratigraphic architecture and observational scale jointly control increment behavior. For $\Delta t < \text{unit thickness}$, interface-driven intermittency produces heavy tails and persistent dynamics ($\beta > 1$), with possible smooth-limit behavior at the finest sampling. As Δt approaches stratigraphic cycle thickness, aggregation of finite lithofacies yields truncated Lévy behavior, increasing effective truncation scale (l_c). For $\Delta t > \text{unit thickness}$, increments integrate background sedimentation and may approach near-Gaussian forms (conditional on background stability), with decreasing β and large fitted l_c reflecting the scale-window effect. The coherent yet dispersed evolution of α , l_c , and β across scales indicates non-stationary increments arising from scale-dependent data-generating mechanisms rather than a single stationary Lévy process.

Tables

Table 1. Summary of IODP well gamma-ray datasets used in this study. well identifiers (well id), sampling intervals (dt), depth range, and total number of GR measurements for the 23 wells analyzed.

Well id	Well	Sample interval	Depth range (m)	Total samples
1	349-U1431E	0.0254	[0, 440.8424]	17357
2	349-U1433B	0.1524	[0, 813.5112]	5339
3	353-U1445A	0.1524	[0, 420.0144]	2757
4	355-U1456C	0.1524	[0, 459.9432]	3019
5	356-U1459C	0.1524	[0.1304, 378.3872]	2483
6	356-U1461D	0.1524	[0, 992.5812]	6514
7	356-U1462A	0.1524	[0, 779.9832]	5119
8	356-U1463B	0.1524	[0, 432.9684]	2842
9	356-U1464C	0.1524	[0.1524, 747.0648]	4902
10	359-U1466B	0.1524	[0, 342.5952]	2249
11	359-U1467C	0.1524	[0, 695.2448]	4563
12	359-U1468B	0.1524	[0.1381, 799.9333]	5249
13	359-U1471E	0.1524	[0, 983.5896]	6455
14	367-U1499B	0.1524	[0, 987.3996]	6480
15	367-U1500B	0.1524	[0, 1088.593]	7144
16	368-U1501D	0.1524	[0.038, 267.6524]	1757
17	368-U1502B	0.1524	[0, 853.44]	5601
18	368-U1505C	0.1524	[0.0806, 306.7094]	2013
19	369-U1512A	0.1524	[0.1239, 691.2579]	4536
20	369-U1513D	0.1524	[0.0451, 339.8971]	2231
21	375-U1520C	0.1524	[0.072, 910.2051]	5973
22	392-U1579D	0.1524	[0.0709, 722.4459]	4741
23	392-U1580A	0.1524	[0.1404, 502.7556]	3299

Table 2. Summary of crossover scales for the studied wells. The table lists the crossover values for individual wells, along with the calculated mean and standard deviation (Std) for the entire dataset ("all wells") and for specific expedition groups (Exp 356, Exp 359, and Exp 367/368) containing multiple well records.

Well	Crossover (m)	Mean	Std	Mean	Std	Mean	Std	Mean	Std
		all wells		Exp 356		Exp 359		Exp 367/368	
349-U1431E	0.771								
349-U1433B	1.287								
353-U1445A	0.913								
355-U1456C	0.896								
356-U1459C	0.515								
356-U1461D	1.022								
356-U1462A	1.119			0.999	0.282				
356-U1463B	1.239								
356-U1464C	1.102								
359-U1466B	1.239								
359-U1467C	1.223								
359-U1468B	1.147	0.968	0.234			1.215	0.046		
359-U1471E	1.248								
367-U1499B	1.043								
367-U1500B	0.968								
368-U1501D	1.159							0.985	0.119
368-U1502B	0.882								
368-U1505C	0.874								
369-U1512A	0.972								
369-U1513D	0.547								
375-U1520C	0.557								
392-U1579D	0.730								
392-U1580A	0.800								

Supplementary Files

This is a list of supplementary files associated with this preprint. Click to download.

- [TableS1.csv](#)