

Appendix

The 0D ionization model is derived using the conservation of mass, momentum, and energy equations. It is governed by the following system of eight nonlinear ODEs, which consists of five plasma states (N_i, N_n, T_e, U_i, U_e), three electrical states (V_{CF}, I_{LH}, V_D), one output (I_D), and one control (V_M). It is based on Troyetsky's 0D model, but contains additions with respect to the RLC harness model and voltage control [?]:

$$\frac{dN_i}{dt} = -\frac{N_i U_i}{L_{ch}} - \frac{2N_i U_{i,w}}{R_\Delta} + N_i N_n \zeta_{ion} \quad (1)$$

$$\frac{dN_n}{dt} = -\frac{(N_n - N_{int})U_n}{L_{ch}} - N_i N_n \zeta_{ion} \quad (2)$$

$$\frac{dT_e}{dt} = \frac{2}{3N_i} (-N_i U_e E - N_i \varepsilon_w \nu_w - N_i N_n \zeta_{ion} \chi \varepsilon_{ion} - \frac{3}{2} T_e \frac{dN_i}{dt} - \frac{5}{2} \frac{N_i U_e T_e}{L_{ch}}) \quad (3)$$

$$\frac{dU_i}{dt} = \frac{eE}{M_i} + \frac{2U_{i,w}U_i}{R_\Delta} + N_n \zeta_{ion} (U_n - U_i) \quad (4)$$

$$\frac{dU_e}{dt} = 0 \quad (5)$$

$$\frac{dV_{CF}}{dt} = -\frac{I_{LH}}{C_F} + \frac{V_P - V_{CF}}{R_F C_F} \quad (6)$$

$$\frac{dI_{LH}}{dt} = \frac{V_{CF} - V_M - R_H I_{LH} - V_D}{L_H} \quad (7)$$

$$\frac{dV_D}{dt} = \frac{I_{LH}}{C_H} - \frac{e A_C N_i (U_i - U_e)}{C_H} \quad (8)$$

The discharge current is governed by the following relation that links the plasma and electrical circuit differential equations:

$$I_D = e A_C N_i (U_i - U_e) \quad (9)$$