

1 **Supplementary materials for Assessing the accuracy of kernel-fitting**
2 **and projections of spatial risk at key early timepoints in outbreaks**

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11 **Table of Contents**

12 A. Descriptive analysis of the 2001 outbreak of foot-and-mouth disease in the United Kingdom..... 2
13 B. Materials and methods 4
14 Section 1. Details of required data and an example 4
15 Section 2. Maximum likelihood estimation equation for estimating transmission kernel parameters. 4
16 C. Determining the earliest point at which the overall transmission kernel could be estimated 6
17 D. Short-term projection of spatial risk using a kernel-derived infection pressure metric..... 25
18 Reference 39

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21 A. Descriptive analysis of the 2001 outbreak of foot-and-mouth disease in the
22 United Kingdom

23 In most affected regions, the number of outbreak cases continued to increase during the first four weeks,
24 followed by a gradual decline after reaching a peak (**Figure S1**). Cumbria experienced the largest FMD
25 outbreak in 2001, which lasted for about 200 days. The other four regions experienced relatively small
26 outbreaks, with peak case numbers not exceeding 10. North Yorkshire showed a relatively low and sporadic
27 pattern of FMD occurrence.

28 The spatial patterns of FMD spread during the first four weeks of the outbreak varied across regions (**Figure**
29 **S2**). In Cumbria, transmission between farms intensified early, forming dense clusters from the outset. In
30 contrast, Devon, Dumfries and Galloway, and Durham began to show increasing numbers of infected
31 premises in weeks 2 and 3, with spatial clustering becoming more evident over time. Meanwhile, North
32 Yorkshire experienced only a small number of sporadic cases. Overall, during the early weeks of the
33 outbreak, FMD predominantly spread through local transmission, with the speed and extent of spread
34 differing by region. These patterns suggest that the spatial dissemination of FMD was strongly influenced by
35 proximity to existing infected premises (IPs), demonstrating a consistent trend of geographically limited
36 spread in the initial phase of the outbreak.

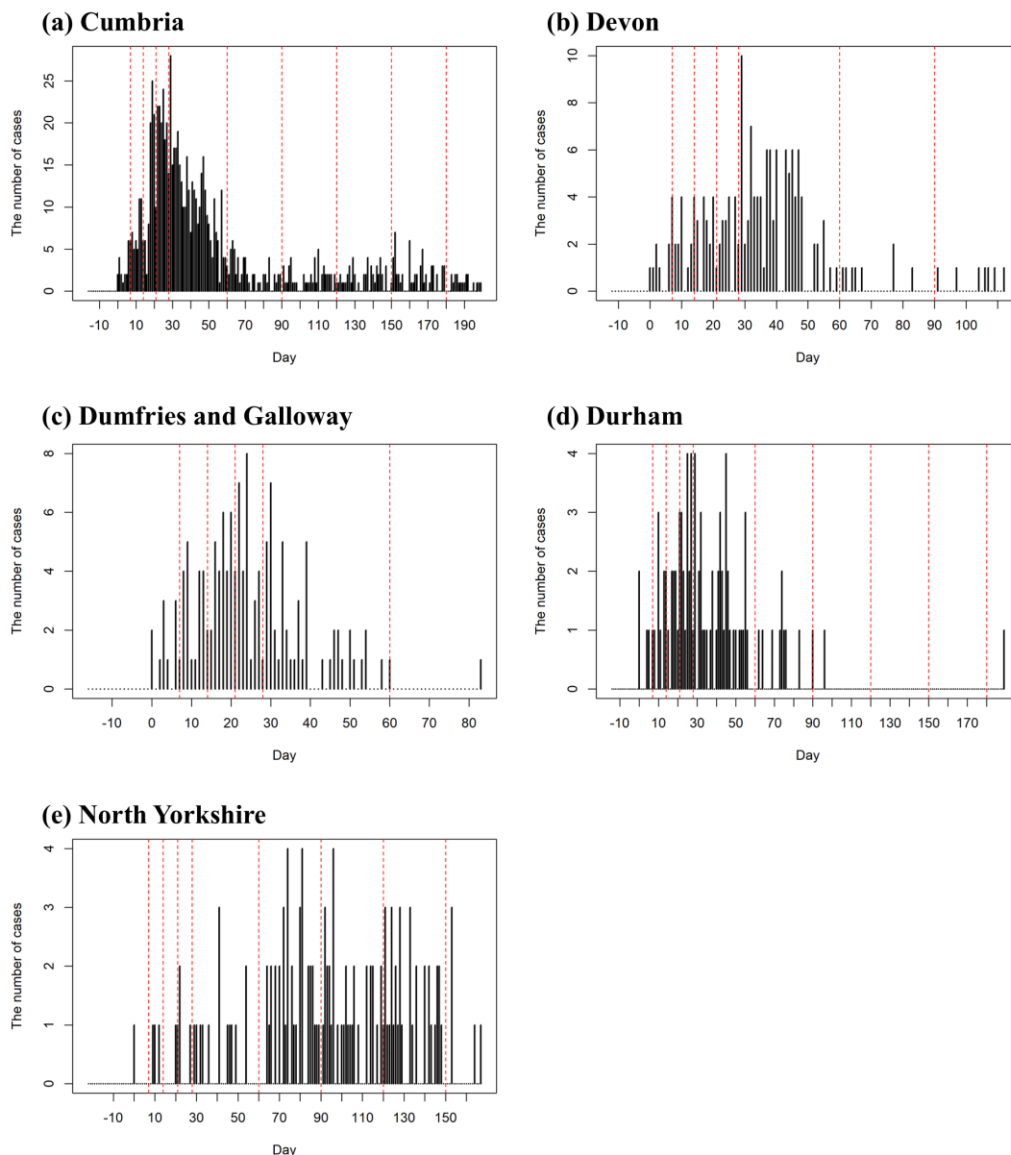


Figure S1. Epidemic curves for (a) Cumbria, (b) Devon, (c) Dumfries and Galloway, (d) Durham, and (e) North Yorkshire. The first notification date for each region is set as day 0. The number of cases is calculated based on the notification date to FMD. Vertical dashed lines represent the specific timepoints for each region (i.e., from left to right: day 7, 14, 21, 28, 60, 90, etc.).

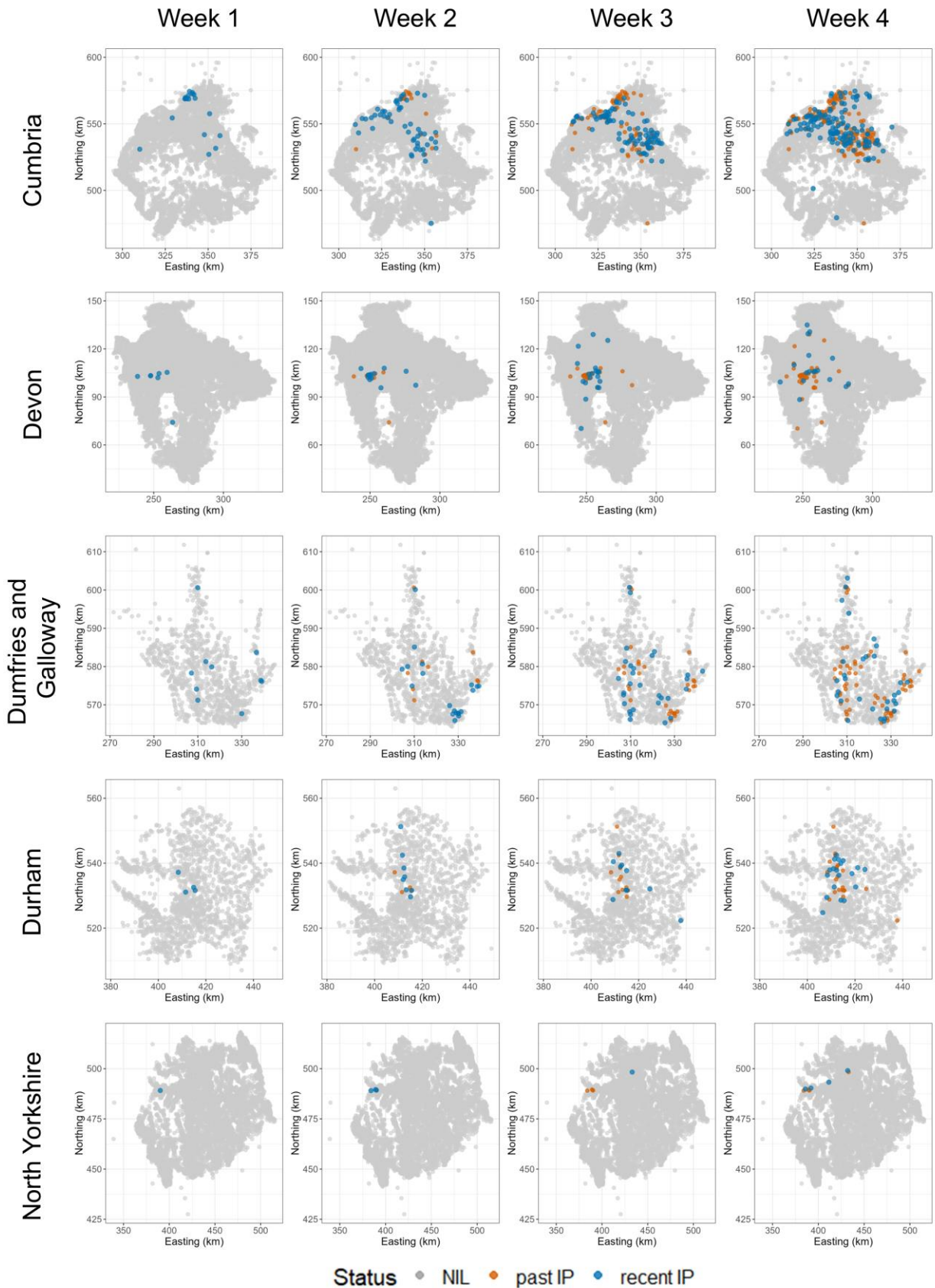


Figure S2. Outbreak maps for the first four weeks, shown in weekly intervals in Cumbria, Devon, Dumfries and Galloway, Durham, and North Yorkshire. Blue dots indicate infected premises (IPs) notified during the corresponding week, while orange dots represent all IPs notified prior to that week.

B. Materials and methods

Section 1. Details of required data and an example

The data required for the analysis includes the coordinates of each premises and two observed dates for infected premises. For calculating Euclidean distances between premises, it is recommended to use projected coordinates instead of latitude and longitude. Two unobserved dates (i.e., ‘the exposure date’ and ‘the infectious onset date’) are back-calculated from the clinical onset date using the latent and incubation periods (1).

Table S1. An example of dataset

ID	X coordinate (meters)	Y coordinate (meters)	Status	Clinical onset date	Depopulation date
1	386600	512900	Infected	01/03/2001	04/03/2001
2	386450	513520	Susceptible	NA	NA
...

The example data and a R script used in the study are available on GitLab (<https://gitlab.unimelb.edu.au/slleee2/kernel-estimation>). The R script includes the process of kernel estimation, calculating relative infection pressure, and visualising it on a map.

Section 2. Maximum likelihood estimation equation for estimating transmission kernel parameters.

The appropriate parameter values were estimated through the following process (2). The infection pressure of a susceptible premises j at day t was calculated as the sum of the kernels from all infectious premises I on that day (eq. (1)). $k(r_{ij})$ denotes the transmission rate from an infectious premises i to a susceptible premises j , dependent on the distance r_{ij} separating them.

$$\lambda_j(t) = \sum_{i \in I} k(r_{ij}) \quad \text{eq. (1)}$$

The likelihood function (L) is calculated as the product of the probabilities that: (i) a premises m in the set M which remain unexposed until the exposed time $t_{exposure,m}$, (ii) a premises m in the set M which become infected is exposed at $t_{exposure,m}$, (iii) a premises n in the set N which do not become infected throughout the entire period, and (iv) a dangerous contact premises or a susceptible premises o in the set O which do not become infected until the depopulation time $t_{depopulation,o}$ (eq. (2)).

$$L = \prod_{m \in M} P_{not_exposed,m}(t_{start} \sim (t_{exposure,m} - 1)) \cdot P_{exposed,m}(t_{exposure,m}) \times \prod_{n \in N} P_{not_exposed,n}(t_{start} \sim t_{end}) \times \prod_{o \in O} P_{not_exposed,o}(t_{start} \sim (t_{depopulation,o} - 1)) \quad \text{eq. (2)}$$

63 The probability that a premises m remains unexposed from t_{start} to t_{end} , denoted
 64 $P_{not_exposed,m}(t_{start} \sim t_{end})$, and the probability that a premises m is exposed at $t_{exposure}$, denoted
 65 $P_{exposed,m}(t_{exposure,m})$, were calculated using a Poisson process with the infection pressure (eqs. (3) and
 66 (4)).

$$P_{not_exposed,m}(t_{start} \sim t_{end}) = e^{-\sum_{t=t_{start}}^{t_{end}} \lambda_m(t)} \quad eq. (3)$$

$$P_{exposed,m}(t_{exposure,m}) = 1 - e^{-\lambda_m(t_{exposure,m})} \quad eq. (4)$$

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68 Therefore, eq. (2) can be expressed as eq. (5).

$$L = \prod_{m \in M} e^{-\sum_{t=t_{start}}^{(t_{exposure,m}-1)} \lambda_m(t)} (1 - e^{-\lambda_m(t_{exposure,m})}) \quad eq. (5)$$

$$\times \prod_{n \in N} e^{-\sum_{t=t_{start}}^{t_{end}} \lambda_n(t)} \times \prod_{o \in O} e^{-\sum_{t=t_{start}}^{(t_{depopulation,o}-1)} \lambda_o(t)}$$

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70 To reduce computational complexity, the likelihood function (L) was transformed into a log-likelihood
 71 function ($\ln(L)$), and parameter values were estimated by maximising this function, i.e., by minimising the
 72 negative log-likelihood function.

$$\ln(L) = \sum_{m \in M} \ln(1 - e^{-\lambda_m(t_{exposure,m})}) - \sum_{m \in M} \sum_{t=t_{start}}^{(t_{exposure,m}-1)} \lambda_m(t) \quad eq. (6)$$

$$- \sum_{n \in N} \sum_{t=t_{start}}^{t_{end}} \lambda_n(t) - \sum_{o \in O} \sum_{t=t_{start}}^{(t_{depopulation,o}-1)} \lambda_o(t)$$

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C. Determining the earliest point at which the overall transmission kernel could be estimated

The best-fitting transmission kernel equation, based on estimation using the entire outbreak dataset in each of the five regions, was consistently the power law type 2 kernel (**Figure S3**). In Cumbria and Devon, the power law type 2 was consistently selected as the best fit from 7 days post-notification in the region through to the end of the outbreak. In Dumfries and Galloway, Durham, and North Yorkshire, although other transmission kernel equation were occasionally selected as the best fit, the power-law type 2 kernel was consistently preferred in most cases. Even when an alternative kernel instead of power law type 2 was chosen, the shape of the alternative kernel curve did not differ noticeably from that of the best-fitting transmission kernel or the overall transmission kernel (**Figures S4, S5, and S6**).

At later timepoints (as more data became available), the shape of the best-fitting transmission kernel curve progressively resembled that of the kernel estimated from the full outbreak dataset for each cluster. Additionally, the 95% and 50% confidential intervals (CIs) of the estimated kernels became more precise (**Figure S5 and S6**). The first 60-day datasets from North Yorkshire did not support CI estimation for kernel parameters (**Figure S5(e) and Table S2**) due to low case numbers (<20 reported IP cases; **Figure S1**). North Yorkshire had only a single IP reported during the first 7 days, making it impossible to estimate a transmission kernel based on the relationships between IPs. This highlights the importance of the number of outbreak cases for reliable transmission kernel estimation.

We compared the shape of the estimated transmission kernel curves to the overall transmission kernel curve. If an estimated kernel was positioned higher than the overall kernel, it was classified as an overestimation (i.e., indicating broader spatial spread of FMD). If lower, it was considered an underestimation (i.e., indicating more restricted spread of FMD). During the first two weeks, underestimation was observed in Cumbria and Dumfries and Galloway (**Figures S5(a) and S5(c)**). However, after this period, the best-fitting kernel curves closely approximated the overall kernel curve. In Devon and Durham, the best-fitting kernel curves were already closely aligned with the overall kernel as early as day 7 (**Figures S5(b) and S5(d)**). In North Yorkshire, the best-fitting transmission kernel overestimated the overall kernel on day 14, underestimated it between days 21 and 60, and closely matched the overall transmission kernel thereafter (**Figure S5(e)**). Overall, discrepancies between estimated and overall best-fitting transmission kernels were resolved within two weeks (**Figure S5**). By day 60, after the epidemic peak, the 95% and 50% CI envelopes of the estimated transmission kernels closely aligned with those of the overall transmission kernel curve (**Figures S1 and S5**).

The estimated 95%, 90%, and 75% risk distances showed substantial variability at early timepoints (i.e., day 7 and day 14), but gradually stabilized over time (**Figure S7**). This suggests that while the spatial spread of FMD was uncertain in the early stages, it later persisted within a more stable range. For Cumbria, the estimated risk distances consistently remained within 10 km at all time points, indicating that the infection primarily spread over relatively short distances. Devon and Durham also experienced wide 95% risk distance confidence intervals during the early stages (Days 7–14), suggesting initial uncertainty in the spread. However, from subsequent time points, both the estimated values and their CIs showed a consistent pattern aligning with the final estimates, indicating a stable trend. Notably, the estimates of 90% and 75% risk distances generally stayed within 10 km, exhibiting relatively small variability. In Dumfries and Galloway, although wide CIs were observed at all time points, the final estimates also exhibited significant uncertainty, which did not pose a major issue for interpretation. The 75% risk distance in this region remained consistently low throughout the entire period, suggesting that FMD transmission primarily occurred over very short distances. Conversely, in North Yorkshire, the 95%, 90%, and 75% risk distances were all estimated to be very small, less than 0.2 km, on Day 14. This indicates that local transmission

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during this period was extremely limited or occurred only over very close distances. However, from Day 21 onwards, the estimated risk distances increased, approaching their final estimates.

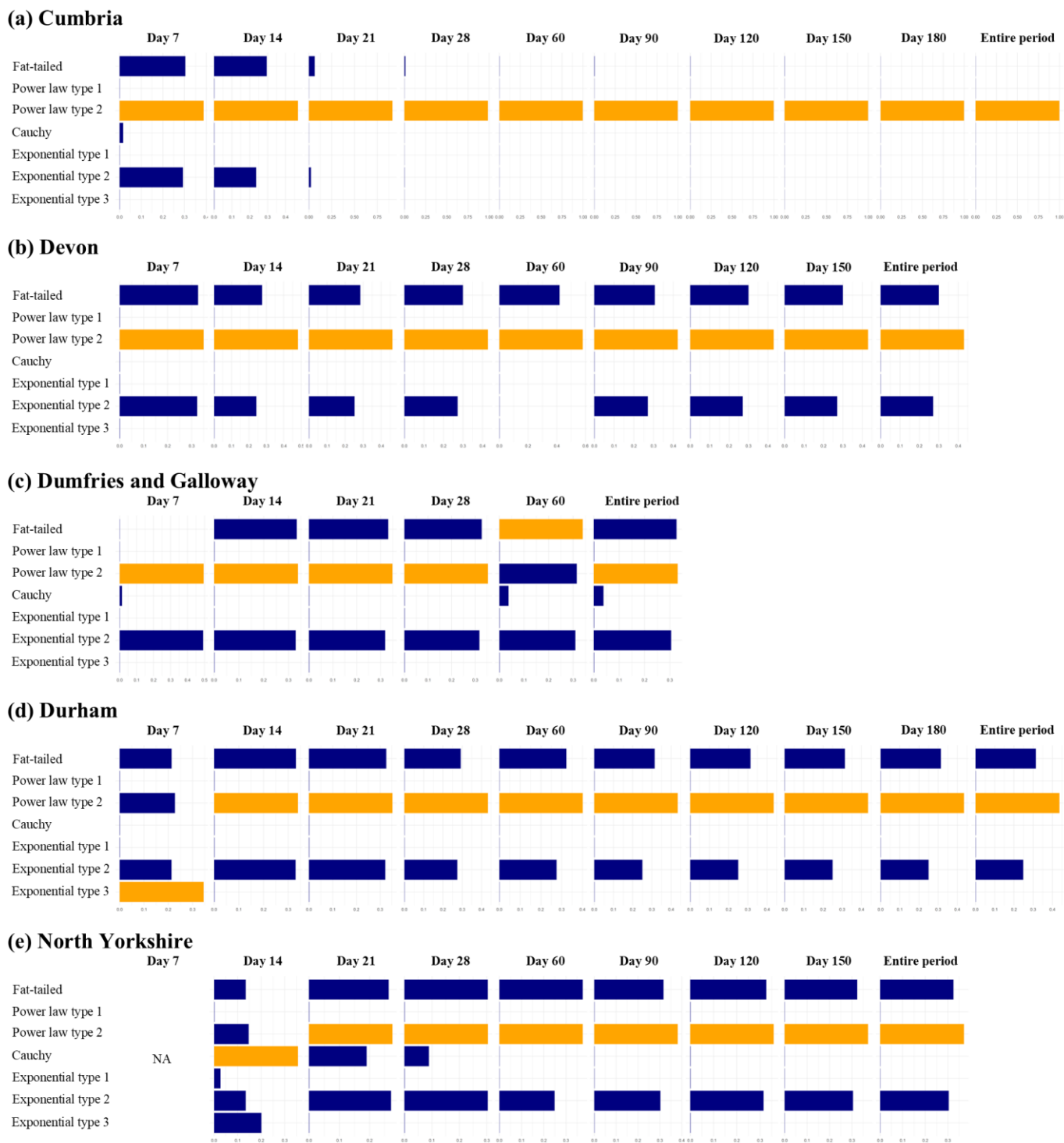


Figure S3. Weight plots of transmission kernels at each timepoint for (a) Cumbria, (b) Devon, (c) Dumfries and Galloway, (d) Durham, and (e) North Yorkshire. The days indicated on the plots represent the timepoint, calculated relative to the first detection date in each region. Entire period refers to the time span from the first detection date to the final depopulation event. The weights indicate the relative fit of each transmission kernel based on AIC values. The transmission kernel with the highest weight (orange) may be preferred over others (navy). In North Yorkshire, kernel estimation was not possible at day 7 due to the presence of only a single case.

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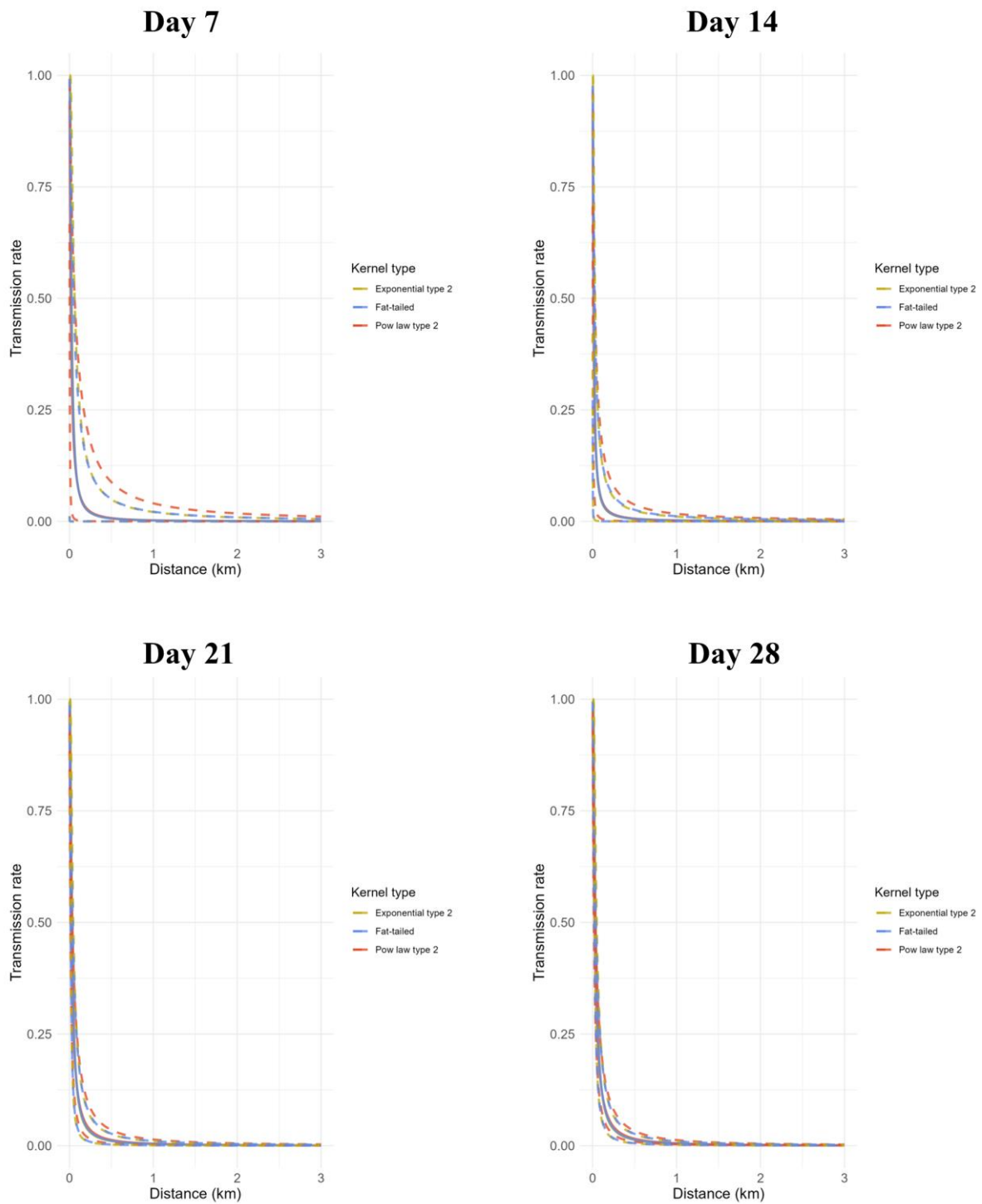


Figure S4(a). Plots of the candidate transmission kernels that could have been selected during the first month for Cumbria. Solid lines represent kernel estimates, while dashed lines indicate lower and upper limits of 95% confidential intervals (CI).

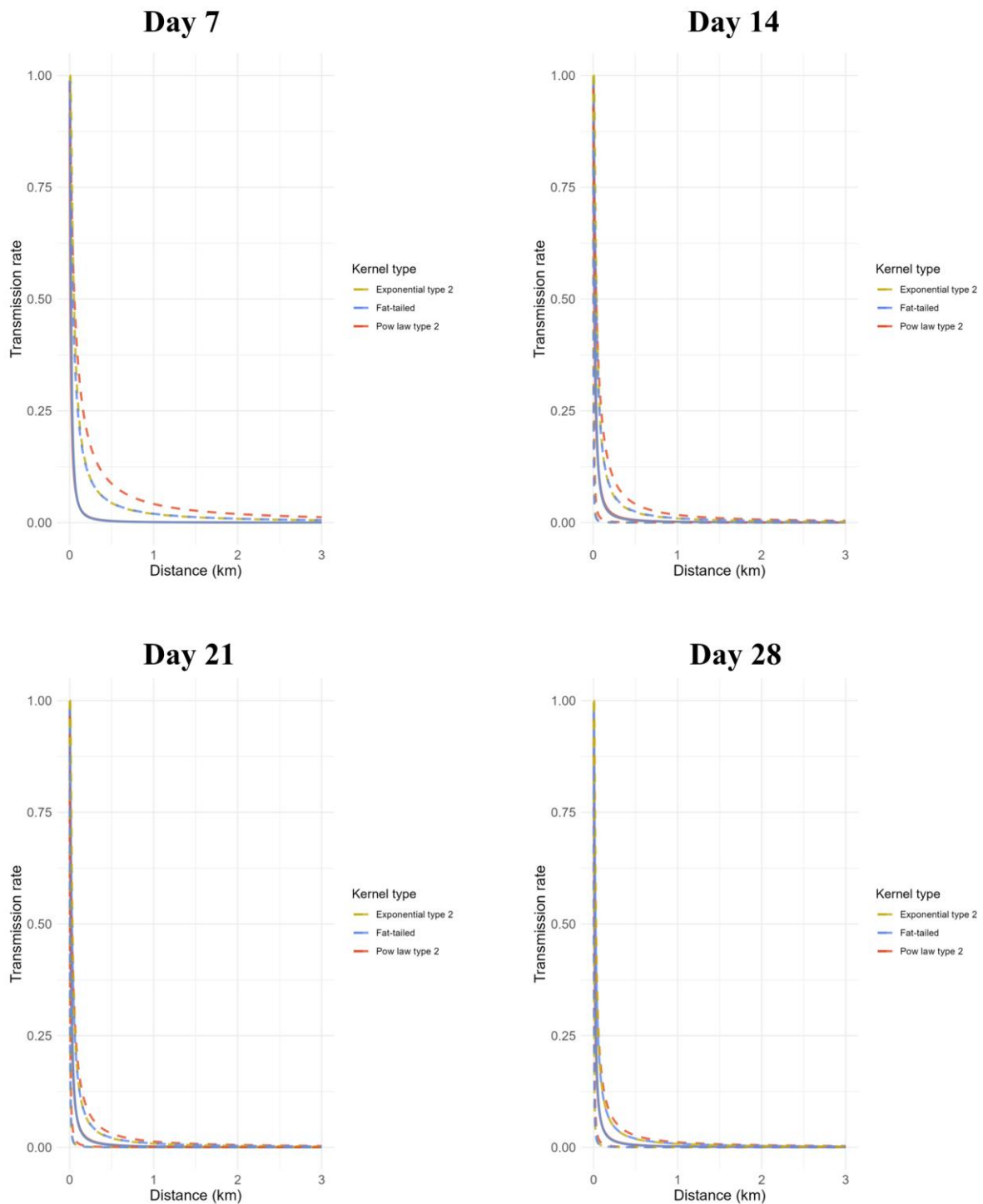


Figure S4(b). Plots of the candidate transmission kernels that could have been selected during the first month for Devon. Solid lines represent kernel estimates, while dashed lines indicate lower and upper limits of 95% confidential intervals (CI). If the 95% CIs for the kernel parameters cannot be estimated or are not visually distinguishable, the corresponding CIs for the transmission kernels are not displayed.

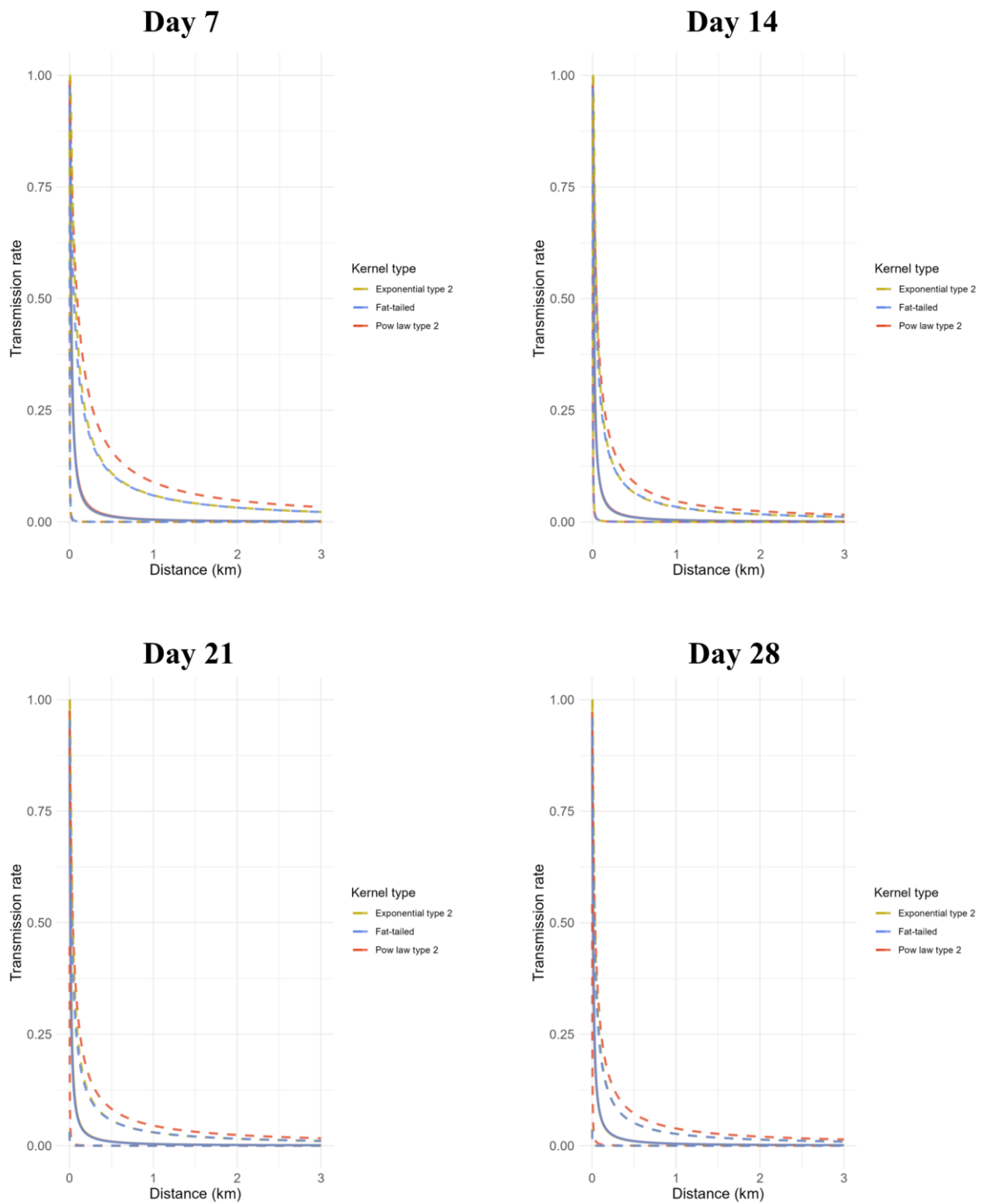


Figure S4(c). Plots of the candidate transmission kernels that could have been selected during the first month for Dumfries and Galloway. Solid lines represent kernel estimates, while dashed lines indicate lower and upper limits of 95% confidential intervals.

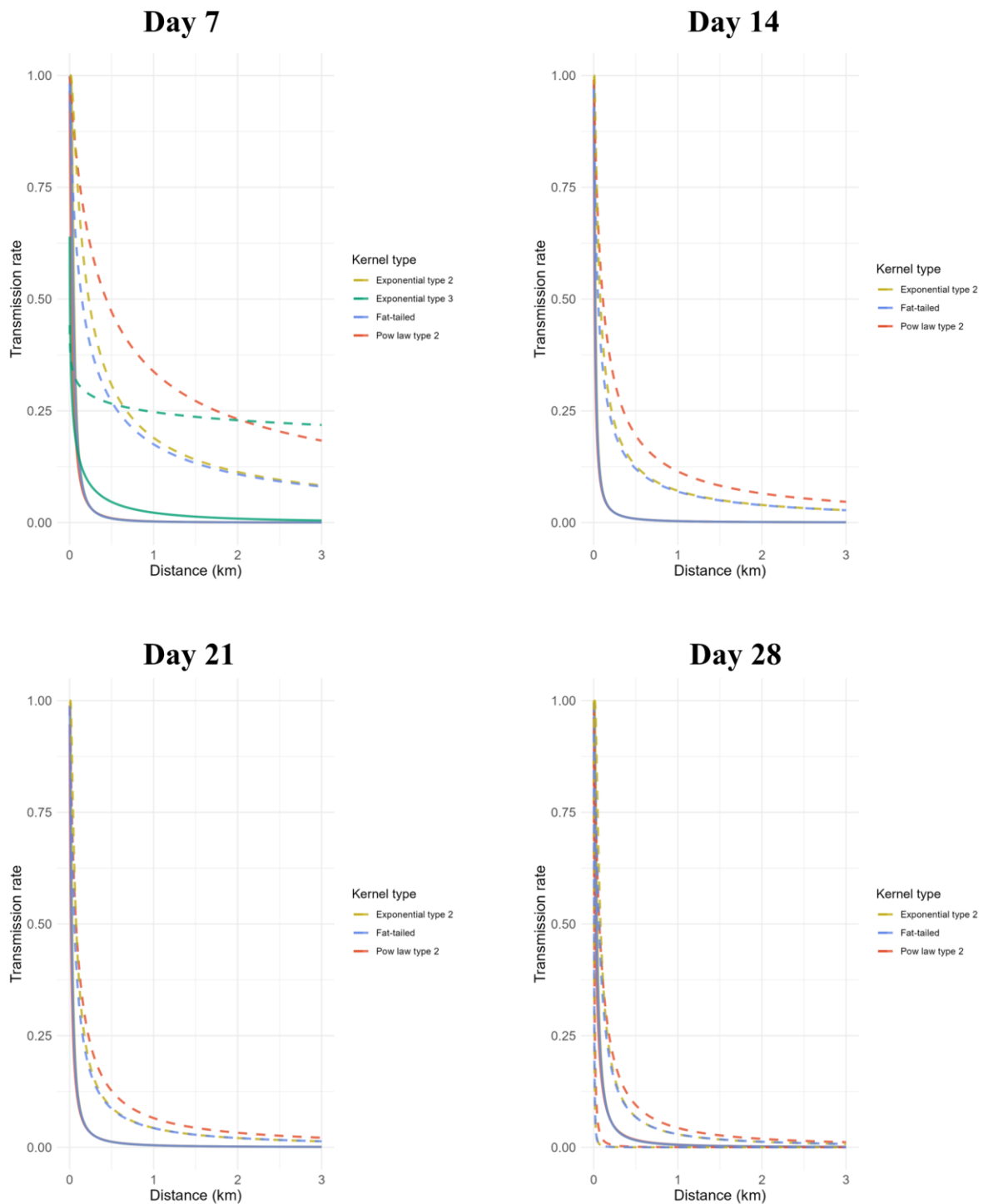
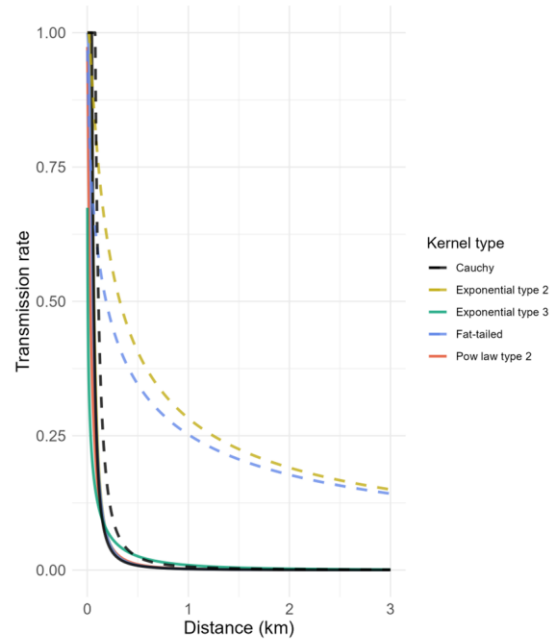


Figure S4(d). Plots of the candidate transmission kernels that could have been selected during the first month for Durham. Solid lines represent kernel estimates, while dashed lines indicate lower and upper limits of 95% confidential intervals (CI). If the 95% CIs for the kernel parameters cannot be estimated or are not visually distinguishable, the corresponding CIs for the transmission kernels are not displayed.

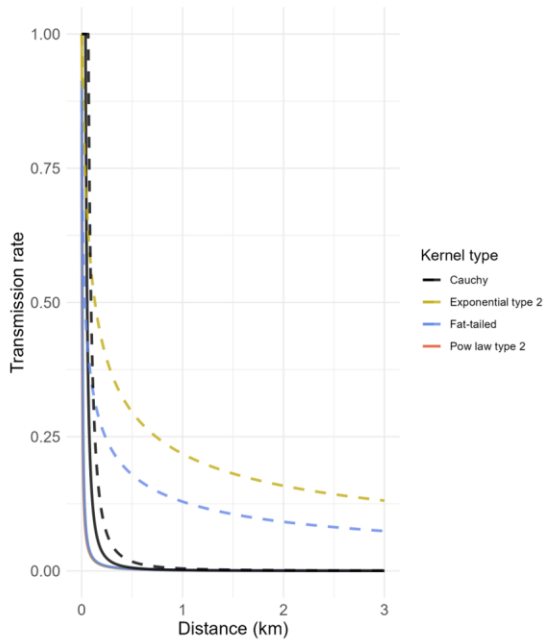
Day 7

N/A

Day 14



Day 21



Day 28

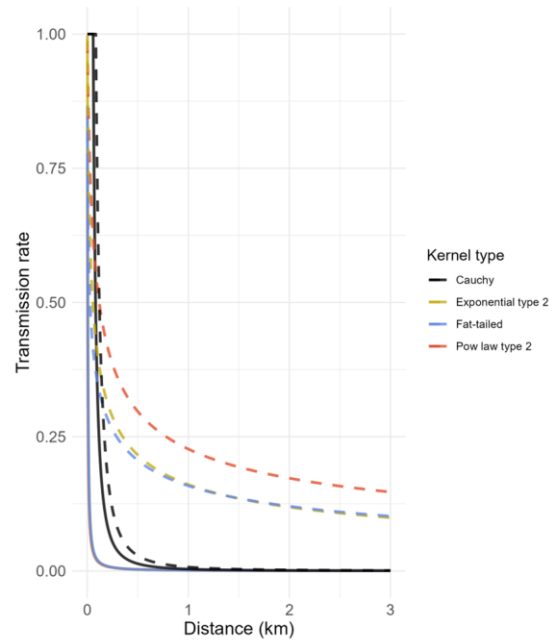


Figure S4(e). Plots of the candidate transmission kernels that could have been selected during the first month for North Yorkshire. Solid lines represent kernel estimates, while dashed lines indicate lower and upper limits of 95% confidential intervals (CI). If the 95% CIs for the kernel parameters cannot be estimated or are not visually distinguishable, the corresponding CIs for the transmission kernels are not displayed. Kernel estimation was not possible at day 7 due to the presence of only a single case.

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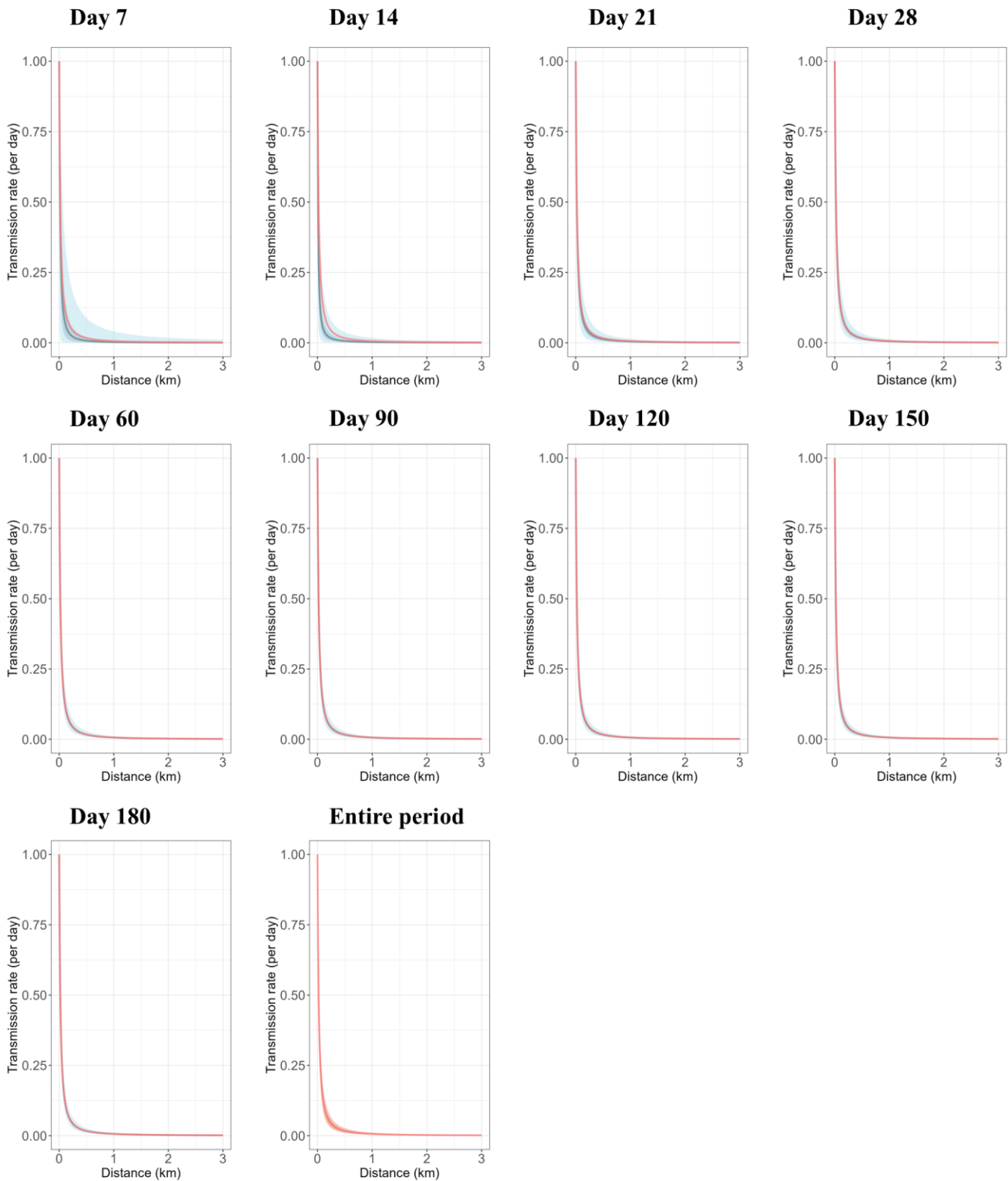


Figure S5(a). Plots of the best-fitting transmission kernel (blue line) and overall transmission kernel (red line) for each timepoint up to the end of the outbreak in Cumbria. The blue curve, along with the dark and light blue shaded areas, represents the best-fitting kernel estimated at the timepoint shown in each plot (day 0 = the first notification in each region), along with its 50% and 95% confidence intervals (CIs). The overall transmission kernel estimated using entire data is shown as a red curve, with its 50% and 95% CIs indicated by dark red and light red shading, respectively. The similarity between kernels was assessed by checking whether the overall transmission kernel curve falls within the CI envelopes of the best-fitting kernel.

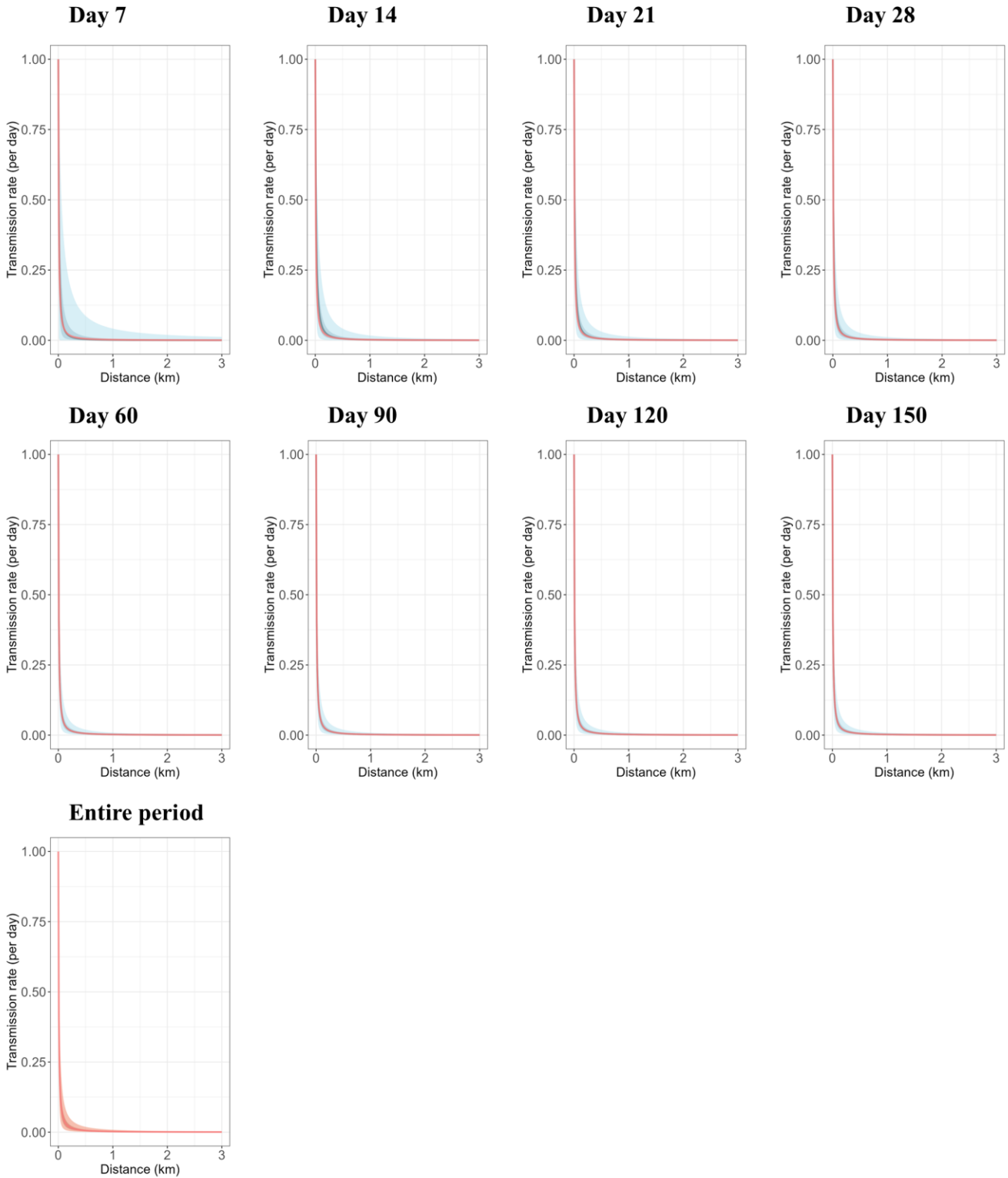


Figure S5(b). Plots of the best-fitting transmission kernel (blue line) and overall transmission kernel (red line) for each timepoint up to the end of the outbreak in Devon. The blue curve, along with the dark and light blue shaded areas, represents the best-fitting kernel estimated at the timepoint shown in each plot (day 0 = the first notification in each region), along with its 50% and 95% confidence intervals (CIs). The overall transmission kernel estimated using entire data is shown as a red curve, with its 50% and 95% CIs indicated by dark red and light red shading, respectively. The similarity between kernels was assessed by checking whether the overall transmission kernel curve falls within the CI envelopes of the best-fitting kernel.

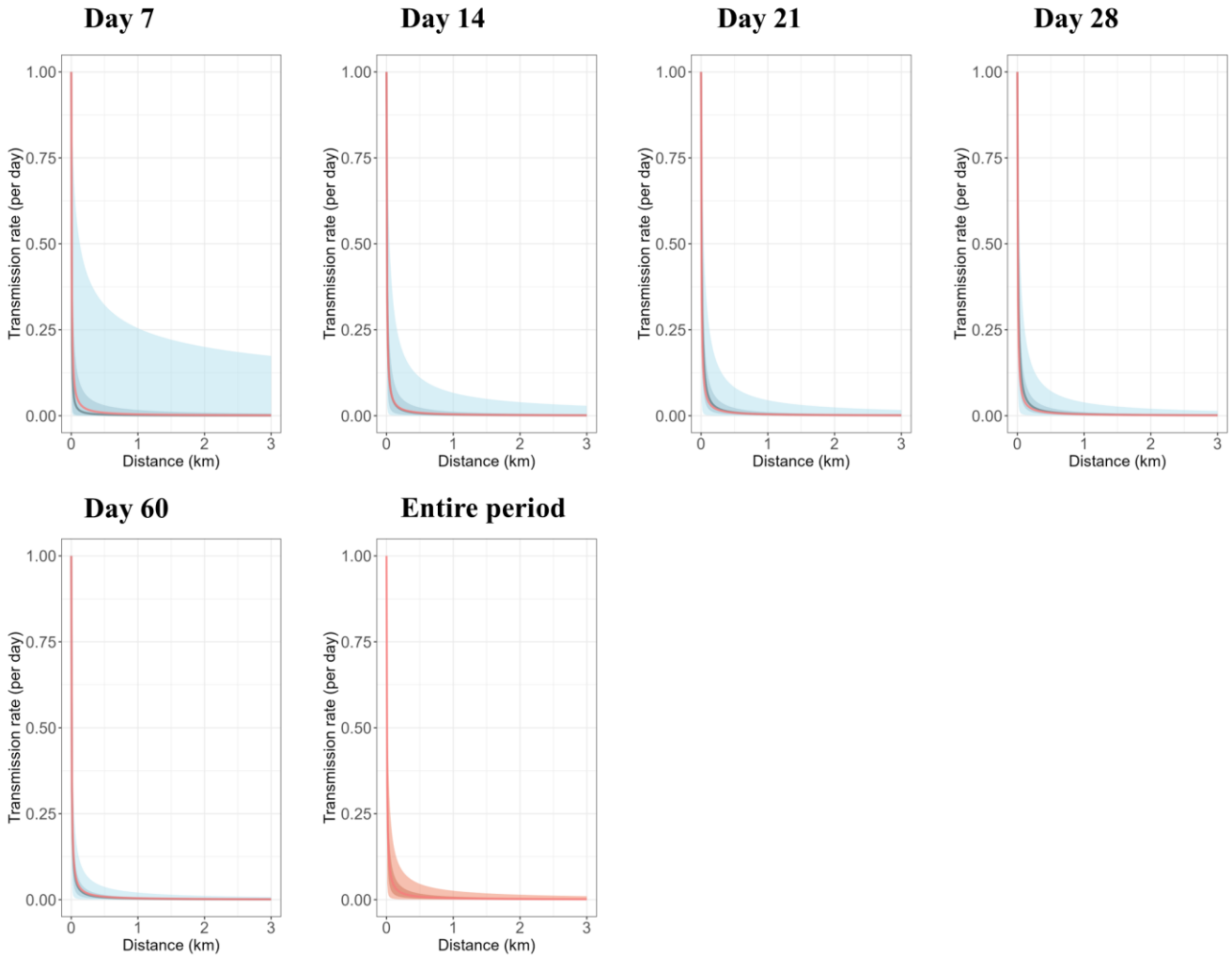


Figure S5(c). Plots of the best-fitting transmission kernel (blue line) and overall transmission kernel (red line) for each timepoint up to the end of the outbreak in Dumfries and Galloway. The blue curve, along with the dark and light blue shaded areas, represents the best-fitting kernel estimated at the timepoint shown in each plot (day 0 = the first notification in each region), along with its 50% and 95% confidence intervals (CIs). The overall transmission kernel estimated using entire data is shown as a red curve, with its 50% and 95% CIs indicated by dark red and light red shading, respectively. The similarity between kernels was assessed by checking whether the overall transmission kernel curve falls within the CI envelopes of the best-fitting kernel.

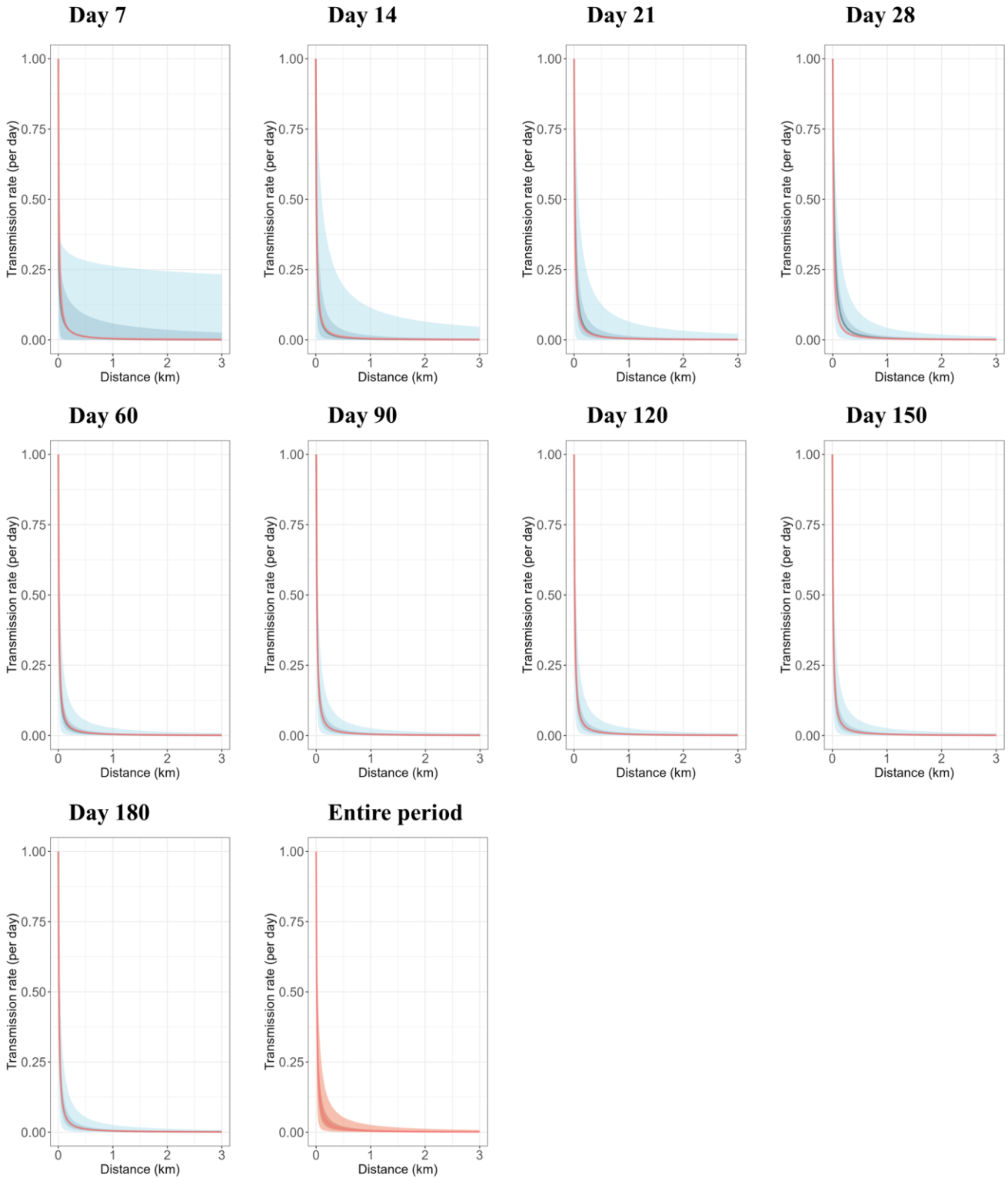


Figure S5(d). Plots of the best-fitting transmission kernel (blue line) and overall transmission kernel (red line) for each timepoint up to the end of the outbreak in Durham. The blue curve, along with the dark and light blue shaded areas, represents the best-fitting kernel estimated at the timepoint shown in each plot (day 0 = the first notification in each region), along with its 50% and 95% confidence intervals (CIs). The overall transmission kernel estimated using entire data is shown as a red curve, with its 50% and 95% CIs indicated by dark red and light red shading, respectively. The similarity between kernels was assessed by checking whether the overall transmission kernel curve falls within the CI envelopes of the best-fitting kernel.

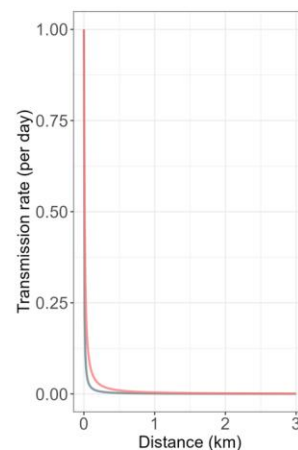
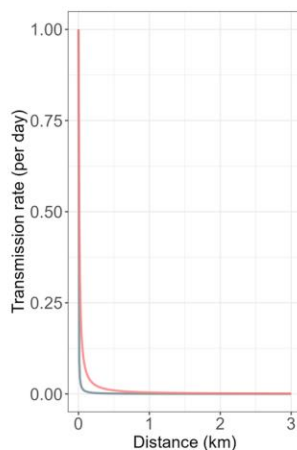
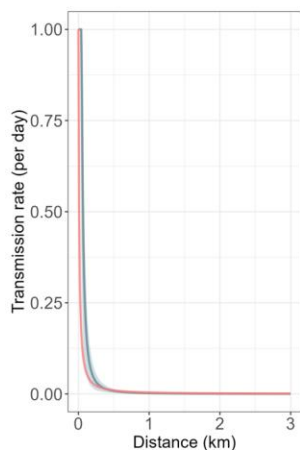
Day 7

Day 14

Day 21

Day 28

N/A

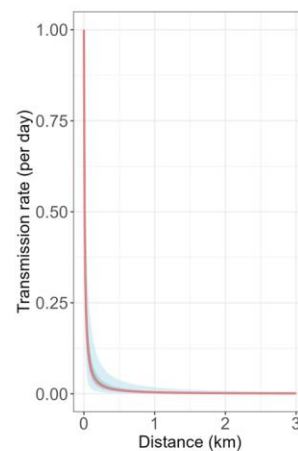
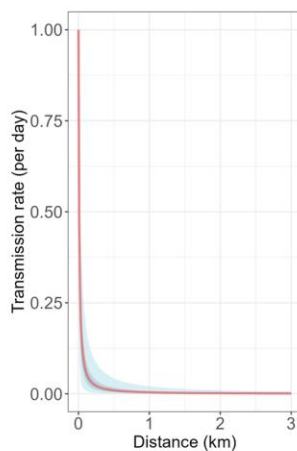
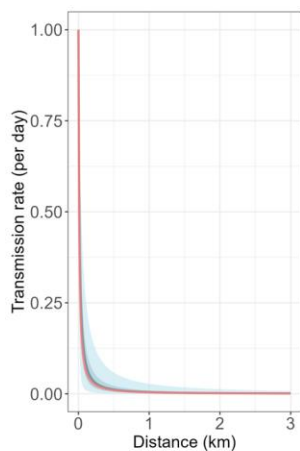
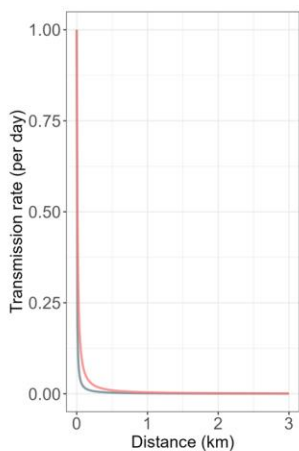


Day 60

Day 90

Day 120

Day 150



Entire period

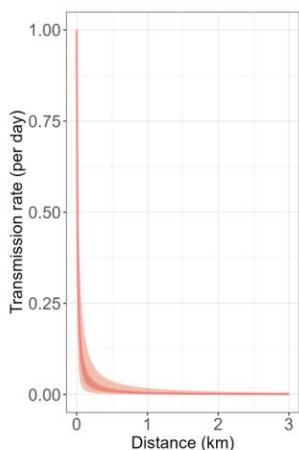


Figure S5(e). Plots of the best-fitting transmission kernel (blue line) and overall transmission kernel (red line) for each timepoint up to the end of the outbreak in North Yorkshire. The blue curve, along with the dark and light blue shaded areas, represents the best-fitting kernel estimated at the timepoint shown in each plot (day 0 = the first notification in each region), along with its 50% and 95% confidence intervals (CIs). The overall transmission kernel estimated using entire data is shown as a red curve, with its 50% and 95% CIs indicated by dark red and light red shading, respectively. The similarity between kernels was assessed by checking whether the overall transmission kernel curve falls within the CI envelopes of the best-fitting kernel. If 50% or 95% CIs for the kernel parameters cannot be estimated, the corresponding CIs for the transmission kernel are not displayed.

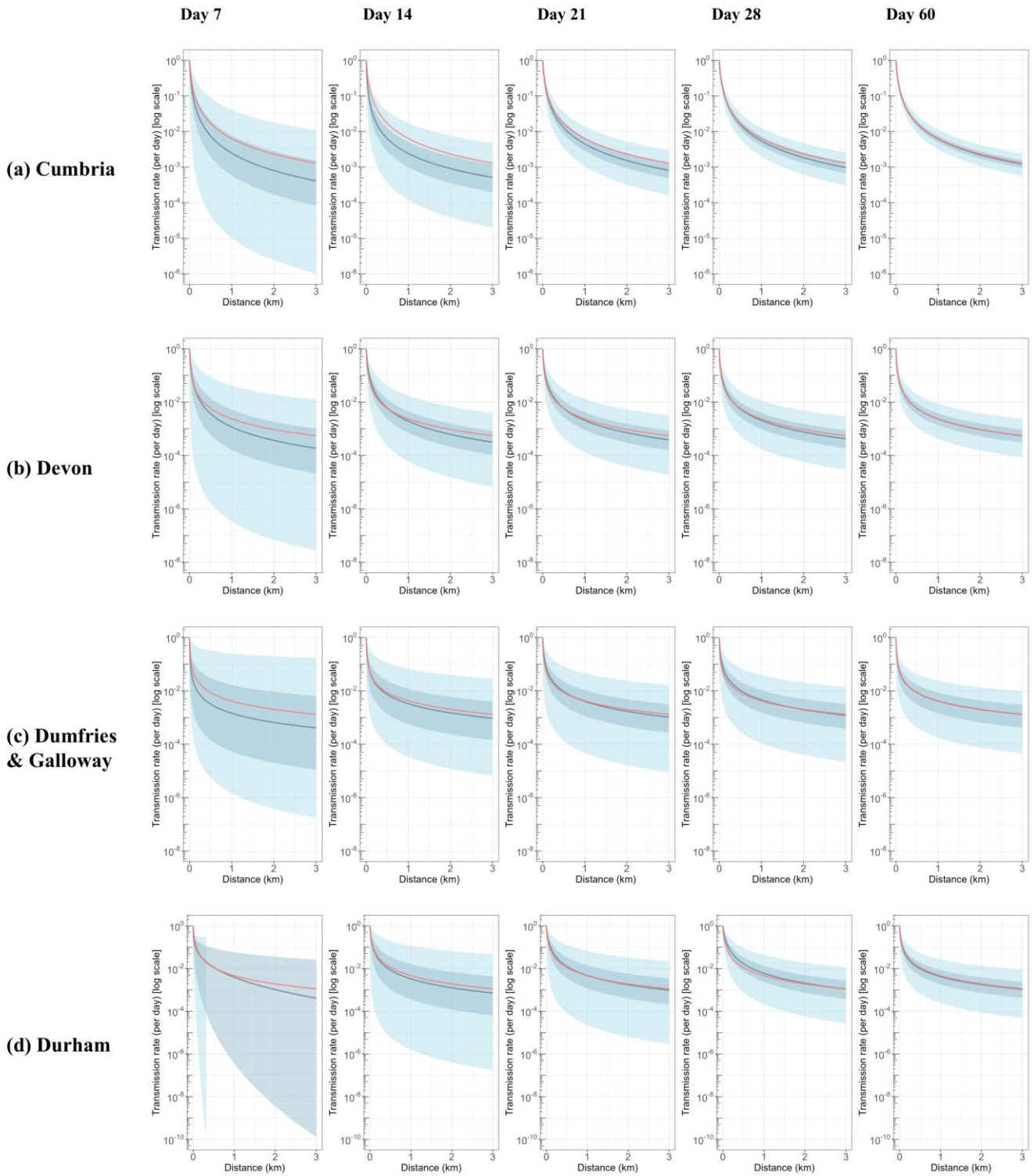


Figure S6. Comparison of the best-fitting and overall transmission kernels for (a) Cumbria, (b) Devon, (c) Dumfries and Galloway, and (d) Durham. The blue curve and associated dark and light blue shaded areas represent the best-fitting kernel and its 50% and 95% confidence intervals (CIs) estimated at each timepoint, respectively (day 0 = the first notification in each region). The red line represents the overall transmission kernel estimated using the full dataset. The y-axis is presented on a logarithmic scale for clarity. Results for North Yorkshire are excluded due to the inability to calculate CIs.

Table S2. Types of the best-fitting transmission kernels and estimated parameter values along with their 95% and 50% confidence intervals (CIs) for each timepoint in Cumbria, Devon, Dumfries and Galloway, Durham, and North Yorkshire. All parameter values are rounded to three decimal places. If CIs could not be obtained during the maximum likelihood estimation, they were marked as NA.

(a) Cumbria							
Days	The best-fitting kernel type	Ψ (kernel power)	95% CIs	50% CIs	ρ_0 (kernel scale)	95% CIs	50% CIs
7	power law type 2	1.659	1.253-2.100	1.518-1.802	0.028	0.004-0.084	0.016-0.042
14	power law type 2	1.415	1.192-1.633	1.337-1.488	0.014	0.004-0.034	0.010-0.020
21	power law type 2	1.593	1.448-1.739	1.542-1.642	0.035	0.020-0.055	0.029-0.041
28	power law type 2	1.606	1.496-1.716	1.568-1.643	0.041	0.027-0.057	0.036-0.046
60	power law type 2	1.515	1.441-1.587	1.489-1.537	0.036	0.027-0.046	0.033-0.039
90	power law type 2	1.484	1.415-1.553	1.460-1.507	0.033	0.025-0.043	0.030-0.036
120	power law type 2	1.477	1.410-1.541	1.453-1.498	0.033	0.025-0.042	0.030-0.036
150	power law type 2	1.465	1.400-1.526	1.441-1.485	0.032	0.024-0.040	0.029-0.034
180	power law type 2	1.477	1.415-1.536	1.454-1.496	0.034	0.026-0.042	0.031-0.036
entire data	power law type 2	1.478	1.418-1.537	1.457-1.497	0.034	0.027-0.042	0.031-0.036

(b) Devon							
Days	The best-fitting kernel type	Ψ (kernel power)	95% CIs	50% CIs	ρ_0 (kernel scale)	95% CIs	50% CIs
7	power law type 2	1.695	1.172-2.294	1.514-1.880	0.019	0.002-0.071	0.010-0.032
14	power law type 2	1.673	1.384-1.986	1.571-1.774	0.024	0.008-0.056	0.017-0.033
21	power law type 2	1.540	1.318-1.766	1.462-1.615	0.018	0.006-0.039	0.013-0.024
28	power law type 2	1.460	1.263-1.644	1.388-1.519	0.015	0.005-0.030	0.011-0.019
60	power law type 2	1.329	1.194-1.453	1.280-1.370	0.011	0.005-0.019	0.008-0.013
90	power law type 2	1.323	1.193-1.448	1.278-1.366	0.011	0.005-0.019	0.008-0.013
120	power law type 2	1.328	1.197-1.450	1.281-1.369	0.011	0.005-0.019	0.008-0.013
150	power law type 2	1.328	1.197-1.450	1.281-1.369	0.011	0.005-0.019	0.008-0.013
entire data	power law type 2	1.328	1.197-1.450	1.281-1.369	0.011	0.005-0.018	0.008-0.013

(c) Dumfries and Galloway

Days	The best-fitting kernel type	Ψ (kernel power)	95% CIs	50% CIs	ρ_0 (kernel scale)	95% CIs	50% CIs
7	power law type 2	1.148	0.351-1.944	0.874-1.422	0.003	0.001-0.021	0.001-0.009
14	power law type 2	1.152	0.776-1.489	1.005-1.258	0.007	0.001-0.031	0.003-0.012
21	power law type 2	1.248	0.922-1.515	1.125-1.331	0.012	0.001-0.036	0.006-0.018
28	power law type 2	1.219	0.940-1.449	1.113-1.291	0.012	0.002-0.033	0.007-0.017
60	fat-tailed	1.050	0.850-1.249	0.981-1.118	0.005	0.001-0.013	0.003-0.008
entire data	power law type 2	1.054	0.847-1.261	0.983-1.125	0.006	0.001-0.014	0.003-0.008

(d) Durham

Days	The best-fitting kernel type	Ψ (kernel power)	95% CIs	50% CIs	ρ_0 (kernel scale)	95% CIs	50% CIs
7	exponential type 3	0.310	0.078-0.543	0.230-0.390	0.004	0.001-0.024	0.001-0.011
14	power law type 2	1.412	0.865-1.976	1.218-1.597	0.018	0.001-0.089	0.007-0.035
21	power law type 2	1.464	1.064-1.869	1.325-1.598	0.026	0.003-0.084	0.015-0.042
28	power law type 2	1.584	1.276-1.908	1.477-1.692	0.041	0.012-0.094	0.029-0.056
60	power law type 2	1.257	1.040-1.470	1.182-1.328	0.013	0.003-0.032	0.009-0.018
90	power law type 2	1.306	1.094-1.509	1.228-1.371	0.017	0.005-0.037	0.011-0.022
120	power law type 2	1.305	1.095-1.509	1.229-1.372	0.017	0.005-0.037	0.011-0.022
150	power law type 2	1.305	1.095-1.509	1.229-1.372	0.017	0.005-0.037	0.011-0.022
180	power law type 2	1.305	1.095-1.509	1.229-1.372	0.017	0.005-0.037	0.011-0.022
entire data	power law type 2	1.307	1.097-1.512	1.232-1.374	0.017	0.005-0.037	0.012-0.022

(e) North Yorkshire

Days	The best-fitting kernel type	Ψ (kernel power)	95% CIs	50% CIs	ρ_0 (kernel scale)	95% CIs	50% CIs
7	NA						
14	Cauchy	0.002	NA -0.006	0.001-0.003			
21	power law type 2	1.173	0.526-2.010	1.015-1.513	0.002	NA -0.077	0.001-0.019
28	power law type 2	1.227	0.402-1.996	0.943-1.486	0.004	NA -0.069	0.001-0.015
60	power law type 2	1.122	0.669-1.575	0.966-1.278	0.003	NA -0.013	0.002-0.007

90	power law type 2	1.372	1.155-1.583	1.296-1.443	0.020	0.006-0.047	0.014-0.028
120	power law type 2	1.254	1.078-1.425	1.194-1.313	0.012	0.004-0.028	0.009-0.017
150	power law type 2	1.291	1.133-1.441	1.235-1.341	0.015	0.006-0.030	0.011-0.019
entire data	power law type 2	1.269	1.116-1.418	1.217-1.320	0.013	0.005-0.027	0.010-0.018

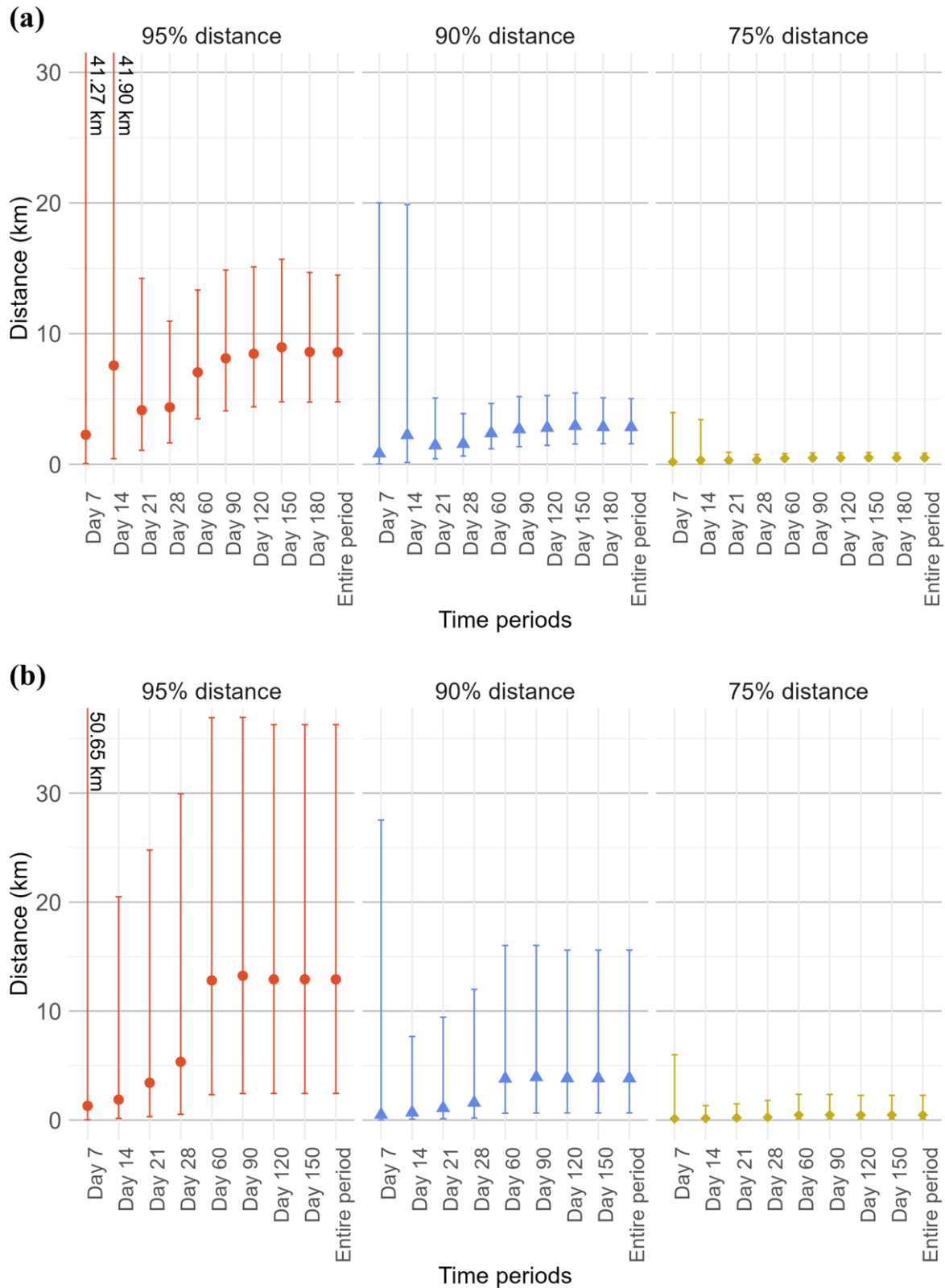


Figure S7. The estimated 95%, 90%, and 75% distances at each timepoint for (a) Cumbria and (b) Devon. The 95%, 90%, and 75% distances represent the radii within which 95%, 90%, and 75% of disease spread from a single infected premises are expected to occur. Shorter distances indicate that most transmissions occur over shorter ranges. When the 95% confidence intervals (CIs) of the estimates exceeded the y-axis range, the upper limit of the 95% CIs was recorded next to the corresponding cropped error bar.

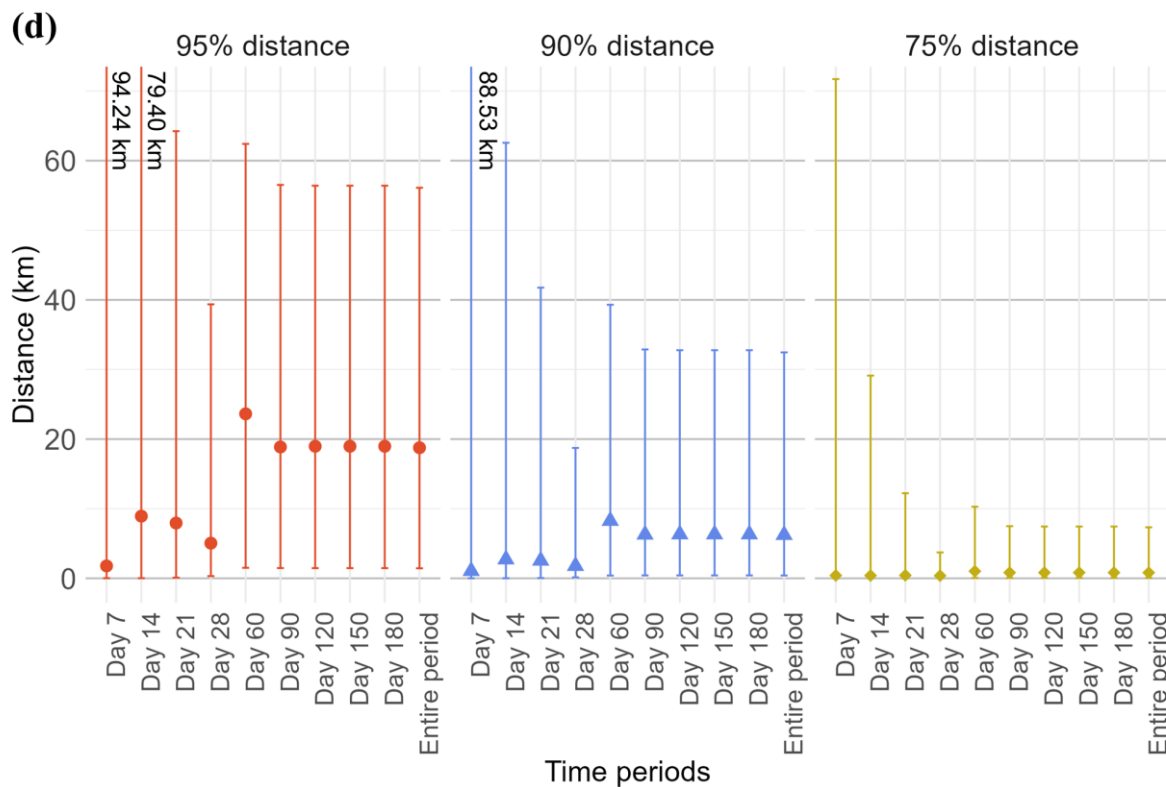
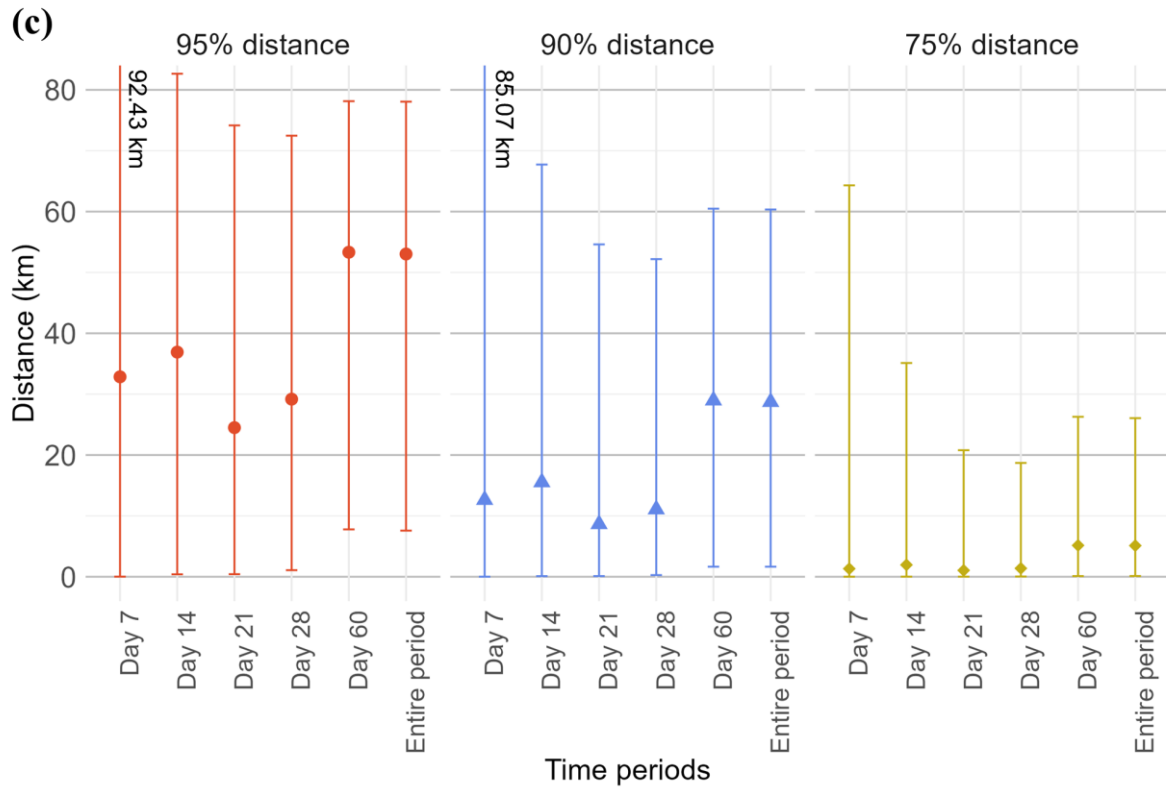


Figure S7. The estimated distance encompassing 95%, 90%, and 75% of the risk density at each timepoint for (c) Dumfries and Galloway and (d) Durham. The 95%, 90%, and 75% distances represent the radii within which 95%, 90%, and 75% of disease spread from a single infected premises are expected to occur. Shorter distances indicate that most transmissions occur over shorter ranges. When the 95% confidence intervals (CIs) of the estimates exceeded the y-axis range, the upper limit of the 95% CIs was recorded next to the corresponding cropped error bar.

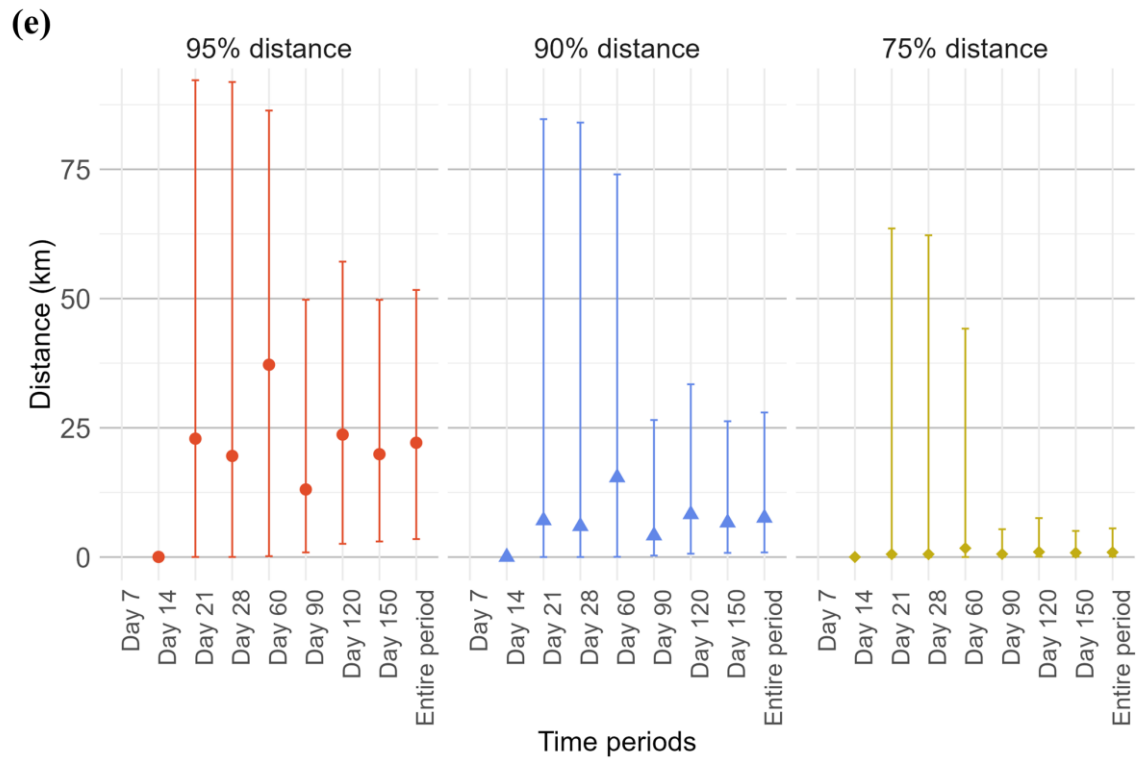


Figure S7. The estimated distance encompassing 95%, 90%, and 75% of the risk density at each timepoint for (e) North Yorkshire. The 95%, 90%, and 75% distances represent the radii within which 95%, 90%, and 75% of disease spread from a single infected premises are expected to occur. Shorter distances indicate that most transmissions occur over shorter ranges. Kernel estimation was not possible at day 7 due to the presence of only a single case.

D. Short-term projection of spatial risk using a kernel-derived infection pressure metric

Figure S8 display the locations of infectious and susceptible premises and their corresponding relative infection pressures at each timepoint. Contour lines allow for the grouping of susceptible premises with the same relative infection pressure. During the first month, premises with a relative infection pressure above 1 had a higher risk of FMD infection within the subsequent 7 days compared to those with a relative infection pressure of 1 or below (**Table S3 incidence risk ratios**).

The ROC curve results indicate that using infection pressure to predict FMD exposure outperforms random guessing (**Figure S9 ROC curves**), as also shown by the strong correlation between infection pressure and FMD exposure history in the Cox proportional hazard model (**Figures S11 and S12**). When the risk groups were defined using the cut-off value of 1 (i.e., median infection pressure), very high sensitivity and a high negative predictive value (NPV) were achieved (**Table S3 NPV and Figure S9 sensitivity**). This indicates an exceptionally small number of false negatives and a high level of confidence in the negative prediction results.

When the cut-off for the at-risk premises was gradually increased (**Figure S10**), the NPV remained consistently high, exceeding 0.97. Over time, the NPV either stayed stable (Devon, Durham, and North Yorkshire) or showed slight decreases (Cumbria and Dumfries and Galloway), which occurred because more stringent conditions shifted some infected premises into the false-negative category. While the high NPV remained encouraging even at the 90th percentile threshold, the positive predictive value (PPV) exhibited only a marginal increase.

The cut-off value must be chosen carefully. Due to the high morbidity of FMD, it is crucial not to miss a single infected premise. On the other hand, applying a lower cut-off value (i.e., the median) means controlling 50% of the premises in a region, which requires significant time and resources. Therefore, the cut-off should be determined based on the country's capacity and the severity of the situation.

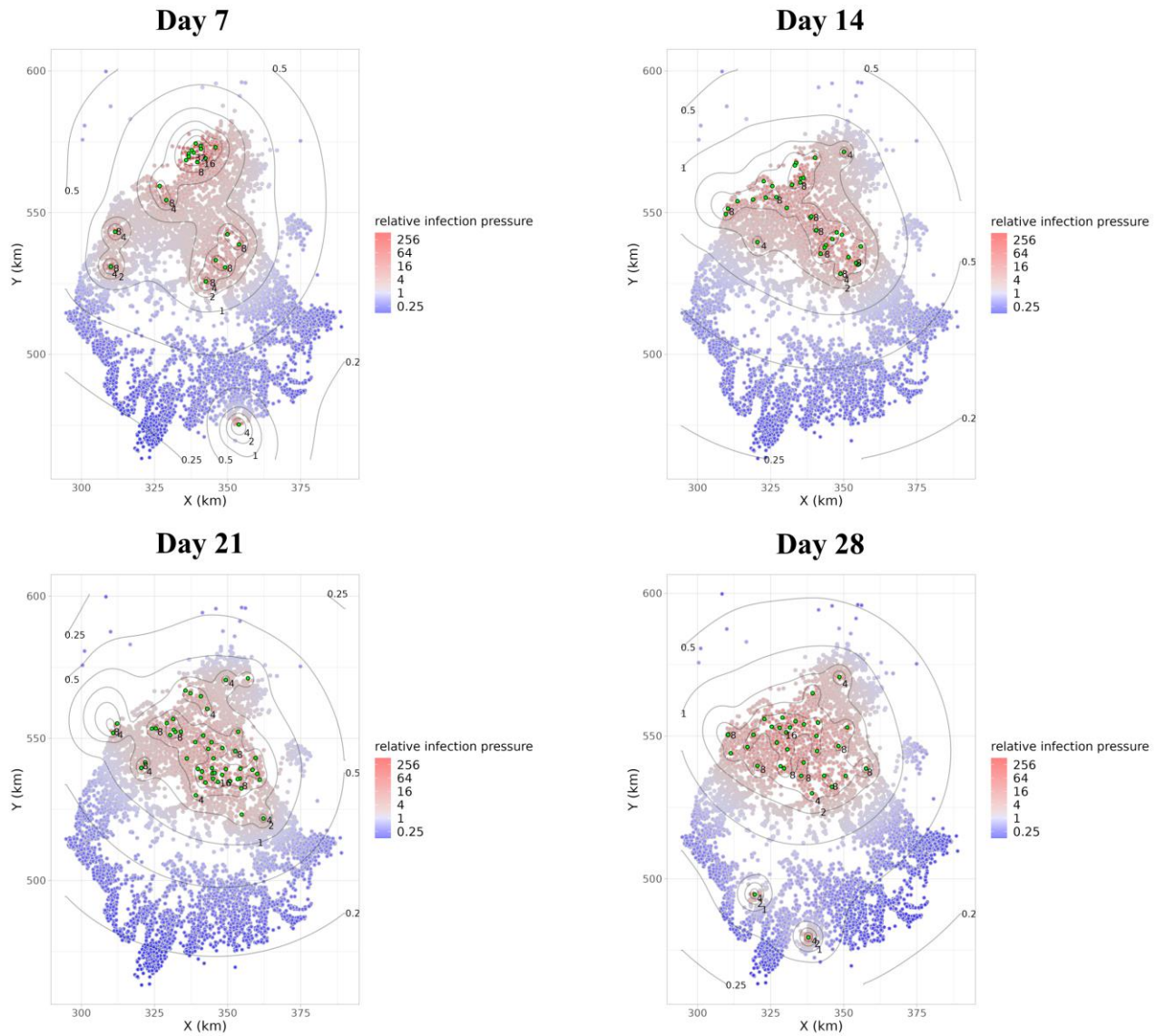


Figure S8(a). Maps of relative infection pressure calculated using the best-fitting transmission kernels at each timepoint (days 7, 14, 21, and 28) for Cumbria. Contour lines, generated using a thin-plate spline, connect locations with identical relative infection pressures. Premises are represented by circles, with green circles denoting infected premises (those with clinical signs onset but not yet depopulated) at the corresponding timepoint. Premises with relative infection pressure above one are shaded in a gradient of red, while those below one are shaded in a gradient of blue.

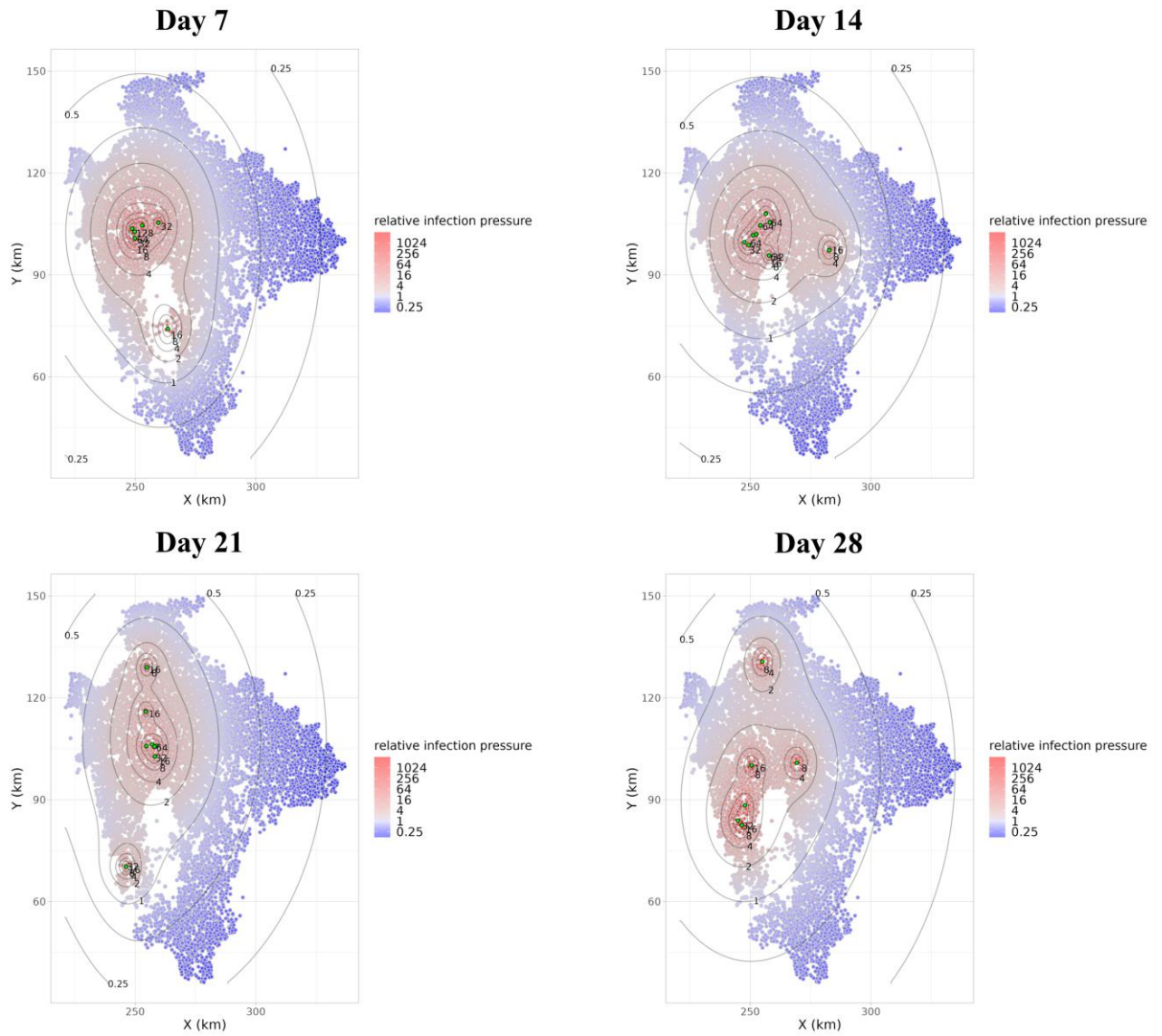


Figure S8(b). Maps of relative infection pressure calculated using the best-fitting transmission kernels at each timepoint (days 7, 14, 21, and 28) for Devon. Contour lines, generated using a thin-plate spline, connect locations with identical relative infection pressures. Premises are represented by circles, with green circles denoting infected premises (those with clinical signs onset but not yet depopulated) at the corresponding timepoint. Premises with relative infection pressure above one are shaded in a gradient of red, while those below one are shaded in a gradient of blue.

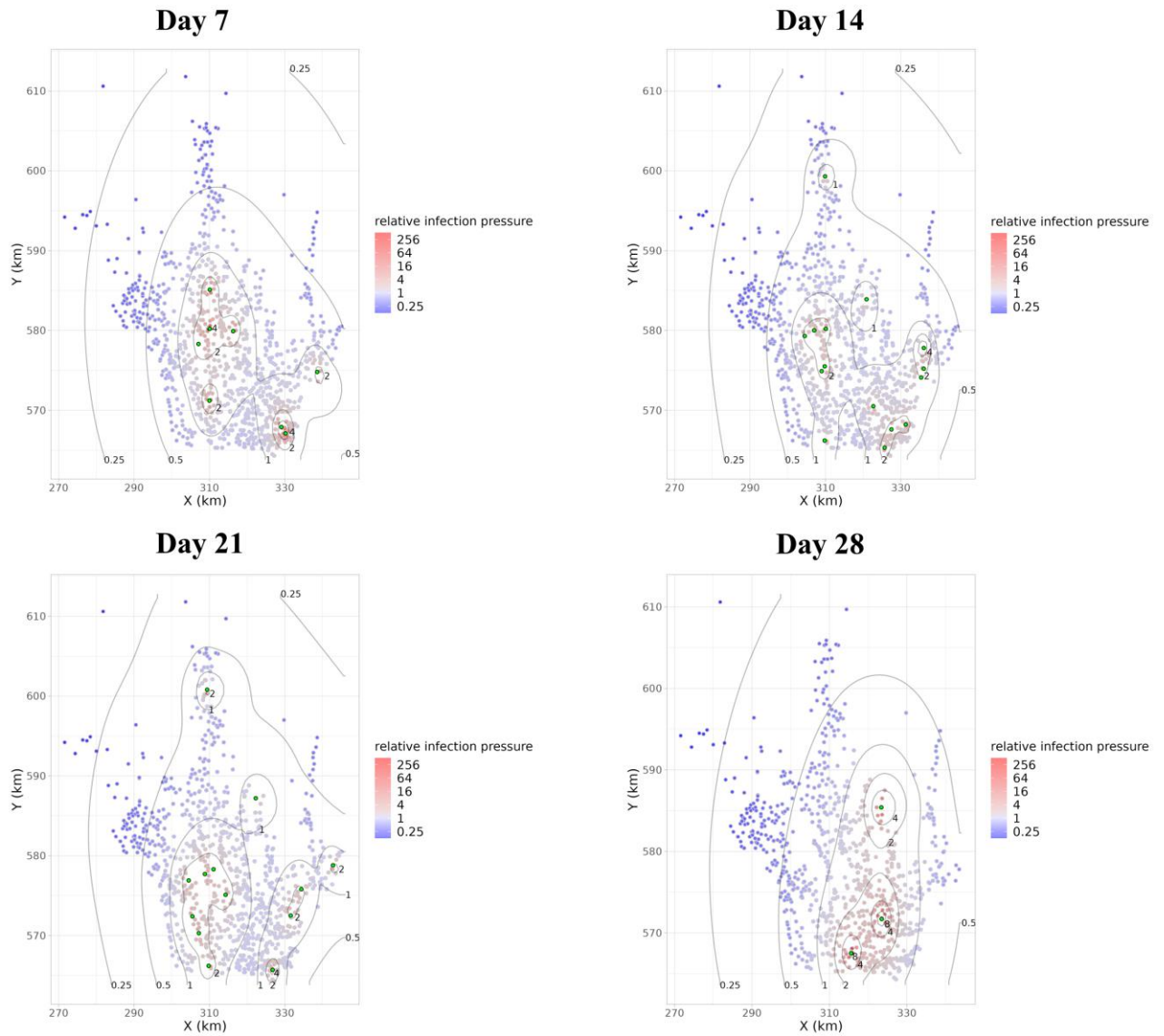


Figure S8(c). Maps of relative infection pressure calculated using the best-fitting transmission kernels at each timepoint (days 7, 14, 21, and 28) for Dumfries and Galloway. Contour lines, generated using a thin-plate spline, connect locations with identical relative infection pressures. Premises are represented by circles, with green circles denoting infected premises (those with clinical signs onset but not yet depopulated) at the corresponding timepoint. Premises with relative infection pressure above one are shaded in a gradient of red, while those below one are shaded in a gradient of blue.

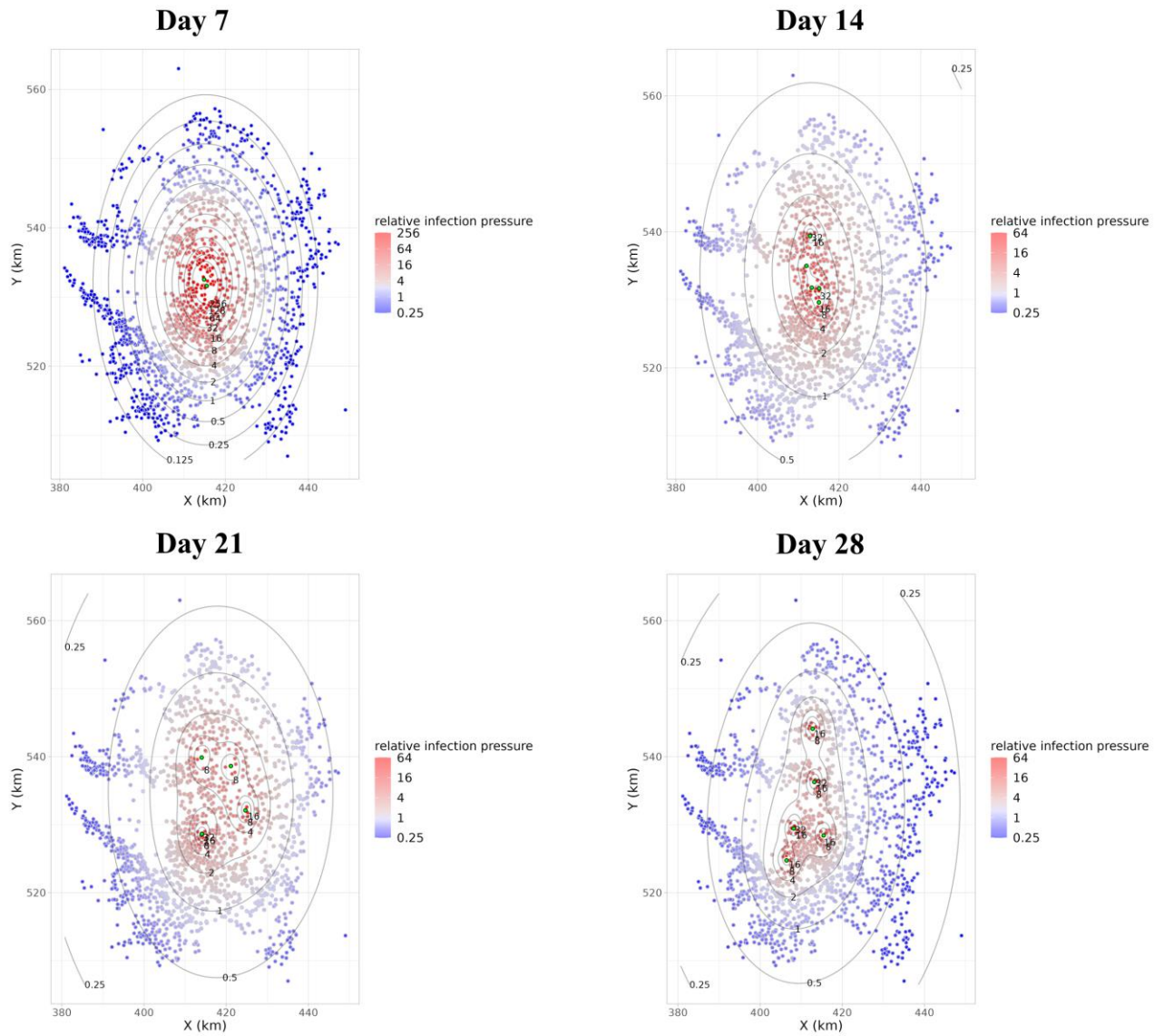


Figure S8(d). Maps of relative infection pressure calculated using the best-fitting transmission kernels at each timepoint (days 7, 14, 21, and 28) for Durham. Contour lines, generated using a thin-plate spline, connect locations with identical relative infection pressures. Premises are represented by circles, with green circles denoting infected premises (those with clinical signs onset but not yet depopulated) at the corresponding timepoint. Premises with relative infection pressure above one are shaded in a gradient of red, while those below one are shaded in a gradient of blue.

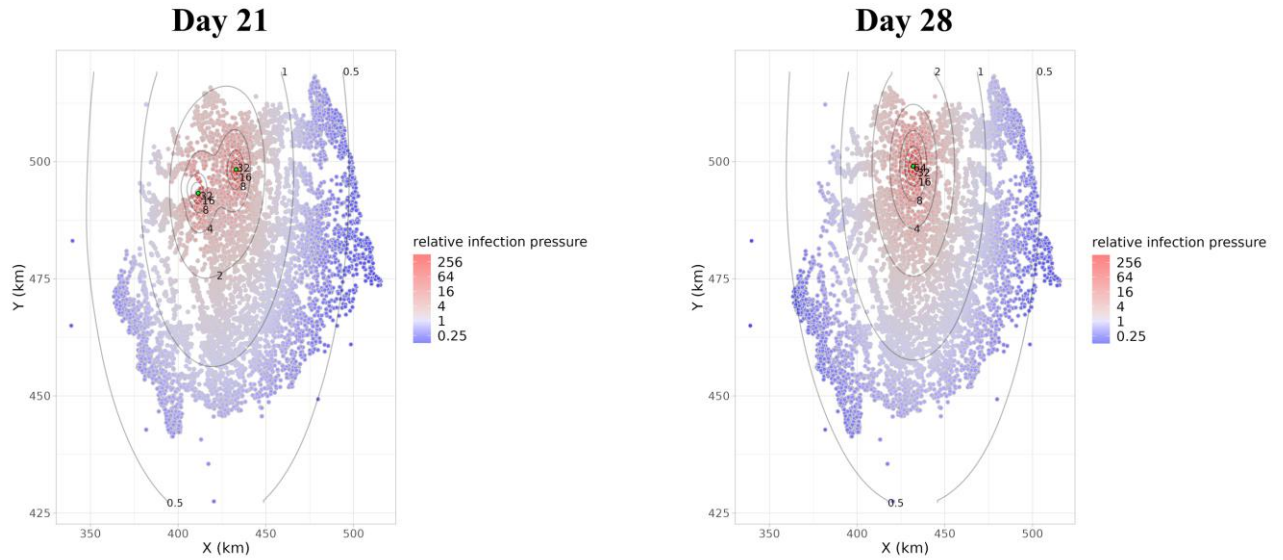


Figure S8(e). Maps of relative infection pressure calculated using the best-fitting transmission kernels at each timepoint (days 21 and 28) for North Yorkshire. Contour lines, generated using a thin-plate spline, connect locations with identical relative infection pressures. Premises are represented by circles, with green circles denoting infected premises (those with clinical signs onset but not yet depopulated) at the corresponding timepoint. Premises with relative infection pressure above one are shaded in a gradient of red, while those below one are shaded in a gradient of blue. For North Yorkshire, infection pressure could not be calculated at days 7 and 14 due to the absence of infected premises.

Table S3. 2×2 contingency tables for relative infection pressure and FMD infection across regions and timepoints. The table displays the relationship between the relative infection pressure (categorised using a cut-off value of 1, i.e., the median) and the subsequent 7-day FMD infection history (classified based on clinical symptom onset) at each timepoint (days 7, 14, 21, and 28). Calculated incidence risk ratios are shown alongside the tables and are rounded to two decimal places. Calculated positive and negative predictive values (PPV and NPV) are shown alongside the tables and are rounded to three decimal places.

(a) Cumbria

Day 7				Day 14			
	FMD infected (+)	FMD not infected (-)	PPV, NPV		FMD infected (+)	FMD not infected (-)	PPV, NPV
Relative infection pressure > 1	52	2875	0.017	Relative infection pressure > 1	94	2802	0.032
Relative infection pressure ≤ 1	1	2927	1.000	Relative infection pressure ≤ 1	1	2896	1.000
Incidence risk ratio	52.02 (7.20 – 376.02)			Incidence risk ratio	94.03 (13.12 – 674.07)		
Day 21				Day 28			
	FMD infected (+)	FMD not infected (-)	PPV, NPV		FMD infected (+)	FMD not infected (-)	PPV, NPV
Relative infection pressure > 1	132	2700	0.047	Relative infection pressure > 1	118	2617	0.043
Relative infection pressure ≤ 1	3	2829	0.999	Relative infection pressure ≤ 1	7	2729	0.997
Incidence risk ratio	44.00 (14.03 – 138.02)			Incidence risk ratio	16.86 (7.88 – 36.08)		

(b) Devon

Day 7				Day 14			
	FMD infected (+)	FMD not infected (-)	PPV, NPV		FMD infected (+)	FMD not infected (-)	PPV, NPV
Relative infection pressure > 1	17	4347	0.004	Relative infection pressure > 1	16	4335	0.004
Relative infection pressure ≤ 1	1	4363	1.000	Relative infection pressure ≤ 1	1	4350	1.000
Incidence risk ratio	17.00 (2.26 – 127.68)			Incidence risk ratio	16.00 (2.12 – 120.59)		
Day 21				Day 28			
	FMD infected (+)	FMD not infected (-)	PPV, NPV		FMD infected (+)	FMD not infected (-)	PPV, NPV
Relative infection pressure > 1	17	4308	0.004	Relative infection pressure > 1	31	4265	0.007
Relative infection pressure ≤ 1	1	4324	1.000	Relative infection pressure ≤ 1	3	4294	1.000
Incidence risk ratio	17.00 (2.26 – 127.68)			Incidence risk ratio	10.34 (3.16 – 33.78)		

(c) Dumfries and Galloway

Day 7				Day 14			
	FMD infected (+)	FMD not infected (-)	PPV, NPV		FMD infected (+)	FMD not infected (-)	PPV, NPV
Relative infection pressure > 1	19	557	0.033	Relative infection pressure > 1	24	536	0.043
Relative infection pressure ≤ 1	5	572	0.991	Relative infection pressure ≤ 1	3	558	0.995
Incidence risk ratio	3.81 (1.43 – 10.13)			Incidence risk ratio	8.01 (2.43 – 26.46)		
Day 21				Day 28			
	FMD infected (+)	FMD not infected (-)	PPV, NPV		FMD infected (+)	FMD not infected (-)	PPV, NPV
Relative infection pressure > 1	20	519	0.037	Relative infection pressure > 1	12	482	0.024
Relative infection pressure ≤ 1	6	533	0.989	Relative infection pressure ≤ 1	10	485	0.980
Incidence risk ratio	3.33 (1.35 – 8.24)			Incidence risk ratio	1.20 (0.52 – 2.76)		

(d) Durham

Day 7				Day 14			
	FMD infected (+)	FMD not infected (-)	PPV, NPV		FMD infected (+)	FMD not infected (-)	PPV, NPV
Relative infection pressure > 1	8	857	0.009	Relative infection pressure > 1	11	849	0.013
Relative infection pressure ≤ 1	1	864	0.999	Relative infection pressure ≤ 1	1	859	0.999
Incidence risk ratio	8.00 (1.00 – 63.83)			Incidence risk ratio	11.00 (1.42 – 85.02)		
Day 21				Day 28			
	FMD infected (+)	FMD not infected (-)	PPV, NPV		FMD infected (+)	FMD not infected (-)	PPV, NPV
Relative infection pressure > 1	17	837	0.019	Relative infection pressure > 1	13	823	0.014
Relative infection pressure ≤ 1	1	853	1.000	Relative infection pressure ≤ 1	1	835	1.000
Incidence risk ratio	17.00 (2.27 – 127.46)			Incidence risk ratio	13.00 (1.70 – 99.15)		

(e) North Yorkshire

Day 21				Day 28			
	FMD infected (+)	FMD not infected (-)	PPV, NPV		FMD infected (+)	FMD not infected (-)	PPV, NPV
Relative infection pressure > 1	4	3030	0.001	Relative infection pressure > 1	3	3014	0.001
Relative infection pressure ≤ 1	1	3033	1.000	Relative infection pressure ≤ 1	1	3016	1.000
Incidence risk ratio	4.00 (0.45 – 35.77)			Incidence risk ratio	3.00 (0.31 – 28.82)		

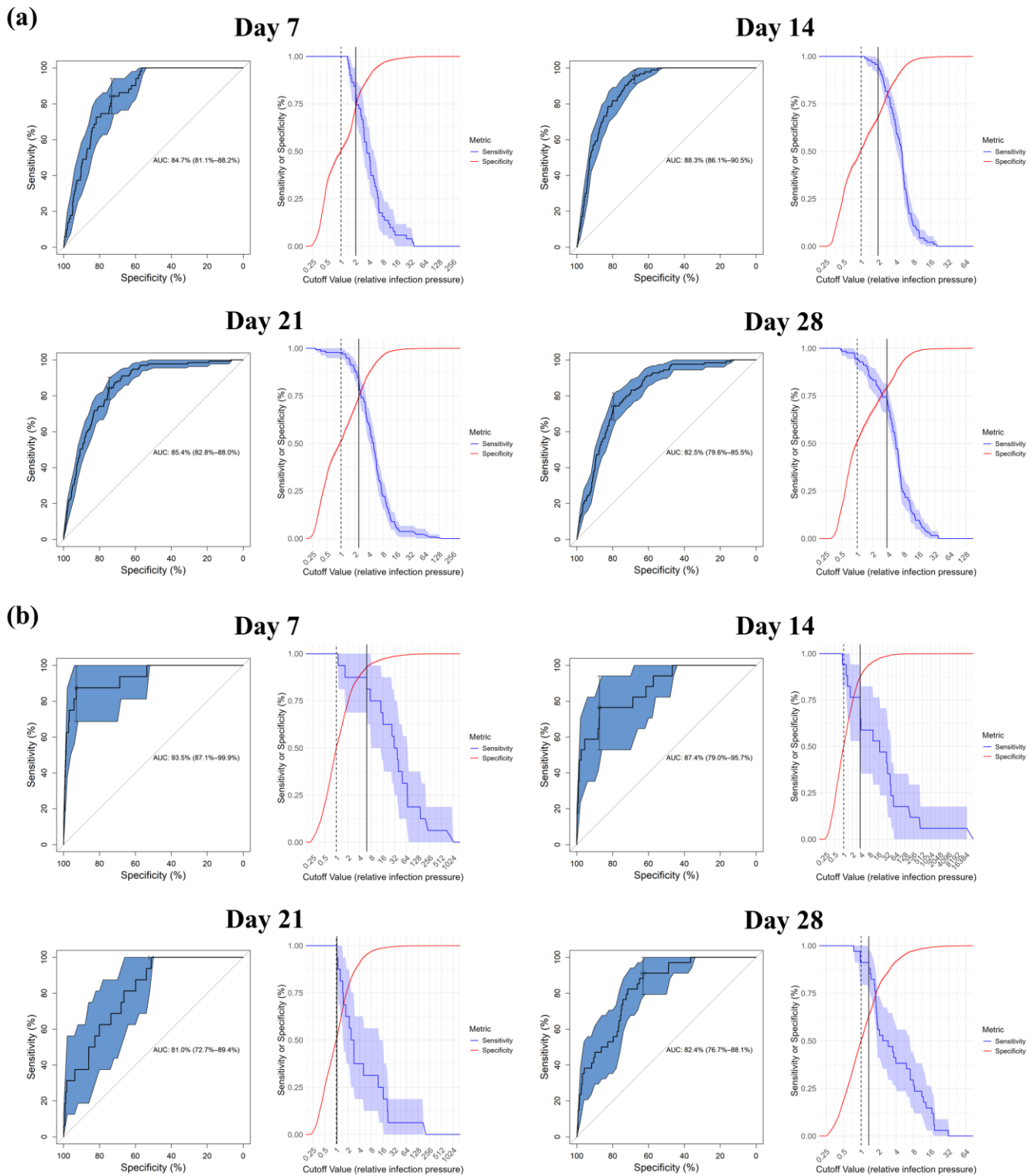


Figure S9. Receiver operating characteristic (ROC) curves with their areas under the curve and graphs displaying sensitivity (blue line) and specificity (red line) across different cut-off values for predicting FMD infection within the next 7 days in (a) Cumbria and (b) Devon. The diagonal dashed line on the ROC curves represents the performance of the random guessing. In the sensitivity and specificity graphs, the vertical dashed line indicates a relative infection pressure of 1 (i.e., median), while the vertical solid line indicates the optimal cut-off value based on Youden's statistic.

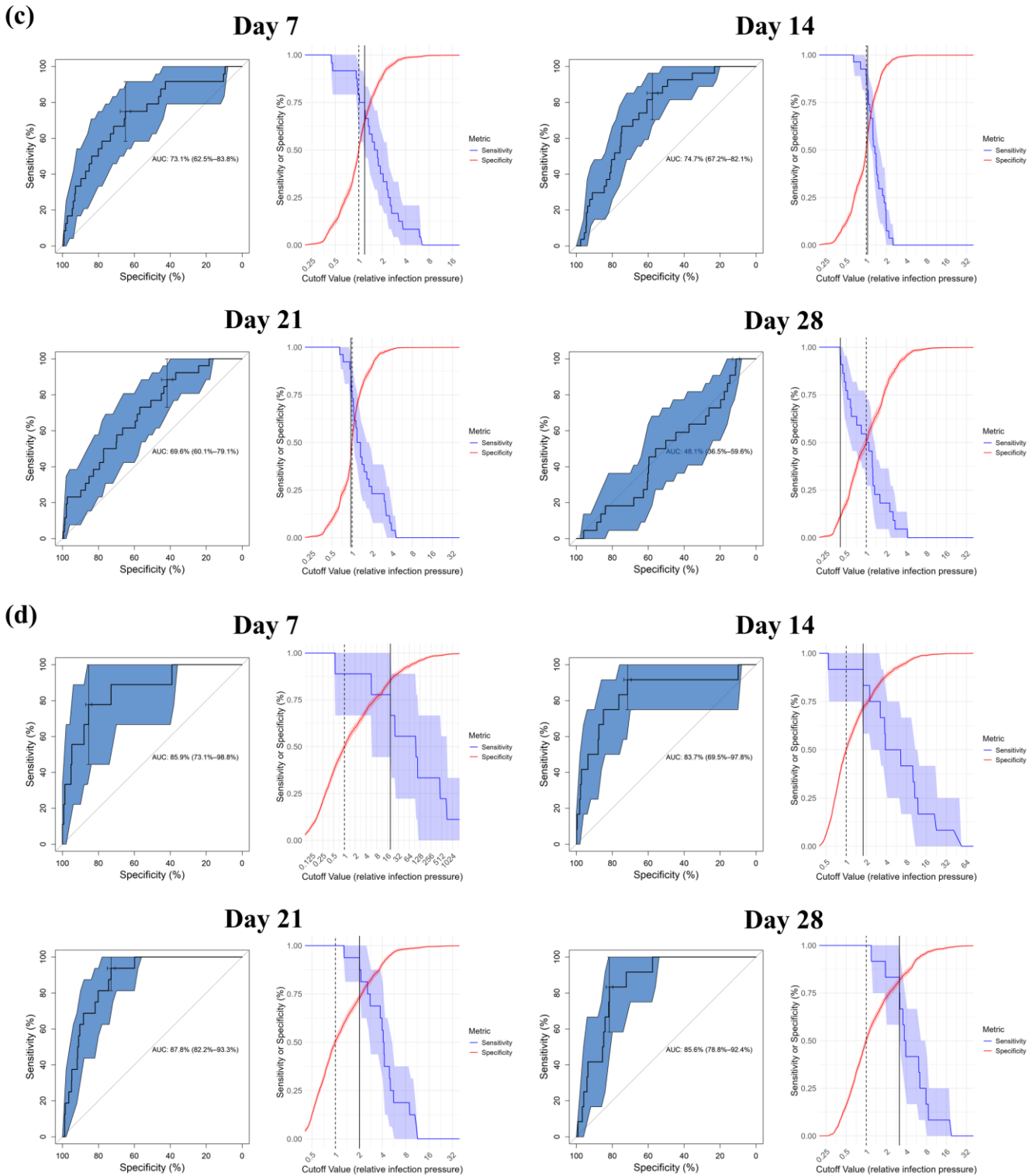


Figure S9. Receiver operating characteristic (ROC) curves with their areas under the curve and graphs displaying sensitivity (blue line) and specificity (red line) across different cut-off values for predicting FMD infection within the next 7 days in (c) Dumfries and Galloway and (d) Durham. The diagonal dashed line on the ROC curves represents the performance of the random guessing. In the sensitivity and specificity graphs, the vertical dashed line indicates a relative infection pressure of 1 (i.e., median), while the vertical solid line indicates the optimal cut-off value based on Youden's statistic.

(e)

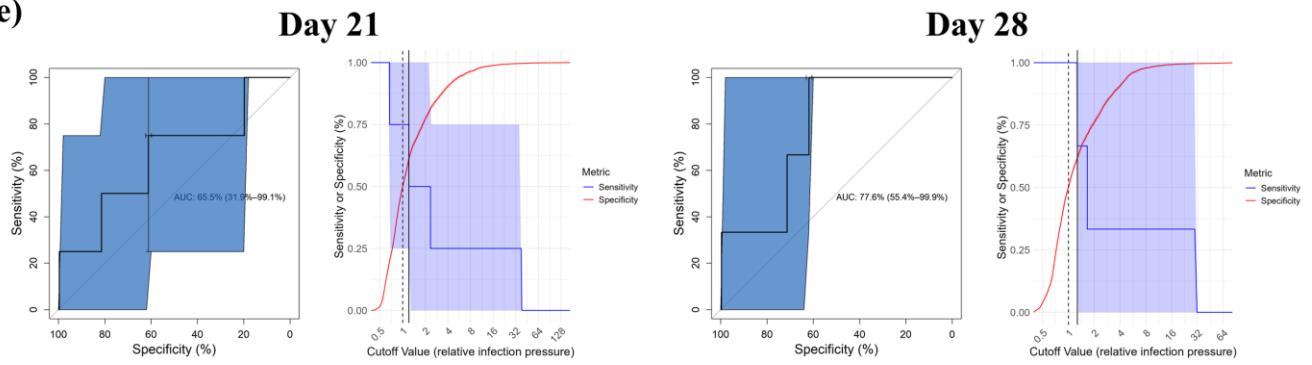
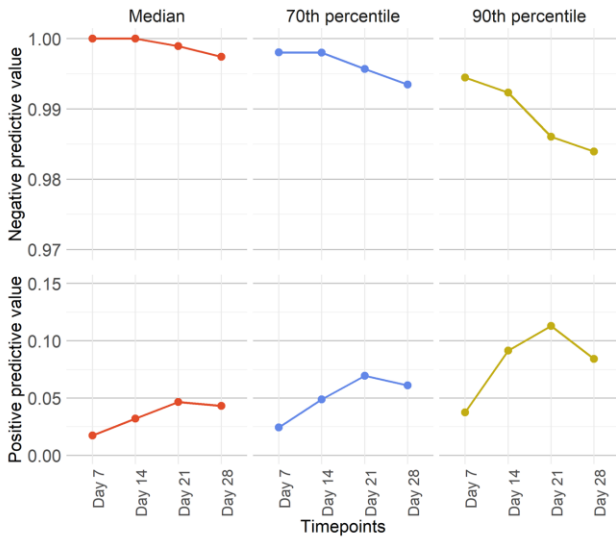
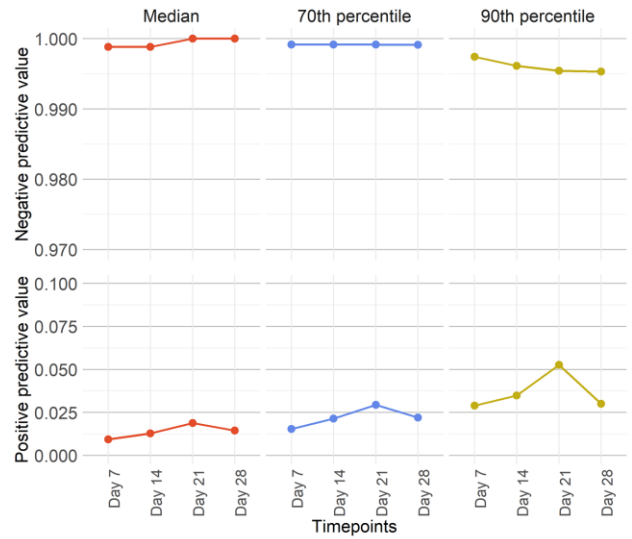


Figure S9(e). Receiver operating characteristic (ROC) curves with their areas under the curve and graphs displaying sensitivity (blue line) and specificity (red line) across different cut-off values for predicting FMD infection within the next 7 days in North Yorkshire. The diagonal dashed line on the ROC curves represents the performance of the random guessing. In the sensitivity and specificity graphs, the vertical dashed line indicates a relative infection pressure of 1 (i.e., median), while the vertical solid line indicates the optimal cut-off value based on Youden's statistic.

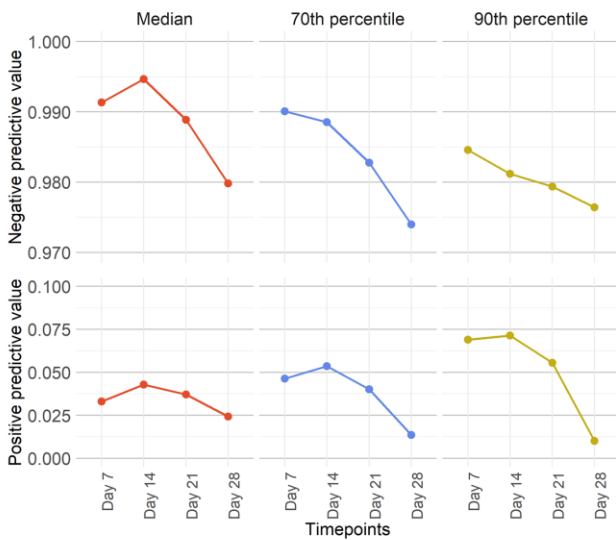
(a) Cumbria



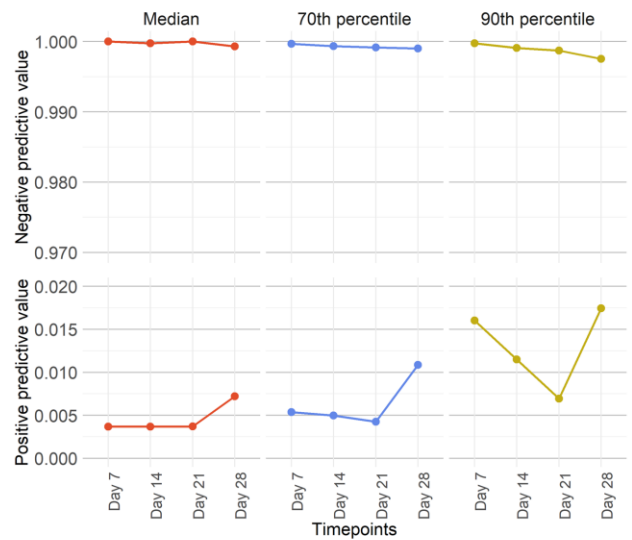
(b) Devon



(c) Dumfries and Galloway



(d) Durham



(e) North Yorkshire

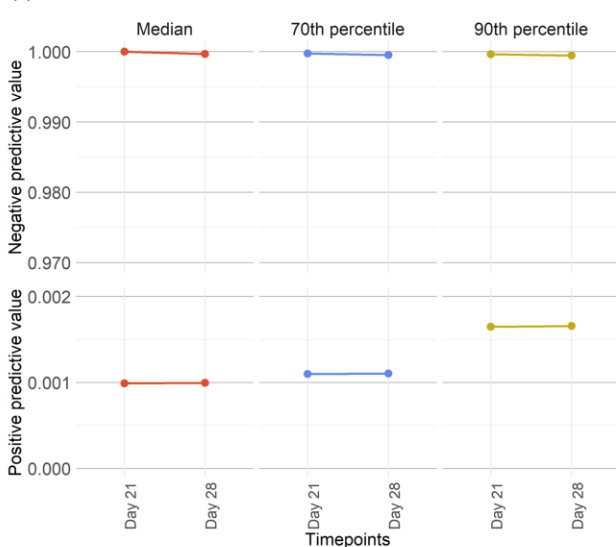


Figure S10. Negative and positive predictive values (NPV and PPV) for (a) Cumbria, (b) Devon, (c) Dumfries and Galloway, (d) Durham, and (e) North Yorkshire. NPV and PPV were calculated at each timepoint (days 7, 14, 21, and 28) with three cut-off values: median, 70th percentile, and 90th percentile of relative infection pressure.

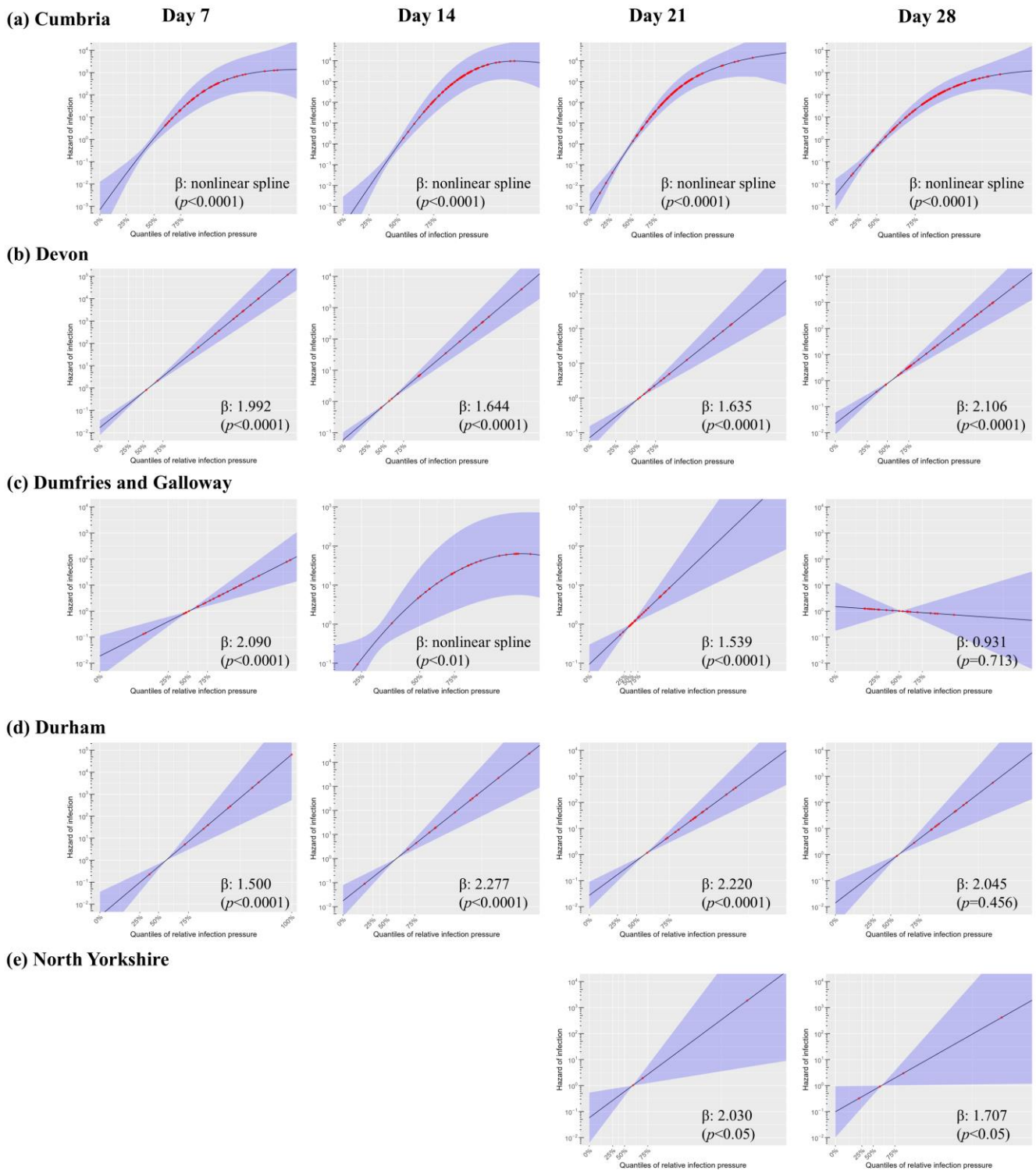


Figure S11. The crude relationship between hazard of infection and infection pressure of susceptible premises at each timepoint (day 7, 14, 21, and 28) for (a) Cumbria, (b) Devon, (c) Dumfries and Galloway, (d) Durham, and (e) North Yorkshire. The blue shaded areas represent the 95% confidence intervals. Relative infection pressure on the x-axis is shown as quantiles. Red points indicate the infection pressures of premises infected to FMD within 7 days following each timepoint and their corresponding infection hazards calculated using the Cox proportional hazard model. The exponential of coefficient β (hazard ratio) corresponding to relative infection pressure from the Cox proportional hazard model and its significance are displayed in the bottom right corner.

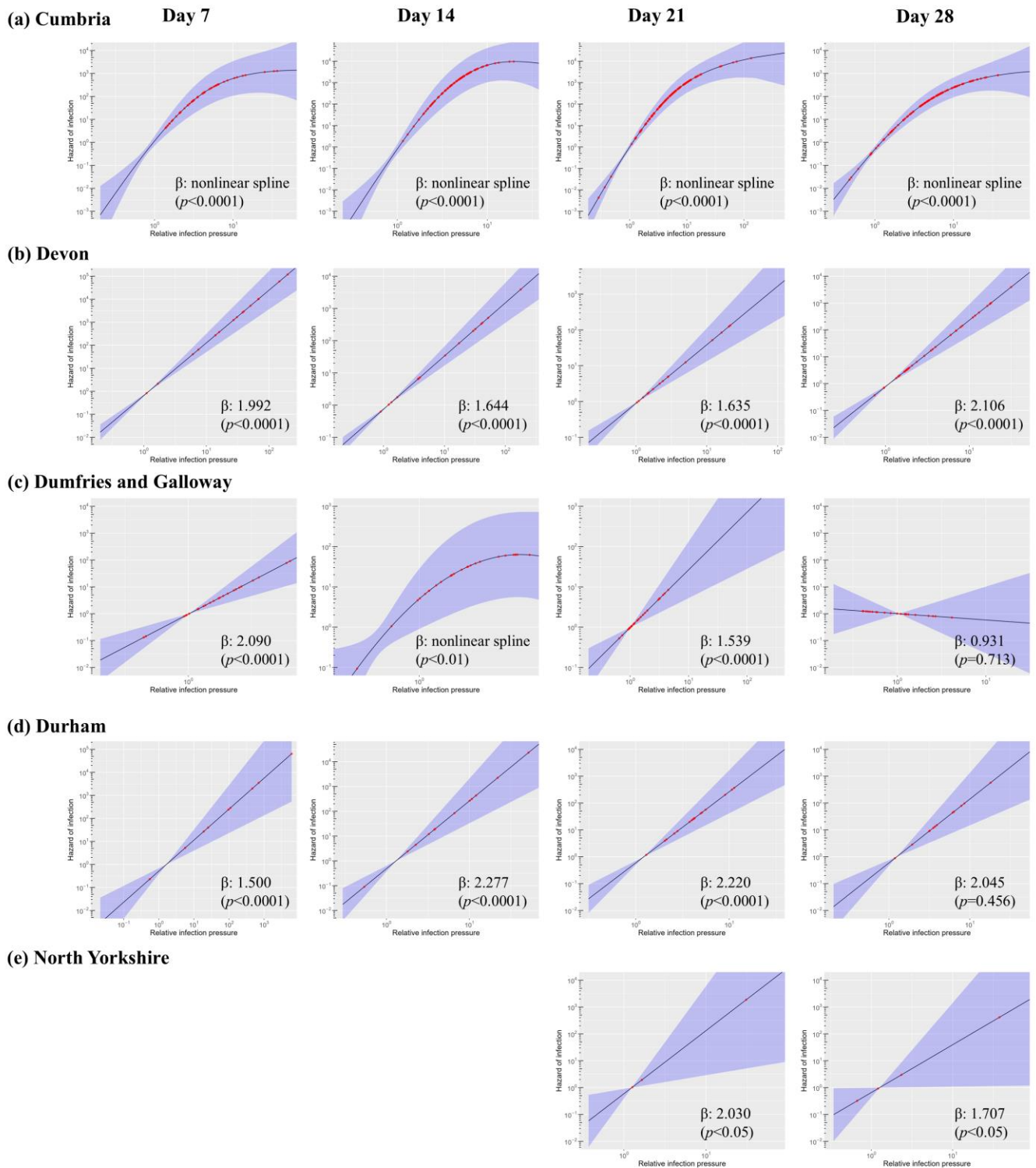


Figure S12. The crude relationship between hazard of infection and infection pressure of susceptible premises at each timepoint (day 7, 14, 21, and 28) for (a) Cumbria, (b) Devon, (c) Dumfries and Galloway, (d) Durham, and (e) North Yorkshire. The blue shaded areas represent the 95% confidence intervals. Red points indicate the infection pressures of premises infected to FMD within 7 days following each timepoint and their corresponding infection hazards calculated using the Cox proportional hazard model. The exponential of coefficient β (hazard ratio) corresponding to relative infection pressure from the Cox proportional hazard model and its significance are displayed in the bottom right corner.

Reference

1. Yadav S, Stenfeldt C, Branan MA, Moreno-Torres KI, Holmstrom LK, Delgado AH, Arzt J. Parameterization of the Durations of Phases of Foot-And-Mouth Disease in Cattle. *Front Vet Sci.* 2019;6:263.
2. Chanchaidechachai T, de Jong MCM, Fischer EAJ. Spatial model of foot-and-mouth disease outbreak in an endemic area of Thailand. *Prev Vet Med.* 2021;195:105468.