Ice Giants Revisited: Uranus and Neptune as Magma Ocean Worlds

Supplemental Information

April 3, 2026

Neptune Summary

Table 1: Neptune interior model summary. The model has two fixed inputs (Section 1) and two free parameters optimized by MCMC (Section 2). Section 3 compares model predictions to observations.

Fixed Inputs (set by Neptune)

Parameter	Value	Source
Total mass M	1.024×10^{26} kg	Jacobson (2009)[a]
Equilibrium temperature T_{eq}	47 K	Voyager 2[b]

Free Parameters (MCMC best fit)

Parameter	Best-fit value	68% interval
Binodal surface pressure P_{binodal}	10.12 GPa	$9.20^{+0.67}_{-1.27}$ GPa
Bulk H_2 mass fraction $f_{\mathrm{H_2}}$	10.07 wt%	$9.73^{+0.25}_{-0.37}$ wt%

Model Predictions vs. Neptune Observations

Observable	Model	Neptune observed
1-bar radius R_{1bar}	24,759 km	24,764 km[c]
Bulk density ρ	1610 kg m^{-3}	1638 kg m^{-3}[c]
$J_2 \times 10^6$	3535.5	3528.9 ± 48.9[†,d]
$J_4 \times 10^6$	-35.84	-35.83 ± 10.76[†,d]
Norm. moment of inertia C/MR^2	0.2313	0.2315 ± 0.0046[e]
Internal luminosity L_{int}	1.58×10^{15} W	~ 2–3×10^{15} W[f]

[†] 1σ uncertainty used in the MCMC cost function (`cal` sigma mode): LOO-weighted in-sample calibration residual for J_2 (48.9×10^{-6}, 1.4%), floored at the observational sigma for J_4 (10.76×10^{-6}, 30%). J_2 and J_4 are consistency checks; all four observables are within 0.13σ at the maximum-likelihood point.

[a] (Jacobson, 2009).

[b] (Conrath et al., 1989) (Voyager 2 IRIS; equilibrium temperature derived from infrared measurements).

[c] (Lindal, 1992) (Voyager 2 radio occultation; 1-bar equatorial radius 24 766 km adopted; bulk density computed from mass and volumetric mean radius).

[d] J_2 and J_4 from (Wang et al., 2023), originally at $R_{\mathrm{ref}} = 25{,}225$ km, renormalized to $R_{\mathrm{eq}} = 24{,}766$ km following (Morf and Helled, 2025).

[e] (Nettelmann et al., 2013); 2% uncertainty assumed.

[f] (Pearl and Conrath, 1991).

MCMC Estimation for Two Free Parameters

Table 2: Posterior parameter estimates and maximum-likelihood solution.

Parameter	Posterior Median	68% Interval
Surface pressure P_{binodal} (GPa)	9.197	$+0.666$ / -1.266
Bulk H_2 fraction f_{H_2} (wt%)	9.726	$+0.252$ / -0.373

Maximum likelihood solution	
P_{binodal} (GPa)	10.1240
f_{H_2} (wt%)	10.0659
$\ln p$	-0.0104
χ^2_{total}	0.0207
$\chi^2_{\mathrm{reduced}}$	0.0104 (d.o.f. = 2)

Predicted Observables vs. Neptune (Cost Function)

Table 3: Best-fit predicted observables compared to Neptune observations. All residuals are in units of the total 1σ uncertainty used in the cost function.

Observable	Predicted	Observed	Residual	Δ/σ
$J_2 \times 10^6$	3535.51	3528.91	$+6.60$	$+0.13\sigma$
$J_4 \times 10^6$	-35.84	-35.83	-0.01	-0.00σ
C/MR^2	0.2313	0.2315	-0.0002	-0.05σ
R_{1bar} (km)	24759.1	24764.0	-4.9	-0.02σ

Table 4: Observational uncertainties used in the MCMC cost function (`cal` sigma mode). σ_{cal} for J_2 is the LOO-weighted in-sample residual (1.4%). σ_{cal} for J_4 equals the observational sigma (obs floor applied). σ_{total} is the value actually used in the cost function.

Quantity	σ_{obs}	σ_{cal}	σ_{total}	Basis
$J_2 \times 10^6$	4.14	48.94 (1.4%)	48.94	LOO-wt
$J_4 \times 10^6$	10.76	10.76 (30.0%)	10.76	obs floor
C/MR^2	±0.0046 (assumed 2%)			—
R_{1bar} (km)	±247.6 (assumed 1%)			—

Predicted Bulk Planet Properties

Table 5: Bulk planet properties at the best-fit parameters.

Property	Value
Total mass, input Neptune value	1.024×10^{26} kg (17.15 $M_\oplus$)
Radius at 1-bar level ($R_\oplus$)	3.886
Radius at 1-bar level (km)	24,759.2
Neptune observed bulk density	≈ 1638 kg m^{-3}
Normalized moment of inertia C/MR^2 (1-bar)	0.231

Predicted Thermal Properties

Table 6: Thermal properties of the best-fit model.

Property	Value
Equilibrium temperature $T_{\rm eq}$, input Neptune value	47.0 K[b]
Radiation temperature $T_{\rm rad}$	43.74 K
Internal luminosity $L_{\rm int}$	1.580×10^{15} W
Neptune observed $L_{\rm int}$	$\sim$ 2–3 $\times 10^{15}$ W[f]
Central temperature	7271 K
Surface temperature (base of atmosphere)	2750 K
Mass-weighted temperature	1704 K
Temperature at radiative–convective boundary $T_{\rm rcb}$	650 K

Predicted Core Structure

Table 7: Core (silicate + H_2–silicate mixture) properties.

Property	Value
Core mass ($M_\oplus$)	16.196
Core radius ($R_\oplus$)	3.042
Metal core radius ($R_\oplus$)	0.000
Core volume (m^3)	3.049×10^{22}
Bulk core density (kg m^{-3})	3172.3
H_2 mass fraction in core	0.048
Thermal energy, core (J)	8.741×10^{32}
Gravitational potential energy, core (J)	-2.034×10^{34}

Predicted Atmosphere Structure

Table 8: Atmospheric envelope properties.

Property	Value
Atmosphere mass (kg)	5.700×10^{24}
Atmosphere mass fraction ($M_{\rm atm}/M_p$)	5.565×10^{-2}
H_2 gas mass fraction of planet	5.272%
H_2 condensate mass fraction of planet	2.342×10^{-3}%
H_2 mass fraction in atmosphere (gas)	0.9995
H_2 mass fraction whole planet	0.1007
Surface pressure at base of atmosphere	1.012×10^5 bar (= 10.12 GPa)
Pressure at base of atmosphere (weight)	5.175×10^5 bar
Radiative–convective boundary height	4805 km
$R_{\rm rcb}/R_c$	1.248
Pressure at rcb	201.4 bar
Optical depth at rcb	9.52×10^4
$\lambda_{\rm rcb}$	120.12
Convection inhibited layer thickness	1.02×10^5 m
Boundary layer thickness, envelope	70.4 m
$R(1\,\text{bar})/R_c$	1.278
Atmosphere volume (1-bar, m^3)	3.309×10^{22}
Mean atm. density (1-bar, kg m^{-3})	172.2
Planet volume fraction: core	0.474
Planet volume fraction: atmosphere	0.526

Predicted Total Energy Budget

Table 9: Integrated energy budget for the full planet.

Quantity	Value (J)
Thermal energy, core	$+8.741 \times 10^{32}$
Thermal energy, planet	$+9.249 \times 10^{32}$
Gravitational potential energy, core	-2.034×10^{34}
Gravitational potential energy, planet	-2.151×10^{34}
Total energy (planet)	-2.058×10^{34}

Uranus Summary

Table 10: Uranus interior model summary. The model has two fixed inputs (Section 1) and two free parameters optimized by MCMC (Section 2). Section 3 compares model predictions to observations.

Fixed Inputs (set by Uranus)

Parameter	Value	Source
Total mass M	8.681×10^{25} kg	Jacobson & Park (2025) [A]
Equilibrium temperature T_{eq}	58 K	Pearl & Conrath (1990) [B]

Free Parameters (MCMC best fit)

Parameter	Best-fit value	68% interval
Binodal surface pressure P_{binodal}	9.84 GPa	$9.23^{+0.61}_{-1.13}$ GPa
Bulk H_2 mass fraction f_{H_2}	10.90 wt%	$10.47^{+0.35}_{-0.68}$ wt%

Model Predictions vs. Uranus Observations

Observable	Model	Uranus observed
1-bar radius R_{1bar}	25,432 km	25,559 km [C]
$J_2 \times 10^6$	3507.30	3509.29 ± 58.9 [†,D]
$J_4 \times 10^6$	-35.56	-35.52 ± 0.466 [†,D]
Norm. moment of inertia C/MR^2	0.2217	0.2250 ± 0.0045 [E]
Bulk density ρ [‡]	1260 kg m^{-3}	$\approx$ 1270 kg m^{-3}
Internal luminosity L_{int} [§]	5.29×10^{14} W	$\sim 6.4 \times 10^{14}$ W [B]

[†] Uncertainties used in the MCMC cost function (`cal` sigma mode). J_2 sigma (58.871×10^{-6}, 1.68%) is the LOO-weighted in-sample calibration residual. J_4 sigma (0.466×10^{-6}, 1.31%) equals the observational sigma (obs floor applied). For Uranus, $R_{\mathrm{ref}} = R_{\mathrm{eq}} =$ 25,559 km (French et al. 2024 = Lindal et al. 1987), so published gravity values are already normalized to the 1-bar equatorial radius; no renormalization was applied.

[‡] Uranus observed bulk density $\approx$ 1270 kg m^{-3} uses the IAU volumetric mean radius.

[§] Internal luminosity based on $\mathrm{Ra}/\mathrm{Ra}_c = 10^{10}$.

[A] (Jacobson and Park, 2025).
[B] (Pearl and Conrath, 1990) (Voyager 2 IRIS; equilibrium temperature and internal luminosity for Uranus).
[C] (Lindal et al., 1987) (Voyager 2 radio occultation; 1-bar equatorial radius 25 559 km).
[D] (French et al., 2024); reported at $R_{\mathrm{ref}} = R_{\mathrm{eq}} =$ 25,559 km — no renormalization required. Rotation period from (Desch et al., 1986).
[E] (Nettelmann et al., 2013); see also (Podolak and Helled, 2012). 2% uncertainty assumed.

MCMC Estimation for Two Free Parameters

Table 11: Posterior parameter estimates and maximum-likelihood solution.

Parameter	Posterior Median	68% Interval
Surface pressure P_{binodal} (GPa)	9.228	$+0.606$ / -1.125
Bulk H_2 fraction f_{H_2} (wt%)	10.465	$+0.346$ / -0.684

Maximum likelihood solution	
P_{binodal} (GPa)	9.8392
f_{H_2} (wt%)	10.8979
$\ln p$	-0.3948
χ^2_{total}	0.7897
$\chi^2_{\mathrm{reduced}}$	0.3948 (d.o.f. = 2)

Predicted Observables vs. Uranus (Cost Function)

Table 12: Best-fit predicted observables compared to Uranus observations. All residuals are in units of the total 1σ uncertainty used in the cost function.

Observable	Predicted	Observed	Residual	Δ/σ
$J_2 \times 10^6$	3507.29	3509.29	-2.00	-0.03σ
$J_4 \times 10^6$	-35.56	-35.52	-0.03	-0.07σ
C/MR^2	0.2217	0.2250	-0.0033	-0.73σ
R_{1bar} (km)	25431.4	25559.0	-127.6	-0.50σ

Table 13: Observational uncertainties used in the MCMC cost function (`cal` sigma mode). σ_{cal} for J_2 is the LOO-weighted in-sample residual from the all-4-planet fit (Jupiter, Saturn, Uranus, Neptune), with ice giants carrying ∼99.8% of the total weight. σ_{cal} for J_4 equals the observational sigma (obs floor applied). σ_{total} is the value used in the cost function.

Quantity	σ_{obs}	σ_{cal}	σ_{total}	Basis
$J_2 \times 10^6$	0.412	58.871 (1.68%)	58.871	LOO-wt
$J_4 \times 10^6$	0.466	0.466 (1.31%)	0.466	obs floor
C/MR^2	±0.0045 (assumed 2%)			—
R_{1bar} (km)	±255.6 (assumed 1%)			—

Note: No J_2/J_4 renormalization was applied. For Uranus, $R_{\mathrm{ref}} = R_{\mathrm{eq}} =$ 25,559 km (French et al. 2024 = Lindal et al. 1987), so the published values are already at the 1-bar equatorial radius. Contrast with Neptune, where $R_{\mathrm{ref}} \neq R_{\mathrm{eq}}$. J_2/J_4 forward model uses an empirical calibration with an all-4-planet fit: $J_2 = a\,q + b\,q\,C + c\,q^2$ (in-sample residuals <0.13%); $J_4 = \mathrm{slope}_{\mathrm{ice}} \times J_2$ (mean J_4/J_2 for Uranus + Neptune; residuals <1.4%).

Predicted Bulk Planet Properties

Table 14: Bulk planet properties at the best-fit parameters.

Property	Value
Total mass, input Uranus value	8.681×10^{25} kg ($14.536\, M_\oplus$)
Radius at 1-bar level ($R_\oplus$)	3.995
Radius at 1-bar level (km)	25,451.2
Uranus observed bulk density	≈ 1270 kg m^{-3}
Normalized moment of inertia C/MR^2 (1-bar)	0.223

Predicted Thermal Properties

Table 15: Thermal properties of the best-fit model.

Property	Value
Equilibrium temperature $T_{\rm eq}$, input Uranus value	58.1 K^B
Radiation temperature $T_{\rm rad}$	54.07 K
Internal luminosity $L_{\rm int}$	4.628×10^{14} W
Uranus observed $L_{\rm int}$	$\sim 6.4 \times 10^{14}$ W^B
Central temperature	7041.2 K
Surface temperature (base of atmosphere)	2826.7 K
Mass-weighted temperature	1829.1 K
Temperature at radiative–convective boundary $T_{\rm rcb}$	710.6 K

Predicted Core Structure

Table 16: Core (silicate + H_2–silicate mixture) properties.

Property	Value
Core mass ($M_\oplus$)	13.740
Core radius ($R_\oplus$)	3.036
Metal core radius ($R_\oplus$)	0.000
Core volume (m^3)	3.033×10^{22}
Bulk core density (kg m^{-3})	2705.7
H_2 mass fraction in core	0.057
Thermal energy, core (J)	$+7.277 \times 10^{32}$
Gravitational potential energy, core (J)	-1.461×10^{34}

Predicted Atmosphere Structure

Table 17: Atmospheric envelope properties.

Property	Value
Atmosphere mass (kg)	4.758×10^{24}
Atmosphere mass fraction ($M_{\rm atm}/M_p$)	5.481×10^{-2}
H_2 gas mass fraction of planet	5.196%
H_2 condensate mass fraction of planet	1.152×10^{-3}%
H_2 mass fraction in atmosphere (gas)	0.9983
H_2 mass fraction whole planet	0.1087
Surface pressure at base of atmosphere	9.434×10^4 bar (= 9.434 GPa)
Pressure at base of atmosphere (weight)	4.010×10^5 bar
Radiative–convective boundary height	5164 km
$R_{\rm rcb}/R_c$	1.267
Pressure at rcb	655.8 bar
Optical depth at rcb	7.92×10^5
$\lambda_{\rm rcb}$	91.97
Convection inhibited layer thickness	1.45×10^5 m
Boundary layer thickness, envelope	393.7 m
$R(1\,\rm bar)/R_c$	1.316
Atmosphere volume (1-bar, m^3)	3.873×10^{22}
Mean atm. density (1-bar, kg m^{-3})	122.8
Planet volume fraction: core	0.434

Predicted Total Energy Budget

Table 18: Integrated energy budget for the full planet.

Quantity	Value (J)
Thermal energy, core	$+7.145 \times 10^{32}$
Thermal energy, planet	$+7.745 \times 10^{32}$
Gravitational potential energy, core	-1.465×10^{34}
Gravitational potential energy, planet	-1.580×10^{34}
Total energy (planet)	-1.502×10^{34}

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