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An outlier-robust dynamic semiparametric model for jointly forecasting value-at-risk and expected shortfall

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Abstract

A joint dynamic semiparametric framework based on Fissler–Ziegel (FZ) loss minimization has recently received increasing attention for forecasting value-at-risk (VaR) and expected shortfall (ES). However, existing models typically do not account for the outliers, though such observations frequently occur in financial time series and may distort risk forecasts. To address this issue, this paper proposes an outlier-robust joint dynamic semiparametric model by incorporating bounded innovation propagation (BIP) into the conditional autoregressive value-at-risk (CAViaR) framework. By limiting the effect of extreme observations on the updating process, the proposed BIP-CAViaR-FZ model delivers more robust forecasts of both VaR and ES while maintaining semiparametric flexibility. The standard CAViaR-FZ model is nested as a special case. Simulation results show that the proposed model enhances the accuracy of VaR and ES estimation and forecasting in the presence of outliers, with this advantage being particularly pronounced under the skewed heavy-tailed distribution and at lower probability level. Furthermore, empirical findings demonstrate that the proposed model outperforms the standard non-robust competing counterpart in terms of both VaR and ES forecasts, thereby corroborating its robustness and effectiveness for joint VaR and ES prediction in practical applications.

Keywords: value-at-risk, expected shortfall, bounded innovation propagation, CAViaR, Fissler–Ziegel loss

1 Introduction

Value-at-risk (VaR) is a widely used risk measure defined as the return quantile at a given probability level. Under the Basel II Accord, VaR was extensively adopted for the quantification of market risk (Basel Committee on Banking Supervision, 2006). Despite its widespread use, VaR has two well-known limitations. First, it provides no information about losses beyond the quantile threshold. Second, it does not satisfy subadditivity, one of the key axioms of coherent risk measures (Artzner, Delbaen, Eber, & Heath, 1999).

Expected shortfall (ES), also referred to as conditional VaR (CVaR), was adopted under the Basel III framework as a more informative alternative to VaR (Basel Committee on Banking Supervision, 2019). ES is defined as the conditional expectation of losses exceeding the corresponding VaR threshold. Compared with VaR, ES offers several advantages. It captures tail losses beyond the VaR cutoff, satisfies the axioms of coherence (Acerbi & Tasche, 2002), and possesses a number of desirable theoretical properties, including monotonicity, law invariance, prudence, and no reward for concentration (R. Wang & Zitikis, 2021). Accordingly, ES has become increasingly prominent in financial risk measurement and forecasting.

A critical drawback of ES lies in its lack of individual elicitable (Gneiting, 2011), in the sense that it cannot be directly estimated by minimizing a strictly consistent loss function. This feature complicates both estimation and backtesting. However, ES and VaR are jointly elicitable (Fissler & Ziegel, 2016), which makes it possible to estimate them jointly through a strictly consistent joint loss function, commonly known as the Fissler–Ziegel (FZ) loss. Building on this, a growing literature has developed forecasting models (Patton, Ziegel, & Chen, 2019; Storti & Wang, 2022; Taylor, 2019) and backtesting procedures (Dimitriadis & Schnaitmann, 2021; Nolde & Ziegel, 2017) for VaR and ES analysis.

Much of the forecasting literature builds on the CAViaR-FZ framework of Taylor (2019), which combines the conditional autoregressive value-at-risk (CAViaR) model of Engle and Manganelli (2004) with FZ loss minimization. The CAViaR model has been widely used to characterize VaR dynamics (Chen, Gerlach, Hwang, & McAleer, 2012; Gerlach, Chen, & Chan, 2011; Jeon & Taylor, 2013; Meng & Taylor, 2018; Rubia & Sanchis-Marco, 2013; Tsiotas, 2020). Within the CAViaR-FZ framework, a multiplicative specification linking ES to VaR has been shown to be effective in capturing ES dynamics (Taylor, 2019). The framework has subsequently been extended in various directions, including high-frequency settings (Chen, Hsu, & Watanabe, 2023; Gerlach & Wang, 2020; Meng & Taylor, 2020; Peiris, Wang, Gerlach, & Tran, 2024; C. Wang, Gerlach, & Chen, 2023), mixed-frequency models (Le, 2020; Luo, Xue, & Izzeldin, 2025; Xu, Chen, Jiang, & Liu, 2022), multivariate specifications (Merlo, Petrella, & Raponi, 2021; Storti & Wang, 2025), regime-switching dynamics (Gao, Ye, & Guo, 2022), dynamic ES-to-VaR ratios (Li, Clements, & Drovandi, 2023; Taylor, 2022), and long-range dependence (Mitrodima & Oberoi, 2024; J. Wang & Wang, 2025).

Despite these advances, the role of outliers has received limited attention in the CAViaR-FZ literature. This issue is potentially important because outliers are common in financial time series (Dutta, 2018; Galvani & Li, 2023; Preminger & Franck, 2007; Trimbur, 2010; Trucíos, 2019). In practice, outliers often appear as large and isolated

shocks that deviate markedly from the prevailing pattern of the data. Since VaR and ES forecasts are updated recursively, even a small number of extreme observations may propagate through the updating equations and adversely affect subsequent risk forecasts.

Outliers may affect not only the distributional characteristics of returns but also the estimated dynamics of time-varying models. In particular, they may inflate dispersion and exaggerate skewness or tail heaviness (Charles, 2008; Charles & Darné, 2014, 2017). In dynamic settings, they may also result in larger in-sample estimation errors and poorer out-of-sample forecast performance (Boudt, Danielsson, & Laurent, 2013; Carriero, Clark, Marcellino, & Mertens, 2024; Fiszeder, Małacka, & Molnár, 2024; Grané & Veiga, 2014; Koehler, Snyder, Ord, & Beaumont, 2012; Shi, 2022; Trucíos, 2019; Trucíos, Mazzeu, Hotta, Pereira, & Hallin, 2021). These issues are particularly relevant for joint VaR and ES forecasting, where distorted updating may impair the accurate measurement of tail risk.

A widely used approach for handling outliers in dynamic models is bounded innovation propagation (BIP), also known as bounded influence propagation (Muler, Peña, & Yohai, 2009; Muler & Yohai, 2008). By incorporating a bounded function into the updating mechanism, the BIP approach limits the effect of outliers on time-varying processes. It has been successfully applied in robust dynamic modeling of the mean (Britos & Ojeda, 2019; Britos, Ojeda, Astrain, & Bustos, 2020; Lu, Wang, & Gao, 2020; Muler et al., 2009; Muler & Yohai, 2013; Muma & Zoubir, 2017), volatility (Bamba, Hili, Diongue, & N'Guessan, 2021; Boudt & Croux, 2010; Carnero, Peña, & Ruiz, 2012; Fiszeder et al., 2024; Laurent, Lecourt, & Palm, 2016; Muler & Yohai, 2008; Trucíos, Hotta, & Ruiz, 2017), and correlation (Boudt et al., 2013). However, to the best of our knowledge, this idea has not yet been explicitly incorporated into joint dynamic semiparametric models for VaR and ES.

Motivated by this gap, this paper develops a robust extension of the CAViaR-FZ framework by incorporating the BIP approach into the dynamic updating equations. The proposed BIP-CAViaR-FZ model preserves the semiparametric flexibility of the original framework while reducing the sensitivity of VaR and ES forecasts to extreme observations. Moreover, the standard CAViaR-FZ model is nested as a special case, which facilitates implementation and comparison. Evidence from the simulation and empirical analyses suggests that the proposed specification can improve risk forecasting performance in the presence of outliers.

The remainder of the paper is organized as follows. Section 2 reviews the CAViaR-FZ framework for VaR and ES. Section 3 introduces the BIP approach and presents the proposed BIP-CAViaR-FZ model. Section 4 reports the simulation study. Section 5 evaluates the empirical performance of the model. Section 6 concludes.

2 The CAViaR-FZ framework for VaR and ES

Let F_t denote the conditional distribution function of the return y_t given the information set $\mathcal{F}_{t-1}$, that is,

$$y_t \mid \mathcal{F}_{t-1} \sim F_t.$$

Then the conditional VaR and ES of y_t at probability level $\alpha \in (0, 1)$ are defined as

$$\begin{aligned} v_t &= F_t^{-1}(\alpha), \\ e_t &= \mathbb{E}(y_t \mid y_t \leq v_t, \mathcal{F}_{t-1}). \end{aligned} \quad (1)$$

The CAViaR-FZ framework of [Taylor \(2019\)](#) combines the CAViaR process of [Engle and Manganelli \(2004\)](#) with the Fissler-Ziegel (FZ) loss function of [Fissler and Ziegel \(2016\)](#). The framework is semiparametric in the sense that it specifies parametric dynamics for VaR and ES while leaving the conditional distribution of returns unspecified. This feature avoids direct estimation of the conditional density and may therefore reduce the risk of distributional misspecification.

The CAViaR-FZ framework can be expressed as

$$\begin{aligned} v_t &\sim \text{CAViaR}, \\ e_t &= (1 + \gamma)v_t, \quad \gamma > 0, \\ \hat{\theta} &= \arg \min_{\theta} \frac{1}{T} \sum_{t=1}^T L_{FZ}(y_t, v_t, e_t; \alpha), \end{aligned} \quad (2)$$

where $e_t < v_t < 0$, θ denotes the parameter vector, and T is the sample size. In this framework, the CAViaR specification captures the dynamics of VaR, whereas ES is modeled as a scalar multiple of VaR. Parameter estimation is conducted by minimizing the FZ loss function.

2.1 The CAViaR process

The CAViaR process of [Engle and Manganelli \(2004\)](#) is widely used for modeling dynamic VaR. Four commonly used specifications are the symmetric absolute value (SAV), asymmetric slope (AS), indirect GARCH (IG), and indirect GJR-GARCH (IGJR).

SAV:

$$|v_t| = \beta_0 + \beta_1 |v_{t-1}| + \beta_2 |y_{t-1}|. \quad (3)$$

AS:

$$|v_t| = \beta_0 + \beta_1 |v_{t-1}| + [\beta_2^- \mathbb{I}(y_{t-1} < 0) + \beta_2^+ \mathbb{I}(y_{t-1} > 0)] |y_{t-1}|. \quad (4)$$

IG:

$$v_t^2 = \beta_0 + \beta_1 v_{t-1}^2 + \beta_2 y_{t-1}^2. \quad (5)$$

IGJR:

$$v_t^2 = \beta_0 + \beta_1 v_{t-1}^2 + [\beta_2^- \mathbb{I}(y_{t-1} < 0) + \beta_2^+ \mathbb{I}(y_{t-1} > 0)] y_{t-1}^2. \quad (6)$$

Here, $v_t < 0$ for all t , and $\mathbb{I}(\cdot)$ denotes the indicator function. To ensure the positivity of $|v_t|$ or v_t^2 , we impose $\beta_0 > 0$, $0 \leq \beta_1 < 1$, $\beta_2 \geq 0$, $\beta_2^- \geq 0$, and $\beta_2^+ \geq 0$. The SAV and IG specifications respond symmetrically to positive and negative lagged returns, whereas the AS and IGJR specifications allow for asymmetric effects. In addition, when $\beta_2^- = \beta_2^+$, the AS and IGJR models reduce to the SAV and IG models, respectively.

Although these specifications are widely used, they do not explicitly account for outliers. In each case, the effect of y_{t-1} on v_t is unbounded, so an extreme observation may exert undue influence on the dynamic updating of VaR. Since financial return series often contain outliers, such observations may distort parameter estimation and adversely affect the accuracy of subsequent risk forecasts.

2.2 The FZ loss function

The FZ loss function (Fissler & Ziegel, 2016) provides a strictly consistent joint loss function for VaR and ES. Among the available specifications, the FZ0 loss function is commonly used in empirical applications (Patton et al., 2019; Taylor, 2019). It is defined as

$$L_{FZ0}(y, v, e; \alpha) = -\frac{\mathbb{I}(y \leq v)(v - y)}{\alpha e} + \frac{v}{e} + \ln(-e) - 1, \quad (7)$$

where $y \in \mathbb{R}$, $v < 0$, $e < 0$, and $\alpha \in (0, 1)$.

The FZ0 loss function has several convenient properties. First, the loss difference is homogeneous of degree zero (Patton et al., 2019), implying scale invariance. Second, minimizing the FZ0 loss is equivalent to maximizing the likelihood associated with an asymmetric Laplace (AL) distribution (Taylor, 2019). The corresponding density is given by

$$f_{AL}(y) = \frac{\alpha - 1}{e} \exp\left(\frac{[\alpha - \mathbb{I}(y \leq v)](y - v)}{\alpha e}\right), \quad (8)$$

where v , e , and α may be interpreted as location, scale, and skewness parameters, respectively. For this reason, the FZ0 loss function is also referred to as the AL loss function. In this paper, we use the FZ0 loss function for parameter estimation.

3 The BIP-CAViaR-FZ model

To address the lack of robustness in the standard CAViaR-FZ framework, we develop a robust extension, termed the BIP-CAViaR-FZ model. The key idea is to incorporate BIP into the dynamic updating equation for VaR within the CAViaR-FZ framework.

The model is given by

$$\begin{aligned} v_t &\sim \text{BIP-CAViaR}, \\ e_t &= (1 + \gamma)v_t, \quad \gamma > 0, \\ \hat{\theta} &= \arg \min_{\theta} \frac{1}{T} \sum_{t=1}^T L_{FZ}(y_t, v_t, e_t; \alpha), \end{aligned} \quad (9)$$

where v_t follows a BIP-CAViaR process and the ratio e_t/v_t is assumed to be constant. Estimation proceeds by minimizing the FZ loss function with respect to the parameter vector θ . By bounding the effect of extreme innovations in the updating mechanism, the proposed specification is intended to reduce the sensitivity of both VaR and ES forecasts to outliers.

The remainder of this section is organized as follows. Section 3.1 introduces the BIP approach. Section 3.2 develops the BIP-CAViaR process. Section 3.3 presents the estimation procedure.

3.1 The BIP method

The BIP method provides a general approach to constructing robust time-varying parameter models in the presence of outliers. Its central idea is to introduce a bounded function into the dynamic updating equation, thereby limiting the propagation of extreme innovations through the model dynamics.

To illustrate the idea, consider the standard GARCH model

$$\sigma_t^2 = \beta_0 + \beta_1 \sigma_{t-1}^2 + \beta_2 \varepsilon_{t-1}^2, \quad (10)$$

where σ_t^2 denotes the conditional variance, ε_t is a zero-mean innovation, and $(\beta_0, \beta_1, \beta_2)$ is the parameter vector. The usual constraints $\beta_0 > 0$, $\beta_1 \geq 0$, $\beta_2 \geq 0$, and $\beta_1 + \beta_2 < 1$ ensure positivity and covariance stationarity. In this model, the effect of past innovations on the conditional variance is unbounded, so an extreme observation can have a disproportionately large impact on volatility dynamics.

The BIP modification takes the form

$$\sigma_t^2 = \beta_0 + \beta_1 \sigma_{t-1}^2 + \beta_2 \eta\left(\frac{\varepsilon_{t-1}^2}{\sigma_{t-1}^2}; c\right) \sigma_{t-1}^2, \quad (11)$$

where $\eta(x; c)$ is a bounded function with upper bound c . This specification limits the influence of extreme innovations on future conditional variances. When $c \rightarrow \infty$, it reduces to the standard GARCH model.

A commonly used choice is

$$\eta(x; c) = \min(x, c), \quad (12)$$

with $x \geq 0$ and $c > 0$, which yields

$$\sigma_t^2 = \beta_0 + \beta_1 \sigma_{t-1}^2 + \beta_2 \min(\varepsilon_{t-1}^2, c \sigma_{t-1}^2). \quad (13)$$

Hence, when ε_{t-1}^2 exceeds the threshold $c \sigma_{t-1}^2$, its contribution is capped. This feature underlies the robustness of BIP-type dynamic models to outliers ([Muler & Yohai, 2008](#)).

3.2 The BIP-CAViaR process

The BIP-CAViaR process incorporates bounded innovation propagation into the standard CAViaR framework. More specifically, a bounded function is introduced into the VaR updating equation so as to limit the effect of extreme observations.

Starting from the four standard CAViaR specifications, the corresponding BIP-CAViaR models are given as follows.

BIP-SAV:

$$|v_t| = \beta_0 + \beta_1 |v_{t-1}| + \beta_2 \eta\left(\frac{|y_{t-1}|}{|v_{t-1}|}; \beta_3\right) |v_{t-1}|. \quad (14)$$

BIP-AS:

$$|v_t| = \beta_0 + \beta_1 |v_{t-1}| + [\beta_2^- \mathbb{I}(y_{t-1} < 0) + \beta_2^+ \mathbb{I}(y_{t-1} > 0)] \eta\left(\frac{|y_{t-1}|}{|v_{t-1}|}; \beta_3\right) |v_{t-1}|. \quad (15)$$

BIP-IG:

$$v_t^2 = \beta_0 + \beta_1 v_{t-1}^2 + \beta_2 \eta\left(\frac{y_{t-1}^2}{v_{t-1}^2}; \beta_3^2\right) v_{t-1}^2. \quad (16)$$

BIP-IGJR:

$$v_t^2 = \beta_0 + \beta_1 v_{t-1}^2 + [\beta_2^- \mathbb{I}(y_{t-1} < 0) + \beta_2^+ \mathbb{I}(y_{t-1} > 0)] \eta\left(\frac{y_{t-1}^2}{v_{t-1}^2}; \beta_3^2\right) v_{t-1}^2. \quad (17)$$

To ensure positivity of $|v_t|$ or v_t^2 , we impose the same restrictions as in the corresponding CAViaR specifications, together with $\beta_3 > 0$.

We use the bounded function $\eta(x; c) = \min(x, c)$, where $x \geq 0$ and $c > 0$. Under this specification, the BIP-SAV model reduces to

$$|v_t| = \beta_0 + \beta_1 |v_{t-1}| + \beta_2 \min(|y_{t-1}|, \beta_3 |v_{t-1}|), \quad (18)$$

with analogous expressions applying to the remaining specifications.

Hence, when $|y_{t-1}|$ exceeds the threshold $\beta_3 |v_{t-1}|$, its contribution to the updating equation is bounded. This feature limits the sensitivity of VaR dynamics to extreme observations and, through the joint specification, is intended to enhance the robustness of ES forecasts as well.

3.3 Estimation

Following the estimation strategy commonly used in the literature on joint dynamic VaR–ES models (Taylor, 2019), we estimate the parameters of the proposed BIP-CAViaR-FZ model in four stages. First, 10^4 candidate parameter vectors are generated. Second, the 10 candidates associated with the lowest FZ0 loss values are retained. Third, local optimization is performed using each of these 10 candidates as a starting value. Finally, the parameter vector that attains the minimum FZ0 loss is selected as the final estimate.

Estimation of the BIP-CAViaR-FZ model requires the gradient of the objective function. Let

$$L_T = \frac{1}{T} \sum_{t=1}^T L_{FZ0}(y_t, v_t, e_t; \alpha) \quad (19)$$

denote the average FZ0 loss. The gradient with respect to the parameter vector θ is then given by

$$\begin{aligned}\frac{\partial L_T}{\partial \theta} &= \frac{1}{T} \sum_{t=1}^T \frac{\partial L_{FZ0}(y_t, v_t, e_t; \alpha)}{\partial \theta} \\ &= \frac{1}{T} \sum_{t=1}^T \left[\frac{\partial L_{FZ0}(y_t, v_t, e_t; \alpha)}{\partial v_t} \frac{\partial v_t}{\partial \theta} + \frac{\partial L_{FZ0}(y_t, v_t, e_t; \alpha)}{\partial e_t} \frac{\partial e_t}{\partial \theta} \right].\end{aligned}\quad (20)$$

The partial derivatives of L_{FZ0} with respect to v_t and e_t are

$$\begin{aligned}\frac{\partial L_{FZ0}(y_t, v_t, e_t; \alpha)}{\partial v_t} &= -\frac{\mathbb{I}(y_t \leq v_t)}{\alpha e_t} + \frac{1}{e_t}, \\ \frac{\partial L_{FZ0}(y_t, v_t, e_t; \alpha)}{\partial e_t} &= \frac{\mathbb{I}(y_t \leq v_t)(v_t - y_t)}{\alpha e_t^2} - \frac{v_t}{e_t^2} + \frac{1}{e_t}.\end{aligned}\quad (21)$$

The derivative of v_t with respect to θ depends on the specific BIP-CAViaR specification. Since $e_t = (1 + \gamma)v_t$, its derivative with respect to θ can be written as

$$\frac{\partial e_t}{\partial \theta} = v_t \frac{\partial \gamma}{\partial \theta} + (1 + \gamma) \frac{\partial v_t}{\partial \theta}. \quad (22)$$

The degree of robustness in the BIP-CAViaR-FZ model is governed by β_3 , which determines the upper bound $\beta_3|v_{t-1}|$ applied to $|y_{t-1}|$. If β_3 is too small, the bound may be overly restrictive and dampen not only outlying observations but also regular shocks. To avoid this, we impose $\beta_3 \geq 1$ in estimation, so that ordinary shocks retain their natural influence while unusually large shocks remain bounded. For ease of presentation, we report β_3^{-1} in the tables. Under $\beta_3 \geq 1$, we have $\beta_3^{-1} \in (0, 1]$, where smaller values indicate weaker bounding, and $\beta_3^{-1} \rightarrow 0$ (equivalently, $\beta_3 \rightarrow \infty$) recovers the non-robust model.

4 Simulation study

This section investigates the finite-sample performance of the BIP-CAViaR-FZ model by means of simulation. [Section 4.1](#) presents the data-generating process (DGP), [Section 4.2](#) describes the simulation design, and [Sections 4.3 to 4.5](#) report the results for parameter estimation, risk estimation, and risk forecasting, respectively.

4.1 Data-generating process

The DGP for the BIP-CAViaR-FZ model is given by

$$\begin{aligned}y_t &= \frac{v_t}{v_z} z_t, & z_t &\stackrel{\text{i.i.d.}}{\sim} D(0, 1), \\ v_z &= F_z^{-1}(\alpha), & e_z &= \mathbb{E}(z_t \mid z_t \leq v_z), \\ v_t &\sim \text{BIP-CAViaR}, & e_t &= \frac{e_z}{v_z} v_t,\end{aligned}\quad (23)$$

where z_t is an i.i.d. random variable drawn from a parametric distribution $D(0, 1)$ with zero mean and unit variance, and the ratio v_t/v_z acts as a time-varying scale factor. Here, v_z and e_z denote the VaR and ES of z_t at level α , respectively, while v_t and e_t denote the corresponding VaR and ES of y_t . To mitigate initialization effects, each simulation generates $10^4 + T$ observations, of which the first 10^4 are discarded.

4.2 Simulation design

The simulation settings include five aspects: CAViaR specification, innovation distribution, probability level, sample size, and simulation replication. The detailed settings are specified as follows:

1. **CAViaR specification:** To evaluate the impact of different dynamics, eight CAViaR types are considered: SAV, AS, IG, IGJR, BIP-SAV, BIP-AS, BIP-IG, and BIP-IGJR. The true parameter values are specified as follows:
 - (a) SAV: $(\beta_0, \beta_1, \beta_2) = (0.05 |v_z|, 0.8, 0.15 |v_z|)$;
 - (b) AS: $(\beta_0, \beta_1, \beta_2^-, \beta_2^+) = (0.05 |v_z|, 0.8, 0.25 |v_z|, 0.05 |v_z|)$;
 - (c) IG: $(\beta_0, \beta_1, \beta_2) = (0.05 v_z^2, 0.8, 0.15 v_z^2)$;
 - (d) IGJR: $(\beta_0, \beta_1, \beta_2^-, \beta_2^+) = (0.05 v_z^2, 0.8, 0.25 v_z^2, 0.05 v_z^2)$;
 - (e) BIP-SAV: $(\beta_0, \beta_1, \beta_2, \beta_3) = (0.05 |v_z|, 0.8, 0.15 |v_z|, F_z^{-1}(0.01)/v_z)$;
 - (f) BIP-AS: $(\beta_0, \beta_1, \beta_2^-, \beta_2^+, \beta_3) = (0.05 |v_z|, 0.8, 0.25 |v_z|, 0.05 |v_z|, F_z^{-1}(0.01)/v_z)$;
 - (g) BIP-IG: $(\beta_0, \beta_1, \beta_2, \beta_3) = (0.05 v_z^2, 0.8, 0.15 v_z^2, F_z^{-1}(0.01)/v_z)$;
 - (h) BIP-IGJR: $(\beta_0, \beta_1, \beta_2^-, \beta_2^+, \beta_3) = (0.05 v_z^2, 0.8, 0.25 v_z^2, 0.05 v_z^2, F_z^{-1}(0.01)/v_z)$.
 These specifications ensure comparability across CAViaR types and maintain return dynamics independent of the probability level. The specification $\beta_2^- > \beta_2^+$ reflects the greater impact of negative shocks compared to equally sized positive shocks. The specification $\beta_3 = F_z^{-1}(0.01)/v_z$ treats observations below the 1% quantile as potential outliers.
2. **Innovation distribution:** To examine the impact of different distributional characteristics, three distributions are considered: the normal distribution (symmetric, thin-tailed), the t -distribution with 3 degrees of freedom (symmetric, fat-tailed), and Hansen's skew- t distribution with a shape parameter vector of $(3, -0.5)$ (asymmetric, fat-tailed).
3. **Probability level:** To assess the impact of different risk levels, two probability levels are considered: 1% and 2.5%. The 1% level captures more extreme tail risk than the 2.5% level. At the 1% level, the true values of β_0 , β_2 , β_2^- , and β_2^+ are higher. The true value of β_1 remains unchanged across probability levels. The true value of β_3 equals one at the 1% level and exceeds one at the 2.5% level.
4. **Sample size:** To analyze the effect of data volume, two sample sizes are used: $T = 500$ and $T = 5000$, corresponding to approximately 2 and 20 years of daily returns, respectively.
5. **Simulation replication:** A total of 500 simulation replications are performed.

Fig. 1 reports the simulated return, VaR, and ES series under the skew- t innovation distribution at the 2.5% probability level. This setting provides the strongest contrast between the standard CAViaR-FZ models and their BIP counterparts, as it combines

heavy tails and asymmetry. Following the large negative shock near the middle of the sample, the standard specifications exhibit pronounced downward shifts in VaR and ES, succeeded by persistent recoveries, whereas the BIP models respond more mildly and revert more rapidly. The contrast is particularly pronounced for the quadratic specifications.

The corresponding results under the t and normal innovation distributions are shown in Fig. 2 and Fig. 3, respectively. The effect of robustification remains evident under the t distribution, though less pronounced than in the skew- t case, while under the normal distribution the differences between the standard and BIP specifications become comparatively small.

Overall, the findings suggest that the benefit of bounded innovation propagation becomes more pronounced as the innovation environment becomes more severe. Across all cases, the BIP-CAViaR-FZ framework limits the effect of extreme shocks on future VaR and ES dynamics while preserving the main features of the underlying specifications.

4.3 Parameter estimation

Table 1 reports the parameter estimation results under the skew- t innovation distribution. Estimation accuracy is assessed using the root mean square error (RMSE) of the parameter estimates across simulation replications, where lower RMSE values indicate better performance.

Three patterns stand out. First, RMSE values generally decline as the in-sample size increases from $T = 500$ to $T = 5000$, indicating more precise estimation in larger samples. Second, estimation is typically less precise at the 1% probability level than at the 2.5% level, consistent with the greater difficulty of recovering tail-related parameters in a more extreme tail region. Third, the deterioration in estimation precision is particularly marked for the quadratic specifications.

The corresponding results under the t and normal innovation distributions are reported in Table 2 and Table 3, respectively. Relative to the skew- t case, RMSE values under the t innovation distribution are generally smaller, indicating that the removal of asymmetry improves parameter recovery to some extent, although heavy tails continue to hinder accurate estimation. Compared with the t case, RMSE values under the normal innovation distribution tend to be smaller still, suggesting that parameter estimation becomes more stable when both asymmetry and heavy tails are absent.

Overall, parameter estimation tends to be less precise in settings with smaller in-sample sizes, more extreme probability levels, heavier tails, and greater asymmetry.

4.4 Risk estimation

To assess the accuracy of risk estimation, we examine the RMSE of the estimated VaR and ES across all in-sample dates and simulation replications.

Table 4 reports the results for the skew- t innovation distribution. Four main patterns emerge from the analysis. First, when the data are generated from the standard

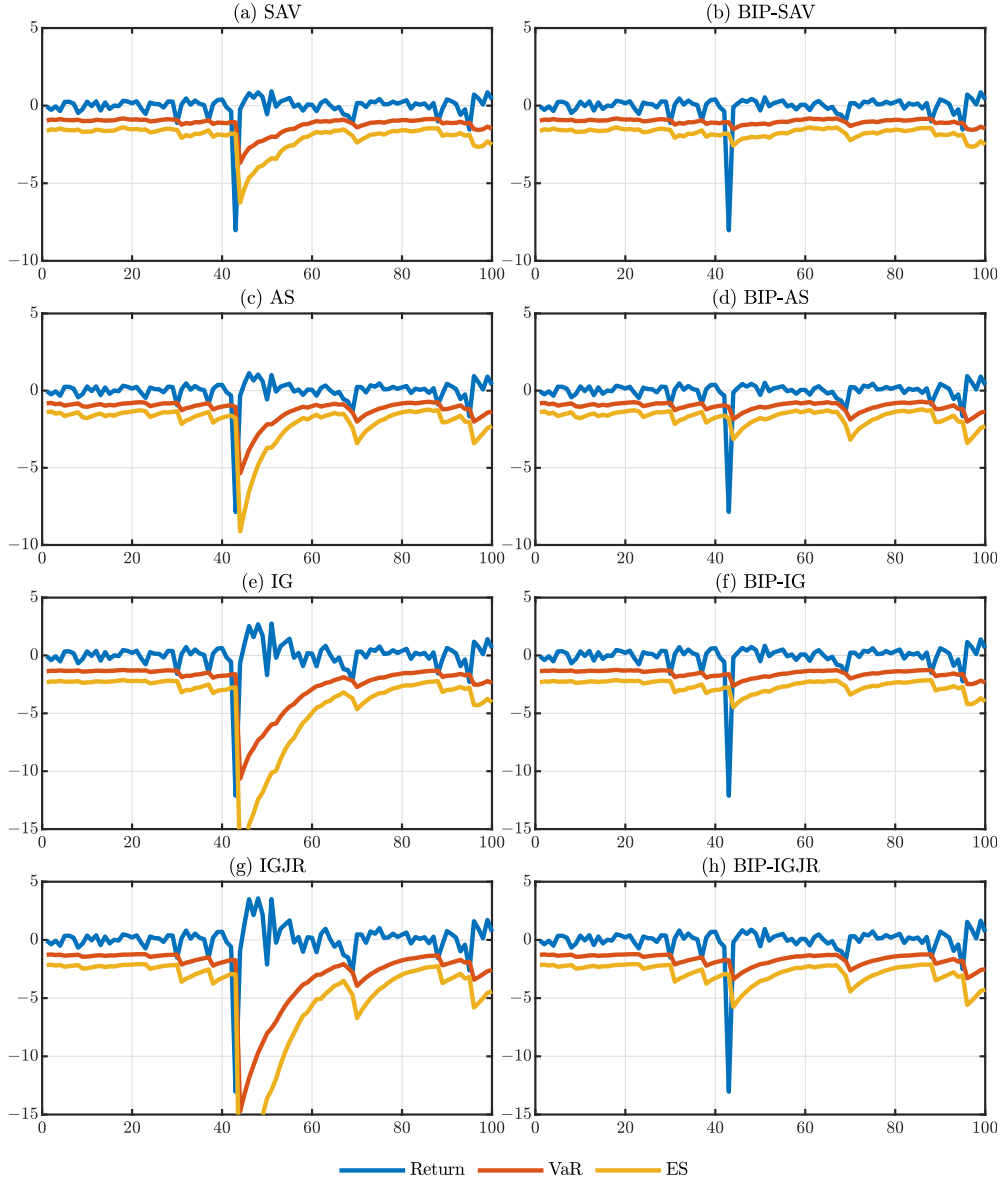


Fig. 1 Simulated series generated under the skew- t innovation distribution at the 2.5% probability level.

CAViaR-FZ model, the BIP specification generally delivers performance comparable to that of the standard specification. Second, when the data are generated from the BIP-CAViaR-FZ model, the BIP specification consistently produces lower RMSE values, with the most substantial gains observed under the IG and IGJR dynamics. Third, ES exhibits greater estimation difficulty than VaR, as evidenced by its larger

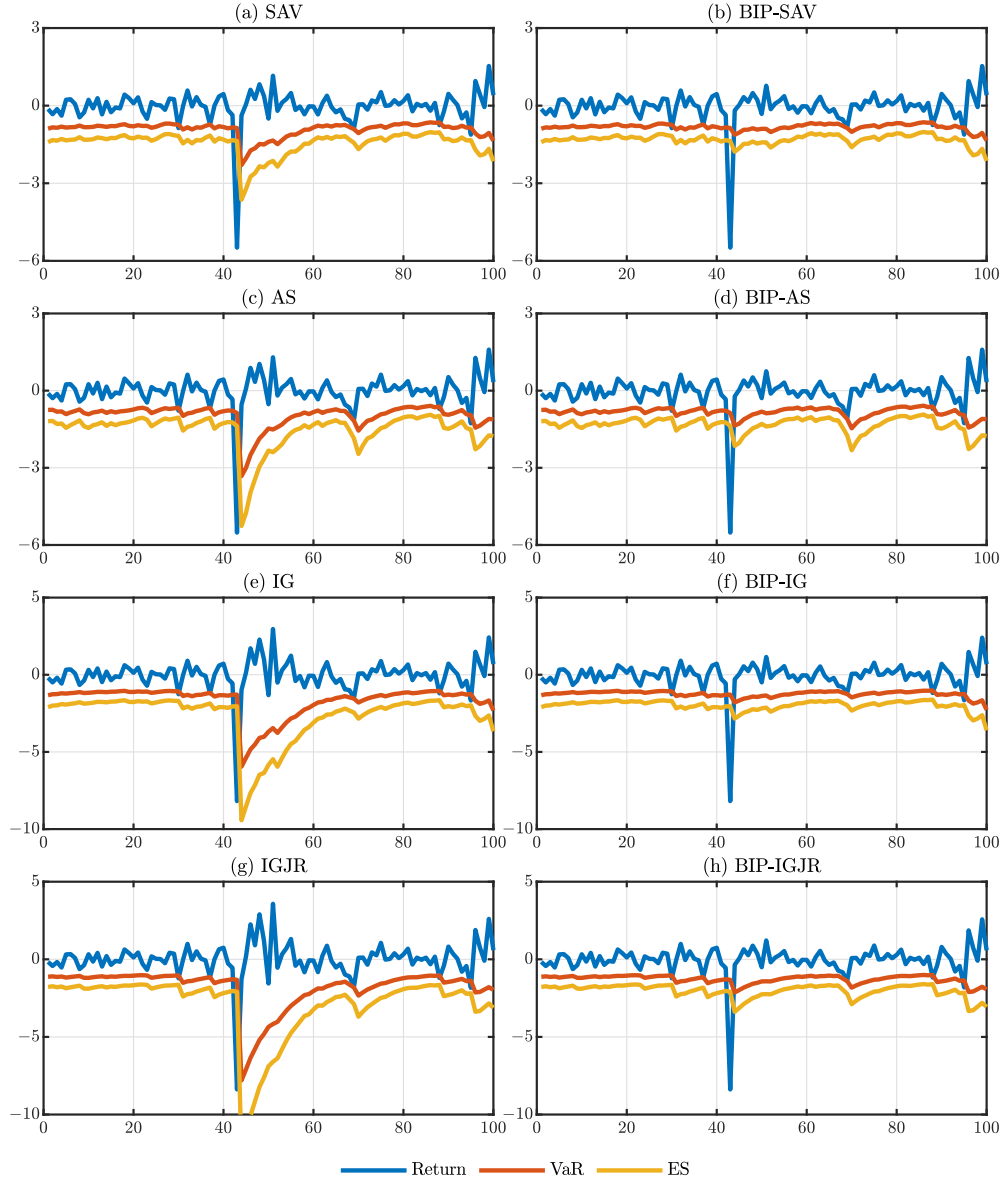


Fig. 2 Simulated series generated under the t innovation distribution at the 2.5% probability level.

RMSE values. Fourth, estimation accuracy improves with increasing sample size but deteriorates as the probability level becomes more extreme.

The corresponding results under the t and normal innovation distributions are reported in Table 5 and Table 6, respectively. A clear ranking emerges across distributions: RMSE values are typically highest under the skew- t distribution, lower

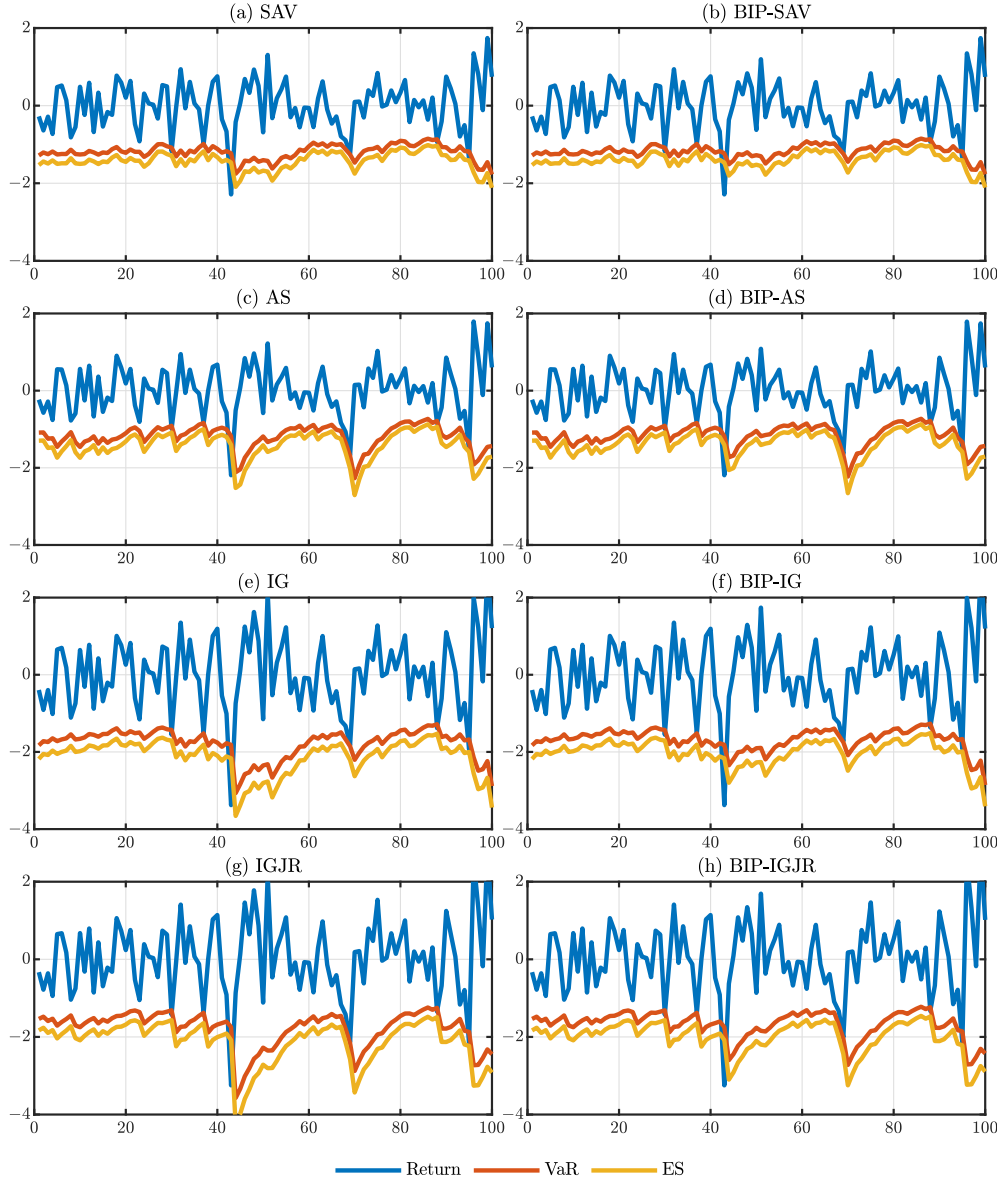


Fig. 3 Simulated series generated under the normal innovation distribution at the 2.5% probability level.

under the t distribution, and lowest under the normal distribution. This ordering indicates that both asymmetry and heavy tails contribute to estimation difficulty in risk measurement. Furthermore, the relative advantage of the BIP specification appears limited under normal distribution but becomes more pronounced under t and skew- t distribution.

Table 1 Parameter estimation results for simulated return series under the skew- t distribution.

Sim. & Est.	Par.	$\alpha = 2.5\%$					$\alpha = 1\%$				
		True	$T = 500$		$T = 5000$		True	$T = 500$		$T = 5000$	
			Mean	RMSE	Mean	RMSE		Mean	RMSE	Mean	RMSE
BIP-SAV	β_0	0.112	0.308	0.371	0.150	0.140	0.169	0.514	0.643	0.267	0.294
	β_1	0.800	0.559	0.404	0.752	0.167	0.800	0.510	0.447	0.721	0.227
	β_2	0.337	0.579	0.618	0.381	0.186	0.508	1.014	1.394	0.591	0.378
	β_3^{-1}	0.663	0.742	0.361	0.726	0.311	1.000	0.789	0.391	0.841	0.324
	γ	0.699	0.599	0.297	0.680	0.098	0.618	0.435	0.365	0.588	0.138
BIP-AS	β_0	0.112	0.254	0.318	0.130	0.068	0.169	0.343	0.439	0.213	0.156
	β_1	0.800	0.599	0.381	0.770	0.098	0.800	0.597	0.382	0.754	0.143
	β_2^-	0.561	0.901	0.915	0.625	0.228	0.846	1.614	2.267	0.955	0.486
	β_2^+	0.112	0.387	0.694	0.156	0.180	0.169	0.821	1.576	0.262	0.373
	β_3^{-1}	0.663	0.752	0.370	0.727	0.283	1.000	0.820	0.389	0.869	0.277
BIP-IG	β_0	0.252	0.775	1.011	0.326	0.259	0.573	1.794	2.228	0.807	0.726
	β_1	0.800	0.532	0.420	0.754	0.148	0.800	0.509	0.434	0.740	0.190
	β_2	0.756	1.754	2.676	0.933	0.564	1.719	5.435	12.701	2.116	1.784
	β_3^{-1}	0.663	0.718	0.317	0.719	0.273	1.000	0.756	0.437	0.852	0.290
	γ	0.699	0.593	0.291	0.678	0.097	0.618	0.422	0.363	0.587	0.136
BIP-IGJR	β_0	0.252	0.669	0.946	0.299	0.167	0.573	1.505	1.945	0.752	0.586
	β_1	0.800	0.565	0.398	0.767	0.099	0.800	0.549	0.402	0.746	0.146
	β_2^-	1.259	3.326	7.312	1.479	0.737	2.865	11.010	32.372	3.415	2.361
	β_2^+	0.252	1.784	4.611	0.425	0.611	0.573	6.408	18.686	1.254	2.117
	β_3^{-1}	0.663	0.764	0.314	0.704	0.227	1.000	0.717	0.498	0.898	0.226
	γ	0.699	0.549	0.302	0.673	0.098	0.618	0.354	0.373	0.582	0.137

Note: Sim. is the simulated model, Est. is the estimated model, Par. is the estimated parameter, True is the true value of the parameter in the simulation, Mean is the mean of the parameter estimates, and RMSE is the root mean square error of the parameter estimates.

Figs. 4 and 5 report the upper-tail quantiles of absolute estimation errors for the skew- t case with $\alpha = 2.5\%$ and $T = 5000$. The figures are broadly consistent with the RMSE evidence. When the data are generated from the standard model, the BIP specification is only slightly less accurate, suggesting limited efficiency loss in the absence of outliers. When the data are generated from the BIP model, however, the BIP specification produces clearly smaller errors. As in the RMSE results, the errors are generally larger for ES than for VaR.

4.5 Risk forecasting

To assess out-of-sample forecasting performance, we examine the RMSE of the risk forecasts across all out-of-sample dates and simulation replications, with lower values indicating better forecasting accuracy.

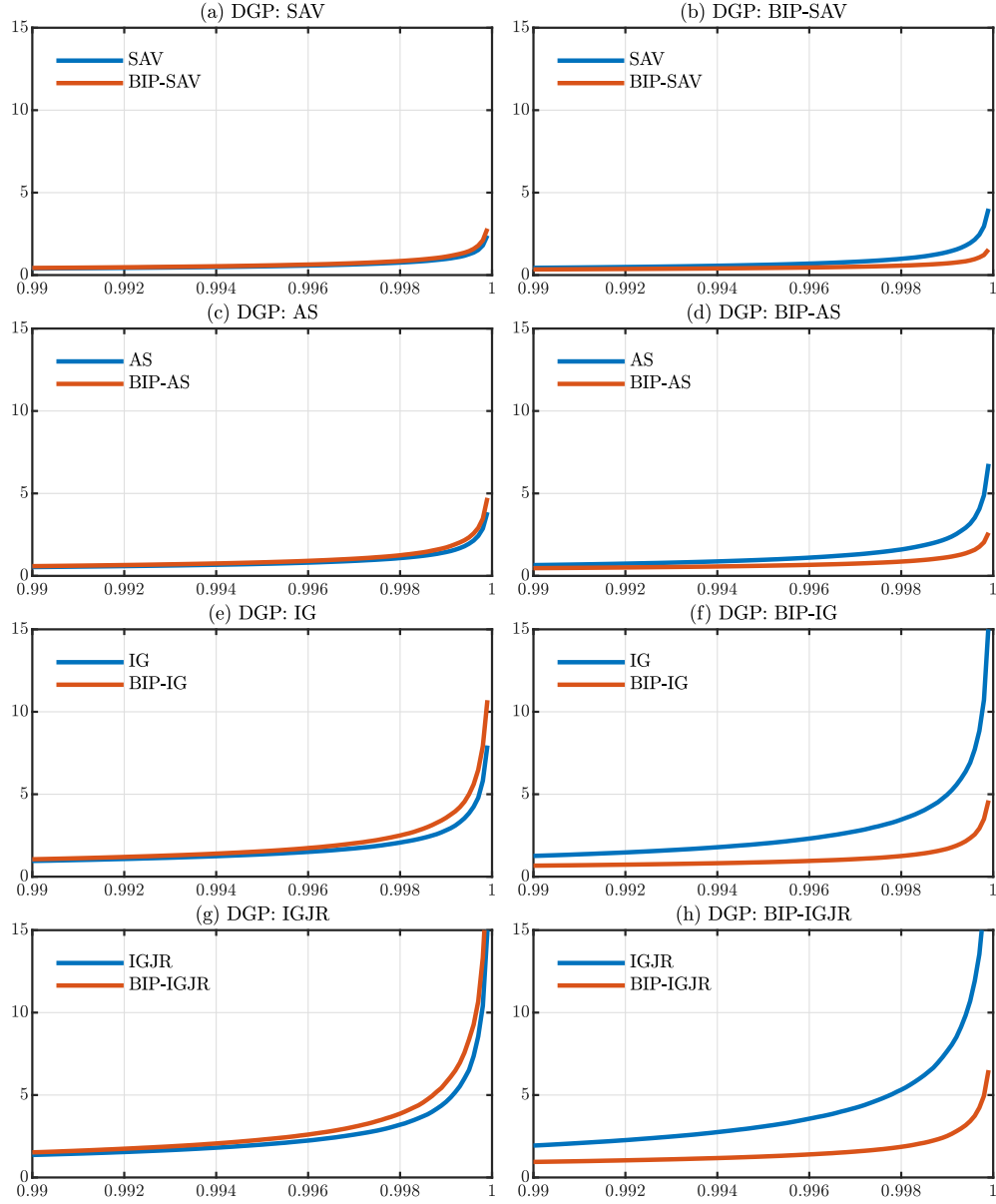


Fig. 4 Upper-tail quantiles of absolute errors in VaR estimates. Note: The simulation is based on the skew- t distribution with a 2.5% probability level and a sample size of 5000. The horizontal axis represents quantile levels, and the vertical axis shows the corresponding quantiles of absolute errors.

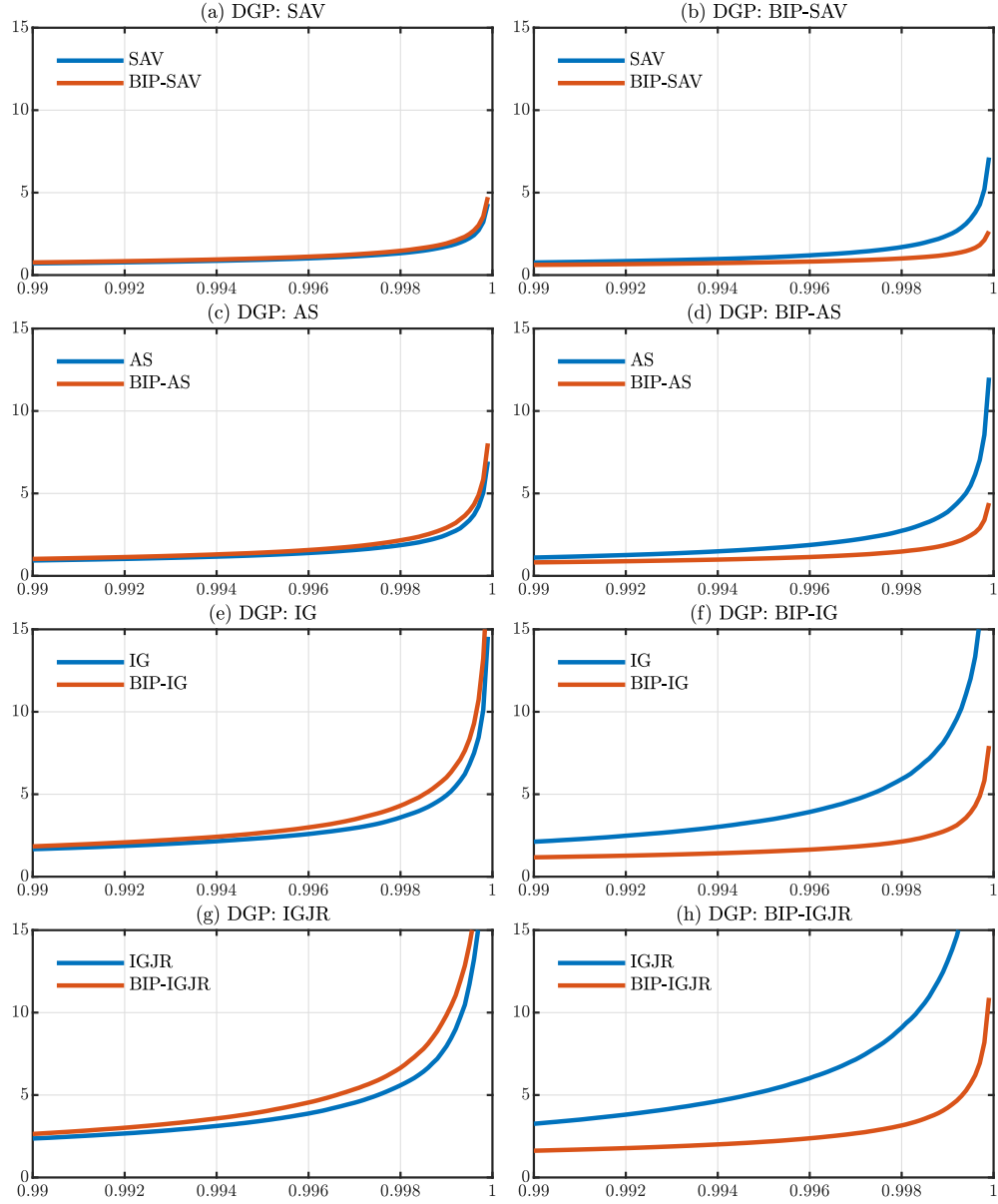


Fig. 5 Upper-tail quantiles of absolute errors in ES estimates. Note: The simulation is based on the skew- t distribution with a 2.5% probability level and a sample size of 5000. The horizontal axis represents quantile levels, and the vertical axis shows the corresponding quantiles of absolute errors.

Table 2 Parameter estimation results for simulated return series under the t distribution.

Sim. & Est.	Par.	True	$\alpha = 2.5\%$				$\alpha = 1\%$				
			$T = 500$		$T = 5000$		$T = 500$			$T = 5000$	
			Mean	RMSE	Mean	RMSE	Mean	RMSE		Mean	RMSE
BIP-SAV	β_0	0.092	0.249	0.323	0.113	0.081	0.131	0.328	0.416	0.187	0.191
	β_1	0.800	0.579	0.403	0.767	0.119	0.800	0.596	0.370	0.743	0.192
	β_2	0.276	0.410	0.395	0.305	0.118	0.393	0.639	0.859	0.437	0.248
	β_3^{-1}	0.701	0.748	0.341	0.721	0.331	1.000	0.804	0.373	0.825	0.318
	γ	0.584	0.496	0.252	0.569	0.081	0.542	0.380	0.324	0.514	0.117
BIP-AS	β_0	0.092	0.202	0.263	0.106	0.052	0.131	0.390	0.502	0.164	0.122
	β_1	0.800	0.631	0.351	0.776	0.086	0.800	0.536	0.432	0.762	0.135
	β_2^-	0.459	0.666	0.560	0.498	0.164	0.655	1.128	1.413	0.726	0.346
	β_2^+	0.092	0.216	0.384	0.110	0.099	0.131	0.462	0.996	0.165	0.199
	β_3^{-1}	0.701	0.810	0.307	0.729	0.289	1.000	0.817	0.364	0.856	0.278
	γ	0.584	0.456	0.252	0.564	0.082	0.542	0.306	0.321	0.507	0.118
BIP-IG	β_0	0.169	0.335	0.422	0.207	0.141	0.344	1.079	1.571	0.472	0.425
	β_1	0.800	0.664	0.303	0.767	0.113	0.800	0.551	0.419	0.750	0.168
	β_2	0.506	0.906	1.154	0.597	0.294	1.031	2.671	9.678	1.174	0.849
	β_3^{-1}	0.701	0.699	0.331	0.742	0.238	1.000	0.800	0.354	0.858	0.273
	γ	0.584	0.493	0.248	0.567	0.082	0.542	0.375	0.326	0.516	0.118
BIP-IGJR	β_0	0.169	0.474	0.638	0.204	0.125	0.344	0.870	1.106	0.455	0.370
	β_1	0.800	0.558	0.417	0.767	0.108	0.800	0.563	0.403	0.752	0.154
	β_2^-	0.844	1.762	2.641	0.991	0.504	1.718	4.739	11.114	2.061	1.548
	β_2^+	0.169	0.638	1.610	0.227	0.224	0.344	2.679	11.197	0.460	0.654
	β_3^{-1}	0.701	0.799	0.307	0.737	0.235	1.000	0.785	0.390	0.887	0.243
	γ	0.584	0.463	0.250	0.562	0.082	0.542	0.314	0.332	0.507	0.119

Note: See [Table 1](#).

[Tables 7](#) to [9](#) report the forecasting results under the skew- t , t , and normal innovation distributions, respectively. Overall, the findings are generally consistent with those for risk estimation, underscoring the influence of in-sample estimation quality on out-of-sample forecast performance. Moreover, the results indicate that the BIP-CAViaR-FZ model tends to yield greater benefits in more challenging innovation environments.

5 Empirical study

This section investigates the empirical performance of the BIP-CAViaR-FZ model. [Section 5.1](#) describes the stock return data, [Section 5.2](#) reports descriptive statistics, [Section 5.3](#) compares the full-sample risk estimation performance of the BIP-CAViaR-FZ and CAViaR-FZ models, and [Section 5.4](#) examines their out-of-sample risk forecasting performance.

Table 3 Parameter estimation results for simulated return series under the normal distribution.

Sim. & Est.	Par.	$\alpha = 2.5\%$					$\alpha = 1\%$				
		True	$T = 500$		$T = 5000$		True	$T = 500$		$T = 5000$	
			Mean	RMSE	Mean	RMSE		Mean	RMSE	Mean	RMSE
BIP-SAV	β_0	0.098	0.141	0.155	0.102	0.030	0.116	0.183	0.222	0.123	0.042
	β_1	0.800	0.750	0.168	0.795	0.037	0.800	0.735	0.196	0.795	0.044
	β_2	0.294	0.327	0.158	0.298	0.046	0.349	0.392	0.208	0.349	0.063
	β_3^{-1}	0.843	0.774	0.291	0.677	0.408	1.000	0.896	0.284	0.912	0.209
	γ	0.193	0.170	0.057	0.190	0.017	0.146	0.102	0.071	0.142	0.019
BIP-AS	β_0	0.098	0.128	0.113	0.102	0.019	0.116	0.157	0.149	0.121	0.028
	β_1	0.800	0.760	0.126	0.795	0.027	0.800	0.751	0.142	0.796	0.034
	β_2^-	0.490	0.527	0.189	0.496	0.057	0.582	0.641	0.266	0.580	0.086
	β_2^+	0.098	0.122	0.135	0.101	0.045	0.116	0.160	0.194	0.114	0.059
	β_3^{-1}	0.843	0.756	0.320	0.685	0.420	1.000	0.836	0.349	0.628	0.514
BIP-IG	β_0	0.192	0.298	0.354	0.203	0.064	0.271	0.438	0.646	0.289	0.105
	β_1	0.800	0.747	0.167	0.794	0.035	0.800	0.738	0.195	0.794	0.042
	β_2	0.576	0.656	0.344	0.595	0.104	0.812	0.937	0.620	0.812	0.161
	β_3^{-1}	0.843	0.819	0.254	0.854	0.140	1.000	0.805	0.337	0.918	0.202
	γ	0.193	0.170	0.057	0.190	0.017	0.146	0.105	0.069	0.142	0.020
BIP-IGJR	β_0	0.192	0.292	0.322	0.203	0.051	0.271	0.435	0.571	0.286	0.084
	β_1	0.800	0.741	0.161	0.794	0.031	0.800	0.731	0.179	0.795	0.038
	β_2^-	0.960	1.102	0.564	0.974	0.154	1.353	1.593	1.008	1.349	0.261
	β_2^+	0.192	0.286	0.351	0.199	0.094	0.271	0.384	0.517	0.271	0.145
	β_3^{-1}	0.843	0.872	0.233	0.734	0.289	1.000	0.655	0.470	0.880	0.235
	γ	0.193	0.163	0.060	0.189	0.017	0.146	0.089	0.077	0.140	0.020

Note: See [Table 1](#).

5.1 Data

Financial return series frequently exhibit outliers, which may distort estimation and adversely affect risk forecasts. Individual stock returns are particularly susceptible to such observations, as they are driven not only by macroeconomic conditions but also by firm-specific shocks. Consequently, individual stock return series offers an appropriate choice for evaluating the robustness of the proposed joint dynamic model under realistic conditions.

We consider five major U.S. technology stocks: Apple (AAPL), Amazon (AMZN), Alphabet (GOOGL), Intel (INTC), and Microsoft (MSFT). These firms are representative of the technology sector and exhibit substantial heterogeneity in return dynamics. The data are obtained from Yahoo Finance and cover the period from January 1, 2006, to December 31, 2025. Each return series contains 5030 daily observations.

Table 4 Risk estimation results for simulated return series under the skew- t distribution.

Sim.	Est.	$\alpha = 2.5\%$				$\alpha = 1\%$			
		$T = 500$		$T = 5000$		$T = 500$		$T = 5000$	
		VaR	ES	VaR	ES	VaR	ES	VaR	ES
SAV	SAV	0.357	0.697	0.121	0.228	0.800	1.365	0.264	0.507
	BIP-SAV	0.344	0.700	0.129	0.236	0.908	1.840	0.235	0.431
AS	AS	0.688	1.209	0.168	0.302	2.495	3.615	0.365	0.639
	BIP-AS	0.594	1.251	0.187	0.330	2.413	3.506	0.346	0.586
IG	IG	1.012	2.259	0.306	0.558	5.022	9.609	0.635	1.164
	BIP-IG	0.915	2.018	0.350	0.605	1.875	3.498	0.645	1.095
IGJR	IGJR	2.138	4.821	0.509	0.953	5.186	8.693	1.091	2.018
	BIP-IGJR	1.836	4.676	0.544	0.960	5.118	8.097	1.098	1.904
BIP-SAV	SAV	0.377	0.763	0.155	0.283	0.786	1.319	0.310	0.576
	BIP-SAV	0.270	0.535	0.101	0.192	0.661	1.155	0.203	0.384
BIP-AS	AS	0.622	1.071	0.239	0.423	1.462	2.193	0.464	0.791
	BIP-AS	0.425	0.720	0.133	0.240	1.161	1.786	0.277	0.480
BIP-IG	IG	0.852	1.739	0.476	0.842	1.799	3.299	0.847	1.487
	BIP-IG	0.499	0.960	0.199	0.360	1.186	2.065	0.392	0.694
BIP-IGJR	IGJR	1.463	2.550	0.736	1.288	3.527	5.275	1.273	2.153
	BIP-IGJR	0.881	1.427	0.273	0.480	2.281	3.238	0.544	0.915

Note: This table presents the root mean square error (RMSE) of the risk estimates across all in-sample dates and simulation replications. Sim. is the simulated model, and Est. is the estimated model. Bold indicates the best model for each group of two models.

Daily returns are computed as

$$y_t = 100(\ln P_t - \ln P_{t-1}), \quad (24)$$

where P_t denotes the closing price on day t .

5.2 Descriptive statistics

This subsection summarizes the distributional and dynamic features of the selected return series, with particular attention to the presence of potential outliers. To provide a intuitive view of these observations, we construct a time-varying non-outlier interval using a centered moving window together with the boxplot rule of [Tukey \(1977\)](#). This procedure is used solely for descriptive visualization; it is not used in risk estimation or forecasting.

For descriptive purposes, the outlier-filtered return is defined as

$$\max(\min(y_t, \text{ub}_t), \text{lb}_t), \quad (25)$$

Table 5 Risk estimation results for simulated return series under the t distribution.

Sim.	Est.	$\alpha = 2.5\%$				$\alpha = 1\%$			
		$T = 500$		$T = 5000$		$T = 500$		$T = 5000$	
		VaR	ES	VaR	ES	VaR	ES	VaR	ES
SAV	SAV	0.243	0.453	0.078	0.140	0.668	1.116	0.158	0.289
	BIP-SAV	0.247	0.447	0.090	0.156	0.600	0.916	0.162	0.288
AS	AS	0.343	0.587	0.099	0.171	0.838	1.188	0.209	0.357
	BIP-AS	0.294	0.504	0.107	0.180	0.751	1.131	0.207	0.348
IG	IG	0.950	1.840	0.174	0.299	2.251	3.414	0.366	0.642
	BIP-IG	0.827	1.461	0.263	0.428	1.910	3.122	0.474	0.771
IGJR	IGJR	1.048	1.881	0.222	0.375	2.135	3.219	0.477	0.789
	BIP-IGJR	0.920	1.733	0.252	0.408	1.981	2.971	0.482	0.766
BIP-SAV	SAV	0.255	0.471	0.104	0.178	0.605	0.976	0.193	0.336
	BIP-SAV	0.189	0.359	0.070	0.127	0.456	0.774	0.138	0.254
BIP-AS	AS	0.323	0.541	0.138	0.231	0.748	1.109	0.253	0.420
	BIP-AS	0.253	0.425	0.086	0.149	0.661	0.991	0.175	0.297
BIP-IG	IG	0.469	0.831	0.317	0.516	1.017	1.540	0.522	0.853
	BIP-IG	0.312	0.570	0.124	0.216	1.018	1.556	0.247	0.435
BIP-IGJR	IGJR	0.642	1.066	0.361	0.591	1.402	2.057	0.611	0.982
	BIP-IGJR	0.426	0.709	0.154	0.258	1.044	1.537	0.304	0.502

Note: See [Table 4](#).

where lb_t and ub_t denote the lower and upper bounds of the non-outlier interval for the return on day t , respectively. These bounds are calculated using a moving window of $[t - 50, t + 50]$ and the boxplot method. According to the boxplot method, the non-outlier interval is given by

$$[Q_1 - 1.5IQR, Q_3 + 1.5IQR], \quad (26)$$

where Q_1 and Q_3 denote the first and third quartiles, and IQR is the interquartile range. Observations falling outside this interval are classified as potential outliers.

[Table 10](#) reports descriptive statistics for each return series before and after outlier filtering. Several patterns emerge. First, before filtering, four of the five series exhibit statistically significant skewness, while MSFT is close to symmetric. After filtering, the absolute skewness decreases for AAPL, AMZN, GOOGL, and INTC, but not for MSFT. Second, all series display substantial excess kurtosis before filtering, which declines markedly after filtering in every case. Third, the augmented Dickey–Fuller (ADF) statistics strongly reject a unit root both before and after filtering, indicating that the stationarity conclusion is unchanged. Fourth, the Ljung–Box (LB) statistics indicate statistically significant serial dependence in returns, whereas the corresponding statistics for absolute returns are much larger, pointing to pronounced volatility

Table 6 Risk estimation results for simulated return series under the normal distribution.

Sim.	Est.	$\alpha = 2.5\%$				$\alpha = 1\%$			
		$T = 500$		$T = 5000$		$T = 500$		$T = 5000$	
		VaR	ES	VaR	ES	VaR	ES	VaR	ES
SAV	SAV	0.136	0.166	0.044	0.052	0.192	0.229	0.060	0.071
	BIP-SAV	0.136	0.167	0.047	0.055	0.190	0.229	0.062	0.073
AS	AS	0.168	0.201	0.053	0.063	0.238	0.280	0.076	0.087
	BIP-AS	0.167	0.201	0.057	0.067	0.238	0.282	0.077	0.088
IG	IG	0.223	0.270	0.071	0.084	0.316	0.372	0.097	0.114
	BIP-IG	0.224	0.273	0.079	0.094	0.313	0.375	0.106	0.124
IGJR	IGJR	0.261	0.312	0.083	0.099	0.369	0.435	0.117	0.135
	BIP-IGJR	0.261	0.315	0.090	0.106	0.368	0.433	0.122	0.140
BIP-SAV	SAV	0.136	0.166	0.047	0.056	0.192	0.227	0.064	0.074
	BIP-SAV	0.132	0.163	0.046	0.054	0.186	0.224	0.060	0.070
BIP-AS	AS	0.165	0.199	0.058	0.069	0.241	0.281	0.080	0.092
	BIP-AS	0.164	0.197	0.056	0.066	0.230	0.275	0.078	0.090
BIP-IG	IG	0.215	0.262	0.088	0.105	0.305	0.361	0.115	0.134
	BIP-IG	0.207	0.254	0.075	0.089	0.296	0.353	0.096	0.113
BIP-IGJR	IGJR	0.254	0.302	0.101	0.119	0.363	0.423	0.135	0.155
	BIP-IGJR	0.243	0.293	0.090	0.107	0.356	0.418	0.118	0.136

Note: See [Table 4](#).

clustering. This contrast becomes even stronger after filtering, as the LB statistics for $|y_t|$ increase for all five series. Finally, $\text{Corr}(|y_t|, y_{t-1})$ is negative in all cases and statistically significant for most series, although the magnitudes remain small overall.

In addition, [Fig. 6](#) plots the return series together with the corresponding non-outlier intervals. The figure suggests that all five series contain a non-negligible number of potential outliers, although their frequency differs across assets. Such observations appear particularly prominent for AMZN, where some large returns occur at fairly regular intervals, possibly reflecting earnings announcements or other firm-specific news. Filtering these observations tends to remove isolated spikes and produce a more regular return pattern.

Table 7 Risk forecasting results for simulated return series under the skew- t distribution.

Sim.	Est.	$\alpha = 2.5\%$				$\alpha = 1\%$			
		$T = 500$		$T = 5000$		$T = 500$		$T = 5000$	
		VaR	ES	VaR	ES	VaR	ES	VaR	ES
SAV	SAV	0.306	0.556	0.110	0.205	0.618	1.069	0.214	0.409
	BIP-SAV	0.299	0.583	0.126	0.231	0.610	1.120	0.224	0.426
AS	AS	0.447	0.753	0.148	0.256	0.961	1.461	0.297	0.504
	BIP-AS	0.417	0.744	0.183	0.318	0.954	1.488	0.326	0.556
IG	IG	0.680	1.186	0.231	0.414	1.302	2.207	0.459	0.829
	BIP-IG	0.671	1.206	0.352	0.619	1.159	2.109	0.595	1.027
IGJR	IGJR	1.234	2.134	0.347	0.605	2.810	5.075	0.693	1.174
	BIP-IGJR	1.312	2.379	0.474	0.842	3.514	5.229	0.870	1.458
BIP-SAV	SAV	0.302	0.550	0.126	0.226	0.601	1.032	0.231	0.422
	BIP-SAV	0.259	0.517	0.102	0.192	0.539	0.986	0.186	0.366
BIP-AS	AS	0.416	0.692	0.187	0.319	0.908	1.385	0.366	0.593
	BIP-AS	0.346	0.624	0.138	0.243	0.815	1.285	0.269	0.461
BIP-IG	IG	0.630	1.078	0.355	0.595	1.117	1.799	0.618	1.000
	BIP-IG	0.449	0.864	0.199	0.353	0.923	1.622	0.381	0.664
BIP-IGJR	IGJR	0.899	1.490	0.551	0.905	1.854	2.781	0.943	1.450
	BIP-IGJR	0.660	1.181	0.310	0.520	1.413	2.223	0.517	0.857

Note: This table presents the root mean square error (RMSE) of the risk forecasts across all out-of-sample dates and simulation replications. Sim. is the simulated model, and Est. is the estimated model. Bold indicates the best model for each group of two models.

Table 8 Risk forecasting results for simulated return series under the t distribution.

Sim.	Est.	$\alpha = 2.5\%$				$\alpha = 1\%$			
		$T = 500$		$T = 5000$		$T = 500$		$T = 5000$	
		VaR	ES	VaR	ES	VaR	ES	VaR	ES
SAV	SAV	0.218	0.390	0.073	0.133	0.496	0.847	0.143	0.268
	BIP-SAV	0.218	0.404	0.088	0.156	0.520	0.826	0.158	0.289
AS	AS	0.268	0.447	0.093	0.156	0.637	0.920	0.190	0.317
	BIP-AS	0.263	0.452	0.108	0.181	0.572	0.896	0.200	0.334
IG	IG	0.488	0.876	0.133	0.242	1.057	1.634	0.285	0.522
	BIP-IG	0.497	0.852	0.200	0.346	0.935	1.544	0.387	0.661
IGJR	IGJR	0.564	0.909	0.183	0.295	1.174	1.812	0.383	0.613
	BIP-IGJR	0.566	0.954	0.237	0.388	1.119	1.801	0.467	0.740
BIP-SAV	SAV	0.229	0.404	0.089	0.153	0.477	0.776	0.163	0.285
	BIP-SAV	0.189	0.354	0.068	0.124	0.407	0.716	0.131	0.247
BIP-AS	AS	0.267	0.436	0.114	0.185	0.618	0.895	0.216	0.344
	BIP-AS	0.232	0.398	0.091	0.153	0.573	0.867	0.167	0.287
BIP-IG	IG	0.416	0.675	0.264	0.417	0.743	1.165	0.420	0.663
	BIP-IG	0.303	0.551	0.111	0.199	0.791	1.245	0.234	0.422
BIP-IGJR	IGJR	0.512	0.779	0.298	0.464	1.009	1.459	0.517	0.782
	BIP-IGJR	0.369	0.644	0.160	0.260	0.902	1.383	0.289	0.469

Note: See [Table 7](#).

Table 9 Risk forecasting results for simulated return series under the normal distribution.

Sim.	Est.	$\alpha = 2.5\%$				$\alpha = 1\%$			
		$T = 500$		$T = 5000$		$T = 500$		$T = 5000$	
		VaR	ES	VaR	ES	VaR	ES	VaR	ES
SAV	SAV	0.146	0.177	0.043	0.051	0.194	0.235	0.059	0.069
	BIP-SAV	0.149	0.181	0.046	0.055	0.198	0.239	0.062	0.073
AS	AS	0.170	0.207	0.052	0.061	0.233	0.278	0.072	0.083
	BIP-AS	0.175	0.215	0.056	0.066	0.238	0.286	0.074	0.086
IG	IG	0.243	0.291	0.067	0.079	0.324	0.387	0.092	0.109
	BIP-IG	0.250	0.304	0.076	0.091	0.330	0.401	0.104	0.123
IGJR	IGJR	0.279	0.340	0.079	0.093	0.380	0.457	0.109	0.126
	BIP-IGJR	0.292	0.361	0.086	0.102	0.383	0.461	0.117	0.136
BIP-SAV	SAV	0.145	0.176	0.045	0.053	0.195	0.233	0.061	0.072
	BIP-SAV	0.144	0.176	0.045	0.053	0.190	0.230	0.058	0.068
BIP-AS	AS	0.166	0.202	0.056	0.065	0.239	0.278	0.077	0.088
	BIP-AS	0.169	0.206	0.056	0.066	0.237	0.285	0.075	0.086
BIP-IG	IG	0.230	0.277	0.081	0.096	0.313	0.372	0.109	0.126
	BIP-IG	0.227	0.278	0.073	0.086	0.308	0.367	0.092	0.108
BIP-IGJR	IGJR	0.257	0.309	0.096	0.113	0.365	0.434	0.127	0.145
	BIP-IGJR	0.259	0.320	0.095	0.112	0.366	0.439	0.112	0.128

Note: See [Table 7](#).

Table 10 Descriptive statistics for the empirical return series

	AAPL	AMZN	GOOGL	INTC	MSFT
Panel A: Before outlier filtering					
Mean	0.095	0.091	0.067	0.018	0.065
Std. dev.	2.004	2.367	1.882	2.238	1.728
Skewness	-0.218***	0.404***	0.203***	-0.485***	-0.028
Kurtosis	9.550***	15.344***	10.825***	18.152***	12.211***
ADF	-72.499***	-72.038***	-73.302***	-76.449***	-78.400***
LB: y_t	56.571***	32.307**	33.900**	125.531***	118.059***
LB: $ y_t $	2966.416***	1515.152***	1799.736***	2836.961***	3009.956***
Corr($ y_t , y_{t-1}$)	-0.078***	-0.021	-0.053***	-0.016	-0.066***
Panel B: After outlier filtering					
Mean	0.094	0.077	0.064	0.030	0.063
Std. dev.	1.832	2.072	1.672	1.971	1.558
Skewness	-0.119***	-0.056	-0.168***	0.001	-0.069**
Kurtosis	5.455***	5.867***	5.173***	5.445***	6.307***
ADF	-71.669***	-72.472***	-72.825***	-75.584***	-77.035***
LB: y_t	47.485***	41.510***	37.153**	114.920***	91.584***
LB: $ y_t $	4152.022***	3374.075***	3202.222***	4630.531***	4458.370***
Corr($ y_t , y_{t-1}$)	-0.071***	-0.043***	-0.065***	-0.009	-0.085***

Note: The symbols *, **, and *** indicate significance at the 10%, 5%, and 1% levels, respectively.

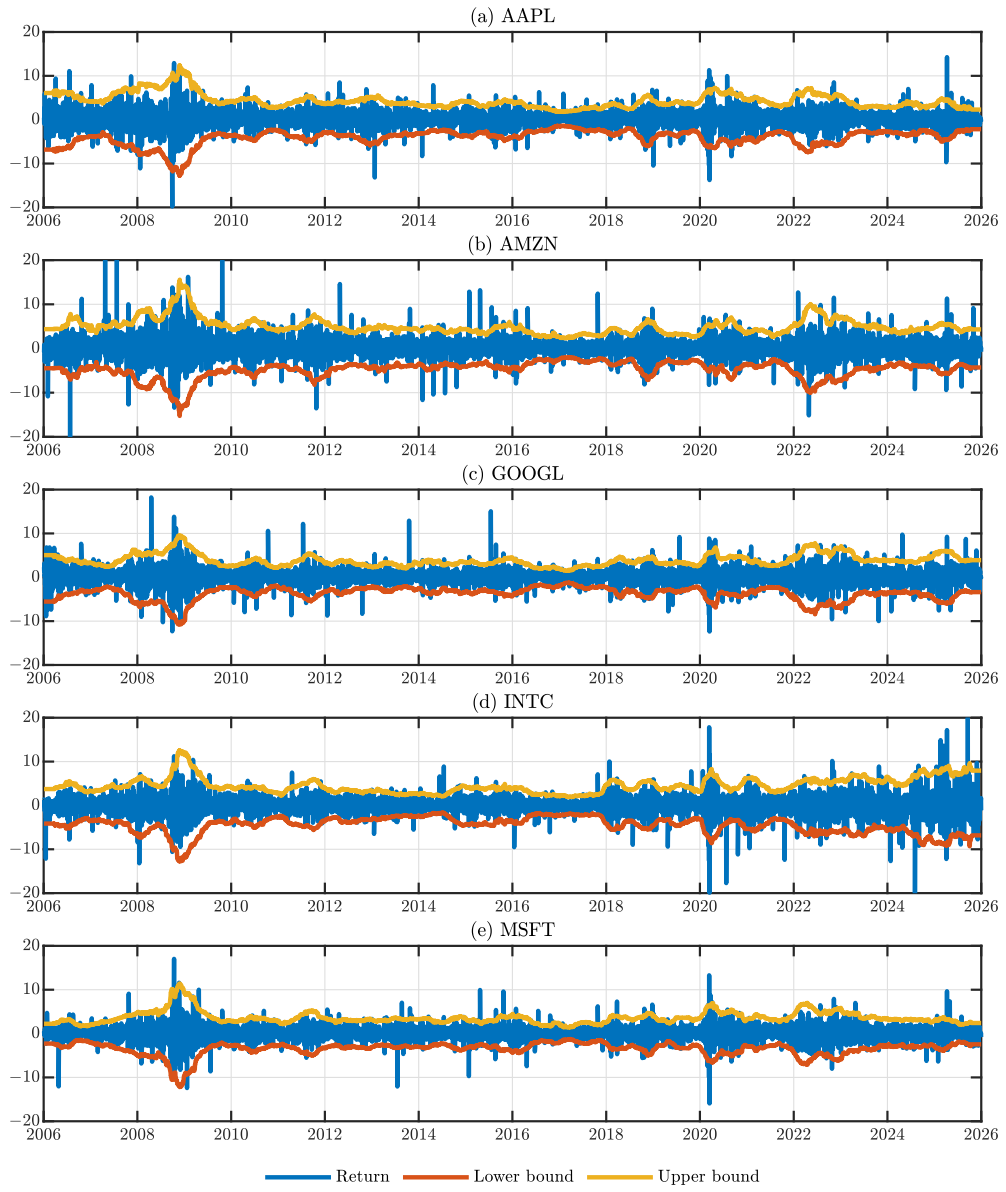


Fig. 6 Return series and non-outlier intervals. Note: Returns that fall outside the non-outlier intervals are identified as potential outliers.

5.3 Risk estimation performance

This subsection evaluates the risk estimation performance of the BIP-CAViaR-FZ model. Table 11 reports full-sample risk estimation rankings based on the FZ0 loss. Smaller ranking values indicate lower FZ0 losses and hence better risk estimation performance. Three main findings emerge. First, the BIP specifications uniformly outperform their non-robust counterparts. For each model pair (SAV versus BIP-SAV, AS versus BIP-AS, IG versus BIP-IG, and IGJR versus BIP-IGJR), the BIP version always achieves a better rank across all assets and both probability levels. Second, the BIP-based specifications generally rank among the better-performing models, whereas the non-robust quadratic models are often ranked near the bottom. These results suggest that the quadratic specifications are more sensitive to extreme observations and therefore benefit more from BIP. Third, although some differences remain between symmetric and asymmetric specifications, these appear less important than the distinction between robust and non-robust updating.

Fig. 7 plots the corresponding 2.5% VaR estimates for AMZN over the full sample. The figure is consistent with the ranking results. The non-robust CAViaR-FZ models react more sharply to potential outliers, producing more abrupt movements in the estimated VaR paths, whereas the BIP-CAViaR-FZ models generate smoother paths while preserving the main dynamic features of the series. The contrast is especially noticeable for the IG and IGJR specifications.

Fig. 8 presents the corresponding 2.5% ES estimates for AMZN. A similar pattern is observed, with the differences appearing even more pronounced for ES. The non-robust models produce sharper fluctuations in the estimated ES paths, whereas the BIP-based models yield smoother estimates.

Table 11 Full-sample risk estimation rankings based on the FZ0 loss

Model	$\alpha = 2.5\%$					$\alpha = 1\%$				
	AAPL	AMZN	GOOGL	INTC	MSFT	AAPL	AMZN	GOOGL	INTC	MSFT
SAV	7	6	5	6	6	7	6	5	6	6
AS	3	5	3	5	5	2	5	4	5	5
IG	8	8	8	7	8	8	8	8	8	8
IGJR	4	7	7	8	7	4	7	7	7	7
BIP-SAV	6	4	4	4	4	5	2	2	4	4
BIP-AS	2	3	1	3	3	1	1	1	3	2
BIP-IG	5	2	6	2	2	6	4	6	2	3
BIP-IGJR	1	1	2	1	1	3	3	3	1	1

Note: Smaller ranking values indicate lower FZ0 losses and hence better risk estimation performance.

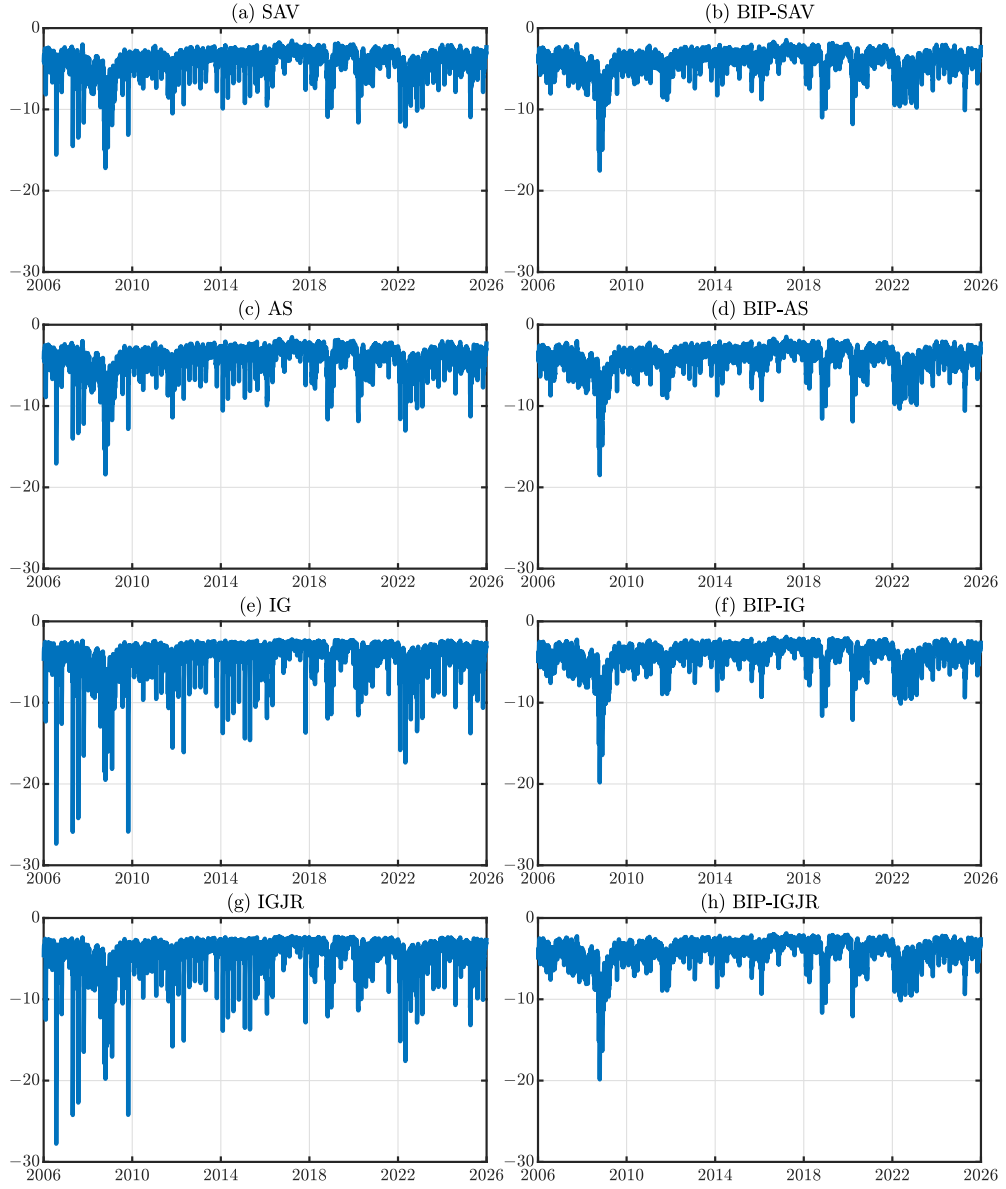


Fig. 7 2.5% VaR estimates for the AMZN stock return series over the full-sample period.

5.4 Risk forecasting performance

This subsection evaluates the one-step-ahead out-of-sample forecasting performance of the BIP-CAViaR-FZ model relative to the original CAViaR-FZ model. For each return series, we use a rolling estimation window of 2000 observations and reserve the final 3030 observations for forecast evaluation. The models are re-estimated every 50

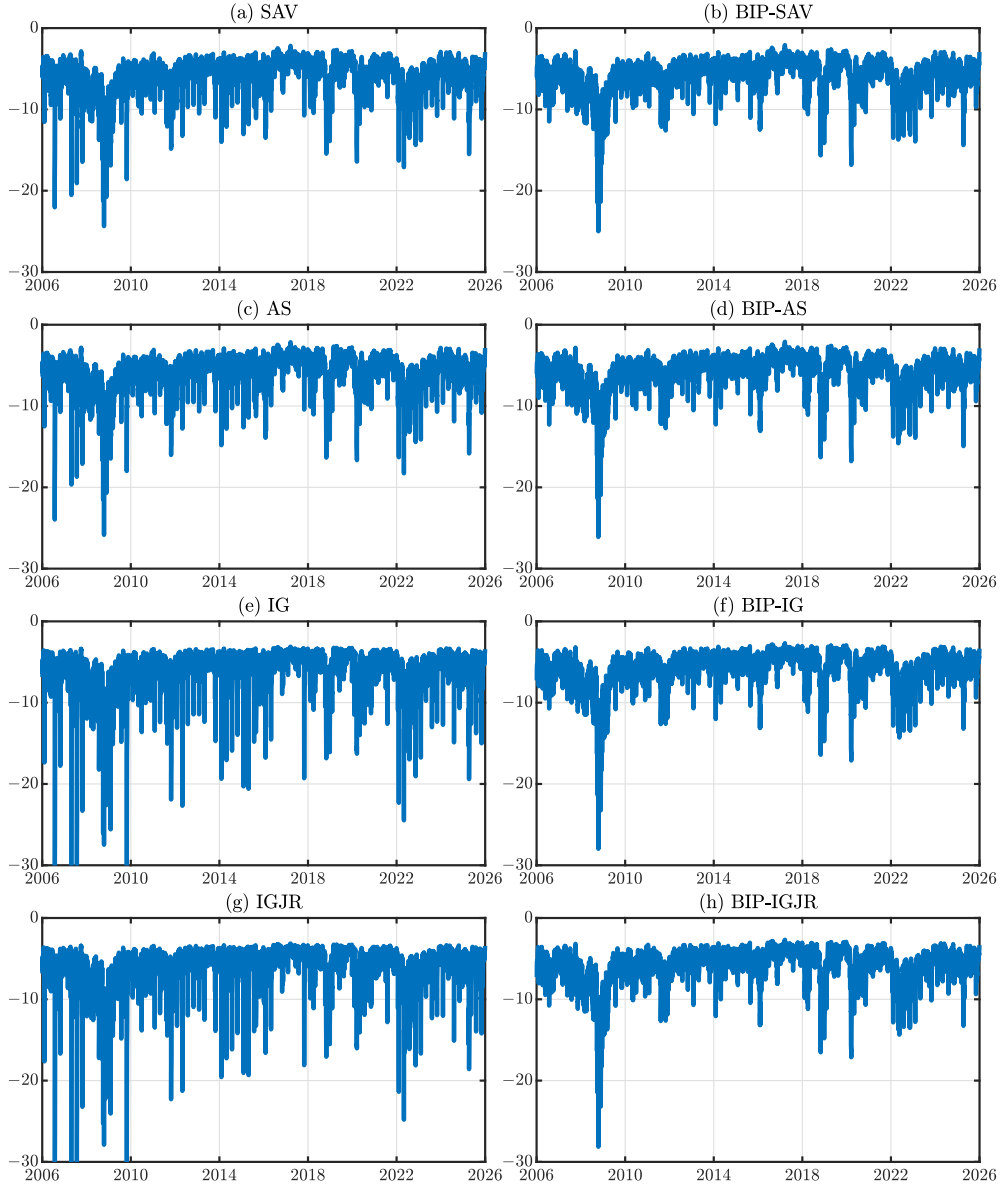


Fig. 8 2.5% ES estimates for the AMZN stock return series over the full-sample period.

observations using the most recent 2000 returns. The resulting parameter estimates are then used to generate 50 successive one-step-ahead VaR and ES forecasts, with the parameter vector held fixed between re-estimation dates. This procedure is repeated until the entire out-of-sample period is covered.

Forecast performance is evaluated using both traditional and comparative backtests. The traditional backtests include the unconditional coverage (UC) test of [Kupiec \(1995\)](#), the conditional coverage (CC) test of [Christoffersen \(1998\)](#), the dynamic quantile (DQ) test of [Engle and Manganelli \(2004\)](#), the VaR quantile regression (VQR) test of [Gaglianone, Lima, Linton, and Smith \(2011\)](#), the dynamic expected shortfall (DES) test of [Patton et al. \(2019\)](#), and the expected shortfall regression (ESR) test of [Bayer and Dimitriadis \(2022\)](#). The first four tests are used to assess the accuracy of VaR forecasts, whereas the latter two focus on the accuracy of ES forecasts.

For comparative backtesting, we consider four loss functions: the one-homogeneous quantile loss (QL1) of [Koenker and Bassett \(1978\)](#), the zero-homogeneous quantile loss (QL0) of [Hassani and Dionne \(2023\)](#), the FZ0 loss of [Patton et al. \(2019\)](#), and the Nolde–Ziegel (NZ) loss of [Nolde and Ziegel \(2017\)](#). The first two are used to evaluate VaR forecast accuracy, whereas the latter two jointly assess the accuracy of VaR and ES forecasts.

[Table 12](#) reports the backtesting results. For each model, the table summarizes the number of rejections across the five return series at the 5% significance level, together with rankings based on average loss values. Lower rejection counts and smaller ranking values indicate better forecasting performance.

Three findings stand out. First, the robust specifications generally perform at least as good as, and often better than, their non-robust counterparts. The gains are most evident for the quadratic specifications. At $\alpha = 2.5\%$, the total number of rejections falls from 12 to 4 when IG is replaced by BIP-IG, and from 9 to 6 when IGJR is replaced by BIP-IGJR. At $\alpha = 1\%$, the corresponding totals decline from 14 to 8 and from 16 to 9. For the linear specifications, the differences are smaller: SAV and BIP-SAV have the same total number of rejections at both probability levels, while BIP-AS improves on AS mainly at $\alpha = 1\%$. Second, the comparative loss rankings point to the same conclusion. At $\alpha = 2.5\%$, BIP-SAV and BIP-IG achieve the best average rankings, equal to 1.25 and 1.75, respectively. At $\alpha = 1\%$, BIP-IG ranks first with an average ranking of 1.0, followed by BIP-SAV with 2.25. In each model pair, the BIP specification attains a better average ranking than its non-robust counterpart. Third, asymmetric specifications tend to perform worse than symmetric ones in most cases. This pattern is largely consistent across both traditional and comparative backtests.

[Figs. 9](#) and [10](#) plot the corresponding 2.5% VaR and ES forecasts for the AMZN return series over the out-of-sample period. The figures are consistent with the backtesting results. Compared with the BIP-based specifications, the non-robust models react more sharply to potential outliers, leading to more abrupt movements and greater variability in the forecast paths. The contrast is particularly clear for the quadratic specifications and is more pronounced for ES than for VaR, consistent with the greater sensitivity of ES to tail events.

Table 12 Out-of-sample backtesting results for the empirical return series.

	Number of rejections							Ranking				
	UC	CC	DQ	VQR	DES	ESR	Sum	QL0	QL1	FZ0	NZ	Avg.
Panel A: $\alpha = 2.5\%$												
SAV	1	1	1	1	1	1	6	4	3	3	3	3.25
AS	1	1	1	0	2	1	6	6	5	6	6	5.75
IG	2	1	2	2	3	2	12	7	7	7	7	7
IGJR	1	1	2	1	2	2	9	8	8	8	8	8
BIP-SAV	1	1	1	0	2	1	6	1	2	1	1	1.25
BIP-AS	1	1	1	0	2	1	6	5	6	5	5	5.25
BIP-IG	1	1	0	0	1	1	4	2	1	2	2	1.75
BIP-IGJR	1	1	0	1	2	1	6	3	4	4	4	3.75
Panel B: $\alpha = 1\%$												
SAV	1	0	3	0	3	1	8	3	2	3	3	2.75
AS	2	2	3	1	3	1	12	6	7	7	7	6.75
IG	1	1	4	3	4	1	14	7	5	6	6	6
IGJR	1	1	5	3	5	1	16	8	8	8	8	8
BIP-SAV	1	1	3	0	3	0	8	2	3	2	2	2.25
BIP-AS	2	2	2	0	2	1	9	5	6	4	5	5
BIP-IG	1	0	3	1	3	0	8	1	1	1	1	1
BIP-IGJR	2	1	2	1	2	1	9	4	4	5	4	4.25

Note: This table reports, for each model, the number of rejections across the five return series at the 5% significance level and the rankings based on the average values of the QL0, QL1, FZ0, and NZ losses. Lower rejection counts and smaller ranking values indicate better out-of-sample forecasting performance.

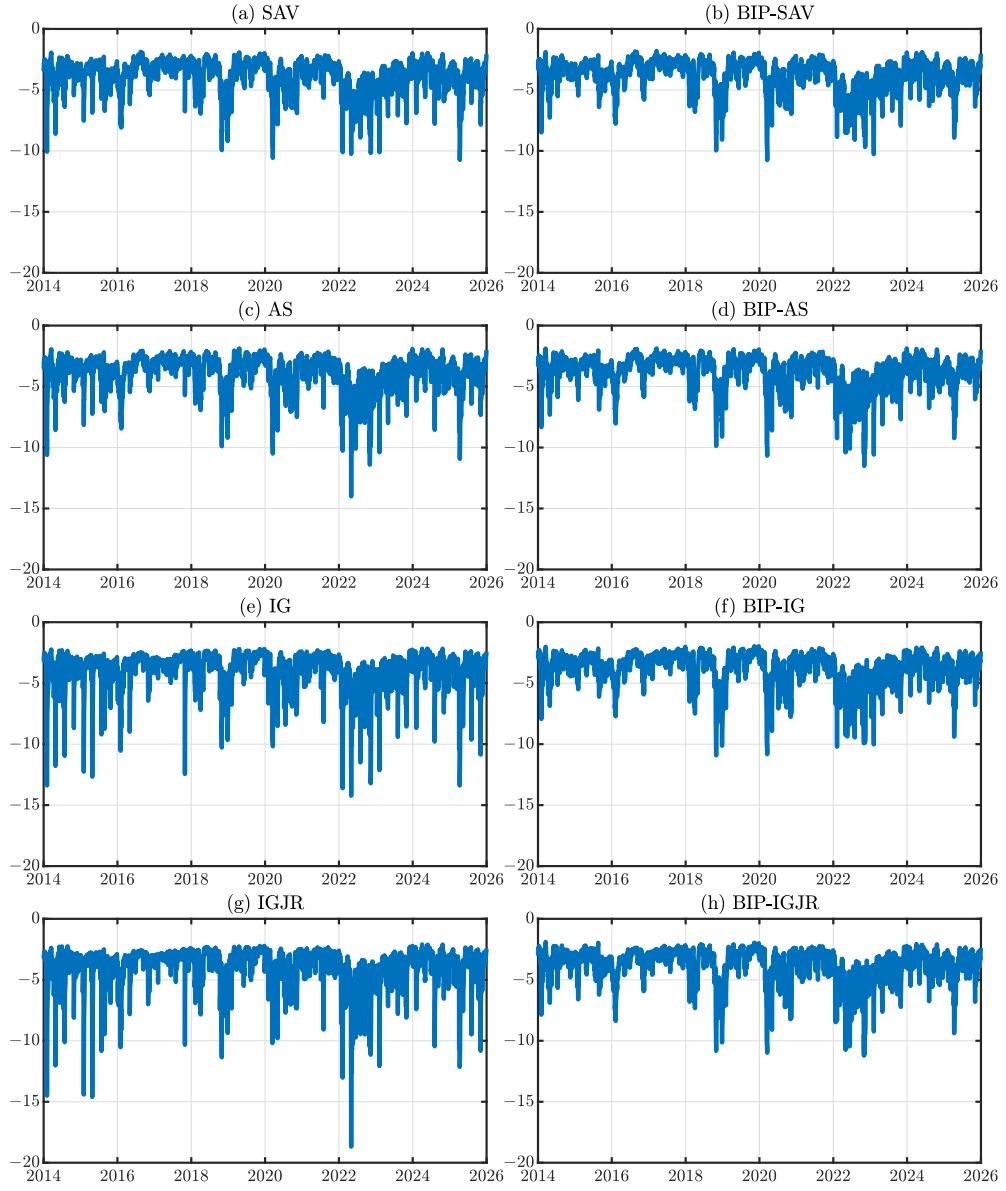


Fig. 9 2.5% VaR forecasts for the AMZN stock return series over the out-of-sample period.

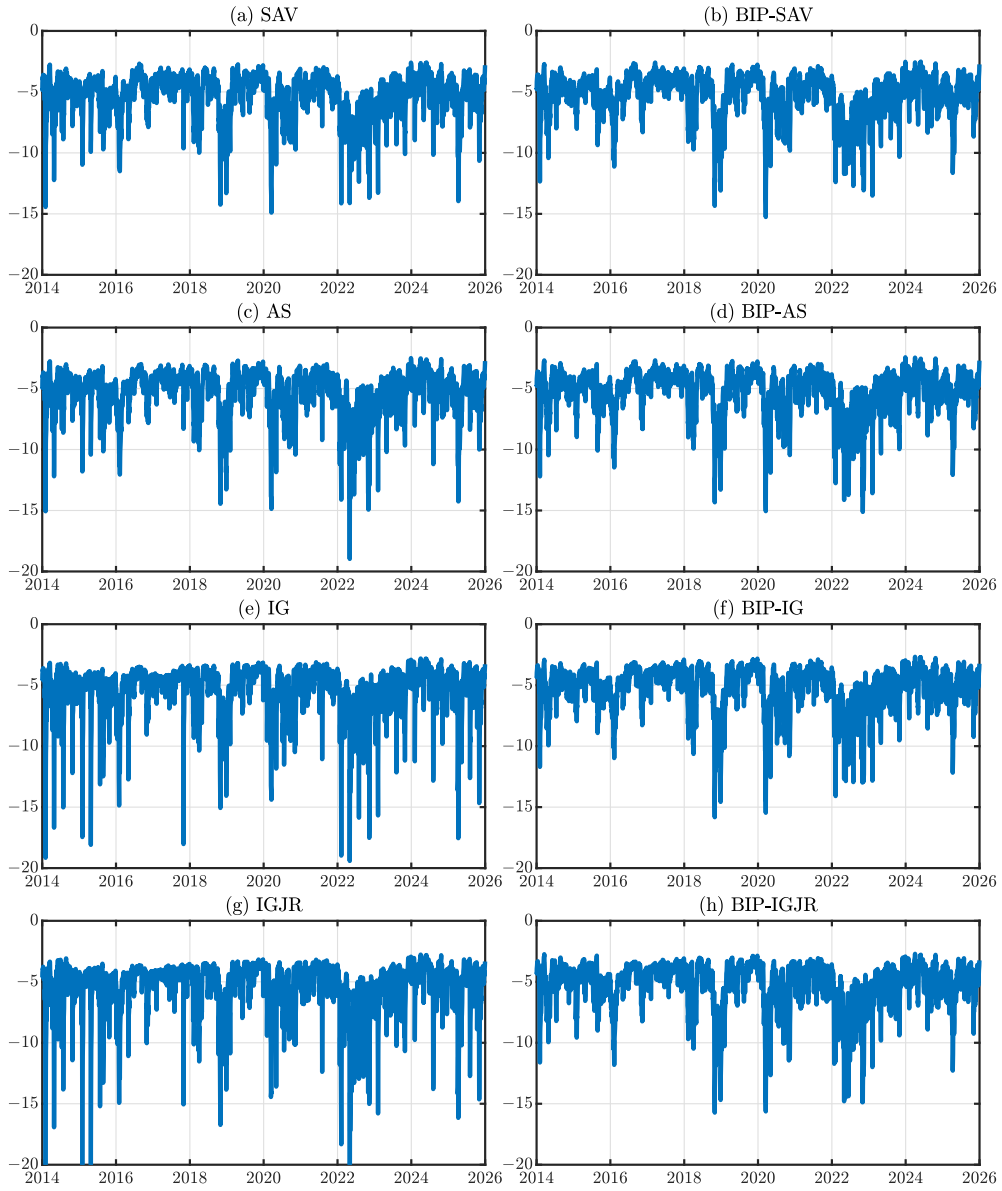


Fig. 10 2.5% ES forecasts for the AMZN stock return series over the out-of-sample period.

6 Conclusion

This paper develops an outlier-robust joint dynamic semiparametric model for forecasting VaR and ES. By incorporating BIP into the CAViaR-FZ framework, the proposed BIP-CAViaR-FZ model is designed to limit the influence of extreme observations on the recursive updating of VaR and ES while preserving the flexibility of the original specification. The standard CAViaR-FZ model is nested as a special case.

The simulation results show that the benefits of the BIP formulation become evident when the return series contains outliers, particularly under challenging conditions such as heavy-tailed or asymmetric innovations, lower probability levels, and smaller sample sizes. In these settings, BIP-based specifications tend to improve the accuracy of risk estimation and forecasting by reducing the propagation of extreme shocks.

The empirical results, based on five major U.S. technology stocks, provide further support for the effectiveness of the proposed approach. These return series exhibit a non-negligible number of potential outliers, and the BIP-CAViaR-FZ model generally performs better than its non-robust counterpart in both risk estimation and forecasting. The gains are especially pronounced for the quadratic specifications, which appear to be more sensitive to outliers under the standard framework.

Overall, the evidence suggests that incorporating bounded innovation propagation into joint VaR–ES modeling provides a useful and practically relevant way to enhance robustness in the presence of outliers. A natural direction for future research would be to extend the proposed framework to settings that incorporate high-frequency financial data and mixed-frequency macroeconomic information.

Declarations

Competing interests. The authors declare that they have no competing interests.

Data availability. The data that support the findings of this study are openly available from Yahoo Finance at <https://finance.yahoo.com/>.

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